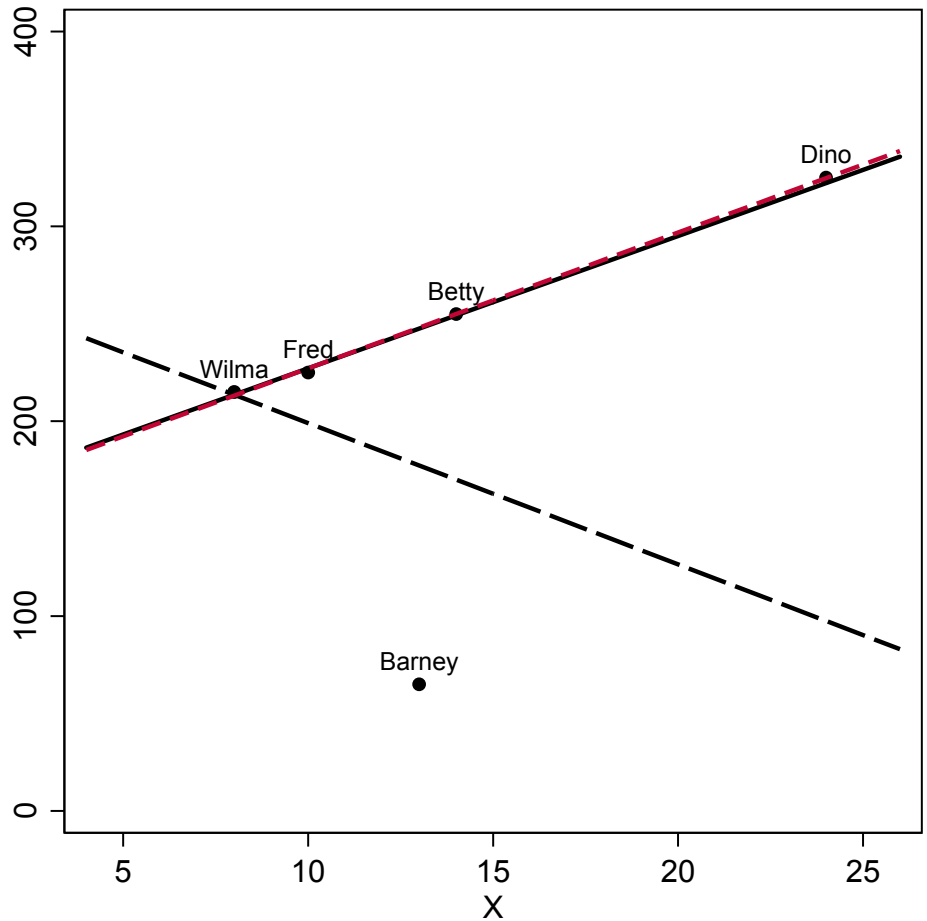
PLSC 503 – Spring 2021
Residuals, Model Fit, and Outliers + Simultaneity

February 23, 2021

Discrepancy, Leverage, and Influence



Note: Solid line is the regression fit for Wilma, Fred, and Betty only.
Long-dashed line is the regression for Wilma, Fred, Betty, and Barney.
Short-dashed (red) line is the regression for Wilma, Fred, Betty and Dino.

Discrepancy, Leverage, and Influence

$$\text{Influence} = \text{Leverage} \times \text{Discrepancy}$$

Leverage

$$\begin{aligned} \hat{\mathbf{Y}} &= \mathbf{X}\hat{\boldsymbol{\beta}} \\ &= \mathbf{X}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}] \\ &= \mathbf{H}\mathbf{Y} \end{aligned}$$

where

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'.$$

$$h_i = \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i'$$

Residuals

Variation:

$$\widehat{\text{Var}(\hat{u}_i)} = \hat{\sigma}^2[1 - \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i'] \tag{1}$$

$$\begin{aligned} \widehat{\text{s.e.}(\hat{u}_i)} &= \hat{\sigma}\sqrt{[1 - \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i']} \\ &= \hat{\sigma}\sqrt{1 - h_i} \end{aligned} \tag{2}$$

"Standardized":

$$\tilde{u}_i = \frac{\hat{u}_i}{\hat{\sigma}\sqrt{1 - h_i}} \tag{3}$$

"Studentized": define

$$\begin{aligned} \hat{\sigma}^2_{-i} &= \text{Variance for the } N-1 \text{ observations} \neq i \\ &= \frac{\hat{\sigma}^2(N-K)}{N-K-1} - \frac{\hat{u}_i^2}{(N-K-1)(1-h_i)}. \end{aligned} \tag{4}$$

Then:

$$\hat{u}'_i = \frac{\hat{u}_i}{\hat{\sigma}_{-i}\sqrt{1-h_i}} \tag{5}$$

Influence

"DFBETA":

$$D_{ki} = \hat{\beta}_k - \hat{\beta}_{k(-i)} \tag{6}$$

"DFBETAS" (the "S" is for "standardized):

$$D^*_{ki} = \frac{D_{ki}}{\widehat{\text{s.e.}(\hat{\beta}_{k(-i)})}} \tag{7}$$

Cook's D:

$$\begin{aligned} D_i &= \frac{\tilde{u}_i^2}{K} \times \frac{h_i}{1-h_i} \\ &= \frac{h_i \hat{u}_i^2}{K\hat{\sigma}^2(1-h_i)^2} \end{aligned} \tag{8}$$

```
> # No Barney OR Dino...
> summary(lm(Y~X,data=subset(flintstones,name!="Dino" & name!="Barney")))

Residuals:
     2      4      5
 0.714 -2.143  1.429

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  159.286      6.776    23.5    0.027 *
X              6.786      0.619    11.0    0.058 .
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.67 on 1 degrees of freedom
Multiple R-squared: 0.992,Adjusted R-squared: 0.984
F-statistic:  120 on 1 and 1 DF,  p-value: 0.0579
```

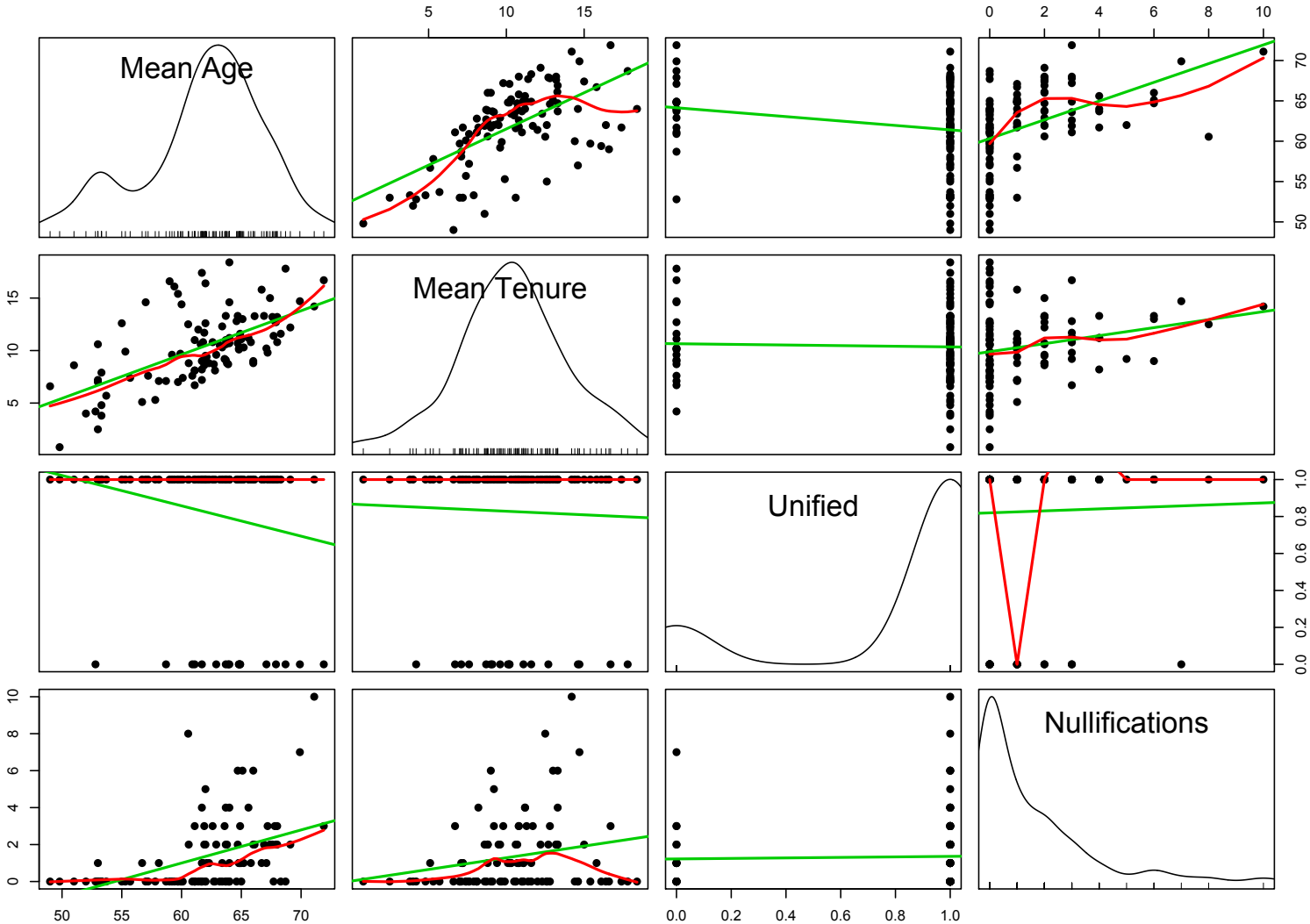
```
> # No Barney (Dino included...)
> summary(lm(Y~X,data=subset(flintstones,name!="Barney")))

Residuals:
        2         3         4         5
-8.88e-16  2.63e-01 -2.11e+00  1.84e+00

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  157.368      2.465    63.8  0.00025 ***
X              6.974      0.161    43.3  0.00053 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.99 on 2 degrees of freedom
Multiple R-squared: 0.999,Adjusted R-squared: 0.998
F-statistic: 1.87e+03 on 1 and 2 DF,  p-value: 0.000534
```

A Variance-Based Statistic

"COVRATIO":

$$\text{COVRATIO}_i = \left[(1 - h_i) \left(\frac{N - K - 1 + \hat{u}_i'^2}{N - K} \right)^K \right]^{-1} \tag{9}$$

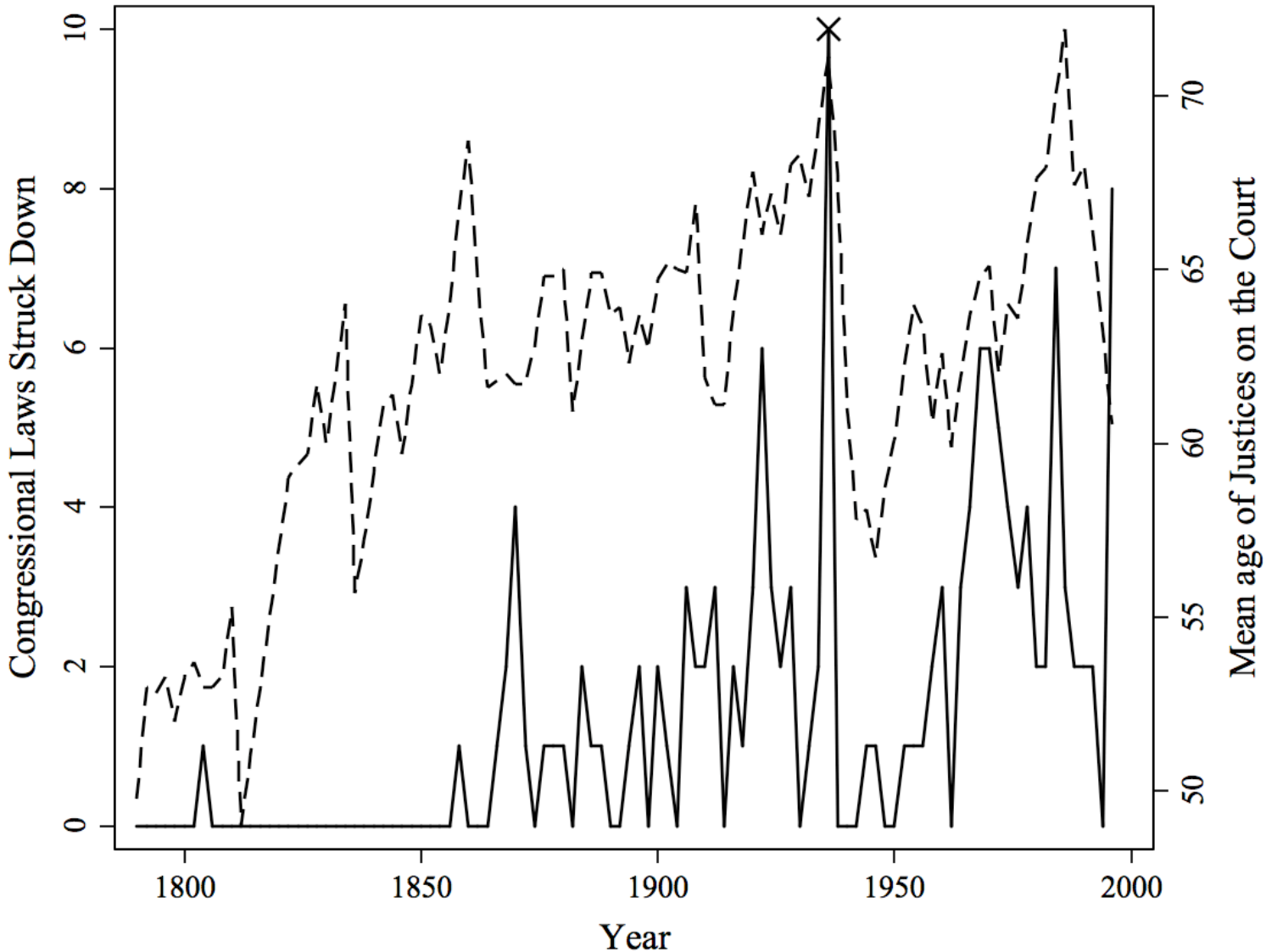
Example: Federal Judicial Review, 1789-1996



Basic Regression...

```
> Fit<-lm(nulls~age+tenure+unified)
> summary(Fit)

Residuals:
    Min      1Q  Median      3Q     Max
-2.7857 -1.0773 -0.3634  0.4238  6.9694

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -12.10340    2.54324  -4.759 6.57e-06 ***
age           0.21886    0.04484   4.881 4.01e-06 ***
tenure       -0.06692    0.06427  -1.041    0.300
unified       0.71760    0.45844   1.565    0.121
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.715 on 100 degrees of freedom
Multiple R-squared: 0.2324,Adjusted R-squared: 0.2093
F-statistic: 10.09 on 3 and 100 DF,  p-value: 7.241e-06
```

Federal Judicial Review and Mean SCOTUS Age



Residuals, etc.

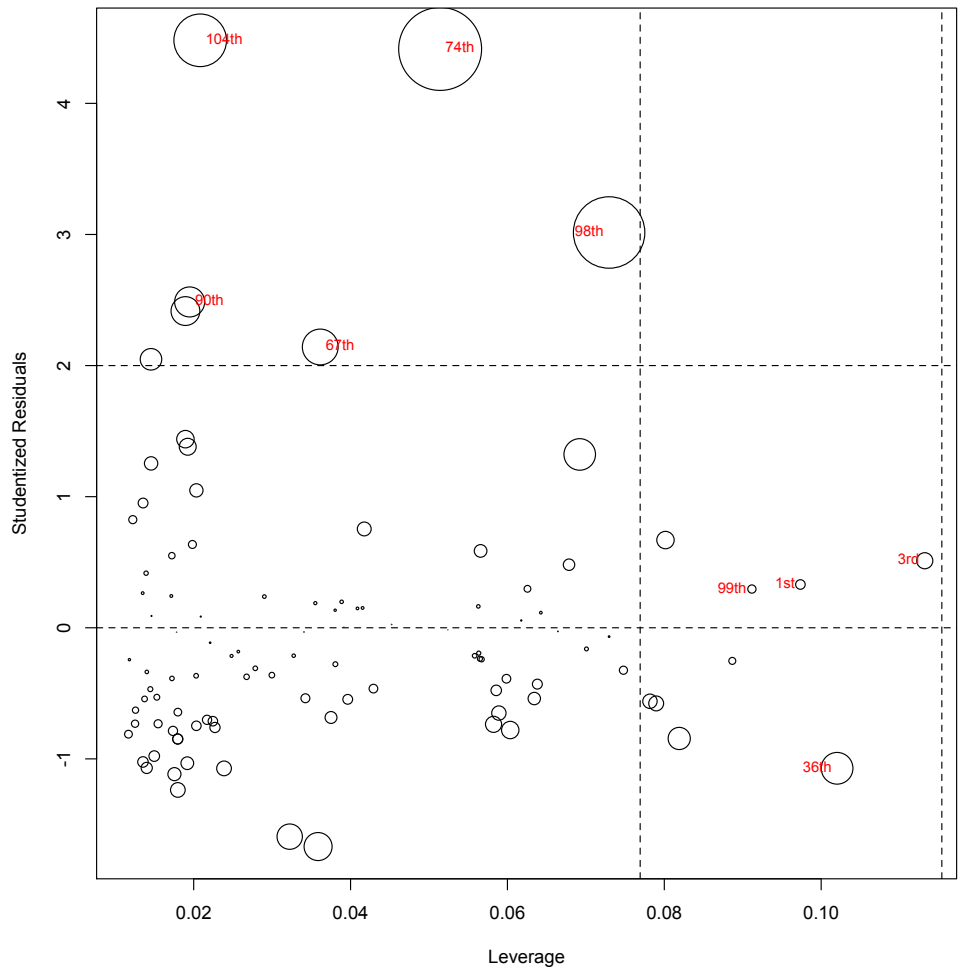
```
> FitResid<-(nulls - predict(Fit)) # residuals
> FitStandard<-rstandard(Fit) # standardized residuals
> FitStudent<-rstudent(Fit) # studentized residuals
> FitCooksD<-cooks.distance(Fit) # Cook's D
> FitDFBeta<-dfbeta(Fit) # DFBeta
> FitDFBetaS<-dfbetas(Fit) # DFBetaS
> FitCOVRATIO<-covratio(Fit) # COVRATIOs
```

Studentized Residuals

```
> FitStudent[74]
      74
4.415151

> Congress74<-rep(0,length=104)
> Congress74[74]<-1

> summary(lm(nulls~age+tenure+unified+Congress74))

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -10.17290    2.37692  -4.280 4.33e-05 ***
age           0.18820    0.04177   4.505 1.82e-05 ***
tenure       -0.06356    0.05905  -1.076    0.284
unified       0.55159    0.42282   1.305    0.195
Congress74    7.14278    1.61779   4.415 2.58e-05 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.576 on 99 degrees of freedom
Multiple R-squared: 0.3586,Adjusted R-squared: 0.3327
F-statistic: 13.84 on 4 and 99 DF,  p-value: 5.304e-09
```

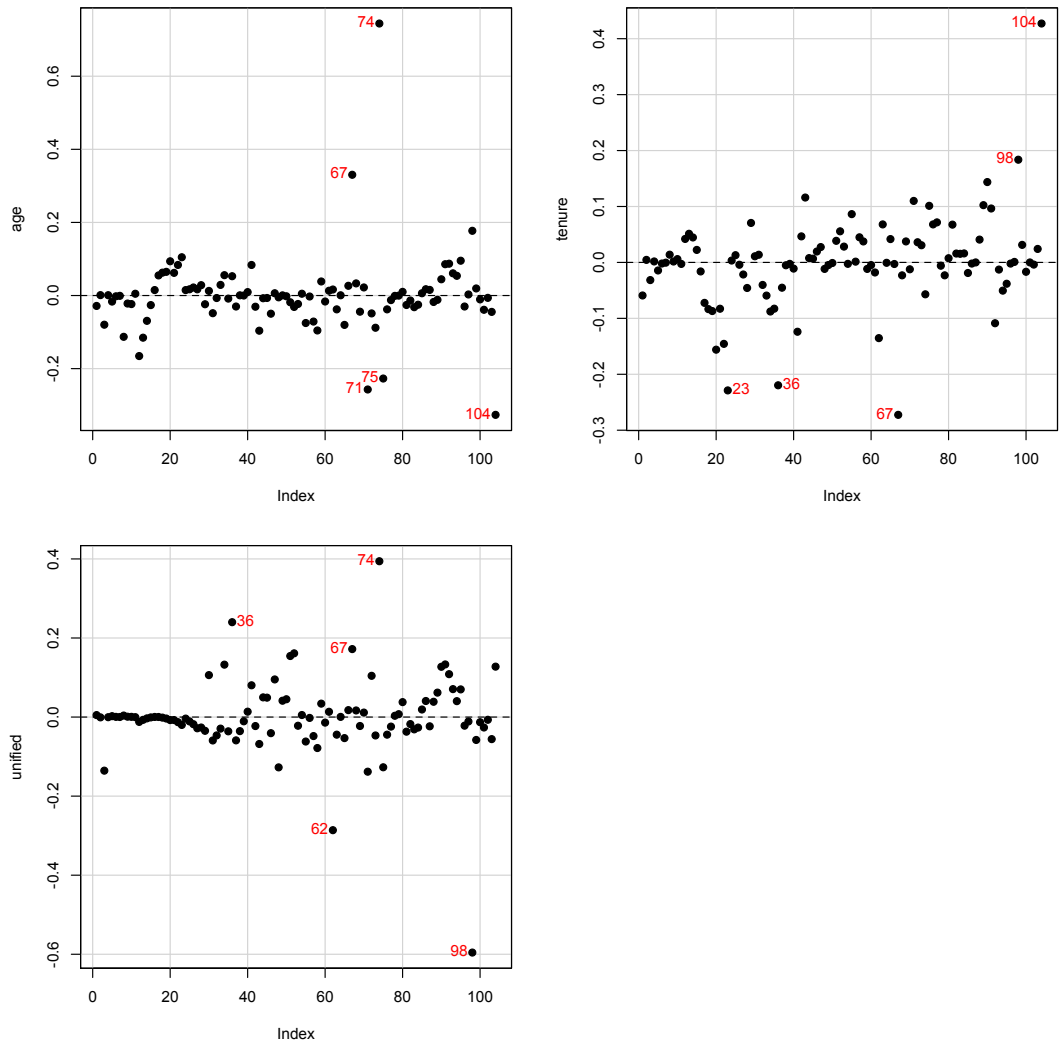
"Bubble Plot"

```
> influencePlot(Fit,id.n=4,labels=Congress,id.cex=0.8,
  id.col="red",xlab="Leverage")
```



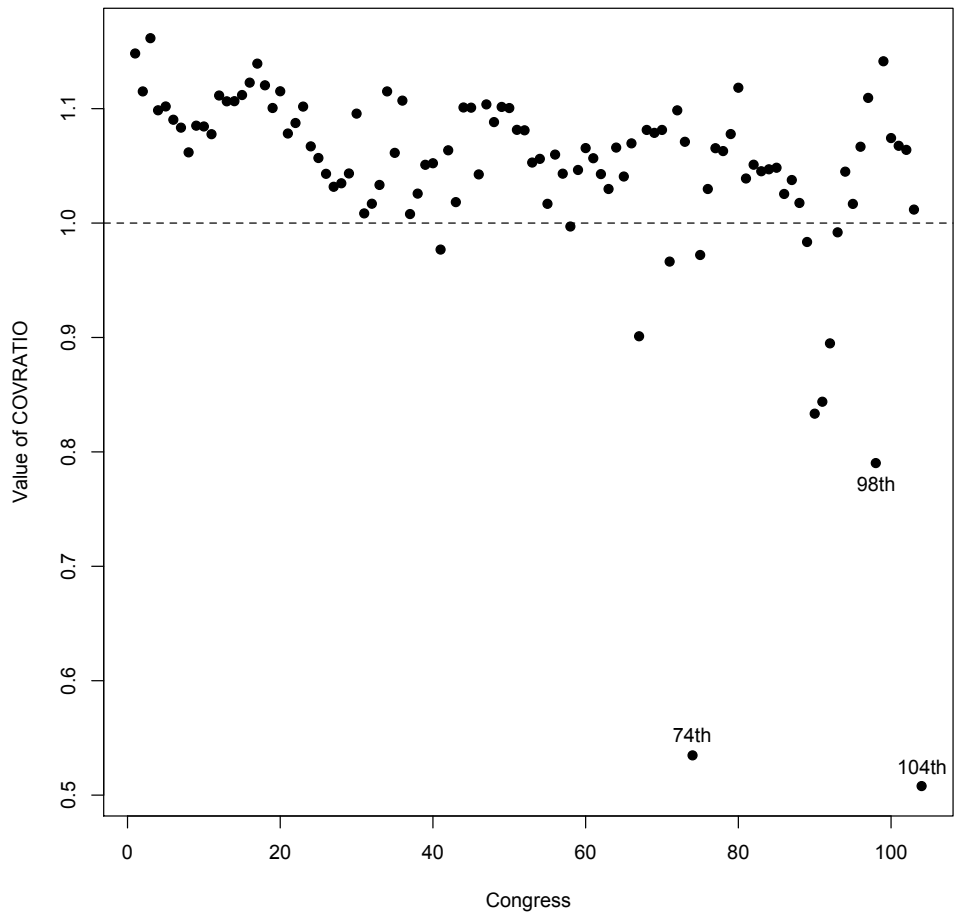
DFBETAS

```
> dfbetasPlots(Fit,id.n=5,id.col="red",main="",pch=19)
```



COVRATIO Plot

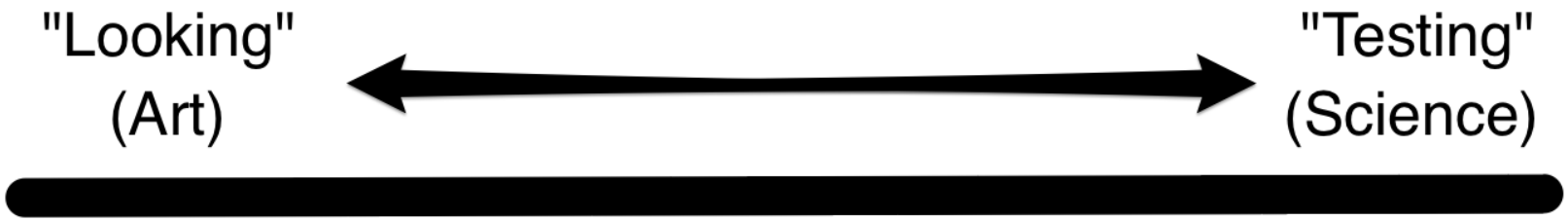
```
> plot(FitCOVRATIO~congress,pch=19,xlab="Congress",ylab="Value of COVRATIO")
> abline(h=1,lty=2)
```



Sensitivity Analyses: Omitting Outliers

```
> Outlier<-rep(0,104)
> Outlier[74]<-1
> Outlier[98]<-1
> Outlier[104]<-1
> DahlSmall<-Dahl[which (Outlier==0),]

> summary(lm(nulls~age+tenure+unified,data=DahlSmall))

Call:
lm(formula = nulls ~ age + tenure + unified, data = DahlSmall)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -10.38536    1.99470  -5.206 1.08e-06 ***
age           0.19302    0.03512   5.496 3.13e-07 ***
tenure       -0.10069    0.04974  -2.024   0.0457 *
unified       0.76645    0.36069   2.125   0.0361 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.319 on 97 degrees of freedom
Multiple R-squared: 0.2578,Adjusted R-squared: 0.2349
F-statistic: 11.23 on 3 and 97 DF,  p-value: 2.167e-06
```

Thinking About Diagnostics



Observational Data
Complex Data
Structure
Informative Missingness
Complex / Uncertain
Causality

Experimental Data
Simple Data Structure
No / Uninformative
Missingness
Simple / Clear Causality

One Approach

Pena, E.A. and E.H. Slate. 2006. "Global Validation of Linear Model Assumptions." *J. American Statistical Association* 101(473):341-354.

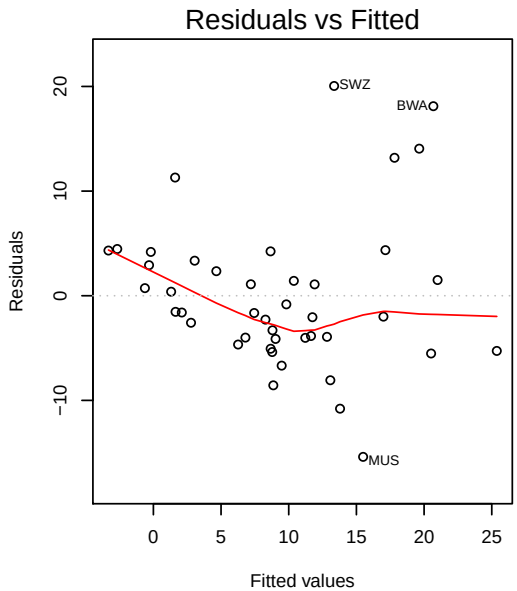
Tests for:

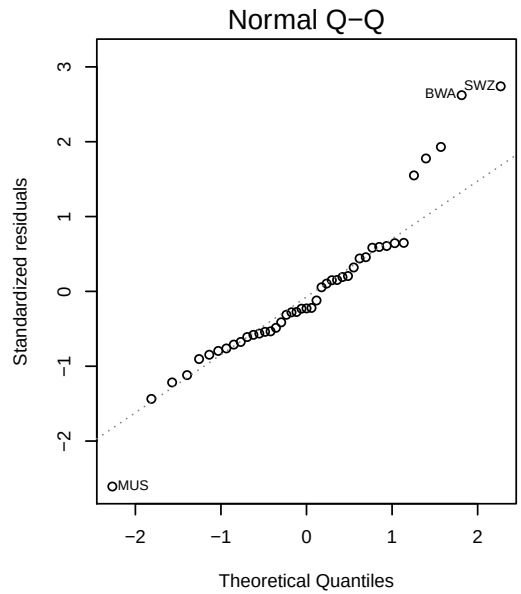
- Normality in $\hat{u}$s (via skewness & kurtosis tests)
- "Link function" (linearity / additivity)
- Constant variance and uncorrelatedness in $\hat{u}$s ("heteroskedasticity" test)

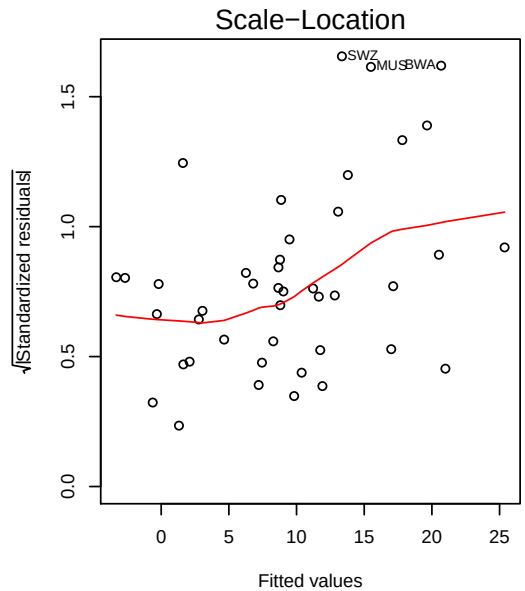
In Action

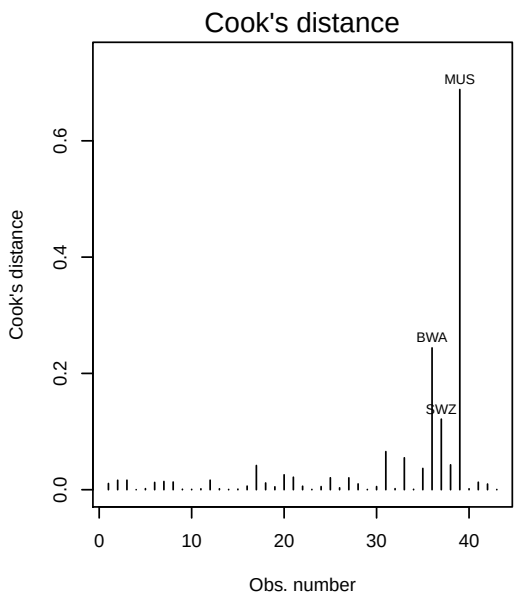
```
> Fit <- with(Africa, lm(adrate~gdppppd+muslperc+subsaharan+healthexp+
                  literacy+internalwar))

> library(gvlma)
> Nope <- gvlma(Fit)
> display.gvlmatests(Nope)

ASSESSMENT OF THE LINEAR MODEL ASSUMPTIONS
USING THE GLOBAL TEST ON 4 DEGREES-OF-FREEDOM:
Level of Significance =  0.05

Call:
 gvlma(x = Fit)

                     Value   p-value                   Decision
Global Stat         21.442 0.0002587 Assumptions NOT satisfied!
Skewness             5.720 0.0167698 Assumptions NOT satisfied!
Kurtosis             2.345 0.1256876    Assumptions acceptable.
Link Function        5.892 0.0152059 Assumptions NOT satisfied!
Heteroscedasticity   7.485 0.0062227 Assumptions NOT satisfied!
```

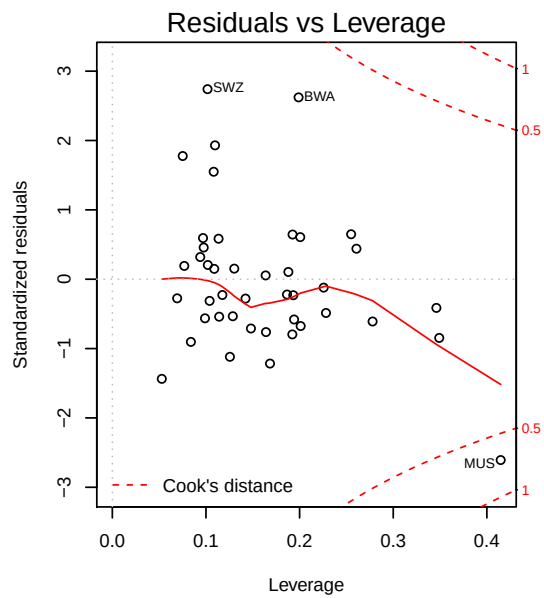
Another Approach: `plot(fit)`

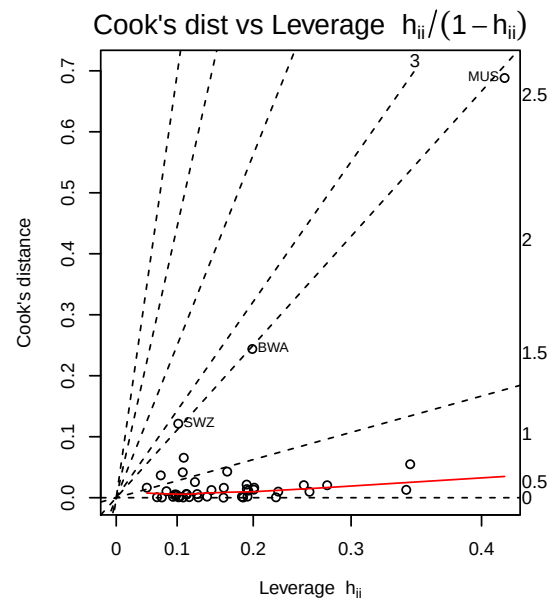




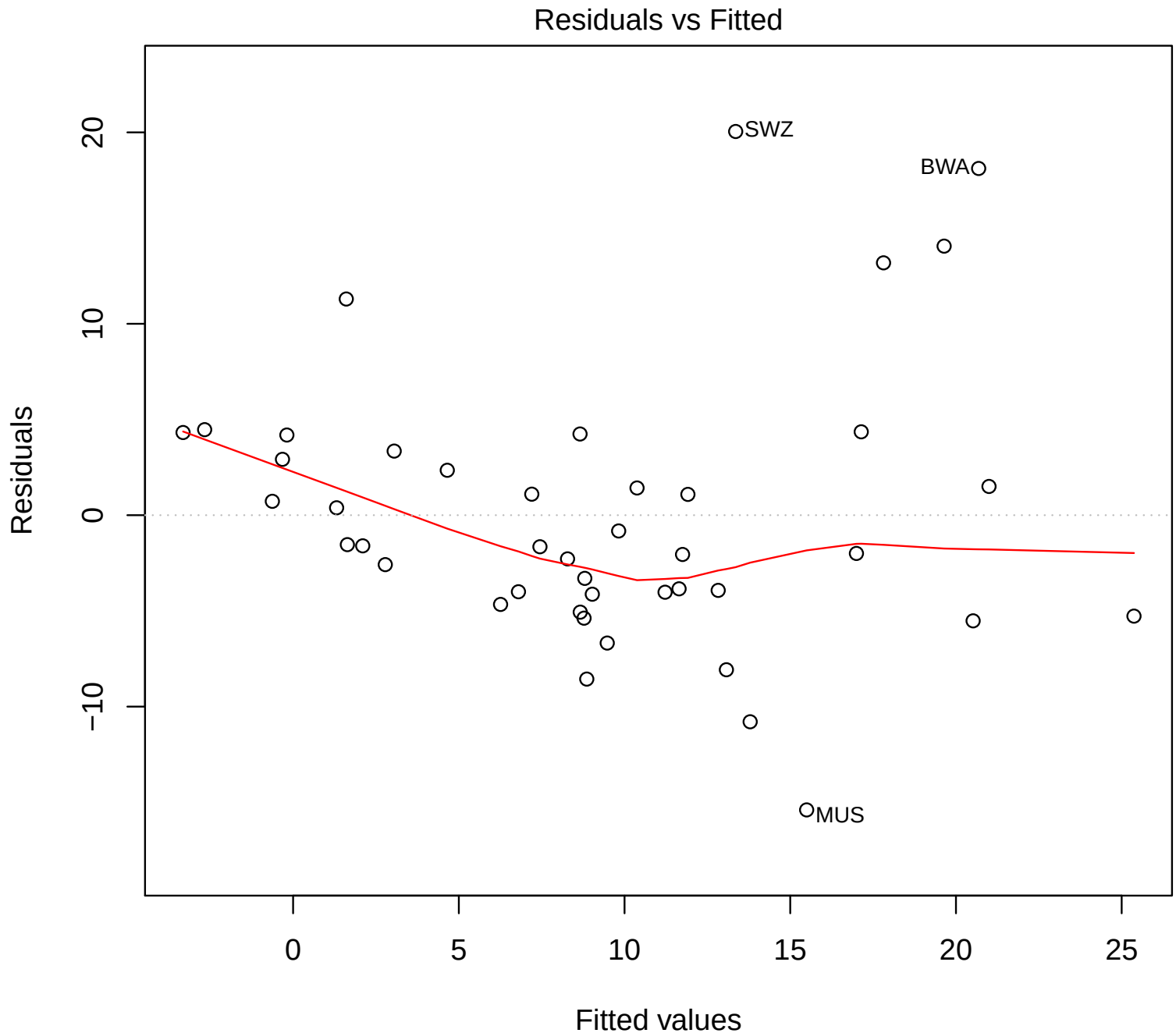




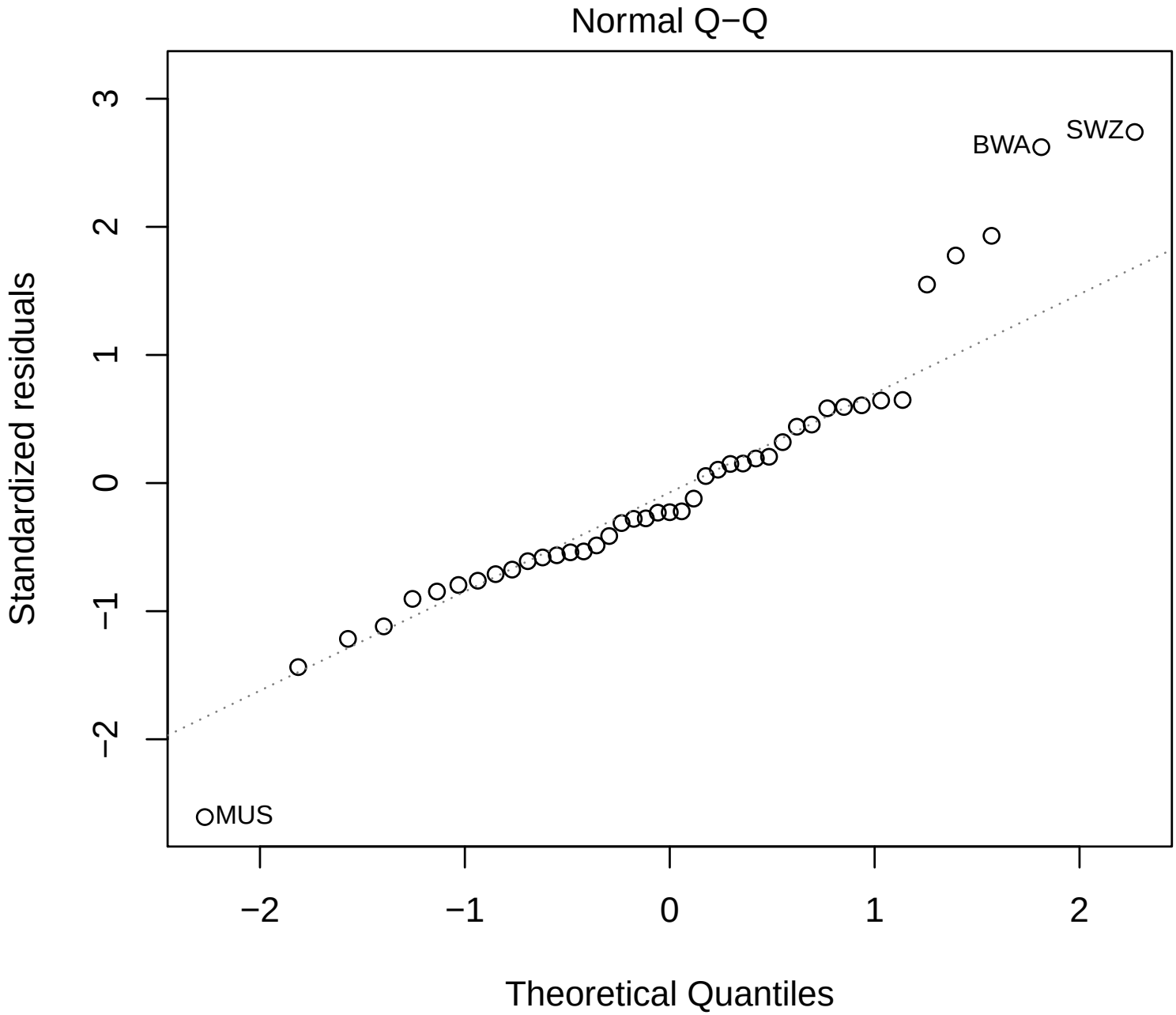




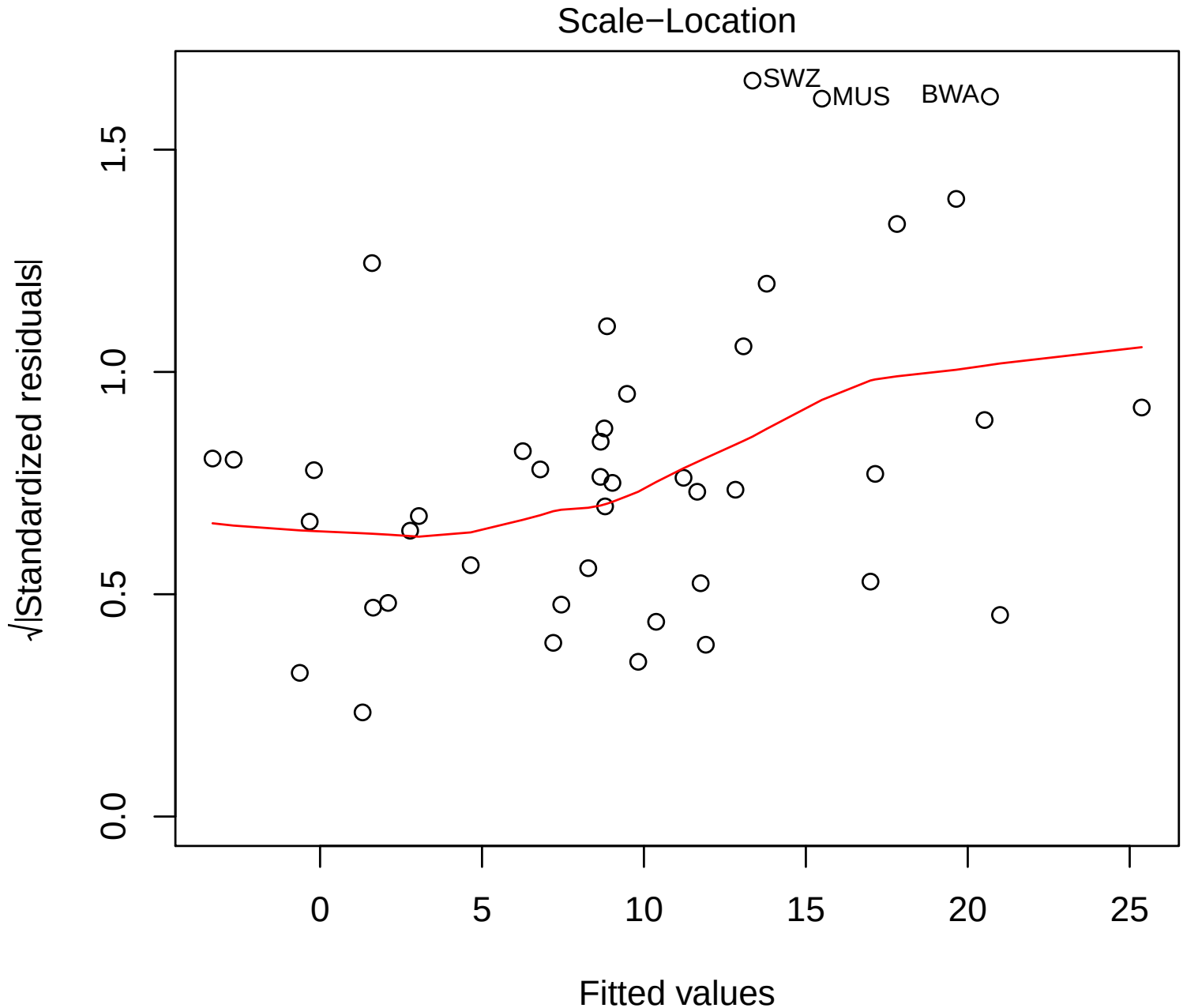
#1: Residuals vs. Fitted Values



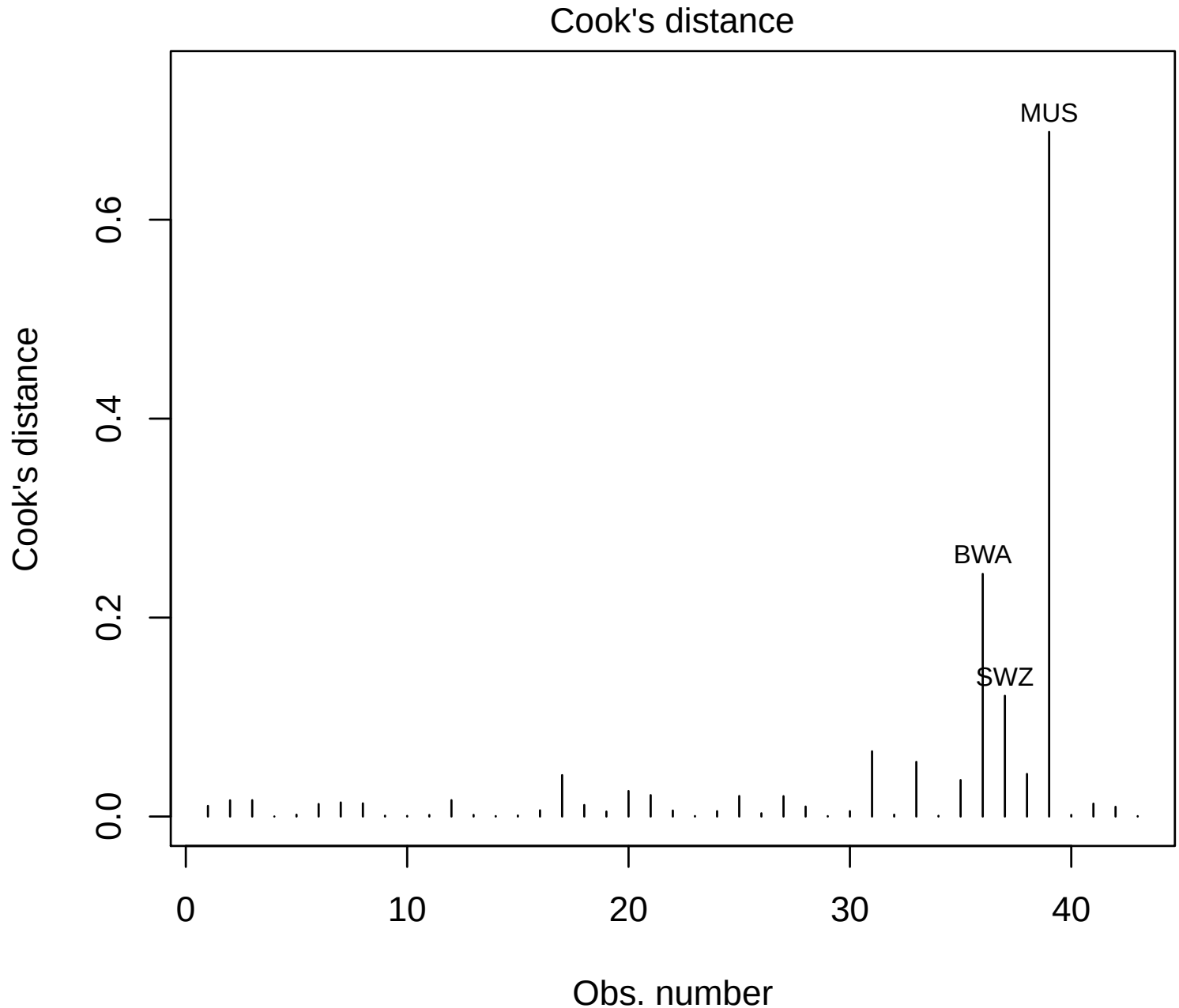
#2: Q-Q Plot of $\hat{u}$s



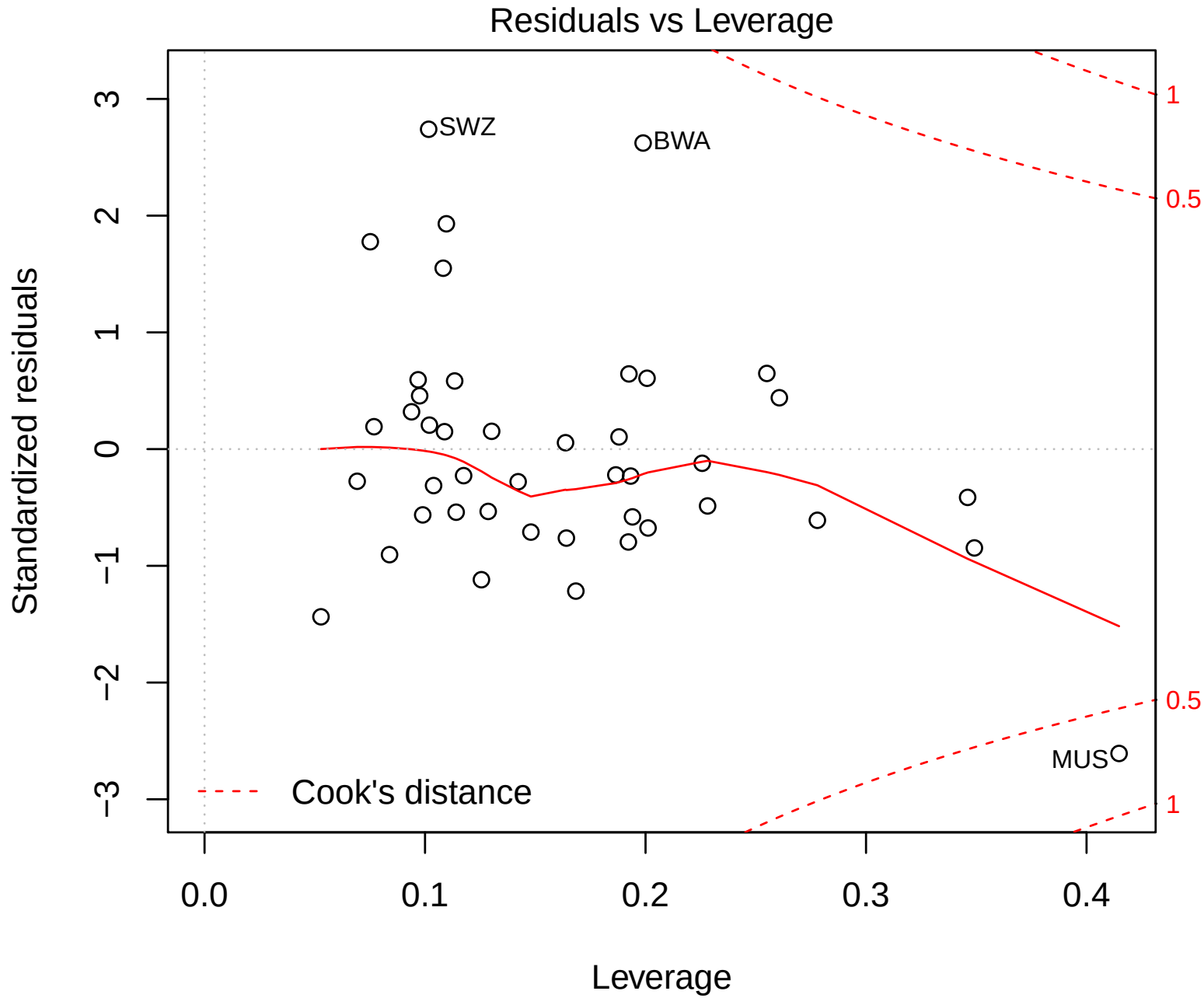
"Scale-Location" Plot



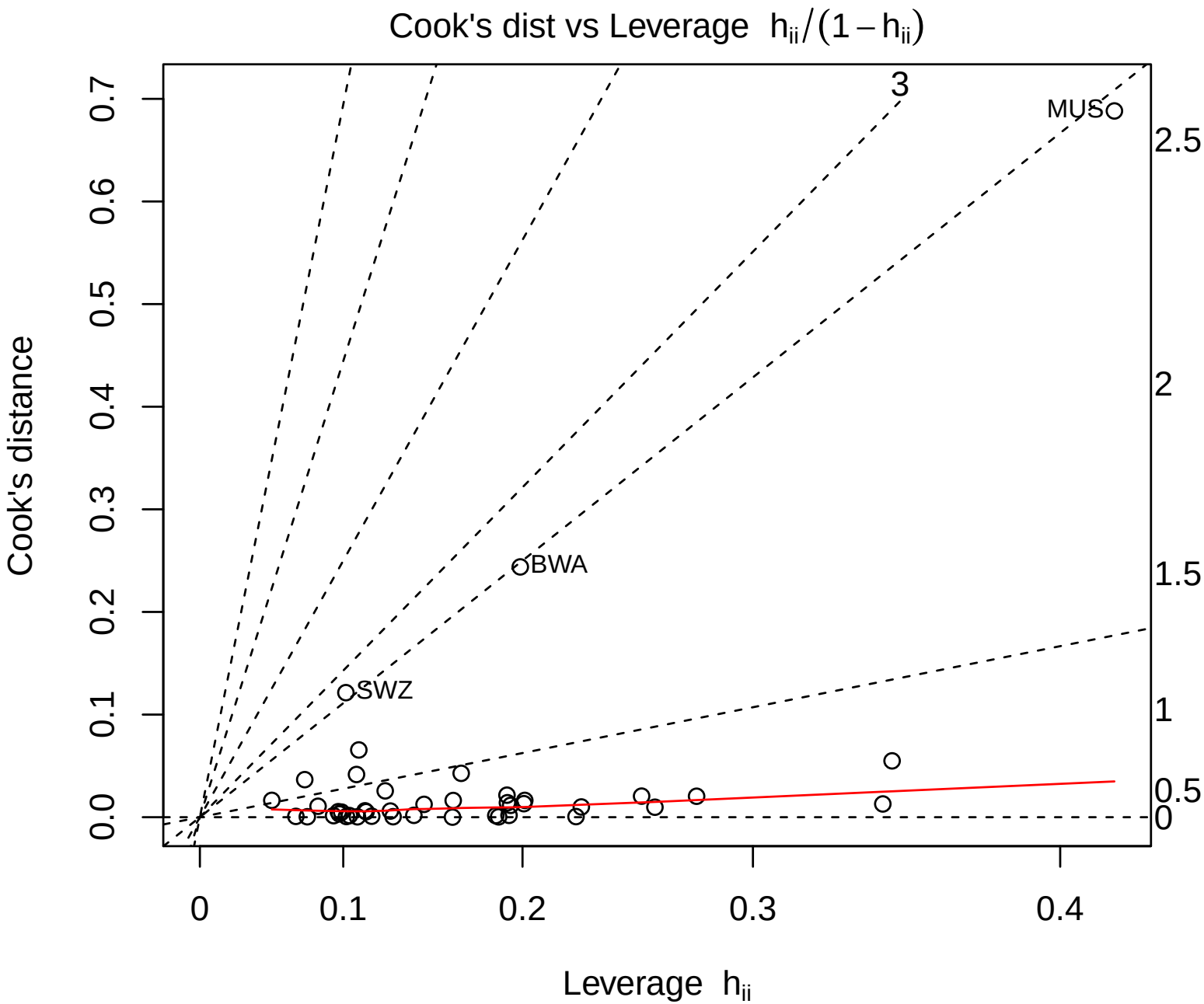
Cook's D


Cook's distance
Cook's distance
0.0
0.2
0.4
0.6
MUS
BWA
SWZ
0
10
20
30
40
Obs. number

Residuals vs. Leverage



Cook's D vs. Leverage



Simultaneity and Endogeneity

Endogeneity

Consider:

$$Y_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

$$Y_2 = \mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2$$

Rewrite:

$$\begin{aligned}
Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1[\mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2] + \mathbf{u}_1 \\
&= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\gamma_2 Y_1 + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 - \gamma_1\gamma_2 Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
(1 - \gamma_1\gamma_2) Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 &= \mathbf{X}_1\left(\frac{1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_1\right) + \mathbf{X}_2\left(\frac{\gamma_1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_2\right) + \left(\frac{\gamma_1\mathbf{u}_2 + \mathbf{u}_1}{1-\gamma_1\gamma_2}\right) \\
&= \Delta_1\mathbf{X}_1 + \Delta_2\mathbf{X}_2 + \mathbf{e}
\end{aligned}$$

"Reduced Form"

$$Y_1 = \mathbf{X}_1 \left(\frac{1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_1 \right) + \mathbf{X}_2 \left(\frac{\gamma_1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_2 \right) + \left(\frac{\gamma_1\mathbf{u}_2 + \mathbf{u}_1}{1-\gamma_1\gamma_2} \right)$$

means

$$\frac{\partial Y_1}{\partial X_\ell} = \frac{\beta_\ell}{1-\gamma_1\gamma_2}.$$

But

$$\hat{\Delta}_1 \neq \hat{\boldsymbol{\beta}}_1.$$

Simultaneity Bias

For (e.g.)

$$Y_1 = \mathbf{X}_1\beta_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

we have:

$$\mathrm{E}(Y_2, \mathbf{u}_1) = \frac{\gamma_2}{1 - \gamma_1\gamma_2}\sigma^2_{\mathbf{u}}$$

Result:

- Bias (unless $\gamma_2 = 0$)
- Inconsistency

What To Do

- OLS
- Lagged Variables
- Two-Stage Least Squares (2SLS)
- Systems of Equations / 3SLS / etc.

Recall that a simple linear model:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

gives us:

$$\hat{\boldsymbol{\beta}}_{OLS} = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}.$$

Suppose $\text{Cov}(\mathbf{X}, \mathbf{u}) \neq \mathbf{0}$, but we have $\mathbf{Z}$ with

- $\text{Cov}(\mathbf{Z}, \mathbf{X}) \neq \mathbf{0}$ and
- $\text{Cov}(\mathbf{Z}, \mathbf{u}) = \mathbf{0}$.

Then:

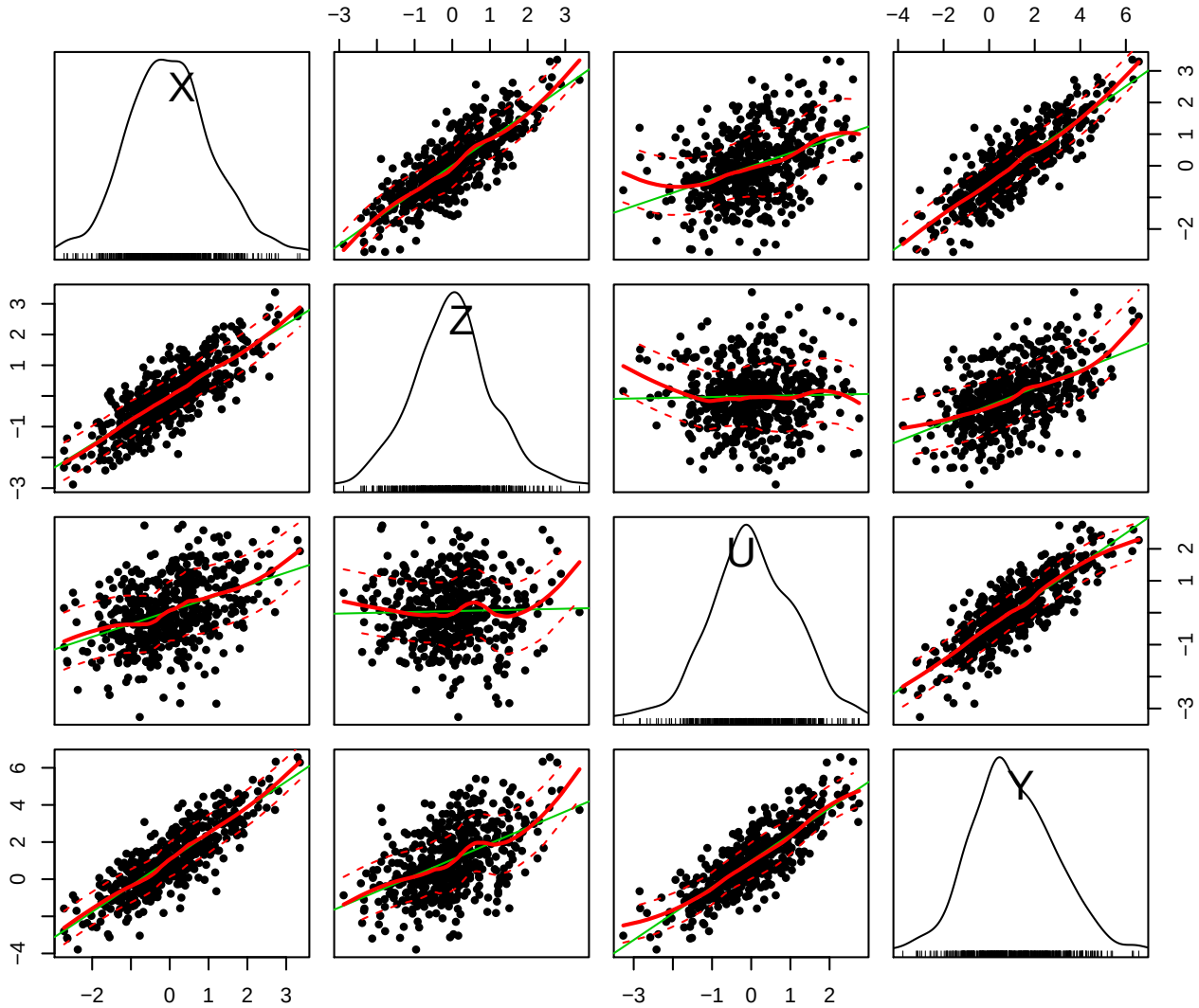
$$\begin{aligned} \hat{\boldsymbol{\beta}}_{IV} &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Y} \\ &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'(\mathbf{X}\boldsymbol{\beta} + \mathbf{u}) \\ &= \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{u} \end{aligned}$$

is consistent.

2SLS: How-To

- Regress endogenous $\mathbf{X}$s variables on $\{\mathbf{Z}, \mathbf{X}\}$
- Generate $\hat{\mathbf{X}}$s
- Regress Y on $\hat{\mathbf{X}}$ to get β_{2SLS}.
- Adjust standard error estimates

```
library(MASS)
library(sem)
library(car)

seed<-1337
set.seed(seed)

mu<-c(0,0,0) # <== X, Z, U
Sigma<-matrix(c(1,0.8,0.4,0.8,1,0,0.4,0,1),
              nrow=3,byrow=TRUE)                # Cor(X,Y)=0.8, etc.
Vars<- mvrnorm(500,mu,Sigma)
colnames(Vars)<-c("X","Z","U")
Vars<-data.frame(Vars)

Vars$Y<- 1 + Vars$X + Vars$U
```


X
Z
U
Y

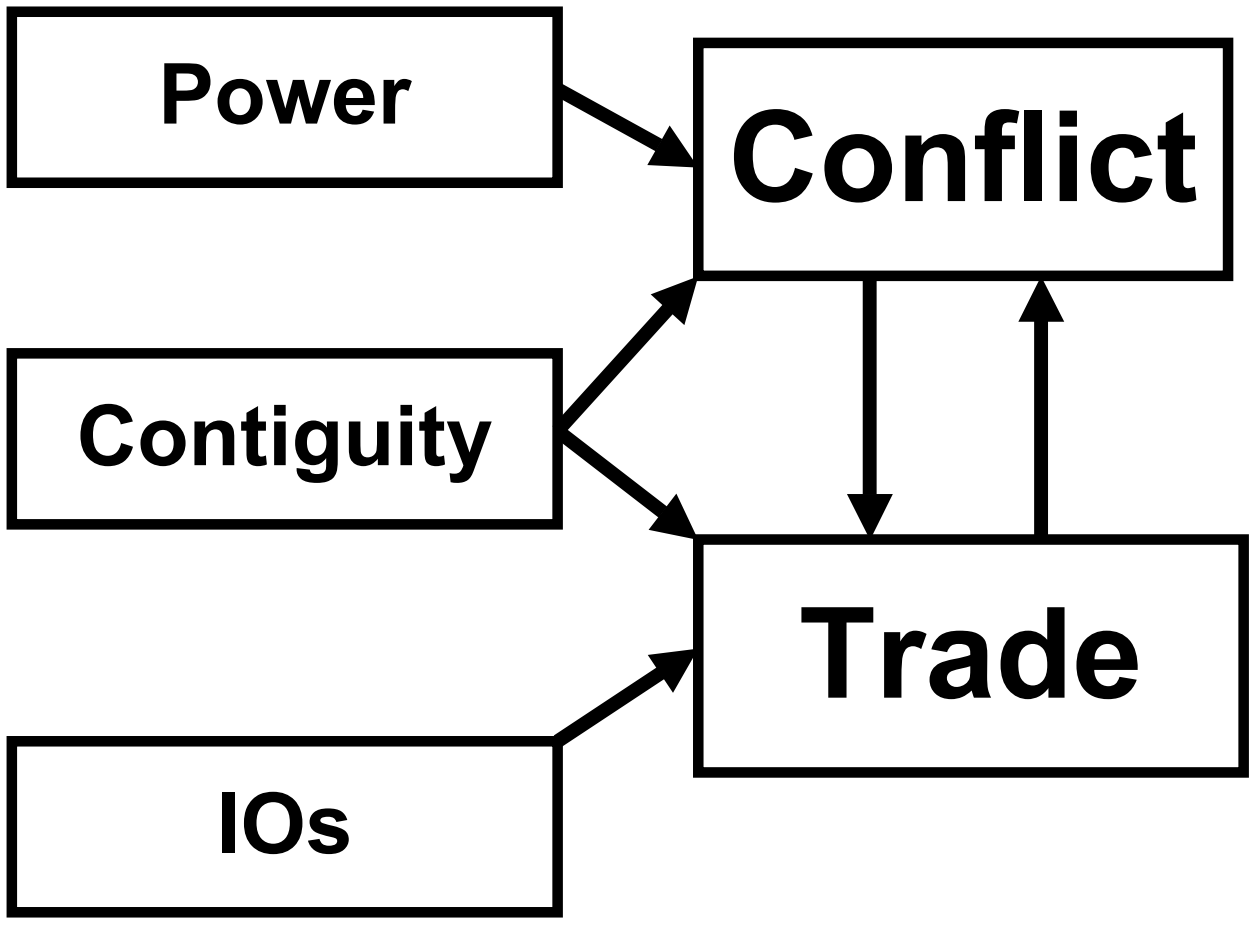
Plain Old OLS...

```
> OLS<- lm(Y~X,data=Vars)
> summary(OLS)

Call:
lm(formula = Y ~ X, data = Vars)

Residuals:
    Min      1Q  Median      3Q     Max
-3.3809 -0.6058 -0.0102  0.6320  2.9470

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.04770    0.04209   24.89   <2e-16 ***
X            1.40254    0.04005   35.02   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.9413 on 498 degrees of freedom
Multiple R-squared:  0.7112,Adjusted R-squared:  0.7106
F-statistic:  1226 on 1 and 498 DF,  p-value: < 2.2e-16
```

Two-Stage Least Squares

```
> TSLS<-tsls(Y~I(X),data=Vars,instruments=~Z)
> summary(TSLS)

2SLS Estimates

Model Formula: Y ~ I(X)

Instruments: ~Z

Residuals:
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-3.29300 -0.68210 -0.06139  0.00000  0.76270  2.70300

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) 1.0491828  0.0456017 23.00754 < 2.22e-16 ***
I(X)        1.0302012  0.0536909 19.18763 < 2.22e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.0196738 on 498 degrees of freedom
```

IV: A (Toy) Example



```
> summary(IRData)
     dyadid         logdisputes         logtrade            IOs        
 Min.   :  2020   Min.   :-0.6931   Min.   :-0.6931   Min.   : 4.579  
 1st Qu.:135155   1st Qu.:-0.6931   1st Qu.: 2.4079   1st Qu.:19.500  
 Median :220484   Median :-0.6931   Median : 5.5786   Median :27.704  
 Mean   :275526   Mean   :-0.2627   Mean   : 4.6518   Mean   :30.891  
 3rd Qu.:385710   3rd Qu.: 0.0000   3rd Qu.: 7.1248   3rd Qu.:39.289  
 Max.   :900920   Max.   : 3.4965   Max.   :11.5037   Max.   :93.700  
   contiguity        capratio         GDPgrowth       
 Min.   :0.0000   Min.   :   1.081   Min.   :-9.0800  
 1st Qu.:0.0000   1st Qu.:   4.849   1st Qu.:-0.2923  
 Median :0.0000   Median :  26.577   Median : 0.8363  
 Mean   :0.3207   Mean   : 196.310   Mean   : 0.5097  
 3rd Qu.:1.0000   3rd Qu.: 144.035   3rd Qu.: 1.7106  
 Max.   :1.0000   Max.   :7451.982   Max.   : 7.0460  
```

Ordinary Regression

```
> OLSWar<-lm(logdisputes~logtrade+contiguity+capratio,data=IRData)
> summary(OLSWar)

Call:
lm(formula = logdisputes ~ logtrade + contiguity + capratio,
    data = IRData)

Residuals:
   Min     1Q Median     3Q    Max
-0.828 -0.326 -0.269 -0.090  3.455

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.4253192  0.0602014   -7.06  3.5e-12 ***
logtrade     0.0085581  0.0105739    0.81    0.419
contiguity   0.4622674  0.0712406    6.49  1.5e-10 ***
capratio    -0.0001296  0.0000647   -2.00    0.045 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.853 on 813 degrees of freedom
Multiple R-squared:  0.083,Adjusted R-squared:  0.0796
F-statistic: 24.5 on 3 and 813 DF,  p-value: 3.35e-15
```

2SLS "By-Hand" (stage one)

```
> ITrade<-lm(logtrade~contiguity+IOs+capratio)
> summary(ITrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.0385 -1.7666  0.4139  1.6154  7.6029

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.7319793  0.1912570   3.827 0.000140 ***
contiguity   1.3386037  0.1816041   7.371 4.17e-13 ***
IOs          0.1218373  0.0055313  22.027  < 2e-16 ***
capratio    -0.0013913  0.0001626  -8.555  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.239 on 813 degrees of freedom
Multiple R-squared: 0.5535,Adjusted R-squared: 0.5519
F-statistic: 335.9 on 3 and 813 DF,  p-value: < 2.2e-16
```

2SLS "By-Hand" (stage two)

```
> IVWarByHand<-with(IRData, lm(logdisputes~capratio+contiguity+
+                          (ITrade$fitted.values)))
> summary(IVWarByHand)

Call:
lm(formula = logdisputes ~ capratio + contiguity + (ITrade$fitted.values))

Residuals:
   Min     1Q Median     3Q    Max
-1.006 -0.362 -0.278 -0.049  3.530

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)          -0.1515180  0.0832287   -1.82  0.06905 .
capratio             -0.0002664  0.0000705   -3.78  0.00017 ***
contiguity            0.6263774  0.0788444    7.94  6.5e-15 ***
ITrade$fitted.values -0.0558374  0.0171921   -3.25  0.00121 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.848 on 813 degrees of freedom
Multiple R-squared:  0.094,Adjusted R-squared:  0.0907
F-statistic: 28.1 on 3 and 813 DF,  p-value: <2e-16
```

2SLS, Automagically

```
> library(sem)
> TwoSLSWar<-tsls(logdisputes~contiguity+capratio+I(logtrade),
   instruments=~contiguity+capratio+IOs)
> summary(TwoSLSWar)

2SLS Estimates

Model Formula: logdisputes ~ contiguity + capratio + I(logtrade)

Instruments: ~contiguity + capratio + IOs

Residuals:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-1.21e+00 -5.24e-01 -2.26e-01 -7.44e-17 -2.10e-02  3.65e+00

              Estimate Std. Error t value  Pr(>|t|)
(Intercept) -0.1515180  8.562e-02  -1.770 7.717e-02
contiguity   0.6263774  8.111e-02   7.722 3.353e-14
capratio    -0.0002664  7.252e-05  -3.674 2.543e-04
I(logtrade) -0.0558374  1.769e-02  -3.157 1.652e-03

Residual standard error: 0.8723 on 813 degrees of freedom
```

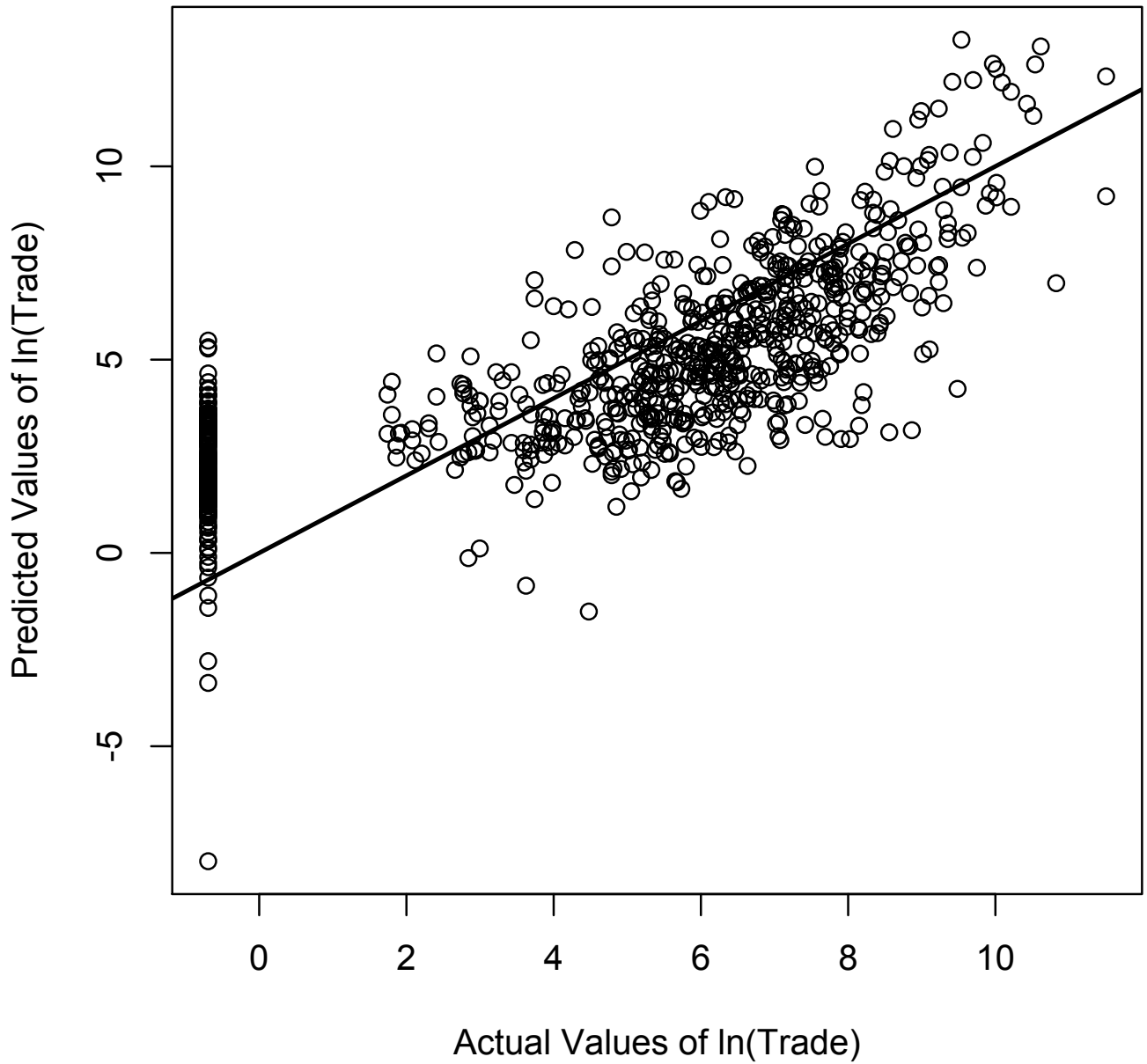
Weak Instruments

```
> OLSTrade<-lm(logtrade~logdisputes+contiguity+IOs)
> summary(OLSTrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.2467 -2.2067  0.4275  1.6659  6.1264

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.191111   0.182875   1.045    0.296
logdisputes 0.408116   0.095067   4.293 1.98e-05 ***
contiguity  1.357557   0.193109   7.030 4.38e-12 ***
IOs         0.133778   0.005614  23.831  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.312 on 813 degrees of freedom
Multiple R-squared: 0.5241,Adjusted R-squared: 0.5223
F-statistic: 298.4 on 3 and 813 DF,  p-value: < 2.2e-16
```

Weak Instruments (continued)

```
> TwoSLSTrade<-tsls(logtrade~contiguity+IOs+I(logdisputes),
    instruments=~contiguity+capratio+IOs)
> summary(TwoSLSTrade)

2SLS Estimates

Model Formula: logtrade ~ contiguity + IOs + I(logdisputes)

Instruments: ~contiguity + capratio + IOs

Residuals:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-2.57e+01 -1.46e+00  1.36e+00  2.84e-14  4.00e+00  1.09e+01

                Estimate Std. Error t value  Pr(>|t|)
(Intercept)        2.150    0.85122   2.526 1.173e-02
contiguity        -2.728    1.52615  -1.787 7.427e-02
IOs                0.172    0.02045   8.408 2.220e-16
I(logdisputes)     7.371    2.45198   3.006 2.727e-03

Residual standard error: 6.3721 on 813 degrees of freedom
```

Pretty Good Instrument (Trade)



Crappy Instrument (War)