PLSC 503 – Spring 2022 Dichotomous Covariates and Transformations

February 9, 2022

"Dummies"...

- ... "naturally" dichotomous, including
  - Structural breaks
  - Proper nouns
- "Factors":

$$\texttt{partyid} = \begin{cases} 0 = \text{Labor} \\ 1 = \text{Liberal} \\ 2 = \text{Conservative} \end{cases}$$

- Ordinal variables...
- Continuous variables...

Coding Dummies

"Dummy coding":

$$\texttt{female} = \begin{cases} 0 \text{ if male} \\ 1 \text{ if female} \end{cases}$$

vs. "Effect coding":

$$\texttt{female} = \begin{cases} -1 \text{ (or } -0.5\text{) if male} \\ \ 1 \text{ (or 0.5) if female} \end{cases}$$

TL;DR: Use the former.

Dichotomous Xs

For

$$Y_i = \beta_0 + \beta_1 D_i + u_i$$

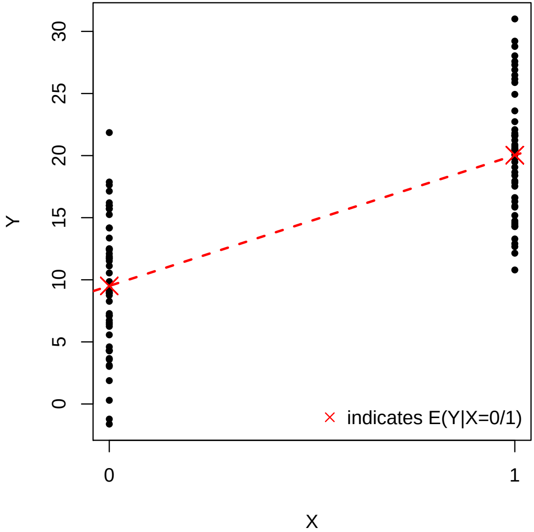
we have

$$\mathrm{E}(Y|D=0) = \beta_0$$

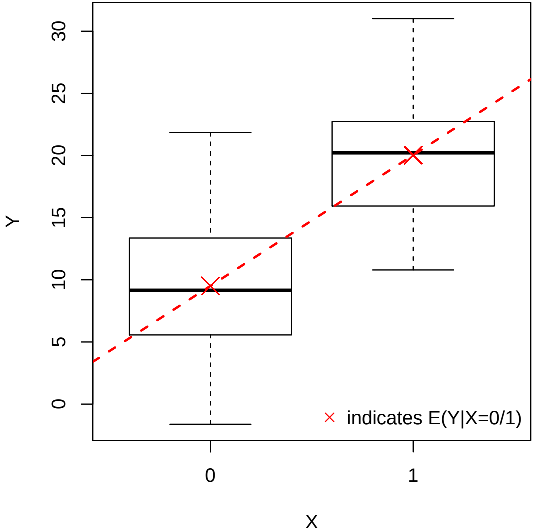
and

$$\mathrm{E}(Y|D=1) = \beta_0 + \beta_1.$$

Dichotomous X, Graphically (No!)



Dichotomous X, Graphically (Yes!)



Many Dummies

For

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \ldots + \beta_\ell D_{\ell i} + u_i$$

- $\mathrm{E}(Y|D_k = 0) \,\forall\, k \in \ell = \beta_0$,
- Otherwise, $\mathrm{E}(Y) = \beta_0 + \sum_{k=1}^{\ell} \beta_k \,\forall\, k \; s.t. \; D_k = 1$.

Note: where the D_ℓ are <u>mutually exclusive and exhaustive</u>:

- The expected values are the same as the within-group means.
- Identification requires that we either
  - omit a "reference category," or
  - omit β_0.

Dummies and Ordinal Xs

Suppose we have:

$$\texttt{gopscale} = \begin{cases} -2 = \text{Strong Democrat} \\ -1 = \text{Weak Democrat} \\ 0 = \text{Independent} \\ 1 = \text{Weak Republican} \\ 2 = \text{Strong Republican} \end{cases}$$

Might estimate:

$$\texttt{closeness}_i = 46.0 + 17.5(\texttt{gopscale}_i) + u_i$$

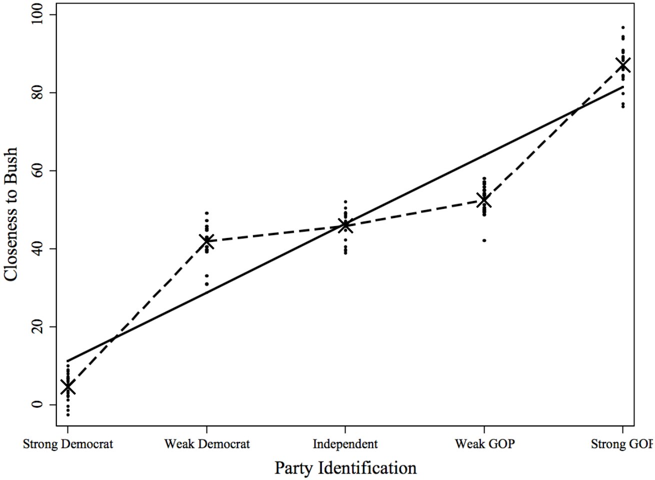
Alternative: "dummy out" `gopscale`:

$$\begin{aligned}\texttt{closeness}_i &= \beta_0 + \beta_1(\texttt{strongdem}_i) + \beta_2(\texttt{weakdem}_i) + \\ &\quad \beta_3(\texttt{weakgop}_i) + \beta_4(\texttt{stronggop}_i) + u_i\end{aligned}$$

yielding:

$$\begin{aligned}\texttt{closeness}_i &= 45.5 - 40(\texttt{strongdem}_i) - 6(\texttt{weakdem}_i) + \\ &\quad 7(\texttt{weakgop}_i) + 42(\texttt{stronggop}_i) + u_i\end{aligned}$$

Ordinal, Illustrated



Dichotomous + Continuous X

E.g.,

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 X_i + u_i$$

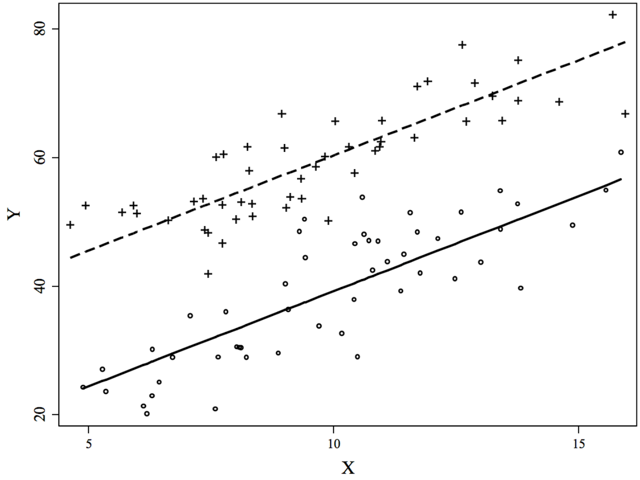
we have

$$\mathrm{E}(Y|X, D = 0) = \beta_0 + \beta_2 X_i$$

and

$$\mathrm{E}(Y|X, D = 1) = (\beta_0 + \beta_1) + \beta_2 X_i.$$

Dichotomous + Continuous X



Examples: SCOTUS (OT1953-1985)

From the “phase II” database...

```
> summary(SCOTUS)
       id             term            Namici           lctdiss          multlaw      
 Min.   :   1   Min.   :53.00   Min.   : 0.000   Min.   :0.0000   Min.   :0.0000  
 1st Qu.:1791   1st Qu.:64.00   1st Qu.: 0.000   1st Qu.:0.0000   1st Qu.:0.0000  
 Median :3581   Median :72.00   Median : 0.000   Median :0.0000   Median :0.0000  
 Mean   :3581   Mean   :71.12   Mean   : 0.842   Mean   :0.1509   Mean   :0.1490  
 3rd Qu.:5371   3rd Qu.:79.00   3rd Qu.: 1.000   3rd Qu.:0.0000   3rd Qu.:0.0000  
 Max.   :7161   Max.   :85.00   Max.   :39.000   Max.   :1.0000   Max.   :1.0000  
                NA's   : 4.00                    NA's   :4.0000   NA's   :5.0000  
    civlibs            econs            constit             lctlib        
 Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :  0.0000  
 1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:  0.0000  
 Median :1.0000   Median :0.0000   Median :0.0000   Median :  0.0000  
 Mean   :0.5009   Mean   :0.1709   Mean   :0.2536   Mean   :  0.3742  
 3rd Qu.:1.0000   3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.:  1.0000  
 Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :  1.0000  
                                                    NA's   :120.0000  
```

Creating Dummies

All civil rights & economics cases:

```
> SCOTUS$civil.econ<-SCOTUS$civlibs + SCOTUS$econs
```

Factors:

```
> SCOTUS$termdummies<-factor(SCOTUS$term)
> is.factor(SCOTUS$termdummies)
[1] TRUE
> summary(SCOTUS$termdummies)
  53   54   55   56   57   58   59   60   61   62   63   64   65   66   67
 126  109  128  162  196  165  157  160  148  189  223  156  187  201  285

  68   69   70   71   72   73   74   75   76   77   78   79   80   81
 207  185  227  262  269  267  223  253  254  244  244  221  255  269

  82   83   84   85 NA's
 277  298  301  309    4
```

Regressions (vs. *t*-tests...)

```
> fit1<-with(SCOTUS, lm(Namici~civlibs))
> summary(fit1)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.91774    0.03661  25.069  < 2e-16 ***
civlibs     -0.15136    0.05173  -2.926  0.00344 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.189 on 7159 degrees of freedom
Multiple R-squared:  0.001195,Adjusted R-squared:  0.001055
F-statistic: 8.563 on 1 and 7159 DF,  p-value: 0.003442
```

```
> with(SCOTUS, t.test(Namici~civlibs))

Welch Two Sample t-test

data:  Namici by civlibs
t = 2.9258, df = 7114.116, p-value = 0.003446
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.04995001 0.25277126
sample estimates:
mean in group 0 mean in group 1
      0.9177392       0.7663786
```

Effect Coding

```
> SCOTUS$civlibeffect<-SCOTUS$civlibs
> SCOTUS$civlibeffect[SCOTUS$civlibs==0]<-(-1)
> fit2<-with(SCOTUS, lm(Namici~SCOTUS$civlibeffect))
> summary(fit2)

Call:
lm(formula = Namici ~ SCOTUS$civlibeffect)

Residuals:
   Min     1Q Median     3Q    Max
-0.918 -0.918 -0.766  0.082 38.234

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)          0.84206    0.02586  32.559  < 2e-16 ***
SCOTUS$civlibeffect -0.07568    0.02586  -2.926  0.00344 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.189 on 7159 degrees of freedom
Multiple R-squared:  0.001195,Adjusted R-squared:  0.001055
F-statistic: 8.563 on 1 and 7159 DF,  p-value: 0.003442
```

Many D_is

```
> fit3<-with(SCOTUS, lm(Namici~lctdiss+multlaw+civlibs+
+                          econs+constit+lctlib))
> summary(fit3)

Call:
lm(formula = Namici ~ lctdiss + multlaw + civlibs + econs + constit +
    lctlib)

Residuals:
   Min     1Q Median     3Q    Max
-2.582 -0.976 -0.472 -0.260 37.086

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.47245    0.05273   8.960  < 2e-16 ***
lctdiss      0.36760    0.07173   5.125 3.06e-07 ***
multlaw      0.61306    0.07445   8.235  < 2e-16 ***
civlibs     -0.21255    0.06022  -3.530 0.000419 ***
econs        0.08772    0.07652   1.146 0.251691
constit      0.53793    0.06372   8.442  < 2e-16 ***
lctlib       0.50309    0.05396   9.323  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.15 on 7033 degrees of freedom
  (121 observations deleted due to missingness)
Multiple R-squared:  0.05013,Adjusted R-squared:  0.04932
F-statistic: 61.86 on 6 and 7033 DF,  p-value: < 2.2e-16
```

Change Over Time: Linear Trend

```
> fit4<-with(SCOTUS, lm(Namici~lctdiss+multlaw+civlibs+
+                          econs+constit+lctlib+term))
> summary(fit4)

Call:
lm(formula = Namici ~ lctdiss + multlaw + civlibs + econs + constit +
    lctlib + term)

Residuals:
   Min     1Q Median     3Q    Max
-2.968 -0.906 -0.428  0.143 36.958

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.726962   0.202367 -13.475  < 2e-16 ***
lctdiss      0.359494   0.070415   5.105 3.39e-07 ***
multlaw      0.649932   0.073109   8.890  < 2e-16 ***
civlibs     -0.289314   0.059295  -4.879 1.09e-06 ***
econs        0.199464   0.075419   2.645  0.00819 **
constit      0.515435   0.062559   8.239  < 2e-16 ***
lctlib       0.339891   0.053901   6.306 3.04e-10 ***
term         0.046142   0.002821  16.354  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.11 on 7032 degrees of freedom
  (121 observations deleted due to missingness)
Multiple R-squared:  0.08493,Adjusted R-squared:  0.08402
F-statistic: 93.24 on 7 and 7032 DF,  p-value: < 2.2e-16
```

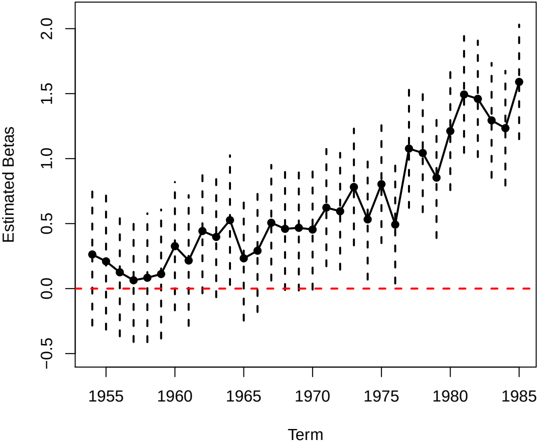
Change Over Time: Using `factor`

```
> fit5<-with(SCOTUS, lm(Namici~lctdiss+multlaw+civlibs+
+                       econs+constit+lctlib+as.factor(term)))
> summary(fit5)

Call:
lm(formula = Namici ~ lctdiss + multlaw + civlibs + econs + constit +
    lctlib + as.factor(term))

Residuals:
   Min     1Q Median     3Q    Max
-3.064 -0.920 -0.384  0.106 36.831

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       -0.16153    0.19530  -0.827 0.408200
lctdiss            0.34558    0.07067   4.890 1.03e-06 ***
multlaw            0.64348    0.07334   8.774  < 2e-16 ***
civlibs           -0.27137    0.05967  -4.548 5.51e-06 ***
econs              0.20039    0.07581   2.643 0.008232 **
constit            0.54280    0.06297   8.620  < 2e-16 ***
lctlib             0.33863    0.05458   6.205 5.80e-10 ***
.
.
.
```

Using `factor` (continued)

```
as.factor(term)54  0.26276    0.27934   0.941 0.346918
as.factor(term)55  0.20958    0.26804   0.782 0.434309
as.factor(term)56  0.12536    0.25126   0.499 0.617859
as.factor(term)57  0.06432    0.24227   0.265 0.790654
as.factor(term)58  0.08353    0.25274   0.331 0.741025
.
.
.
as.factor(term)71  0.62313    0.23019   2.707 0.006806 **
as.factor(term)72  0.59503    0.22929   2.595 0.009476 **
as.factor(term)73  0.78179    0.22918   3.411 0.000650 ***
as.factor(term)74  0.53254    0.23636   2.253 0.024287 *
as.factor(term)75  0.80353    0.23118   3.476 0.000513 ***
as.factor(term)76  0.49269    0.23138   2.129 0.033262 *
as.factor(term)77  1.07725    0.23265   4.630 3.72e-06 ***
as.factor(term)78  1.04335    0.23243   4.489 7.27e-06 ***
as.factor(term)79  0.85363    0.23696   3.602 0.000318 ***
as.factor(term)80  1.21205    0.23183   5.228 1.76e-07 ***
as.factor(term)81  1.49347    0.22925   6.515 7.80e-11 ***
as.factor(term)82  1.46004    0.22858   6.388 1.79e-10 ***
as.factor(term)83  1.29417    0.22549   5.739 9.90e-09 ***
as.factor(term)84  1.23434    0.22517   5.482 4.36e-08 ***
as.factor(term)85  1.59037    0.22491   7.071 1.68e-12 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.108 on 7001 degrees of freedom
  (121 observations deleted due to missingness)
Multiple R-squared:  0.0914,Adjusted R-squared:  0.08647
F-statistic: 18.53 on 38 and 7001 DF,  p-value: < 2.2e-16
```

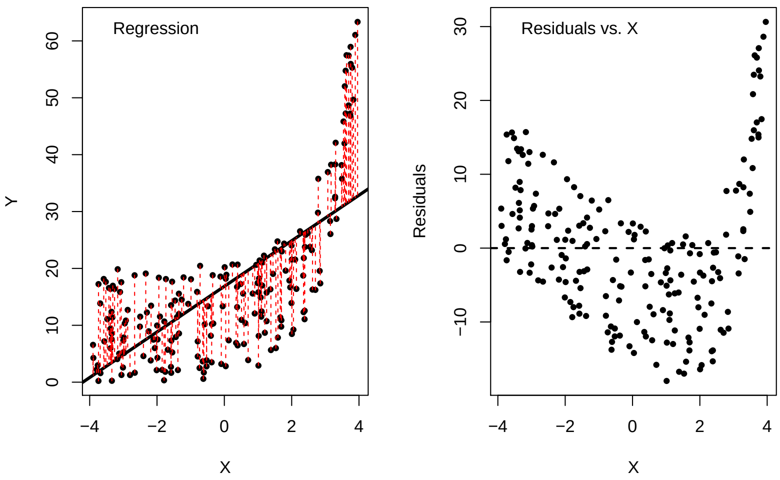
`factor` results, plotted (1953 = 0)



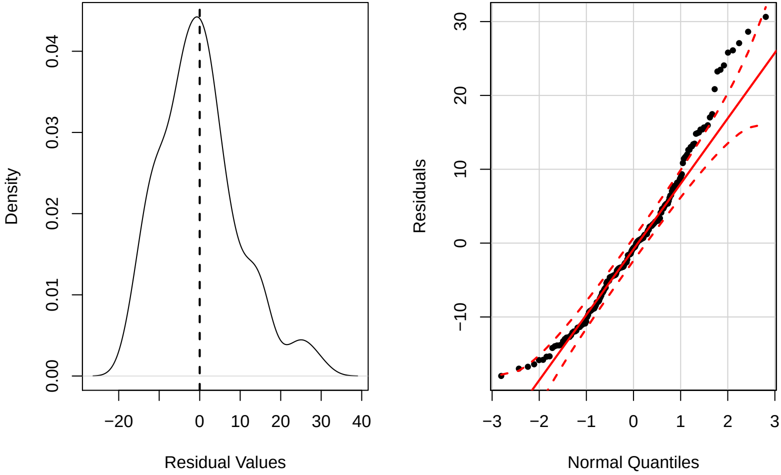
Why Transform?

- Normality (of u_is)
- Linearity
- Additivity
- Interpretation / Model Specification

What Difference Does It Make? (Part I)



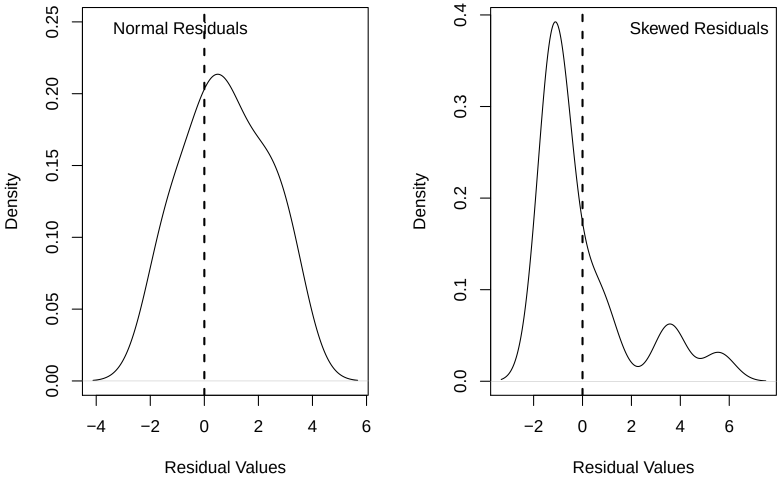
Residuals Are Still (Pretty) Normal...



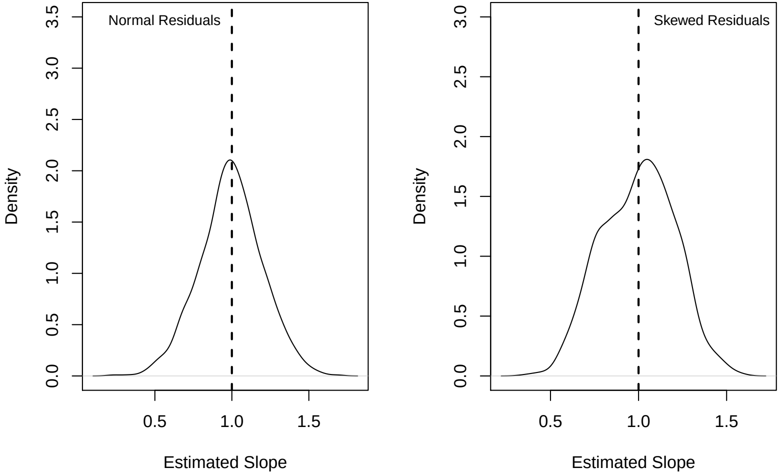
What Difference Does It Make? (Part II)

```
N <- 20 # pretty small sample size
u <- rnorm(N,0,2) # mean zero, s.d = 2

# Exponentiate:
eu <- exp(u)
eu <- eu-mean(eu) # new residuals are mean-zero
eu <- (eu/sd(eu))*2 # and also sd = 2

X <- runif(N,-4,4)
Y1 <- 0 + 1*X + 1*u
Y2 <- 0 + 1*X + 1*eu # same Xs in both
```

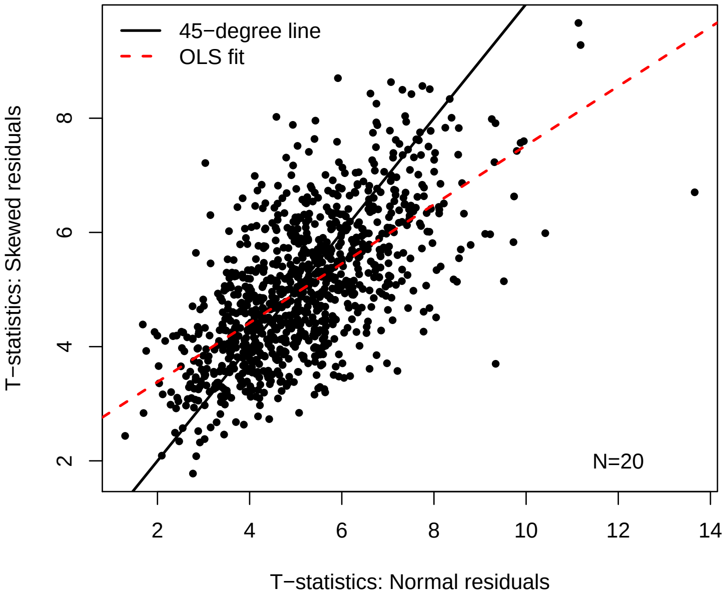
What Difference Does It Make? (Part II)



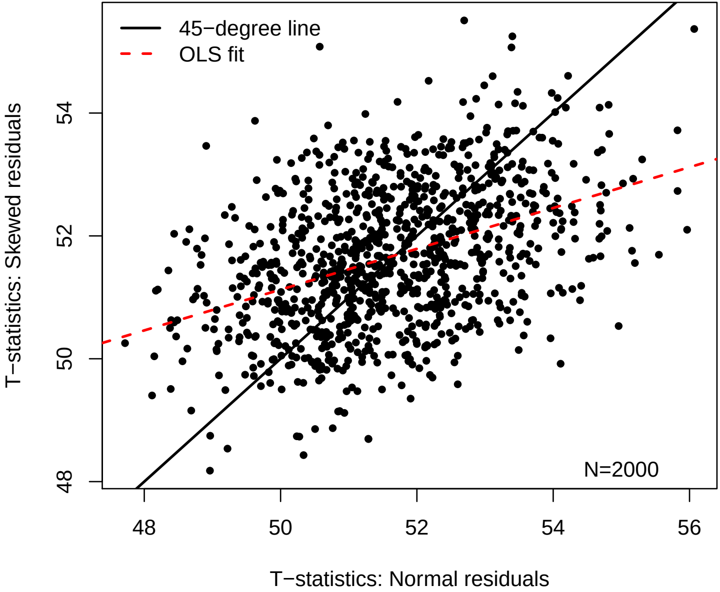
Little Effect On $\hat{\beta}$



Important Differences in Inference



With $N = 2000$? Not So Much...



Examples

This:

$$Y_i = \beta_0 X_i^{\beta_1} u_i$$

becomes this:

$$\ln(Y_i) = \ln(\beta_0) + \beta_1 X_i + \ln(u_i)$$

And this:

$$\exp(Y_i) = \beta_0 + \beta_1 X_i + u_i$$

becomes this:

$$Y_i = \ln(\beta_0) + \beta_1 \ln(X_i) + \ln(u_i)$$

Monotonic Transformations

The "Ladder of Powers":

Transformation	p	$f(X)$	Fox's $f(X)$
Cube	3	X^3	$\frac{X^3-1}{3}$
Square	2	X^2	$\frac{X^2-1}{2}$
(None/Identity)	(1)	(X)	(X)
Square Root	$\frac{1}{2}$	$\sqrt{X}$	$2(\sqrt{X}-1)$
Cube Root	$\frac{1}{3}$	$\sqrt[3]{X}$	$3(\sqrt[3]{X}-1)$
Log	0 (sort of)	$\ln(X)$	$\ln(X)$
Inverse Cube Root	-$\frac{1}{3}$	$\frac{1}{\sqrt[3]{X}}$	$\frac{\left(\frac{1}{\sqrt[3]{X}}-1\right)}{-\frac{1}{3}}$
Inverse Square Root	-$\frac{1}{2}$	$\frac{1}{\sqrt{X}}$	$\frac{\left(\frac{1}{\sqrt{X}}-1\right)}{-\frac{1}{2}}$
Inverse	-1	$\frac{1}{X}$	$\frac{\left(\frac{1}{X}-1\right)}{-1}$
Inverse Square	-2	$\frac{1}{X^2}$	$\frac{\left(\frac{1}{X^2}-1\right)}{-2}$
Inverse Cube	-3	$\frac{1}{X^3}$	$\frac{\left(\frac{1}{X^3}-1\right)}{-3}$

A General Rule

Using higher-order power transformations (e.g. squares, cubes, etc.) “inflates” large values and “compresses” small ones; conversely, using lower-order power transformations (logs, etc.) “compresses” large values and “inflates” (or “expands”) smaller ones.

Power Transformations: Two Issues

1. X must be *positive*; so:

$$X^* = X + (|X_l| + \epsilon)$$

with (CZ's Rule of Thumb):

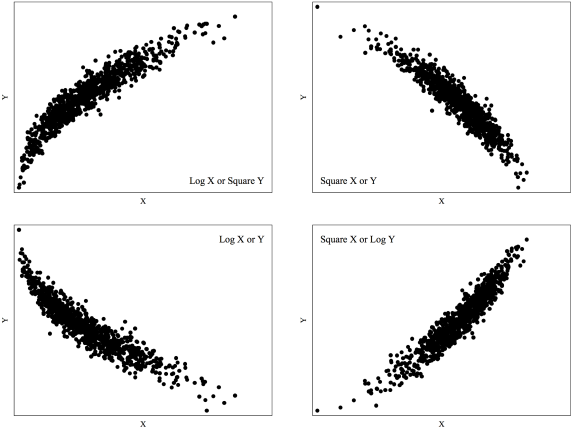
$$\epsilon = \frac{X_{l+1} - X_l}{2}$$

2. Power transformations generally require that:

$$\frac{X_h}{X_l} > 5 \text{ (or so)}$$

Which Transformation?

Mosteller and Tukey's "Bulging Rule":



Nonmonotonicity

Simple solution: Polynomials...

- Second-order / quadratic:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + u_i$$

- Third-order / cubic:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + u_i$$

- pth-order:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 + ... + \beta_p X_i^p + u_i$$

Transformed Xs: Interpretation

For:

$$\ln(Y_i) = \beta_0 + \beta_1 X_i + u_i,$$

then:

$$E(Y) = \exp(\beta_0 + \beta_1 X_i)$$

and so:

$$\frac{\partial E(Y)}{\partial X} = \exp(\beta_1).$$

Transformed Xs: Interpretation

Similarly, for:

$$Y_i = \beta_0 + \beta_1 \ln(X_i) + u_i$$

we have:

$$\frac{\partial \mathrm{E}(Y)}{\partial \ln(X)} = \beta_1.$$

So doubling X (say, from X_ℓ to $2X_\ell$):

$$\begin{aligned}
\Delta \mathrm{E}(Y) &= \mathrm{E}(Y|X = 2X_\ell) - \mathrm{E}(Y|X = X_\ell) \\
&= [\beta_0 + \beta_1 \ln(2X_\ell)] - [\beta_0 + \beta_1 \ln(X_\ell)] \\
&= \beta_1[\ln(2X_\ell) - \ln(X_\ell)] \\
&= \beta_1 \ln(2)
\end{aligned}$$

Log-Log Regressions

Specifying:

$$\ln(Y_i) = \beta_0 + \beta_1 \ln(X_i) + ... + u_i$$

means:

$$\text{Elasticity}_{YX} \equiv \frac{\%\Delta Y}{\%\Delta X} = \beta_1.$$

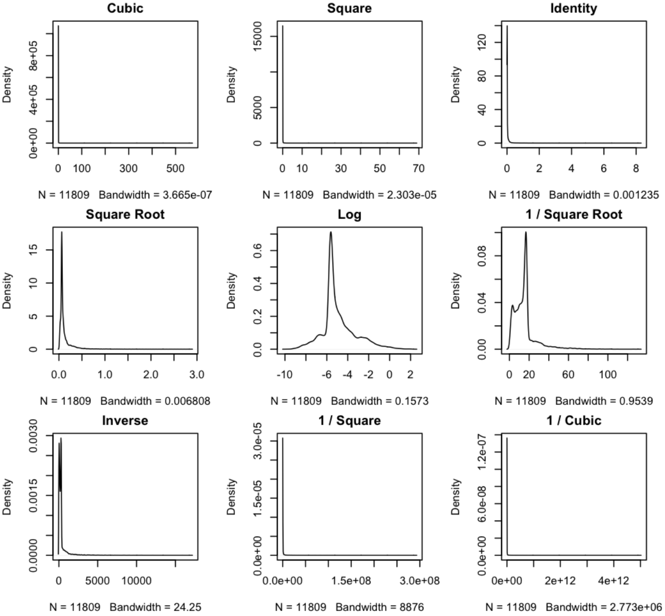
IOW, a one-percent change in X leads to a $\hat{\beta}_1$-percent change in Y.

An Example: Military Spending and GDP

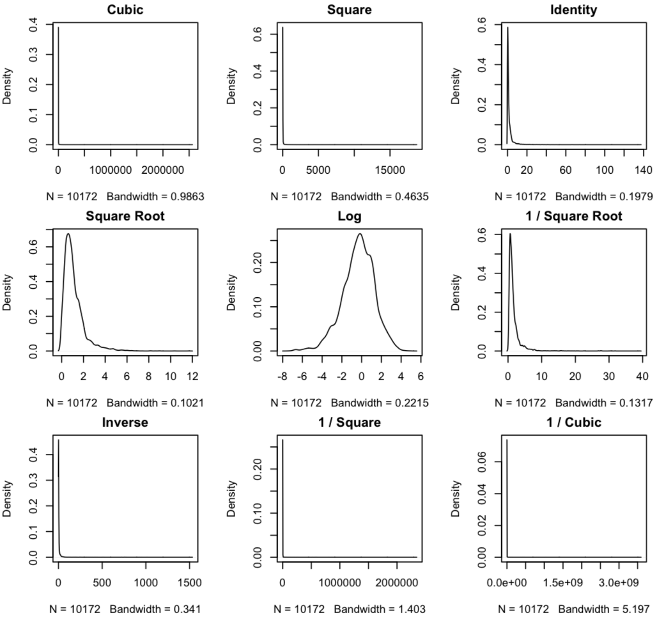
Data are from Fordham and Walker...

```
> with(Data, summary(milgdp))
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
  0.000   0.238   0.749   2.115   2.104 136.900    4327

> with(Data, summary(gdp))
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
 0.0001  0.0033  0.0047  0.0534  0.0153  8.3010    2690
```

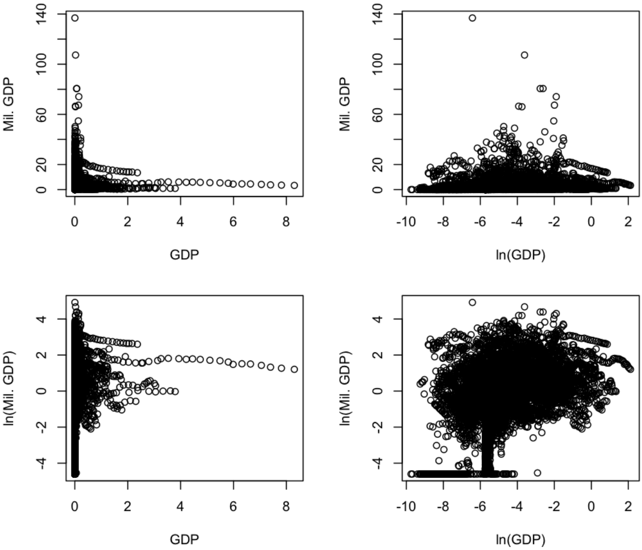
"Ladder of Powers": GDP



"Ladder of Powers": Military Spending



Scatterplots



Untransformed:

```
> with(Data, summary(lm(milgdp~gdp)))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   2.0538     0.0481  42.696  < 2e-16 ***
gdp           1.0038     0.1540   6.518 7.45e-11 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 4.757 on 10170 degrees of freedom
  (4327 observations deleted due to missingness)
Multiple R-squared: 0.00416,Adjusted R-squared: 0.004062
F-statistic: 42.49 on 1 and 10170 DF,  p-value: 7.454e-11
```

Some Regressions

Logging X:

```
> with(Data, summary(lm(milgdp~log(gdp))))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.60137    0.13969   32.94   <2e-16 ***
log(gdp)     0.52196    0.02766   18.87   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 4.686 on 10170 degrees of freedom
  (4327 observations deleted due to missingness)
Multiple R-squared: 0.03384,Adjusted R-squared: 0.03374
F-statistic: 356.2 on 1 and 10170 DF,  p-value: < 2.2e-16
```

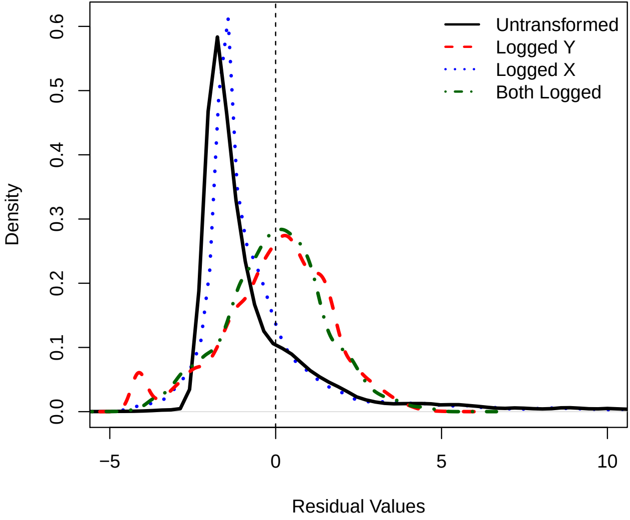
Logging Y:

```
> with(Data, summary(lm(log(milgdp+0.01)~gdp)))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.45918    0.01669  -27.51   <2e-16 ***
gdp          0.75794    0.05343   14.18   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.651 on 10170 degrees of freedom
  (4327 observations deleted due to missingness)
Multiple R-squared: 0.0194,Adjusted R-squared: 0.0193
F-statistic: 201.2 on 1 and 10170 DF,  p-value: < 2.2e-16
```

Logging X and Y:

```
> with(Data, summary(lm(log(milgdp+0.01)~log(gdp))))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.644270   0.044736   36.76   <2e-16 ***
log(gdp)    0.431875   0.008858   48.76   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.501 on 10170 degrees of freedom
  (4327 observations deleted due to missingness)
Multiple R-squared: 0.1895,Adjusted R-squared: 0.1894
F-statistic:  2377 on 1 and 10170 DF,  p-value: < 2.2e-16
```

Density Plots of $\hat{u}_i$s



- **Theory is valuable**.
- **Try different things**.
- **Look at plots**.
- **It takes practice**.