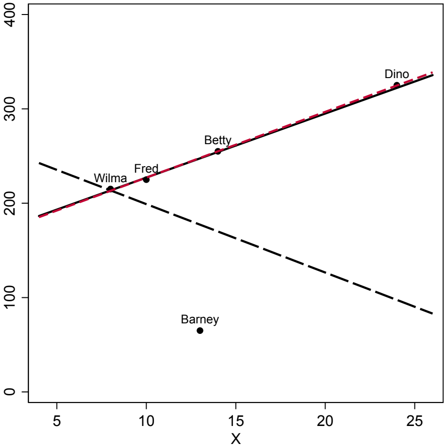
PLSC 503 – Spring 2025
Residuals & Outliers + Instrumental Variables

February 24, 2025

Discrepancy, Leverage, and Influence



Note: Solid line is the regression fit for Wilma, Fred, and Betty only.
Long-dashed line is the regression for Wilma, Fred, Betty, and Barney.
Short-dashed (red) line is the regression for Wilma, Fred, Betty and Dino.

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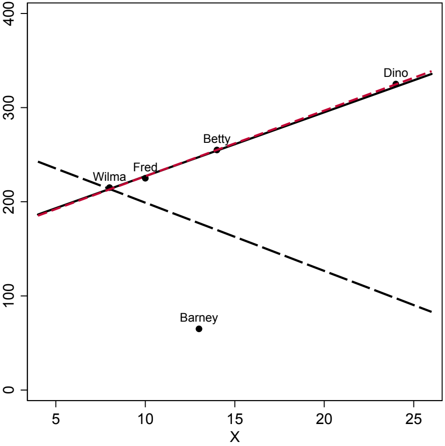

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$$\text{Influence} = \text{Leverage} \times \text{Discrepancy}$$

Leverage

$$\begin{aligned}\hat{\mathbf{Y}} &= \mathbf{X}\hat{\beta} \\ &= \mathbf{X}[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}] \\ &= \mathbf{HY}\end{aligned}$$

where

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'.$$

$$h_i = \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i'$$

Variation:

$$\widehat{\mathsf{Var}(\hat{u}_i)} = \hat{\sigma}^2[1 - \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i'] \tag{1}$$

$$\begin{aligned} \widehat{\mathsf{s.e.}(\hat{u}_i)} &= \hat{\sigma}\sqrt{[1 - \mathbf{X}_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_i']} \\ &= \hat{\sigma}\sqrt{1 - h_i} \end{aligned} \tag{2}$$

"Standardized":

$$\tilde{u}_i = \frac{\hat{u}_i}{\hat{\sigma}\sqrt{1 - h_i}} \tag{3}$$

"Studentized": define

$$
\begin{aligned}
\hat{\sigma}^2_{-i} &= \text{Variance for the } N-1 \text{ observations} \neq i \\
&= \frac{\hat{\sigma}^2(N-K)}{N-K-1} - \frac{\hat{u}_i^2}{(N-K-1)(1-h_i)}. \qquad (4)
\end{aligned}
$$

Then:

$$
\hat{u}_i' = \frac{\hat{u}_i}{\hat{\sigma}_{-i}\sqrt{1-h_i}} \qquad (5)
$$

"DFBETA":

$$D_{ki} = \hat{\beta}_k - \hat{\beta}_{k(-i)} \tag{6}$$

"DFBETAS" (the "S" is for "standardized):

$$D^*_{ki} = \frac{D_{ki}}{\widehat{\text{s.e.}(\hat{\beta}_{k(-i)})}} \tag{7}$$

Cook's D:

$$\begin{aligned} D_i &= \frac{\tilde{u}_i^2}{K} \times \frac{h_i}{1 - h_i} \\ &= \frac{h_i \hat{u}_i^2}{K\hat{\sigma}^2(1 - h_i)^2} \end{aligned} \tag{8}$$

```
> # No Barney OR Dino...
> summary(lm(Y~X,data=subset(flintstones,name!="Dino" & name!="Barney")

Residuals:
     2      4      5
 0.714 -2.143  1.429

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  159.286      6.776    23.5    0.027 *
X              6.786      0.619    11.0    0.058 .
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.67 on 1 degrees of freedom
Multiple R-squared: 0.992, Adjusted R-squared: 0.984
F-statistic:  120 on 1 and 1 DF,  p-value: 0.0579
```

```
> # No Barney (Dino included...)
> summary(lm(Y~X,data=subset(flintstones,name!="Barney")))

Residuals:
        2          3          4          5 
-8.88e-16  2.63e-01 -2.11e+00  1.84e+00 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  157.368      2.465    63.8  0.00025 ***
X              6.974      0.161    43.3  0.00053 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.99 on 2 degrees of freedom
Multiple R-squared: 0.999, Adjusted R-squared: 0.998
F-statistic: 1.87e+03 on 1 and 2 DF,  p-value: 0.000534
```

"COVRATIO":

$$\text{COVRATIO}_i = \left[(1 - h_i)\left(\frac{N - K - 1 + \hat{u}_i'^2}{N - K}\right)^K\right]^{-1} \qquad (9)$$

Example: Federal Judicial Review, 1789-2018

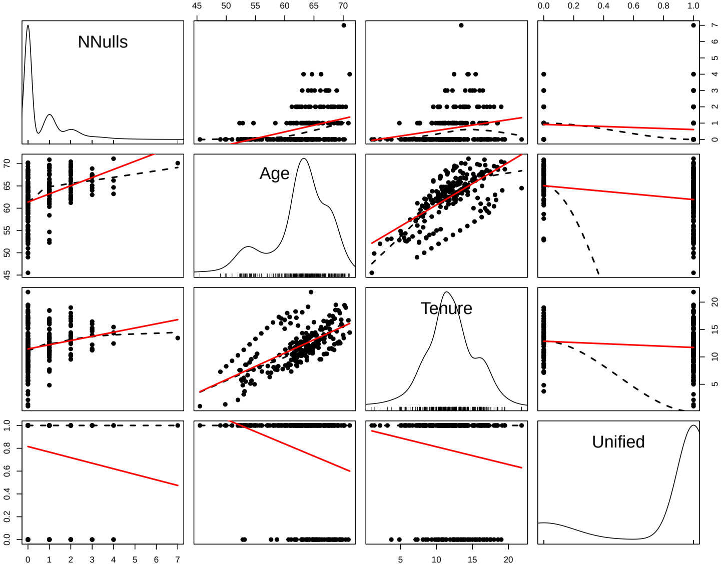
Dahl (1957):

- SCOTUS gets "out of step" with the other branches $\rightarrow$ judicial review
- Older / longer-serving justices will more likely to invalidate legislation

Data:

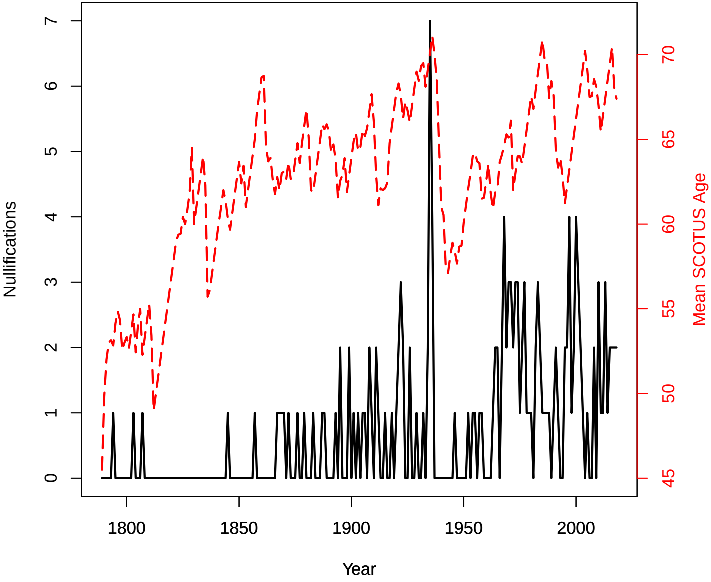
```
> summary(NewDahl)
      Year          NNulls           Age           Tenure          Unified     
 Min.   :1789   Min.   :0.000   Min.   :45.5   Min.   : 1.0   Min.   :0.000  
 1st Qu.:1846   1st Qu.:0.000   1st Qu.:60.7   1st Qu.:10.0   1st Qu.:1.000  
 Median :1904   Median :0.000   Median :63.5   Median :11.8   Median :1.000  
 Mean   :1904   Mean   :0.674   Mean   :62.6   Mean   :12.0   Mean   :0.783  
 3rd Qu.:1961   3rd Qu.:1.000   3rd Qu.:66.0   3rd Qu.:14.1   3rd Qu.:1.000  
 Max.   :2018   Max.   :7.000   Max.   :71.1   Max.   :21.8   Max.   :1.000  
```

Example: Federal Judicial Review, 1789-2018



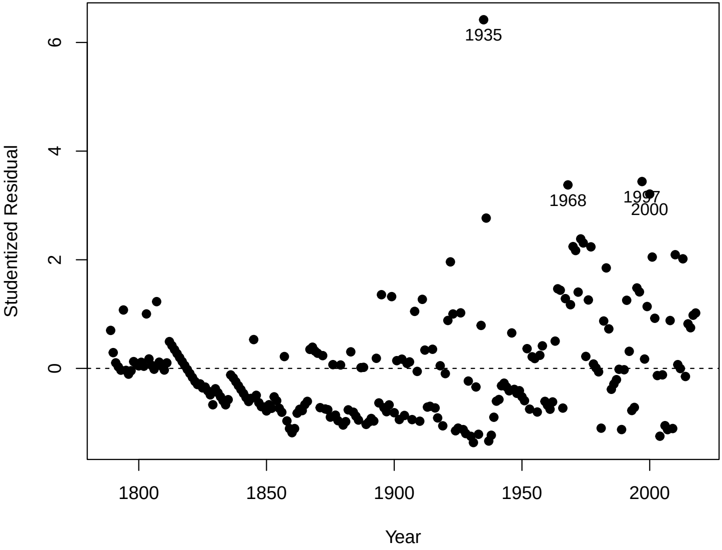
```
> Fit<-with(NewDahl, lm(NNulls~Age+Tenure+Unified))
> summary(Fit)

Call:
lm(formula = NNulls ~ Age + Tenure + Unified)

Residuals:
   Min     1Q Median     3Q    Max
-1.308 -0.700 -0.135  0.308  5.693

Coefficients:
            Estimate Std. Error t value  Pr(>|t|)
(Intercept)  -4.6833     1.0026   -4.67 0.0000051 ***
Age           0.0901     0.0181    4.97 0.0000013 ***
Tenure       -0.0201     0.0248   -0.81      0.42
Unified      -0.0573     0.1613   -0.36      0.72
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.973 on 226 degrees of freedom
  (4 observations deleted due to missingness)
Multiple R-squared:  0.152, Adjusted R-squared:  0.141
F-statistic: 13.6 on 3 and 226 DF,  p-value: 0.0000000365
```

Federal Judicial Review and Mean SCOTUS Age



```
> FitResid<-with(NewDahl, (Fit$model$NNulls - predict(Fit)))
> FitStandard<-rstandard(Fit) # standardized residuals
> FitStudent<-rstudent(Fit) # studentized residuals
> FitCooksD<-cooks.distance(Fit) # Cook?s D
> FitDFBeta<-dfbeta(Fit) # DFBeta
> FitDFBetaS<-dfbetas(Fit) # DFBetaS
> FitCOVRATIO<-covratio(Fit) # COVRATIOs
```

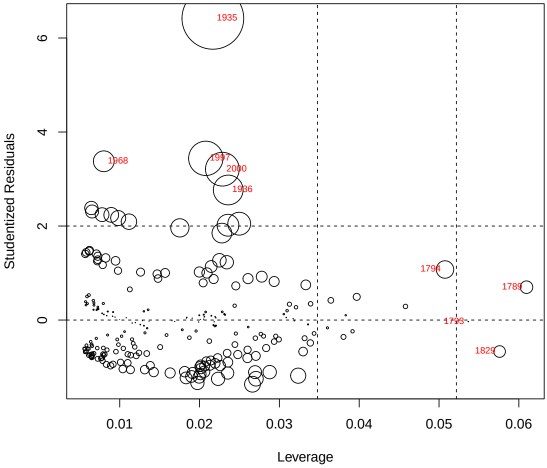
Studentized Residuals



```
> max(FitStudent)
[1] 6.418

> NewDahl$Year1935<-ifelse(NewDahl$Year==1935,1,0)
> summary(with(NewDahl, lm(NNulls~Age+Tenure+Unified+Year1935)))

Residuals:
   Min     1Q Median     3Q    Max
-1.250 -0.652 -0.122  0.302  3.247

Coefficients:
            Estimate Std. Error t value      Pr(>|t|)
(Intercept)  -3.9298     0.9313   -4.22 0.00003546916 ***
Age           0.0768     0.0168    4.56 0.00000846697 ***
Tenure       -0.0113     0.0229   -0.50          0.62
Unified      -0.1210     0.1490   -0.81          0.42
Year1935      5.8186     0.9066    6.42 0.00000000081 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.897 on 225 degrees of freedom
Multiple R-squared:  0.284, Adjusted R-squared:  0.271
F-statistic: 22.3 on 4 and 225 DF,  p-value: 1.65e-15
```

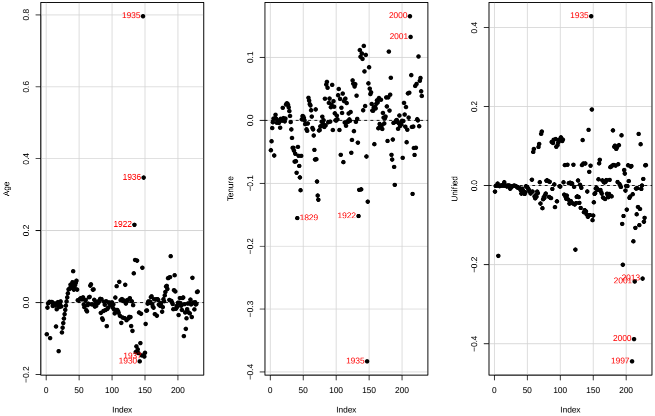
“Bubble Plot”

```
> influencePlot(Fit,id=list(method="noteworthy",n=4,cex=0.7,
                            labels=NewDahl$Year,col="red"),
                xlab="Leverage")
```



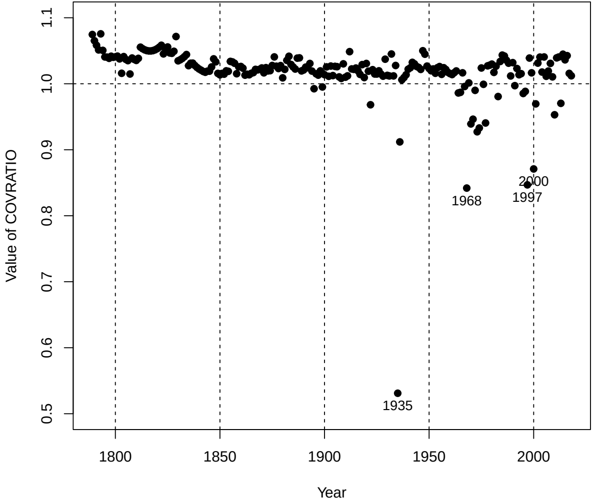
DFBETAS

```
> dfbetasPlots(Fit,id.n=5,id.col="red",main="",pch=19,
               layout=c(1,3),labels=NewDahl$Year)
```



COVRATIO Plot

```
> plot(FitCOVRATIO~NewDahl$Year,pch=19,ylim=c(0.5,1.1),
    xlab="Year",ylab="Value of COVRATIO")
```



Sensitivity Analyses: Omitting Outliers

```
> out1<-c(1935) # one outlier
> LD2<-NewDahl[!(NewDahl$Year %in% out1),]
> out2<-c(1935,1968,1997,2000) # four outliers
> LD3<-NewDahl[!(NewDahl$Year %in% out2),]
> Fit2<-lm(NNulls~Age+Tenure+Unified,data=LD2)
> Fit3<-lm(NNulls~Age+Tenure+Unified,data=LD3)
```

	Dependent variable:		
	NNulls		
	(1)	(2)	(3)
Age	0.090***	0.077***	0.079***
	(0.018)	(0.017)	(0.015)
Tenure	−0.020	−0.011	−0.019
	(0.025)	(0.023)	(0.021)
Unified	−0.057	−0.121	−0.010
	(0.161)	(0.149)	(0.139)
Constant	−4.683***	−3.930***	−4.130***
	(1.003)	(0.931)	(0.855)
Observations	230	229	226
R^2	0.152	0.148	0.158
Adjusted R^2	0.141	0.137	0.147
Residual Std. Error	0.973 (df = 226)	0.897 (df = 225)	0.822 (df = 222)
F Statistic	13.550*** (df = 3; 226)	13.030*** (df = 3; 225)	13.930*** (df = 3; 222)
Note:			*p<0.1; **p<0.05; ***p<0.01

Thinking About Diagnostics

"Looking" (Art) ⟷ "Testing" (Science)

"Looking" (Art)	"Testing" (Science)
Observational Data	Experimental Data
Complex Data Structure	Simple Data Structure
Informative Missingness	No / Uninformative Missingness
Complex / Uncertain Causality	Simple / Clear Causality

Pena, E.A. and E.H. Slate. 2006. "Global Validation of Linear Model Assumptions." J. American Statistical Association 101(473):341-354.

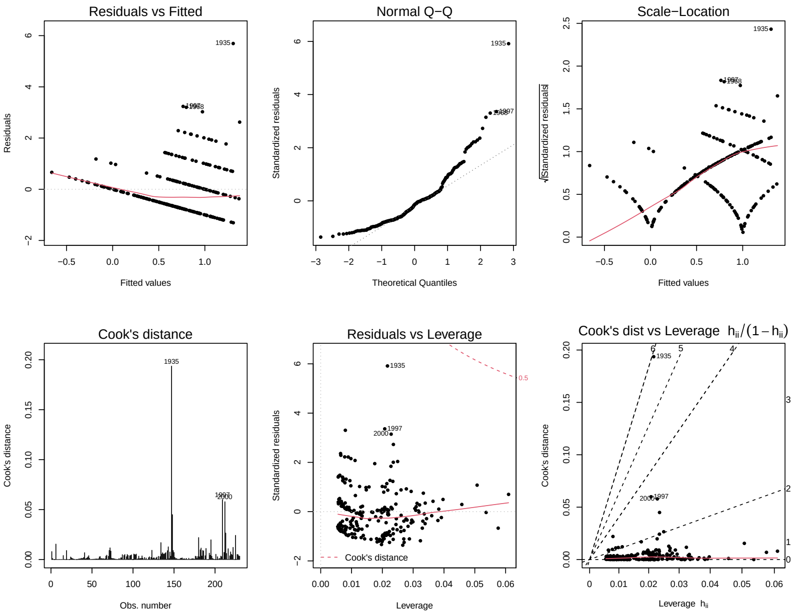
Tests for:

- Normality in $\hat{u}$s (via skewness & kurtosis tests)
- "Link function" (linearity / additivity)
- Constant variance and uncorrelatedness in $\hat{u}$s ("heteroskedasticity" test)

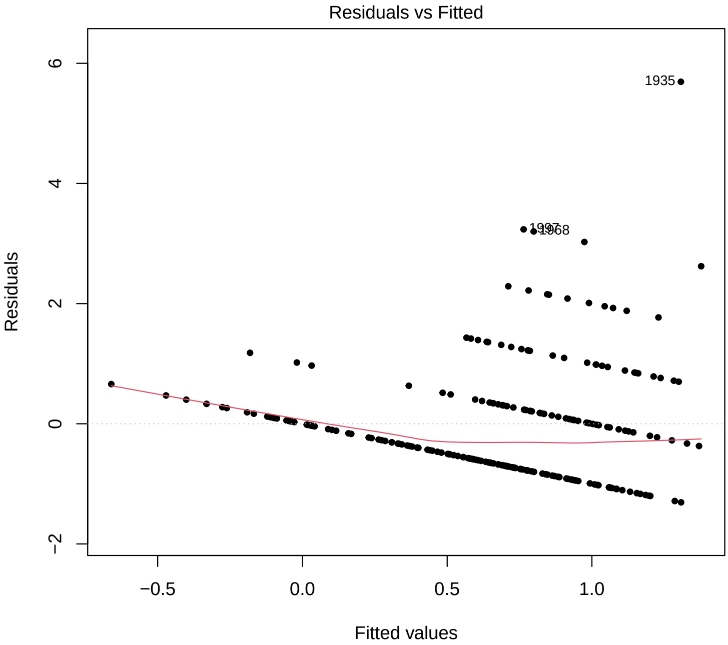
In Action

```
> Fit<-with(NewDahl, lm(NNulls~Age+Tenure+Unified))

> library(gvlma)
> Nope <- gvlma(Fit)
> display.gvlmatests(Nope)

ASSESSMENT OF THE LINEAR MODEL ASSUMPTIONS
USING THE GLOBAL TEST ON 4 DEGREES-OF-FREEDOM:
Level of Significance =  0.05

Call:
 gvlma(x = Fit)

                    Value  p-value                   Decision
Global Stat        454.87 0.00e+00 Assumptions NOT satisfied!
Skewness           122.09 0.00e+00 Assumptions NOT satisfied!
Kurtosis           283.21 0.00e+00 Assumptions NOT satisfied!
Link Function        5.35 2.07e-02 Assumptions NOT satisfied!
Heteroscedasticity  44.23 2.92e-11 Assumptions NOT satisfied!
```

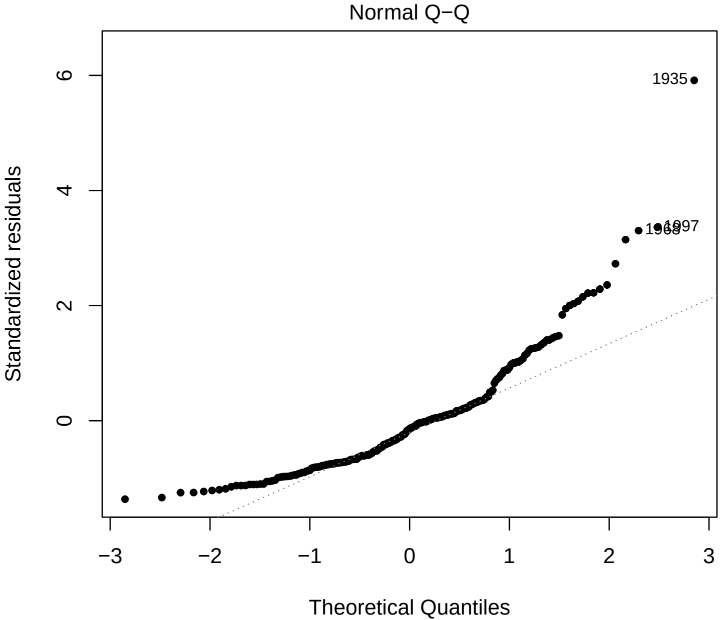
Another Approach: `plot(fit)`



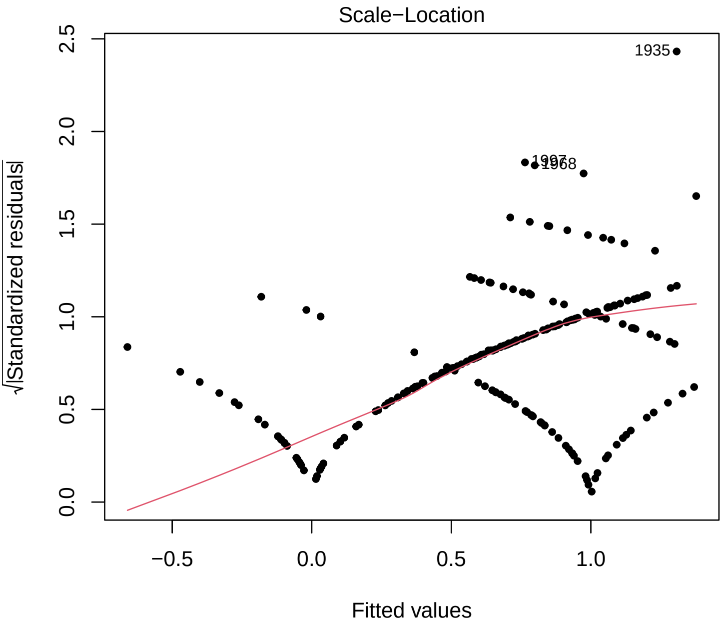
#1: Residuals vs. Fitted Values



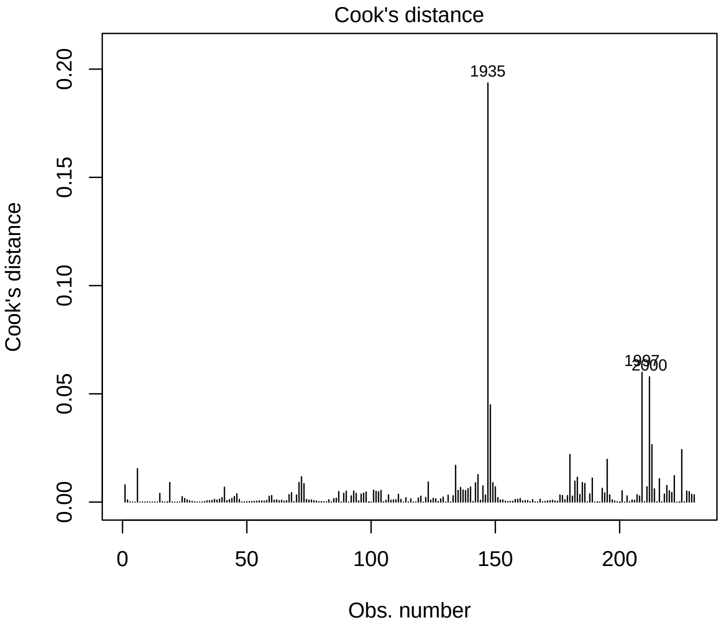
#2: Q-Q Plot of $\hat{u}$s



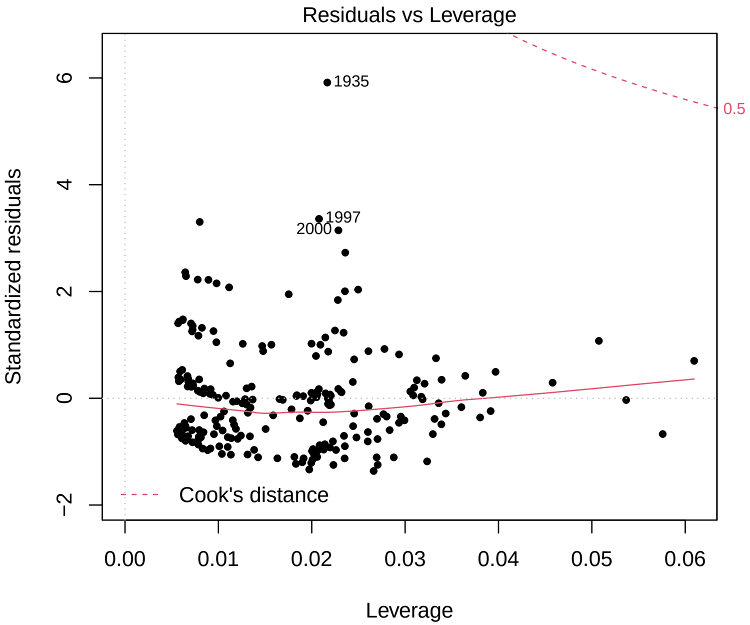
“Scale-Location” Plot



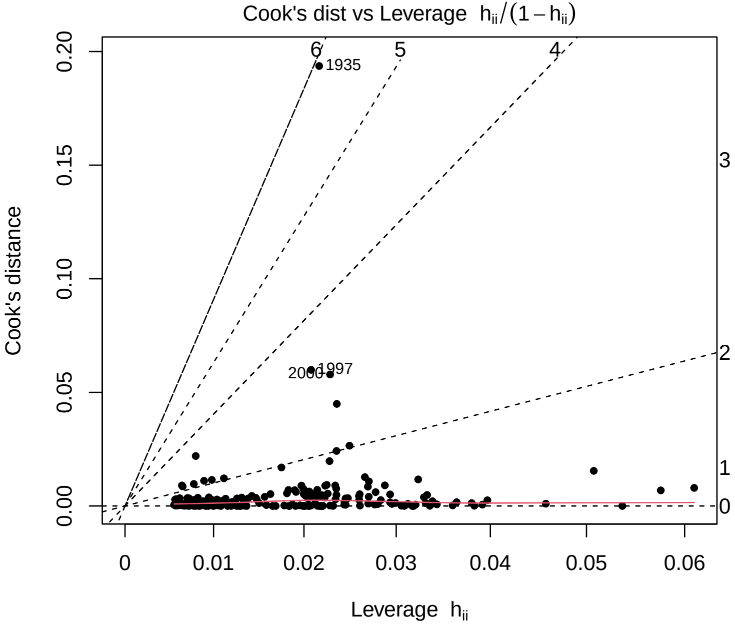
Cook's D


Cook's distance
Cook's distance
0.00
0.05
0.10
0.15
0.20
1935
1997
2000
0
50
100
150
200
Obs. number

Residuals vs. Leverage



Cook's D vs. Leverage



Simultaneity, Endogeneity, and Instrumental Variables

Consider:

$$Y_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

$$Y_2 = \mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2$$

Rewrite:

$$\begin{aligned}
Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1[\mathbf{X}_2\boldsymbol{\beta}_2 + \gamma_2 Y_1 + \mathbf{u}_2] + \mathbf{u}_1 \\
&= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\gamma_2 Y_1 + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 - \gamma_1\gamma_2 Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
(1 - \gamma_1\gamma_2) Y_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1(\mathbf{X}_2\boldsymbol{\beta}_2) + \gamma_1\mathbf{u}_2 + \mathbf{u}_1 \\
Y_1 &= \mathbf{X}_1\left(\frac{1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_1\right) + \mathbf{X}_2\left(\frac{\gamma_1}{1-\gamma_1\gamma_2}\boldsymbol{\beta}_2\right) + \left(\frac{\gamma_1\mathbf{u}_2 + \mathbf{u}_1}{1-\gamma_1\gamma_2}\right) \\
&= \Delta_1\mathbf{X}_1 + \Delta_2\mathbf{X}_2 + \mathbf{e}
\end{aligned}$$

$$Y_1 = \mathbf{X}_1 \left(\frac{1}{1 - \gamma_1 \gamma_2} \boldsymbol{\beta}_1 \right) + \mathbf{X}_2 \left(\frac{\gamma_1}{1 - \gamma_1 \gamma_2} \boldsymbol{\beta}_2 \right) + \left(\frac{\gamma_1 \mathbf{u}_2 + \mathbf{u}_1}{1 - \gamma_1 \gamma_2} \right)$$

means

$$\frac{\partial Y_1}{\partial X_\ell} = \frac{\beta_\ell}{1 - \gamma_1 \gamma_2}.$$

But

$$\hat{\Delta}_1 \neq \hat{\boldsymbol{\beta}}_1.$$

For (e.g.)

$$Y_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \gamma_1 Y_2 + \mathbf{u}_1$$

we have:

$$\mathrm{E}(Y_2, \mathbf{u}_1) = \frac{\gamma_2}{1 - \gamma_1\gamma_2}\sigma^2_{\mathbf{u}}$$

Result:

- Bias (unless $\gamma_2 = 0$)
- Inconsistency

Consider a variable W that affects both X and Y, so that the true data-generating process is:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 W_i + e_i$$

Omitting W from the equation yields:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

where

$$u_i = \beta_2 W_i + e_i.$$

This also results in $\text{Cov}(X_i, u_i) \neq 0$...

Options:

- OLS
- Lagged Variables
- Instrumental Variables (sometimes, 'Two-Stage Least Squares" / "2SLS")
- Systems of Equations / 3SLS / etc.

Instrumental Variables (IV)

Recall that a simple linear model:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

gives us:

$$\hat{\boldsymbol{\beta}}_{OLS} = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}.$$

Suppose $\text{Cov}(\mathbf{X}, \mathbf{u}) \neq \mathbf{0}$, but we have $\mathbf{Z}$ with

- $\text{Cov}(\mathbf{Z}, \mathbf{X}) \neq \mathbf{0}$ and
- $\text{Cov}(\mathbf{Z}, \mathbf{u}) = \mathbf{0}$.

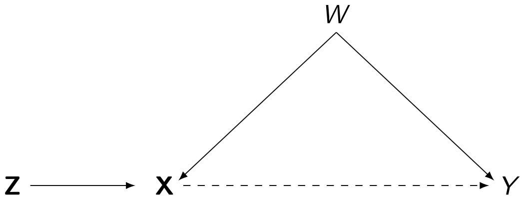
Then:

$$\begin{aligned} \hat{\boldsymbol{\beta}}_{IV} &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{Y} \\ &= (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'(\mathbf{X}\boldsymbol{\beta} + \mathbf{u}) \\ &= \boldsymbol{\beta} + (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{u} \end{aligned}$$

is consistent.

IV In Pictures

For an unmeasured / unmodeled *confounder* W:



Consistency requires that:

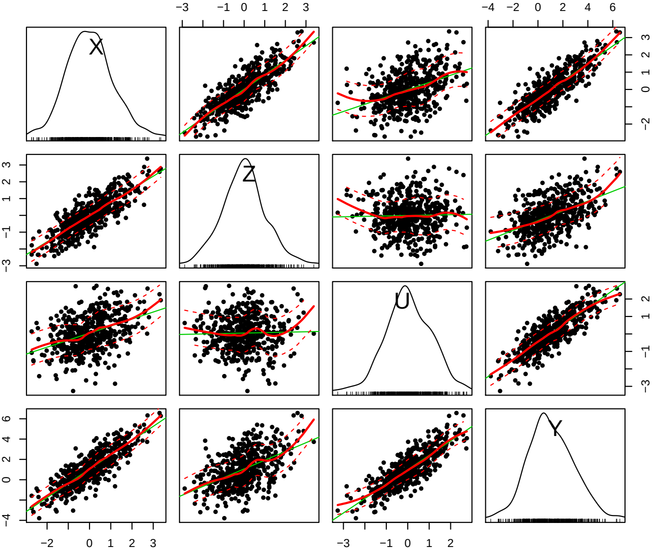
1. $\text{Cov}(\mathbf{X}, \mathbf{Z}) \neq 0$ ($\mathbf{Z}$ is a *strong instrument*)
2. $\mathbf{Z}$ is *exogenous* / *unconfounded* by W [i.e., $\text{Cov}(\mathbf{Z}, W) = 0$]
3. $\mathbf{Z}$ has no independent effect on Y, except through W (the *exclusion restriction*)

"Two-stage least squares":

- Regress endogenous $\mathbf{X}$s variables on $\{\mathbf{Z}, \mathbf{X}\}$
- Generate $\hat{\mathbf{X}}$s
- Regress Y on $\hat{\mathbf{X}}$ to get $\boldsymbol{\beta}_{2SLS}$.
- Adjust standard error estimates

```
library(MASS)
library(sem)
library(car)

seed<-1337
set.seed(seed)

mu<-c(0,0,0) # <== X, Z, U
Sigma<-matrix(c(1,0.8,0.4,0.8,1,0,0.4,0,1),
             nrow=3,byrow=TRUE)           # Cor(X,Y)=0.8, etc.
Vars<- mvrnorm(500,mu,Sigma)
colnames(Vars)<-c("X","Z","U")
Vars<-data.frame(Vars)

Vars$Y<- 1 + Vars$X + Vars$U
```

Plots...

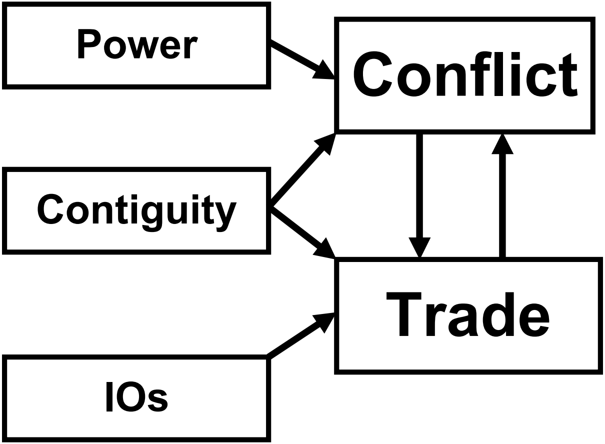


Plain Old OLS...

```
> OLS<- lm(Y~X,data=Vars)
> summary(OLS)

Call:
lm(formula = Y ~ X, data = Vars)

Residuals:
    Min      1Q  Median      3Q     Max
-3.3809 -0.6058 -0.0102  0.6320  2.9470

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.04770    0.04209   24.89   <2e-16 ***
X            1.40254    0.04005   35.02   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.9413 on 498 degrees of freedom
Multiple R-squared:  0.7112, Adjusted R-squared:  0.7106
F-statistic:  1226 on 1 and 498 DF,  p-value: < 2.2e-16
```

```
> TSLS<-tsls(Y~I(X),data=Vars,instruments=~Z)
> summary(TSLS)

2SLS Estimates

Model Formula: Y ~ I(X)

Instruments: ~Z

Residuals:
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
-3.29300 -0.68210 -0.06139  0.00000  0.76270  2.70300

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) 1.0491828  0.0456017 23.00754 < 2.22e-16 ***
I(X)        1.0302012  0.0536909 19.18763 < 2.22e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 1.0196738 on 498 degrees of freedom
```

IV: A (Toy) Example



```
> summary(IRData)

     dyadid        logdisputes          logtrade             IOs        
 Min.   :  2020   Min.   :-0.6931   Min.   :-0.6931   Min.   : 4.579  
 1st Qu.:135155   1st Qu.:-0.6931   1st Qu.: 2.4079   1st Qu.:19.500  
 Median :220484   Median :-0.6931   Median : 5.5786   Median :27.704  
 Mean   :275526   Mean   :-0.2627   Mean   : 4.6518   Mean   :30.891  
 3rd Qu.:385710   3rd Qu.: 0.0000   3rd Qu.: 7.1248   3rd Qu.:39.289  
 Max.   :900920   Max.   : 3.4965   Max.   :11.5037   Max.   :93.700  
   contiguity         capratio          GDPgrowth      
 Min.   :0.0000   Min.   :   1.081   Min.   :-9.0800  
 1st Qu.:0.0000   1st Qu.:   4.849   1st Qu.:-0.2923  
 Median :0.0000   Median :  26.577   Median : 0.8363  
 Mean   :0.3207   Mean   : 196.310   Mean   : 0.5097  
 3rd Qu.:1.0000   3rd Qu.: 144.035   3rd Qu.: 1.7106  
 Max.   :1.0000   Max.   :7451.982   Max.   : 7.0460  
```

```
> OLSWar<-lm(logdisputes~logtrade+contiguity+capratio,data=IRData)
> summary(OLSWar)

Call:
lm(formula = logdisputes ~ logtrade + contiguity + capratio,
    data = IRData)

Residuals:
   Min     1Q Median     3Q    Max
-0.828 -0.326 -0.269 -0.090  3.455

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.4253192  0.0602014   -7.06  3.5e-12 ***
logtrade     0.0085581  0.0105739    0.81    0.419
contiguity   0.4622674  0.0712406    6.49  1.5e-10 ***
capratio    -0.0001296  0.0000647   -2.00    0.045 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.853 on 813 degrees of freedom
Multiple R-squared:  0.083, Adjusted R-squared:  0.0796
F-statistic: 24.5 on 3 and 813 DF,  p-value: 3.35e-15
```

2SLS "By-Hand" (stage one)

```
> ITrade<-lm(logtrade~contiguity+IOs+capratio)
> summary(ITrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.0385 -1.7666  0.4139  1.6154  7.6029

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.7319793  0.1912570   3.827 0.000140 ***
contiguity   1.3386037  0.1816041   7.371 4.17e-13 ***
IOs          0.1218373  0.0055313  22.027  < 2e-16 ***
capratio    -0.0013913  0.0001626  -8.555  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.239 on 813 degrees of freedom
Multiple R-squared: 0.5535, Adjusted R-squared: 0.5519
F-statistic: 335.9 on 3 and 813 DF,  p-value: < 2.2e-16
```

2SLS "By-Hand" (stage two)

```
> IVWarByHand<-with(IRData, lm(logdisputes~capratio+contiguity+
+                         (ITrade$fitted.values)))
> summary(IVWarByHand)

Call:
lm(formula = logdisputes ~ capratio + contiguity + (ITrade$fitted.values))

Residuals:
   Min     1Q Median     3Q    Max
-1.006 -0.362 -0.278 -0.049  3.530

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)         -0.1515180  0.0832287   -1.82  0.06905 .
capratio            -0.0002664  0.0000705   -3.78  0.00017 ***
contiguity           0.6263774  0.0788444    7.94  6.5e-15 ***
ITrade$fitted.values -0.0558374  0.0171921   -3.25  0.00121 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.848 on 813 degrees of freedom
Multiple R-squared:  0.094, Adjusted R-squared:  0.0907
F-statistic: 28.1 on 3 and 813 DF,  p-value: <2e-16
```

```
> library(AER)
> TwoSLSWar<-ivreg(logdisputes~contiguity+capratio+I(logtrade),
   instruments=~contiguity+capratio+IOs)
> summary(TwoSLSWar)

Call:
ivreg(formula = logdisputes ~ contiguity + capratio + I(logtrade) |
    contiguity + capratio + IOs, data = IRData)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.1515180  0.0856218   -1.77  0.07717 .
contiguity   0.6263774  0.0811114    7.72  3.4e-14 ***
capratio    -0.0002664  0.0000725   -3.67  0.00025 ***
I(logtrade) -0.0558374  0.0176864   -3.16  0.00165 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.872 on 813 degrees of freedom
Multiple R-Squared: 0.0412, Adjusted R-squared: 0.0376
Wald test: 26.6 on 3 and 813 DF,  p-value: <2e-16
```

```
> OLSTrade<-lm(logtrade~logdisputes+contiguity+IOs)
> summary(OLSTrade)

Residuals:
    Min      1Q  Median      3Q     Max
-6.2467 -2.2067  0.4275  1.6659  6.1264

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.191111   0.182875   1.045    0.296
logdisputes 0.408116   0.095067   4.293 1.98e-05 ***
contiguity  1.357557   0.193109   7.030 4.38e-12 ***
IOs         0.133778   0.005614  23.831  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2.312 on 813 degrees of freedom
Multiple R-squared: 0.5241, Adjusted R-squared: 0.5223
F-statistic: 298.4 on 3 and 813 DF,  p-value: < 2.2e-16
```

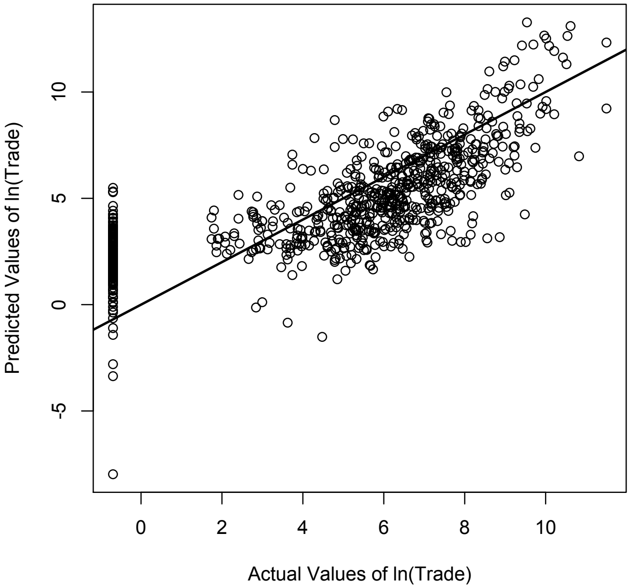
Weak Instruments (continued)

```
> TwoSLSTrade<-ivreg(logtrade~contiguity+IOs+I(logdisputes),
    instruments=~contiguity+capratio+IOs)
> summary(TwoSLSTrade)

Call:
ivreg(formula = logtrade ~ contiguity + IOs + I(logdisputes) |
    contiguity + capratio + IOs, data = IRData)

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      2.1501     0.8512    2.53   0.0117 *
contiguity      -2.7276     1.5262   -1.79   0.0743 .
IOs              0.1720     0.0205    8.41   <2e-16 ***
I(logdisputes)   7.3712     2.4520    3.01   0.0027 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 6.37 on 813 degrees of freedom
Multiple R-Squared: -2.62, Adjusted R-squared: -2.63
Wald test: 41.5 on 3 and 813 DF,  p-value: <2e-16
```

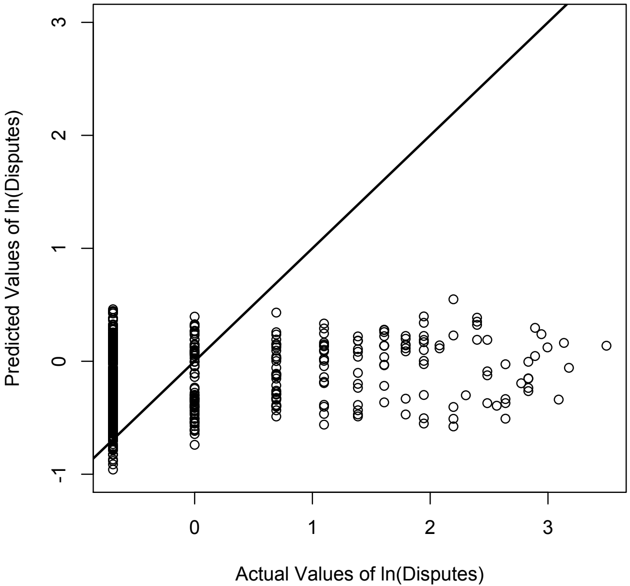
Side-By-Side...

	Dependent variable:			
	ln(Disputes)		ln(Trade)	
	OLS	*IV*	*OLS*	*IV*
ln(Trade)	0.009			
	(0.011)			
ln(Disputes)			0.408^{***}	
			(0.095)	
Contiguity	0.462^{***}	0.626^{***}	1.358^{***}	-2.728^{*}
	(0.071)	(0.081)	(0.193)	(1.526)
Capability Ratio	-0.0001^{**}	-0.0003^{***}		
	(0.0001)	(0.0001)		
I(logtrade)		-0.056^{***}		
		(0.018)		
IOs			0.134^{***}	0.172^{***}
			(0.006)	(0.020)
I(logdisputes)				7.371^{***}
				(2.452)
Constant	-0.425^{***}	-0.152^{*}	0.191	2.150^{**}
	(0.060)	(0.086)	(0.183)	(0.851)
Observations	817	817	817	817
R^2	0.083	0.041	0.524	−2.616
Adjusted R^2	0.080	0.038	0.522	−2.630
Residual Std. Error (df = 813)	0.853	0.872	2.312	6.372
F Statistic (df = 3; 813)	24.530^{***}		298.400^{***}	
Note:				$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Pretty Good Instrument (Trade)



Crappy Instrument (War)



IV Questions

Things to ask yourself when you see someone attempting an instrumental variables analysis:

1. How strong is the relationship between the instrument (Z) and the instrumented predictor (X)?
2. Is the instrument (Z) plausibly unconfounded with X?
3. Does the instrument meet the exclusion restriction? That is, is the only way that Z influences Y via its effect on X?