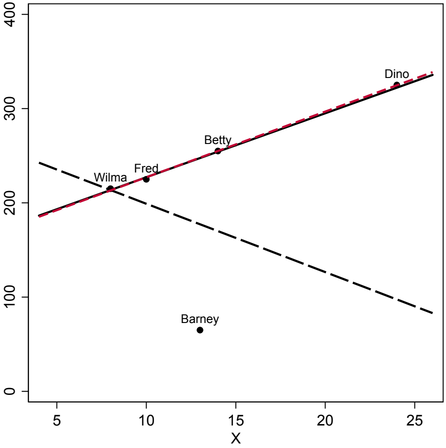
PLSC 503 – Spring 2022 Residuals, Model Fit, and Outliers + Simultaneity

February 23, 2022

Discrepancy, Leverage, and Influence



Note: Solid line is the regression fit for Wilma, Fred, and Betty only.
Long-dashed line is the regression for Wilma, Fred, Betty, and Barney.
Short-dashed (red) line is the regression for Wilma, Fred, Betty and Dino.

Discrepancy, Leverage, and Influence

$$\text{Influence} = \text{Leverage} \times \text{Discrepancy}$$

Leverage

$$\begin{aligned}\hat{\mathbf{Y}} &= \mathbf{X}\hat{\boldsymbol{\beta}} \\ &= \mathbf{X}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}] \\ &= \mathbf{HY}\end{aligned}$$

where

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'.$$

$$h_i = \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i'$$

Variation:

$$\widehat{\text{Var}(\hat{u}_i)} = \hat{\sigma}^2[1 - \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i'] \tag{1}$$

$$\begin{aligned} \widehat{\text{s.e.}(\hat{u}_i)} &= \hat{\sigma}\sqrt{[1 - \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i']} \\ &= \hat{\sigma}\sqrt{1 - h_i} \end{aligned} \tag{2}$$

"Standardized":

$$\tilde{u}_i = \frac{\hat{u}_i}{\hat{\sigma}\sqrt{1 - h_i}} \tag{3}$$

"Studentized": define

$$\begin{aligned}\hat{\sigma}^2_{-i} &= \text{Variance for the } N-1 \text{ observations} \neq i \\ &= \frac{\hat{\sigma}^2(N-K)}{N-K-1} - \frac{\hat{u}_i^2}{(N-K-1)(1-h_i)}. \qquad (4)\end{aligned}$$

Then:

$$\hat{u}'_i = \frac{\hat{u}_i}{\hat{\sigma}_{-i}\sqrt{1-h_i}} \qquad (5)$$

"DFBETA":

$$D_{ki} = \hat{\beta}_k - \hat{\beta}_{k(-i)} \tag{6}$$

"DFBETAS" (the "S" is for "standardized):

$$D^*_{ki} = \frac{D_{ki}}{\widehat{\text{s.e.}(\hat{\beta}_{k(-i)})}} \tag{7}$$

Cook's D:

$$\begin{aligned} D_i &= \frac{\tilde{u}_i^2}{K} \times \frac{h_i}{1 - h_i} \\ &= \frac{h_i \hat{u}_i^2}{K \hat{\sigma}^2 (1 - h_i)^2} \end{aligned} \tag{8}$$

Variance

```
> # No Barney OR Dino...
> summary(lm(Y~X,data=subset(flintstones,name!="Dino" & name!="Barney")))

Residuals:
     2      4      5 
 0.714 -2.143  1.429 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  159.286      6.776    23.5    0.027 *
X              6.786      0.619    11.0    0.058 .
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.67 on 1 degrees of freedom
Multiple R-squared: 0.992,Adjusted R-squared: 0.984 
F-statistic:  120 on 1 and 1 DF,  p-value: 0.0579
```

```
> # No Barney (Dino included...)
> summary(lm(Y~X,data=subset(flintstones,name!="Barney")))

Residuals:
        2         3         4         5
-8.88e-16  2.63e-01 -2.11e+00  1.84e+00

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  157.368      2.465    63.8  0.00025 ***
X              6.974      0.161    43.3  0.00053 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.99 on 2 degrees of freedom
Multiple R-squared: 0.999,Adjusted R-squared: 0.998
F-statistic: 1.87e+03 on 1 and 2 DF,  p-value: 0.000534
```

A Variance-Based Statistic

"COVRATIO":

$$\text{COVRATIO}_i = \left[(1 - h_i)\left(\frac{N - K - 1 + \hat{u}_i'^2}{N - K}\right)^K\right]^{-1} \tag{9}$$

Example: Federal Judicial Review, 1789-2018

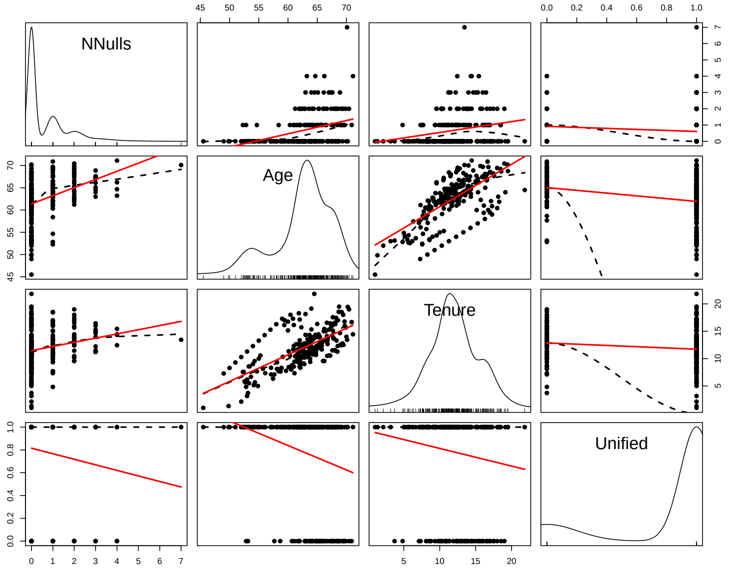
Dahl (1957):

- SCOTUS gets "out of step" with the other branches → judicial review
- Older / longer-serving justices will more likely to invalidate legislation

Data:

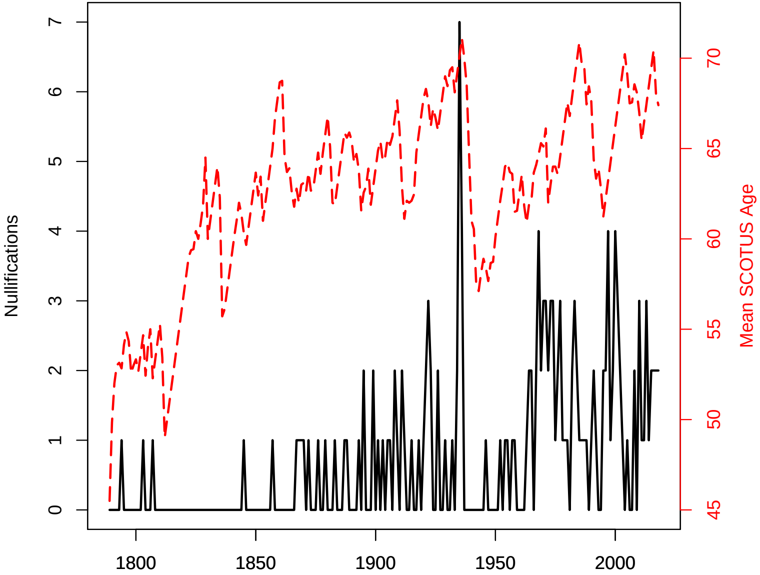
```
> summary(NewDahl)
      Year          NNulls            Age           Tenure         Unified     
 Min.   :1789   Min.   :0.000   Min.   :45.5   Min.   : 1.0   Min.   :0.000  
 1st Qu.:1846   1st Qu.:0.000   1st Qu.:60.7   1st Qu.:10.0   1st Qu.:1.000  
 Median :1904   Median :0.000   Median :63.5   Median :11.8   Median :1.000  
 Mean   :1904   Mean   :0.674   Mean   :62.6   Mean   :12.0   Mean   :0.783  
 3rd Qu.:1961   3rd Qu.:1.000   3rd Qu.:66.0   3rd Qu.:14.1   3rd Qu.:1.000  
 Max.   :2018   Max.   :7.000   Max.   :71.1   Max.   :21.8   Max.   :1.000  
```

Example: Federal Judicial Review, 1789-2018



```
> Fit<-with(NewDahl, lm(NNulls~Age+Tenure+Unified))
> summary(Fit)

Call:
lm(formula = NNulls ~ Age + Tenure + Unified)

Residuals:
   Min     1Q Median     3Q    Max
-1.308 -0.700 -0.135  0.308  5.693

Coefficients:
            Estimate Std. Error t value  Pr(>|t|)
(Intercept)  -4.6833     1.0026   -4.67 0.0000051 ***
Age           0.0901     0.0181    4.97 0.0000013 ***
Tenure       -0.0201     0.0248   -0.81      0.42
Unified      -0.0573     0.1613   -0.36      0.72
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.973 on 226 degrees of freedom
  (4 observations deleted due to missingness)
Multiple R-squared:  0.152,Adjusted R-squared:  0.141
F-statistic: 13.6 on 3 and 226 DF,  p-value: 0.0000000365
```

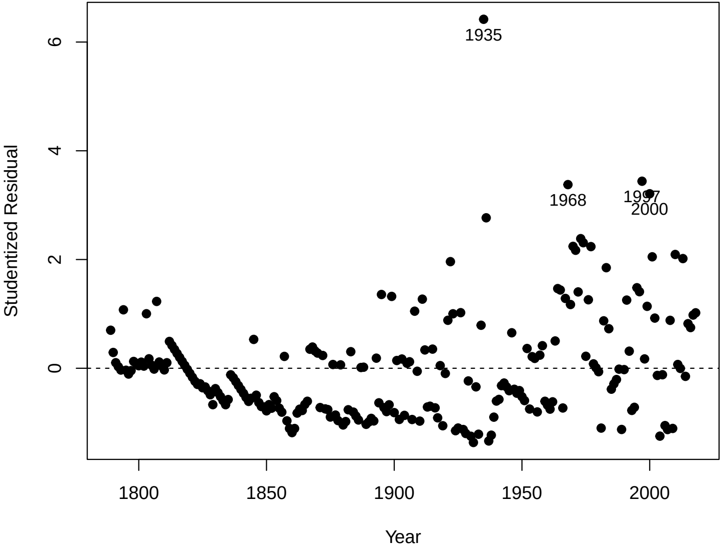
Federal Judicial Review and Mean SCOTUS Age



Residuals, etc.

```
> FitResid<-with(NewDahl, (Fit$model$NNulls - predict(Fit)))
> FitStandard<-rstandard(Fit) # standardized residuals
> FitStudent<-rstudent(Fit) # studentized residuals
> FitCooksD<-cooks.distance(Fit) # Cook?s D
> FitDFBeta<-dfbeta(Fit) # DFBeta
> FitDFBetaS<-dfbetas(Fit) # DFBetaS
> FitCOVRATIO<-covratio(Fit) # COVRATIOs
```

Studentized Residuals

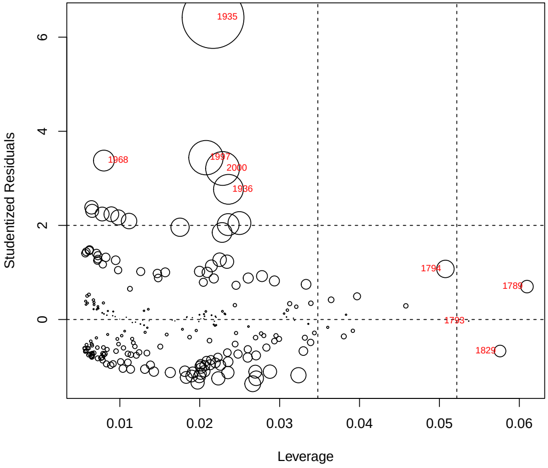


More About Studentized Residuals

```
> max(FitStudent)
[1] 6.418

> NewDahl$Year1935<-ifelse(NewDahl$Year==1935,1,0)
> summary(with(NewDahl, lm(NNulls~Age+Tenure+Unified+Year1935)))

Residuals:
   Min     1Q Median     3Q    Max
-1.250 -0.652 -0.122  0.302  3.247

Coefficients:
            Estimate Std. Error t value      Pr(>|t|)
(Intercept)  -3.9298     0.9313   -4.22 0.00003546916 ***
Age           0.0768     0.0168    4.56 0.00000846697 ***
Tenure       -0.0113     0.0229   -0.50          0.62
Unified      -0.1210     0.1490   -0.81          0.42
Year1935      5.8186     0.9066    6.42 0.00000000081 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.897 on 225 degrees of freedom
Multiple R-squared:  0.284,Adjusted R-squared:  0.271
F-statistic: 22.3 on 4 and 225 DF,  p-value: 1.65e-15
```

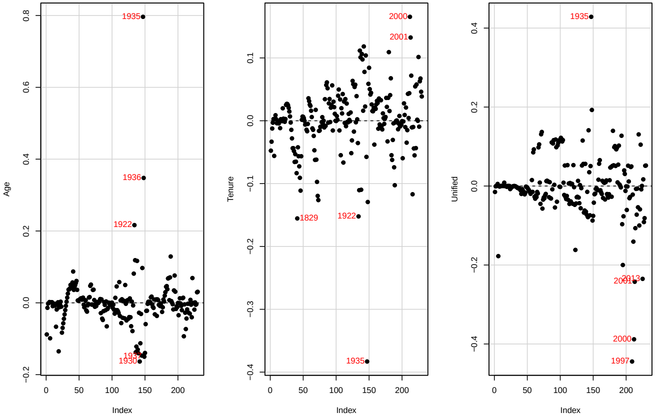
“Bubble Plot”

```
> influencePlot(Fit,id=list(method="noteworthy",n=4,cex=0.7,
                       labels=NewDahl$Year,col="red"),
              xlab="Leverage")
```



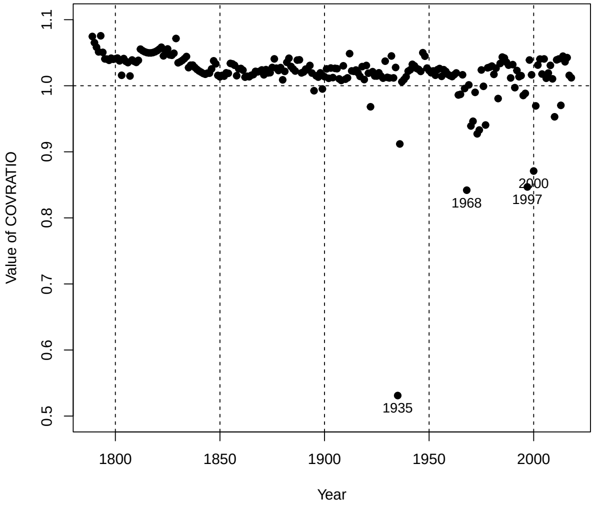
DFBETAS

```
> dfbetasPlots(Fit,id.n=5,id.col="red",main="",pch=19,
               layout=c(1,3),labels=NewDahl$Year)
```



COVRATIO Plot

```
> plot(FitCOVRATIO~NewDahl$Year,pch=19,ylim=c(0.5,1.1),
    xlab="Year",ylab="Value of COVRATIO")
```

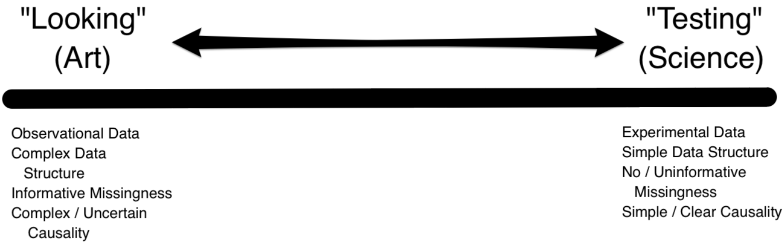


Sensitivity Analyses: Omitting Outliers

```
> out1<-c(1935) # one outlier
> LD2<-NewDahl[!(NewDahl$Year %in% out1),]
> out2<-c(1935,1968,1997,2000) # four outliers
> LD3<-NewDahl[!(NewDahl$Year %in% out2),]
> Fit2<-lm(NNulls~Age+Tenure+Unified,data=LD2)
> Fit3<-lm(NNulls~Age+Tenure+Unified,data=LD3)
```

	Dependent variable:		
	NNulls		
	(1)	(2)	(3)
Age	0.090***	0.077***	0.079***
	(0.018)	(0.017)	(0.015)
Tenure	−0.020	−0.011	−0.019
	(0.025)	(0.023)	(0.021)
Unified	−0.057	−0.121	−0.010
	(0.161)	(0.149)	(0.139)
Constant	−4.683***	−3.930***	−4.130***
	(1.003)	(0.931)	(0.855)
Observations	230	229	226
R^2	0.152	0.148	0.158
Adjusted R^2	0.141	0.137	0.147
Residual Std. Error	0.973 (df = 226)	0.897 (df = 225)	0.822 (df = 222)
F Statistic	13.550*** (df = 3; 226)	13.030*** (df = 3; 225)	13.930*** (df = 3; 222)
Note:			*p<0.1; **p<0.05; ***p<0.01

Thinking About Diagnostics



One Approach

Pena, E.A. and E.H. Slate. 2006. "Global Validation of Linear Model Assumptions." *J. American Statistical Association* 101(473):341-354.

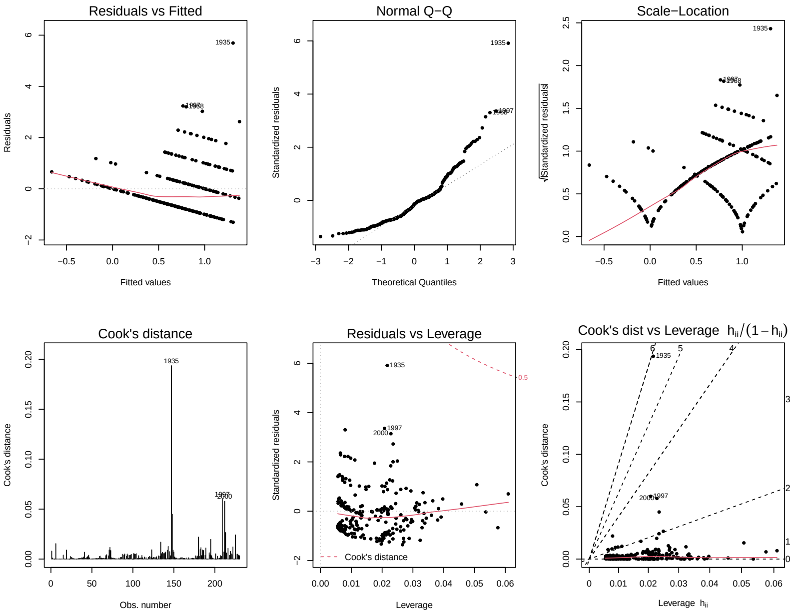
Tests for:

- Normality in $\hat{u}$s (via skewness & kurtosis tests)
- "Link function" (linearity / additivity)
- Constant variance and uncorrelatedness in $\hat{u}$s ("heteroskedasticity" test)

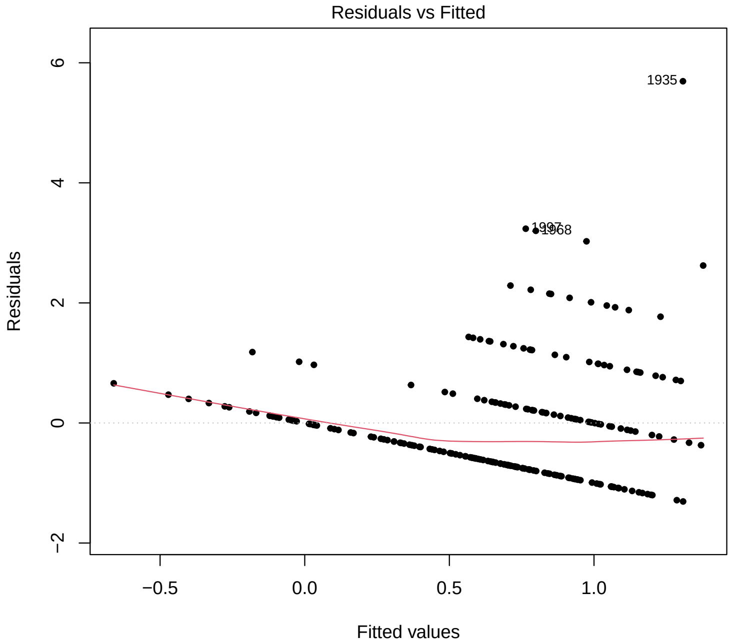
In Action

```
> Fit<-with(NewDahl, lm(NNulls~Age+Tenure+Unified))

> library(gvlma)
> Nope <- gvlma(Fit)
> display.gvlmatests(Nope)

ASSESSMENT OF THE LINEAR MODEL ASSUMPTIONS
USING THE GLOBAL TEST ON 4 DEGREES-OF-FREEDOM:
Level of Significance =  0.05

Call:
 gvlma(x = Fit)

                    Value  p-value                   Decision
Global Stat        454.87 0.00e+00 Assumptions NOT satisfied!
Skewness           122.09 0.00e+00 Assumptions NOT satisfied!
Kurtosis           283.21 0.00e+00 Assumptions NOT satisfied!
Link Function        5.35 2.07e-02 Assumptions NOT satisfied!
Heteroscedasticity  44.23 2.92e-11 Assumptions NOT satisfied!
```

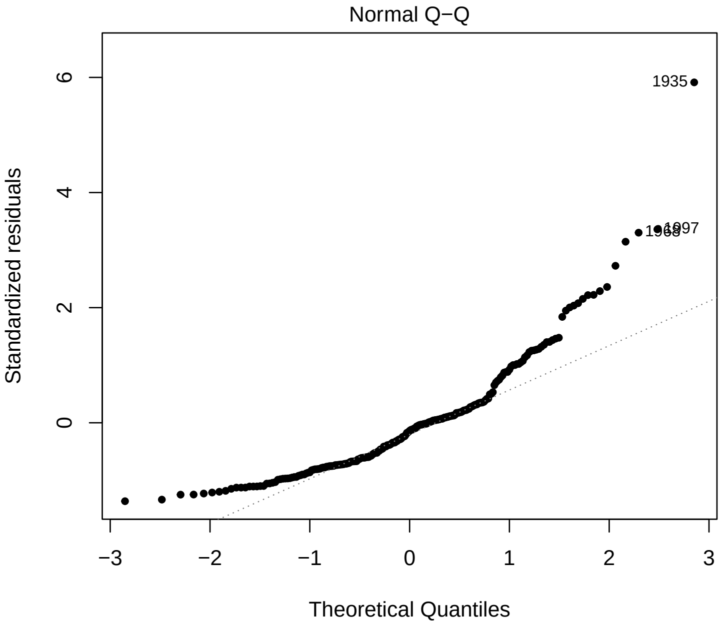
Another Approach: `plot(fit)`



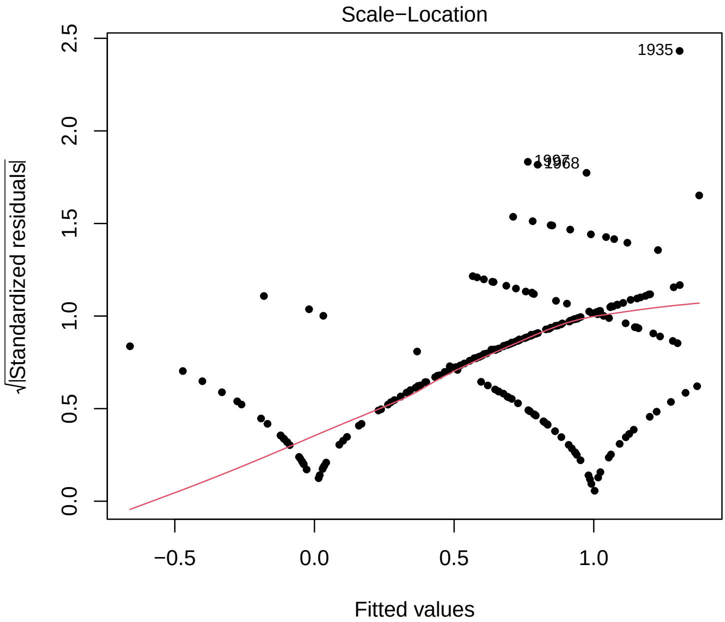
#1: Residuals vs. Fitted Values



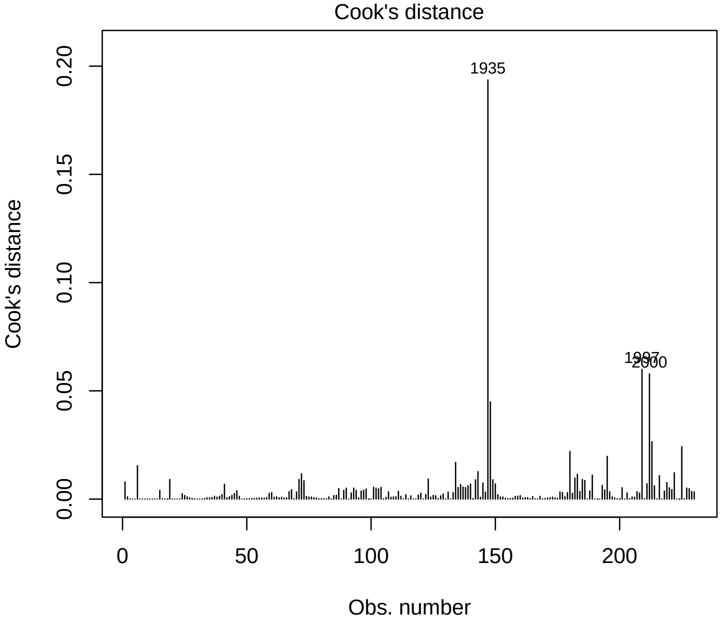
#2: Q-Q Plot of $\hat{u}$s



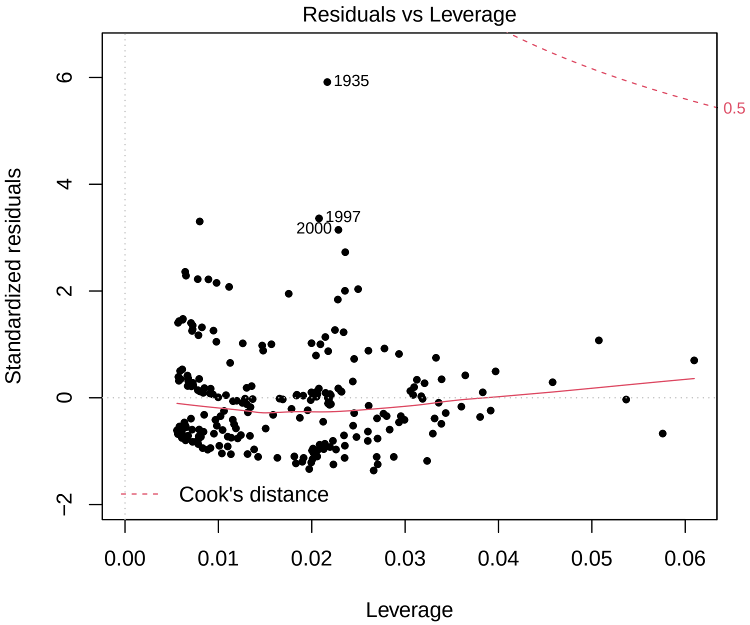
"Scale-Location" Plot



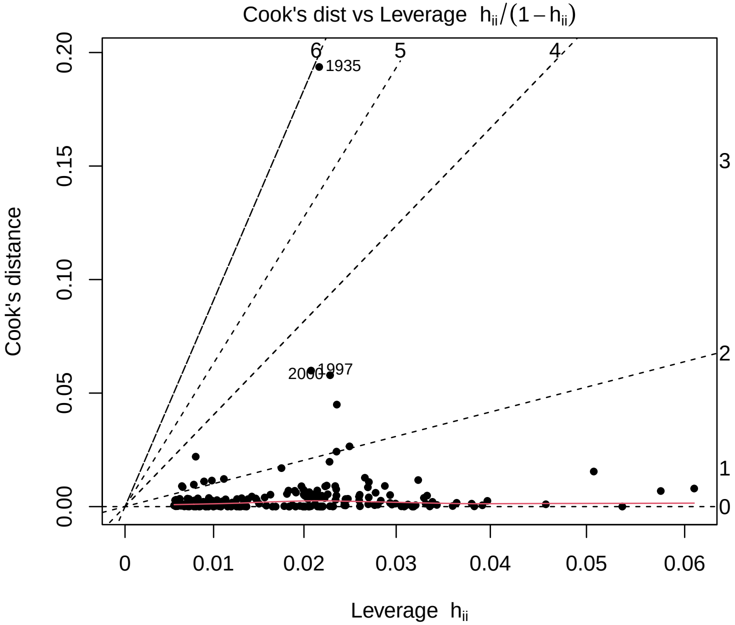
Cook's D


Cook's distance
Cook's distance
0.00
0.05
0.10
0.15
0.20
0
50
100
150
200
Obs. number
1935
1997
2000

Residuals vs. Leverage



Cook's D vs. Leverage



Simultaneity and Endogeneity

Consider:

$$Y_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

$$Y_2 = \mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2$$

Rewrite:

$$\begin{aligned}
Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1[\mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2] + \mathbf{u}_1 \\
&= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\gamma_2 Y_1 + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 - \gamma_1\gamma_2 Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
(1 - \gamma_1\gamma_2) Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 &= \mathbf{X}_1\left(\frac{1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_1\right) + \mathbf{X}_2\left(\frac{\gamma_1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_2\right) + \left(\frac{\gamma_1\mathbf{u}_2 + \mathbf{u}_1}{1-\gamma_1\gamma_2}\right) \\
&= \Delta_1\mathbf{X}_1 + \Delta_2\mathbf{X}_2 + \mathbf{e}
\end{aligned}$$

"Reduced Form"

$$Y_1 = \mathbf{X}_1 \left(\frac{1}{1 - \gamma_1 \gamma_2} \boldsymbol{\beta}_1 \right) + \mathbf{X}_2 \left(\frac{\gamma_1}{1 - \gamma_1 \gamma_2} \boldsymbol{\beta}_2 \right) + \left(\frac{\gamma_1 \mathbf{u}_2 + \mathbf{u}_1}{1 - \gamma_1 \gamma_2} \right)$$

means

$$\frac{\partial Y_1}{\partial X_\ell} = \frac{\beta_\ell}{1 - \gamma_1 \gamma_2}.$$

But

$$\hat{\Delta}_1 \neq \hat{\boldsymbol{\beta}}_1.$$

Simultaneity Bias

For (e.g.)

$$Y_1 = \mathbf{X}_1 \boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

we have:

$$\mathrm{E}(Y_2, \mathbf{u}_1) = \frac{\gamma_2}{1 - \gamma_1 \gamma_2} \sigma^2_{\mathbf{u}}$$

Result:

- Bias (unless $\gamma_2 = 0$)
- Inconsistency

What To Do

- OLS
- Lagged Variables
- Two-Stage Least Squares (2SLS)
- Systems of Equations / 3SLS / etc.

Recall that a simple linear model:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

gives us:

$$\hat{\boldsymbol{\beta}}_{OLS} = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}.$$

Suppose $\text{Cov}(\mathbf{X}, \mathbf{u}) \neq \mathbf{0}$, but we have $\mathbf{Z}$ with

- $\text{Cov}(\mathbf{Z}, \mathbf{X}) \neq \mathbf{0}$ and
- $\text{Cov}(\mathbf{Z}, \mathbf{u}) = \mathbf{0}$.

Then:

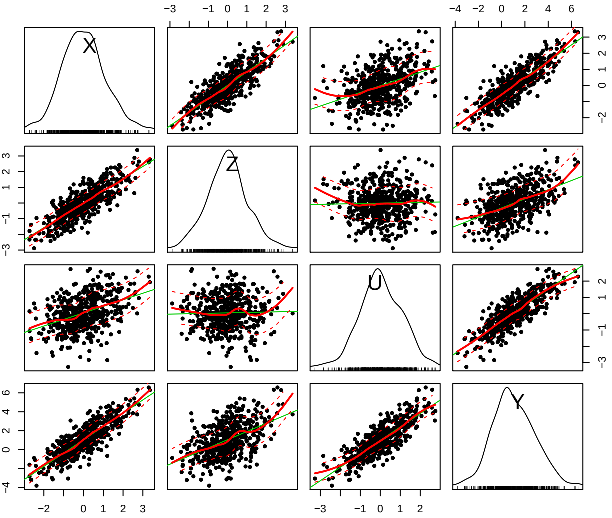
$$\begin{aligned} \hat{\boldsymbol{\beta}}_{IV} &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Y} \\ &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'(\mathbf{X}\boldsymbol{\beta} + \mathbf{u}) \\ &= \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{u} \end{aligned}$$

is consistent.

2SLS: How-To

- Regress endogenous $\mathbf{X}$s variables on $\{\mathbf{Z}, \mathbf{X}\}$
- Generate $\hat{\mathbf{X}}$s
- Regress Y on $\hat{\mathbf{X}}$ to get β_{2SLS}.
- Adjust standard error estimates

```
library(MASS)
library(sem)
library(car)

seed<-1337
set.seed(seed)

mu<-c(0,0,0) # <== X, Z, U
Sigma<-matrix(c(1,0.8,0.4,0.8,1,0,0.4,0,1),
             nrow=3,byrow=TRUE)                # Cor(X,Y)=0.8, etc.
Vars<- mvrnorm(500,mu,Sigma)
colnames(Vars)<-c("X","Z","U")
Vars<-data.frame(Vars)

Vars$Y<- 1 + Vars$X + Vars$U
```

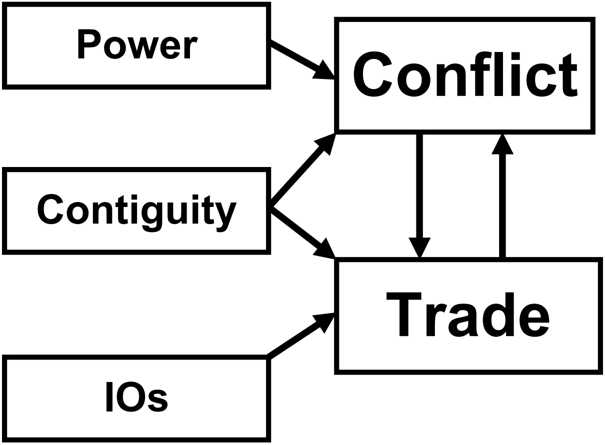

X
Z
U
Y
−3 −1 0 1 2 3
−4 −2 0 2 4 6
−2 0 1 2 3
−3 −1 0 1 2
−3 −1 1 2 3
−4 0 2 4 6
−2 0 1 2 3
−3 −1 1 2

Plain Old OLS...

```
> OLS<- lm(Y~X,data=Vars)
> summary(OLS)

Call:
lm(formula = Y ~ X, data = Vars)

Residuals:
    Min      1Q  Median      3Q     Max
-3.3809 -0.6058 -0.0102  0.6320  2.9470

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.04770    0.04209   24.89   <2e-16 ***
X            1.40254    0.04005   35.02   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.9413 on 498 degrees of freedom
Multiple R-squared:  0.7112,Adjusted R-squared:  0.7106
F-statistic:  1226 on 1 and 498 DF,  p-value: < 2.2e-16
```

```
> TSLS<-tsls(Y~I(X),data=Vars,instruments=~Z)
> summary(TSLS)

2SLS Estimates

Model Formula: Y ~ I(X)

Instruments: ~Z

Residuals:
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-3.29300 -0.68210 -0.06139  0.00000  0.76270  2.70300

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) 1.0491828  0.0456017 23.00754 < 2.22e-16 ***
I(X)        1.0302012  0.0536909 19.18763 < 2.22e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.0196738 on 498 degrees of freedom
```

IV: A (Toy) Example



```
> summary(IRData)
     dyadid        logdisputes          logtrade             IOs
 Min.   :  2020   Min.   :-0.6931   Min.   :-0.6931   Min.   : 4.579
 1st Qu.:135155   1st Qu.:-0.6931   1st Qu.: 2.4079   1st Qu.:19.500
 Median :220484   Median :-0.6931   Median : 5.5786   Median :27.704
 Mean   :275526   Mean   :-0.2627   Mean   : 4.6518   Mean   :30.891
 3rd Qu.:385710   3rd Qu.: 0.0000   3rd Qu.: 7.1248   3rd Qu.:39.289
 Max.   :900920   Max.   : 3.4965   Max.   :11.5037   Max.   :93.700
   contiguity         capratio          GDPgrowth
 Min.   :0.0000   Min.   :   1.081   Min.   :-9.0800
 1st Qu.:0.0000   1st Qu.:   4.849   1st Qu.:-0.2923
 Median :0.0000   Median :  26.577   Median : 0.8363
 Mean   :0.3207   Mean   : 196.310   Mean   : 0.5097
 3rd Qu.:1.0000   3rd Qu.: 144.035   3rd Qu.: 1.7106
 Max.   :1.0000   Max.   :7451.982   Max.   : 7.0460
```

Ordinary Regression

```
> OLSWar<-lm(logdisputes~logtrade+contiguity+capratio,data=IRData)
> summary(OLSWar)

Call:
lm(formula = logdisputes ~ logtrade + contiguity + capratio,
    data = IRData)

Residuals:
   Min     1Q Median     3Q    Max
-0.828 -0.326 -0.269 -0.090  3.455

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.4253192  0.0602014   -7.06  3.5e-12 ***
logtrade     0.0085581  0.0105739    0.81    0.419
contiguity   0.4622674  0.0712406    6.49  1.5e-10 ***
capratio    -0.0001296  0.0000647   -2.00    0.045 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.853 on 813 degrees of freedom
Multiple R-squared:  0.083,Adjusted R-squared:  0.0796
F-statistic: 24.5 on 3 and 813 DF,  p-value: 3.35e-15
```

2SLS "By-Hand" (stage one)

```
> ITrade<-lm(logtrade~contiguity+IOs+capratio)
> summary(ITrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.0385 -1.7666  0.4139  1.6154  7.6029

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.7319793  0.1912570   3.827 0.000140 ***
contiguity   1.3386037  0.1816041   7.371 4.17e-13 ***
IOs          0.1218373  0.0055313  22.027  < 2e-16 ***
capratio    -0.0013913  0.0001626  -8.555  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.239 on 813 degrees of freedom
Multiple R-squared: 0.5535,Adjusted R-squared: 0.5519
F-statistic: 335.9 on 3 and 813 DF,  p-value: < 2.2e-16
```

2SLS “By-Hand” (stage two)

```
> IVWarByHand<-with(IRData, lm(logdisputes~capratio+contiguity+
+                        (ITrade$fitted.values)))
> summary(IVWarByHand)

Call:
lm(formula = logdisputes ~ capratio + contiguity + (ITrade$fitted.values))

Residuals:
   Min     1Q Median     3Q    Max
-1.006 -0.362 -0.278 -0.049  3.530

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)         -0.1515180  0.0832287   -1.82  0.06905 .
capratio            -0.0002664  0.0000705   -3.78  0.00017 ***
contiguity           0.6263774  0.0788444    7.94  6.5e-15 ***
ITrade$fitted.values -0.0558374  0.0171921   -3.25  0.00121 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.848 on 813 degrees of freedom
Multiple R-squared:  0.094,Adjusted R-squared:  0.0907
F-statistic: 28.1 on 3 and 813 DF,  p-value: <2e-16
```

2SLS, Automagically

```
> library(AER)
> TwoSLSWar<-ivreg(logdisputes~contiguity+capratio+I(logtrade),
   instruments=~contiguity+capratio+IOs)
> summary(TwoSLSWar)

Call:
ivreg(formula = logdisputes ~ contiguity + capratio + I(logtrade) |
    contiguity + capratio + IOs, data = IRData)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.1515180  0.0856218   -1.77  0.07717 .
contiguity   0.6263774  0.0811114    7.72  3.4e-14 ***
capratio    -0.0002664  0.0000725   -3.67  0.00025 ***
I(logtrade) -0.0558374  0.0176864   -3.16  0.00165 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.872 on 813 degrees of freedom
Multiple R-Squared: 0.0412,Adjusted R-squared: 0.0376
Wald test: 26.6 on 3 and 813 DF,  p-value: <2e-16
```

Weak Instruments

```
> OLSTrade<-lm(logtrade~logdisputes+contiguity+IOs)
> summary(OLSTrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.2467 -2.2067  0.4275  1.6659  6.1264

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.191111   0.182875   1.045    0.296
logdisputes 0.408116   0.095067   4.293 1.98e-05 ***
contiguity  1.357557   0.193109   7.030 4.38e-12 ***
IOs         0.133778   0.005614  23.831  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.312 on 813 degrees of freedom
Multiple R-squared: 0.5241,Adjusted R-squared: 0.5223
F-statistic: 298.4 on 3 and 813 DF,  p-value: < 2.2e-16
```

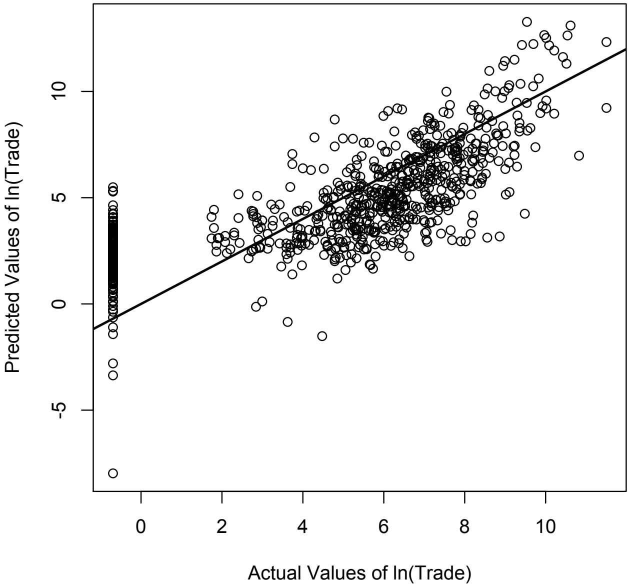
Weak Instruments (continued)

```
> TwoSLSTrade<-ivreg(logtrade~contiguity+IOs+I(logdisputes),
    instruments=~contiguity+capratio+IOs)
> summary(TwoSLSTrade)

Call:
ivreg(formula = logtrade ~ contiguity + IOs + I(logdisputes) |
    contiguity + capratio + IOs, data = IRData)

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      2.1501     0.8512    2.53   0.0117 *
contiguity      -2.7276     1.5262   -1.79   0.0743 .
IOs              0.1720     0.0205    8.41   <2e-16 ***
I(logdisputes)   7.3712     2.4520    3.01   0.0027 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 6.37 on 813 degrees of freedom
Multiple R-Squared: -2.62,Adjusted R-squared: -2.63
Wald test: 41.5 on 3 and 813 DF,  p-value: <2e-16
```

Side-By-Side...

	Dependent variable:			
	logdisputes		logtrade	
	OLS	*IV*	*OLS*	*IV*
logtrade	0.009			
	(0.011)			
logdisputes			0.408***	
			(0.095)	
contiguity	0.462***	0.626***	1.358***	−2.728*
	(0.071)	(0.081)	(0.193)	(1.526)
capratio	−0.0001**	−0.0003***		
	(0.0001)	(0.0001)		
I(logtrade)		−0.056***		
		(0.018)		
IOs			0.134***	0.172***
			(0.006)	(0.020)
I(logdisputes)				7.371***
				(2.452)
Constant	−0.425***	−0.152*	0.191	2.150**
	(0.060)	(0.086)	(0.183)	(0.851)
Observations	817	817	817	817
R^2	0.083	0.041	0.524	−2.616
Adjusted R^2	0.080	0.038	0.522	−2.630
Residual Std. Error (df = 813)	0.853	0.872	2.312	6.372
F Statistic (df = 3; 813)	24.530***		298.400***	
Note:				*p<0.1; **p<0.05; ***p<0.01

Pretty Good Instrument (Trade)



Crappy Instrument (War)