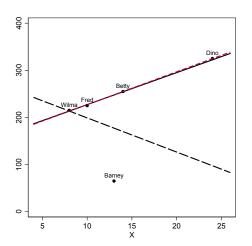
PLSC 503 – Spring 2021 Residuals, Model Fit, and Outliers + Simultaneity

February 23, 2021

Discrepancy, Leverage, and Influence



Note: Solid line is the regression fit for Wilma, Fred, and Betty only. Long-dashed line is the regression for Wilma, Fred, Betty, and Barney. Short-dashed (red) line is the regression for Wilma, Fred, Betty and Dino.

Discrepancy, Leverage, and Influence

 $Influence = Leverage \times Discrepancy$

Leverage

$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\beta}}
= \mathbf{X}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}]
= \mathbf{H}\mathbf{Y}$$

where

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'.$$

$$h_i = \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i'$$

Residuals

Variation:

$$\widehat{\mathsf{Var}(\hat{u}_i)} = \hat{\sigma}^2 [1 - \mathsf{X}_i(\mathsf{X}'\mathsf{X})^{-1} \mathsf{X}_i'] \tag{1}$$

$$\widehat{\mathsf{s.e.}(\hat{u}_i)} = \hat{\sigma}\sqrt{[1-\mathsf{X}_i(\mathsf{X}'\mathsf{X})^{-1}\mathsf{X}_i']}$$

$$= \hat{\sigma}\sqrt{1-h_i}$$
(2)

"Standardized":

$$\tilde{u}_i = \frac{\hat{u}_i}{\hat{\sigma}\sqrt{1 - h_i}} \tag{3}$$

Residuals

"Studentized": define

$$\hat{\sigma}_{-i}^{2} = \text{Variance for the } N-1 \text{ observations } \neq i$$

$$= \frac{\hat{\sigma}^{2}(N-K)}{N-K-1} - \frac{\hat{u}_{i}^{2}}{(N-K-1)(1-h_{i})}. \tag{4}$$

Then:

$$\hat{u}_i' = \frac{\hat{u}_i}{\hat{\sigma}_{-i}\sqrt{1 - h_i}} \tag{5}$$

Influence

"DFBETA":

$$D_{ki} = \hat{\beta}_k - \hat{\beta}_{k(-i)} \tag{6}$$

"DFBETAS" (the "S" is for "standardized):

$$D_{ki}^* = \frac{D_{ki}}{\widehat{\mathsf{s.e.}}(\widehat{\beta}_{k(-i)})} \tag{7}$$

Cook's D:

$$D_{i} = \frac{\tilde{u}_{i}^{2}}{K} \times \frac{h_{i}}{1 - h_{i}}$$

$$= \frac{h_{i}\hat{u}_{i}^{2}}{K\hat{\sigma}^{2}(1 - h_{i})^{2}}$$
(8)

Variance

```
> # No Barney OR Dino...
> summary(lm(Y~X,data=subset(flintstones,name!="Dino" & name!="Barney")))
Residuals:
    2 4 5
0.714 -2.143 1.429
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 159.286 6.776 23.5 0.027 *
Х
              6.786 0.619 11.0 0.058 .
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 2.67 on 1 degrees of freedom
Multiple R-squared: 0.992, Adjusted R-squared: 0.984
F-statistic: 120 on 1 and 1 DF, p-value: 0.0579
```

Variance

```
> # No Barney (Dino included...)
> summary(lm(Y~X,data=subset(flintstones,name!="Barney")))
Residuals:
       2
-8.88e-16 2.63e-01 -2.11e+00 1.84e+00
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 157.368 2.465 63.8 0.00025 ***
Х
              6.974
                        0.161 43.3 0.00053 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 1.99 on 2 degrees of freedom
Multiple R-squared: 0.999, Adjusted R-squared: 0.998
F-statistic: 1.87e+03 on 1 and 2 DF, p-value: 0.000534
```

A Variance-Based Statistic

"COVRATIO":

$$\mathsf{COVRATIO}_i = \left[(1 - h_i) \left(\frac{N - K - 1 + \hat{u}_i'^2}{N - K} \right)^K \right]^{-1} \tag{9}$$

Example: Federal Judicial Review, 1789-2018

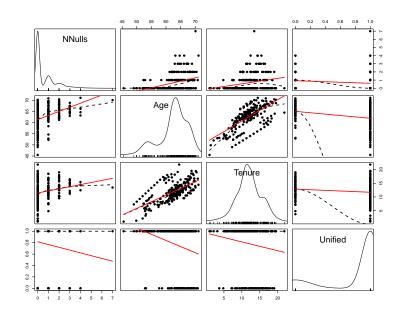
Dahl (1957):

- \bullet SCOTUS gets "out of step" with the other branches \to judicial review
- Older / longer-serving justices will more likely to invalidate legislation

Data:

> summary(NewDahl) Year NNulls Tenure Unified Age Min. :1789 Min. :0.000 Min. :45.5 Min. : 1.0 Min. :0.000 1st Qu.:1846 1st Qu.:0.000 1st Qu.:60.7 1st Qu.:10.0 1st Qu.:1.000 Median:1904 Median :0.000 Median:63.5 Median:11.8 Median :1.000 :1904 Mean :0.674 :62.6 Mean :12.0 Mean Mean Mean :0.783 3rd Qu.:1961 3rd Qu.:1.000 3rd Qu.:66.0 3rd Qu.:14.1 3rd Qu.:1.000 Max. :2018 Max. :7.000 Max. :71.1 Max. :21.8 Max. :1.000

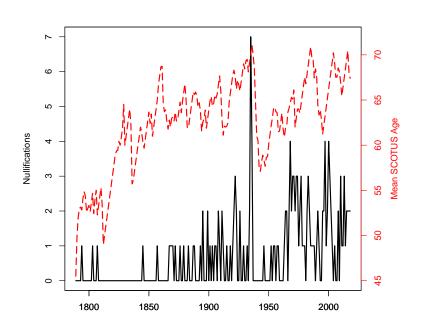
Example: Federal Judicial Review, 1789-2018



Basic Regression...

```
> Fit<-with(NewDahl, lm(NNulls~Age+Tenure+Unified))
> summary(Fit)
Call:
lm(formula = NNulls ~ Age + Tenure + Unified)
Residuals:
  Min
         10 Median 30
                            Max
-1.308 - 0.700 - 0.135 0.308 5.693
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -4.6833 1.0026 -4.67 0.0000051 ***
Age
          0.0901 0.0181 4.97 0.0000013 ***
Tenure -0.0201 0.0248 -0.81 0.42
Unified -0.0573 0.1613 -0.36 0.72
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.973 on 226 degrees of freedom
  (4 observations deleted due to missingness)
Multiple R-squared: 0.152, Adjusted R-squared: 0.141
F-statistic: 13.6 on 3 and 226 DF, p-value: 0.0000000365
```

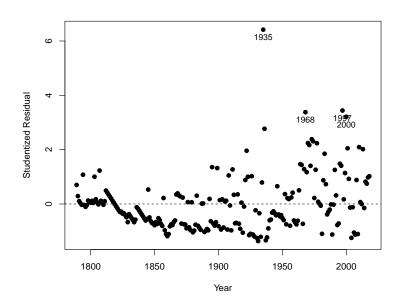
Federal Judicial Review and Mean SCOTUS Age



Residuals, etc.

- > FitResid<-with(NewDahl, (Fit\$model\$NNulls predict(Fit)))
- > FitStandard<-rstandard(Fit) # standardized residuals
- > FitStudent<-rstudent(Fit) # studentized residuals
- > FitCooksD<-cooks.distance(Fit) # Cook?s D
- > FitDFBeta<-dfbeta(Fit) # DFBeta
- > FitDFBetaS<-dfbetas(Fit) # DFBetaS
- > FitCOVRATIO<-covratio(Fit) # COVRATIOs

Studentized Residuals

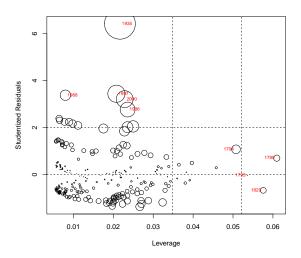


More About Studentized Residuals

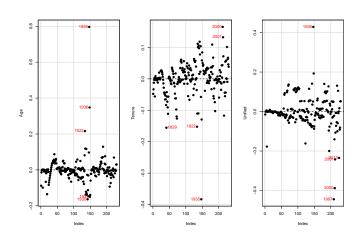
```
> max(FitStudent)
[1] 6.418
> NewDahl$Year1935<-ifelse(NewDahl$Year==1935,1,0)</pre>
> summary(with(NewDahl, lm(NNulls~Age+Tenure+Unified+Year1935)))
Residuals:
  Min
         10 Median
                     30 Max
-1.250 -0.652 -0.122 0.302 3.247
Coefficients:
          Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.9298
                     0.9313 -4.22 0.00003546916 ***
         Age
Tenure -0.0113 0.0229 -0.50
                                      0.62
Unified -0.1210 0.1490 -0.81 0.42
Year1935 5.8186 0.9066 6.42 0.00000000081 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.897 on 225 degrees of freedom
Multiple R-squared: 0.284, Adjusted R-squared: 0.271
```

F-statistic: 22.3 on 4 and 225 DF, p-value: 1.65e-15

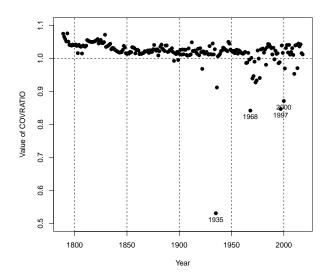
"Bubble Plot"



DFBETAS



COVRATIO Plot



Sensitivity Analyses: Omitting Outliers

- > out1<-c(1935) # one outlier
- > LD2<-NewDahl[!(NewDahl\$Year %in% out1),]
- > out2<-c(1935,1968,1997,2000) # four outliers
- > LD3<-NewDahl[!(NewDahl\$Year %in% out2),]
- > Fit2<-lm(NNulls~Age+Tenure+Unified,data=LD2)
- > Fit3<-lm(NNulls~Age+Tenure+Unified,data=LD3)

		Dependent variable:	
	NNulls		
	(1)	(2)	(3)
Age	0.090***	0.077***	0.079***
	(0.018)	(0.017)	(0.015)
Tenure	-0.020	-0.011	-0.019
	(0.025)	(0.023)	(0.021)
Unified	-0.057	-0.121	-0.010
	(0.161)	(0.149)	(0.139)
Constant	-4.683***	-3.930***	-4.130***
	(1.003)	(0.931)	(0.855)
Observations	230	229	226
R ²	0.152	0.148	0.158
Adjusted R ²	0.141	0.137	0.147
Residual Std. Error	0.973 (df = 226)	0.897 (df = 225)	0.822 (df = 222)
F Statistic	13.550 *** (df = 3; 226)	13.030*** (df = 3; 225)	13.930 *** (df = 3; 222)

*p<0.1; **p<0.05; ***p<0.01

Thinking About Diagnostics



Observational Data Complex Data Structure Informative Missingness Complex / Uncertain Causality Experimental Data Simple Data Structure No / Uninformative Missingness Simple / Clear Causality

One Approach

Pena, E.A. and E.H. Slate. 2006. "Global Validation of Linear Model Assumptions." *J. American Statistical Association* 101(473):341-354.

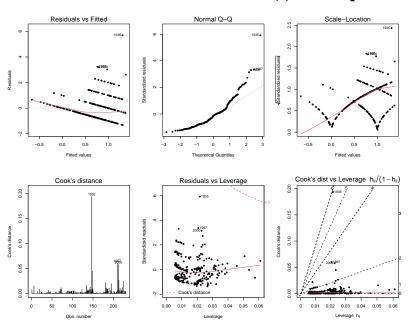
Tests for:

- Normality in ûs (via skewness & kurtosis tests)
- "Link function" (linearity / additivity)
- Constant variance and uncorrelatedness in ûs ("heteroskedasticity" test)

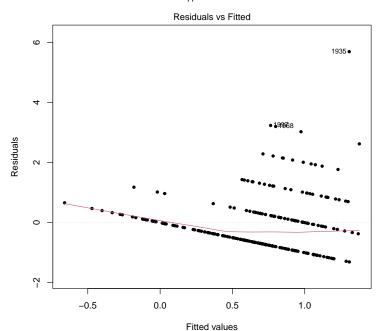
In Action

```
> Fit<-with(NewDahl, lm(NNulls~Age+Tenure+Unified))
> library(gvlma)
> Nope <- gvlma(Fit)
> display.gvlmatests(Nope)
ASSESSMENT OF THE LINEAR MODEL ASSUMPTIONS
USING THE GLOBAL TEST ON 4 DEGREES-OF-FREEDOM:
Level of Significance = 0.05
Call:
 gvlma(x = Fit)
                   Value p-value
                                                     Decision
Global Stat
                   454.87 0.00e+00 Assumptions NOT satisfied!
Skewness
                   122.09 0.00e+00 Assumptions NOT satisfied!
Kurtosis
                   283.21 0.00e+00 Assumptions NOT satisfied!
Link Function
                    5.35 2.07e-02 Assumptions NOT satisfied!
Heteroscedasticity 44.23 2.92e-11 Assumptions NOT satisfied!
```

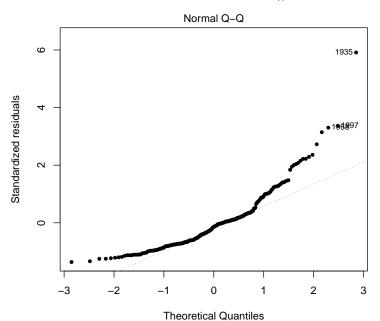
Another Approach: plot(fit)



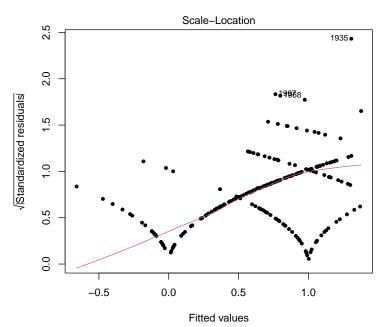
#1: Residuals vs. Fitted Values



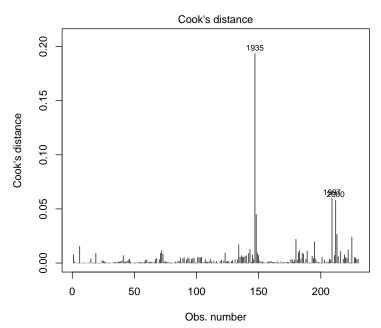
#2: Q-Q Plot of \hat{u} s



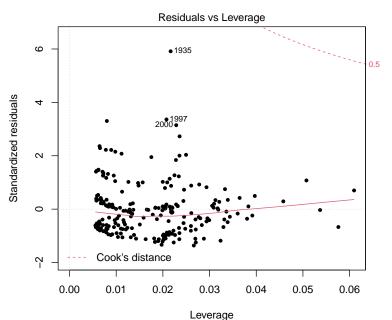
"Scale-Location" Plot



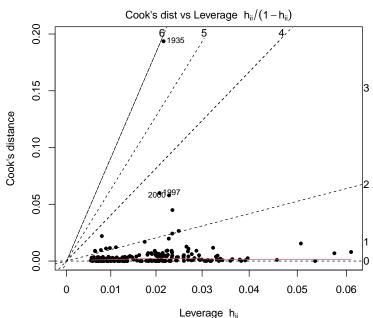
Cook's D



Residuals vs. Leverage



Cook's D vs. Leverage



Simultaneity and Endogeneity

Endogeneity

Consider:

$$\begin{aligned} \mathbf{Y}_1 &= \mathbf{X}_1 \boldsymbol{\beta}_1 + \gamma_1 \mathbf{Y}_2 + \mathbf{u}_1 \\ \\ \mathbf{Y}_2 &= \mathbf{X}_2 \boldsymbol{\beta}_2 + \gamma_2 \mathbf{Y}_1 + \mathbf{u}_2 \end{aligned}$$

Rewrite:

$$\begin{array}{rcl} Y_1 & = & \textbf{X}_1\beta_1 + \gamma_1[\textbf{X}_2\beta_2 + \gamma_2\,Y_1 + \textbf{u}_2] + \textbf{u}_1 \\ & = & \textbf{X}_1\beta_1 + \gamma_1(\textbf{X}_2\beta_2) + \gamma_1\gamma_2\,Y_1 + \gamma_1\textbf{u}_2 + \textbf{u}_1 \\ Y_1 - \gamma_1\gamma_2\,Y_1 & = & \textbf{X}_1\beta_1 + \gamma_1(\textbf{X}_2\beta_2) + \gamma_1\textbf{u}_2 + \textbf{u}_1 \\ (1 - \gamma_1\gamma_2)Y_1 & = & \textbf{X}_1\beta_1 + \gamma_1(\textbf{X}_2\beta_2) + \gamma_1\textbf{u}_2 + \textbf{u}_1 \\ Y_1 & = & \textbf{X}_1\left(\frac{1}{1 - \gamma_1\gamma_2}\beta_1\right) + \textbf{X}_2\left(\frac{\gamma_1}{1 - \gamma_1\gamma_2}\beta_2\right) + \left(\frac{\gamma_1\textbf{u}_2 + \textbf{u}_1}{1 - \gamma_1\gamma_2}\right) \\ & = & \Delta_1\textbf{X}_1 + \Delta_2\textbf{X}_2 + \textbf{e} \end{array}$$

"Reduced Form"

$$\mathbf{\textit{Y}}_{1} = \mathbf{\textit{X}}_{1}\left(\frac{1}{1-\gamma_{1}\gamma_{2}}\boldsymbol{\beta}_{1}\right) + \mathbf{\textit{X}}_{2}\left(\frac{\gamma_{1}}{1-\gamma_{1}\gamma_{2}}\boldsymbol{\beta}_{2}\right) + \left(\frac{\gamma_{1}\mathbf{\textit{u}}_{2} + \mathbf{\textit{u}}_{1}}{1-\gamma_{1}\gamma_{2}}\right)$$

means

$$\frac{\partial Y_1}{\partial X_\ell} = \frac{\beta_\ell}{1 - \gamma_1 \gamma_2}.$$

But

$$\hat{\Delta}_1 \neq \hat{\boldsymbol{\beta}}_1.$$

Simultaneity Bias

For (e.g.)

$$Y_1 = \mathbf{X}_1 \boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

we have:

$$\mathsf{E}(Y_2,\mathbf{u}_1) = \frac{\gamma_2}{1 - \gamma_1 \gamma_2} \sigma_{\mathbf{u}}^2$$

Result:

- Bias (unless $\gamma_2 = 0$)
- Inconsistency

What To Do

- OLS
- Lagged Variables
- Two-Stage Least Squares (2SLS)
- Systems of Equations / 3SLS / etc.

Recall that a simple linear model:

$$Y = X\beta + u$$

gives us:

$$\hat{\boldsymbol{\beta}}_{OLS} = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}.$$

Suppose $Cov(X, u) \neq 0$, but we have Z with

- $Cov(Z, X) \neq 0$ and
- $\bullet \ \, \mathsf{Cov}(\mathbf{Z},\mathbf{u}) = \mathbf{0}. \\$

Then:

$$\begin{split} \hat{\boldsymbol{\beta}}_{IV} &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Y} \\ &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'(\mathbf{X}\boldsymbol{\beta} + \mathbf{u}) \\ &= \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{u} \end{split}$$

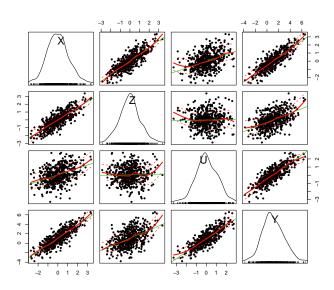
is consistent.

2SLS: How-To

- \bullet Regress endogenous $\boldsymbol{X}s$ variables on $\{\boldsymbol{Z},\boldsymbol{X}\}$
- Generate X̂s
- Regress Y on $\hat{\mathbf{X}}$ to get β_{2SLS} .
- Adjust standard error estimates

IV Estimation

```
library(MASS)
library(sem)
library(car)
seed<-1337
set.seed(seed)
mu < -c(0,0,0) \# <== X, Z, U
Sigma<-matrix(c(1,0.8,0.4,0.8,1,0,0.4,0,1),
                                                # Cor(X,Y)=0.8, etc.
             nrow=3,byrow=TRUE)
Vars<- mvrnorm(500,mu,Sigma)</pre>
colnames(Vars)<-c("X","Z","U")</pre>
Vars<-data.frame(Vars)
Vars$Y<- 1 + Vars$X + Vars$U
```



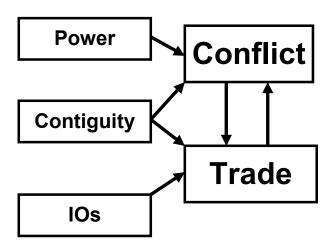
Plain Old OLS...

```
> OLS<- lm(Y~X,data=Vars)</pre>
> summary(OLS)
Call:
lm(formula = Y ~ X, data = Vars)
Residuals:
   Min
            10 Median 30
                                   Max
-3.3809 -0.6058 -0.0102 0.6320 2.9470
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.04770 0.04209 24.89 <2e-16 ***
X
            1.40254   0.04005   35.02   <2e-16 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.9413 on 498 degrees of freedom
Multiple R-squared: 0.7112, Adjusted R-squared: 0.7106
F-statistic: 1226 on 1 and 498 DF, p-value: < 2.2e-16
```

Two-Stage Least Squares

```
> TSLS<-tsls(Y~I(X),data=Vars,instruments=~Z)
> summary(TSLS)
2SLS Estimates
Model Formula: Y ~ I(X)
Instruments: ~7
Residuals:
   Min. 1st Qu. Median Mean 3rd Qu. Max.
-3.29300 -0.68210 -0.06139 0.00000 0.76270 2.70300
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.0491828  0.0456017  23.00754 < 2.22e-16 ***
T(X)
           1.0302012 \quad 0.0536909 \quad 19.18763 < 2.22e-16 ***
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 1.0196738 on 498 degrees of freedom
```

IV: A (Toy) Example



> summary(IRData)

dyadid	logdisputes	logtrade	IOs
Min. : 2020	Min. :-0.6931	Min. :-0.6931	Min. : 4.579
1st Qu.:135155	1st Qu.:-0.6931	1st Qu.: 2.4079	1st Qu.:19.500
Median :220484	Median :-0.6931	Median : 5.5786	Median :27.704
Mean :275526	Mean :-0.2627	Mean : 4.6518	Mean :30.891
3rd Qu.:385710	3rd Qu.: 0.0000	3rd Qu.: 7.1248	3rd Qu.:39.289
Max. :900920	Max. : 3.4965	Max. :11.5037	Max. :93.700
		(IDD 1-	
contiguity	capratio	GDPgrowtn	
contiguity Min. :0.0000	Min.: 1.081	0	
0 0	-	Min. :-9.0800	
Min. :0.0000	Min. : 1.081	Min. :-9.0800	
Min. :0.0000 1st Qu.:0.0000	Min. : 1.081 1st Qu.: 4.849	Min. :-9.0800 1st Qu.:-0.2923	
Min. :0.0000 1st Qu::0.0000 Median :0.0000	Min. : 1.081 1st Qu.: 4.849 Median : 26.577	Min. :-9.0800 1st Qu.:-0.2923 Median : 0.8363 Mean : 0.5097	

Ordinary Regression

```
> OLSWar<-lm(logdisputes~logtrade+contiguity+capratio,data=IRData)
> summary(OLSWar)
Call:
lm(formula = logdisputes ~ logtrade + contiguity + capratio,
   data = TRData)
Residuals:
  Min 10 Median 30 Max
-0.828 -0.326 -0.269 -0.090 3.455
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.4253192  0.0602014  -7.06  3.5e-12 ***
logtrade 0.0085581 0.0105739 0.81 0.419
contiguity 0.4622674 0.0712406 6.49 1.5e-10 ***
capratio -0.0001296 0.0000647 -2.00 0.045 *
___
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.853 on 813 degrees of freedom
Multiple R-squared: 0.083, Adjusted R-squared: 0.0796
F-statistic: 24.5 on 3 and 813 DF, p-value: 3.35e-15
```

2SLS "By-Hand" (stage one)

```
> ITrade<-lm(logtrade~contiguity+IOs+capratio)
> summary(ITrade)
```

Residuals:

```
Min 1Q Median 3Q Max
-6.0385 -1.7666 0.4139 1.6154 7.6029
```

Coefficients:

```
Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.7319793 0.1912570 3.827 0.000140 ***
contiguity 1.3386037 0.1816041 7.371 4.17e-13 ***
IOS 0.1218373 0.0055313 22.027 < 2e-16 ***
capratio -0.0013913 0.0001626 -8.555 < 2e-16 ***
---
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
```

Residual standard error: 2.239 on 813 degrees of freedom Multiple R-squared: 0.5535, Adjusted R-squared: 0.5519 F-statistic: 335.9 on 3 and 813 DF, p-value: < 2.2e-16

2SLS "By-Hand" (stage two)

```
> IVWarByHand<-with(IRData, lm(logdisputes~capratio+contiguity+
                      (ITrade$fitted.values)))
> summary(IVWarByHand)
Call:
lm(formula = logdisputes ~ capratio + contiguity + (ITrade$fitted.values))
Residuals:
  Min 10 Median 30
                            Max
-1.006 -0.362 -0.278 -0.049 3.530
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)
                 -0.1515180 0.0832287 -1.82 0.06905 .
capratio
                  -0.0002664 0.0000705 -3.78 0.00017 ***
contiguity
                  ITrade$fitted.values -0.0558374 0.0171921 -3.25 0.00121 **
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.848 on 813 degrees of freedom
Multiple R-squared: 0.094, Adjusted R-squared: 0.0907
F-statistic: 28.1 on 3 and 813 DF, p-value: <2e-16
```

2SLS, Automagically

```
> library(AER)
> TwoSLSWar<-ivreg(logdisputes~contiguity+capratio+I(logtrade),
   instruments=~contiguity+capratio+IOs)
> summary(TwoSLSWar)
Call:
ivreg(formula = logdisputes ~ contiguity + capratio + I(logtrade) |
   contiguity + capratio + IOs, data = IRData)
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.1515180 0.0856218 -1.77 0.07717.
contiguity 0.6263774 0.0811114 7.72 3.4e-14 ***
capratio -0.0002664 0.0000725 -3.67 0.00025 ***
I(logtrade) -0.0558374  0.0176864  -3.16  0.00165 **
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 0.872 on 813 degrees of freedom
Multiple R-Squared: 0.0412, Adjusted R-squared: 0.0376
Wald test: 26.6 on 3 and 813 DF, p-value: <2e-16
```

Weak Instruments

```
> OLSTrade<-lm(logtrade~logdisputes+contiguity+IOs)
> summary(OLSTrade)
```

Residuals:

```
Min 1Q Median 3Q Max
-6.2467 -2.2067 0.4275 1.6659 6.1264
```

Coefficients:

Residual standard error: 2.312 on 813 degrees of freedom Multiple R-squared: 0.5241, Adjusted R-squared: 0.5223 F-statistic: 298.4 on 3 and 813 DF, p-value: < 2.2e-16

Weak Instruments (continued)

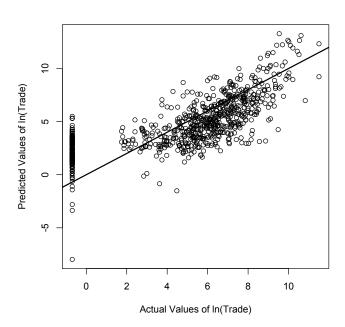
```
> TwoSLSTrade<-ivreg(logtrade~contiguity+IOs+I(logdisputes),</pre>
    instruments=~contiguity+capratio+IOs)
> summary(TwoSLSTrade)
Call:
ivreg(formula = logtrade ~ contiguity + IOs + I(logdisputes) |
   contiguity + capratio + IOs, data = IRData)
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)
              2.1501 0.8512 2.53 0.0117 *
contiguity -2.7276 1.5262 -1.79 0.0743.
I0s
              0.1720 0.0205 8.41 <2e-16 ***
I(logdisputes) 7.3712 2.4520 3.01 0.0027 **
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
Residual standard error: 6.37 on 813 degrees of freedom
Multiple R-Squared: -2.62, Adjusted R-squared: -2.63
Wald test: 41.5 on 3 and 813 DF, p-value: <2e-16
```

Side-By-Side...

	Dependent variable:				
	logdisputes		logtrade		
	OLS	IV	OLS	IV	
logtrade	0.009 (0.011)				
logdisputes			0.408*** (0.095)		
contiguity	0.462*** (0.071)	0.626*** (0.081)	1.358*** (0.193)	-2.728* (1.526)	
capratio	-0.0001** (0.0001)	-0.0003*** (0.0001)			
l(logtrade)		-0.056*** (0.018)			
IOs			0.134*** (0.006)	0.172*** (0.020)	
l(logdisputes)				7.371*** (2.452)	
Constant	-0.425*** (0.060)	-0.152* (0.086)	0.191 (0.183)	2.150** (0.851)	
Observations	817	817	817	817	
R ²	0.083	0.041	0.524	-2.616	
Adjusted R ²	0.080	0.038	0.522	-2.630	
Residual Std. Error (df = 813) F Statistic (df = 3; 813)	0.853 24.530***	0.872	2.312 298.400***	6.372	

Note: *p<0.1; ***p<0.05; ****p<0.01

Pretty Good Instrument (Trade)



Crappy Instrument (War)

