PLSC 503 – Spring 2025 Introduction + Bivariate Regression Review

January 13, 2025

"Multivariate Analysis for Political Research"

- "Regression" course
- Texts: Weisberg (2014) / Gelman-Hill-Vehtari (2020)
- Class materials: `https://github.com/PrisonRodeo/PLSC503-2025-git`
- Preceptor: Morrgan Herlihy
- Software: R > Stata > others
- Grading: Ten homework assignments (@ 50 points), plus a final project (500 points)

Things We Will And Won't Do

Will: "Regression":

$$Y = f(\mathbf{X})$$

Won't: Multivariate regression:

$$\mathbf{Y} = f(\mathbf{X})$$

Won't: Measurement (e.g. PCA, factor analysis, etc.):

$$\mathbf{Y} = \mathbf{W}^{\mathrm{T}}\mathbf{X}$$

Won't: Classification:

- Cluster Analysis
- Classification and Regression Trees $\rightarrow$ Random Forests, etc.
- Pattern Recognition
- Machine Learning, Support Vector Machines, etc.

"Regression," conceptually:

$$\Pr(Y|\mathbf{X}) = f(\mathbf{X})$$

Two important things:

- The distribution of Y is *conditional on all variables in* $\mathbf{X}$, and
- The conditional distribution of Y is conditional on the *joint distribution* of the elements of $\mathbf{X}$.

$\rightarrow$ Regression is <u>hard</u>...

Why regression?

	Description	Explanation	Prediction
Task	Summarize data	Correlation/causation	Forecast OOS / future data
Emphasis	Data	Theory / Hypotheses	Outcomes
Focus	Univariate	Multivariate	Multivariate
Typical Application	Summarize / "reduce" data	Discuss marginal associations between predictors and an outcome of interest	Optimize out-of-sample predictive power / minimize prediction error

Linear Regression...

A *random variable*:

$$Y_i = \mu + u_i \tag{1}$$

A *linear function*:

$$\mu_i = \beta_0 + \beta_1 X_i$$

so:

$$Y_i = \beta_0 + \beta_1 X_i + u_i \tag{2}$$

Goals:

- *Estimate* $\hat{\beta}_0$ and $\hat{\beta}_1$
- Estimate the *variability* $\hat{\beta}_0$ and $\hat{\beta}_1$
- Assess *model fit*

If we have $\hat{\beta}_0$ and $\hat{\beta}_1$, then:

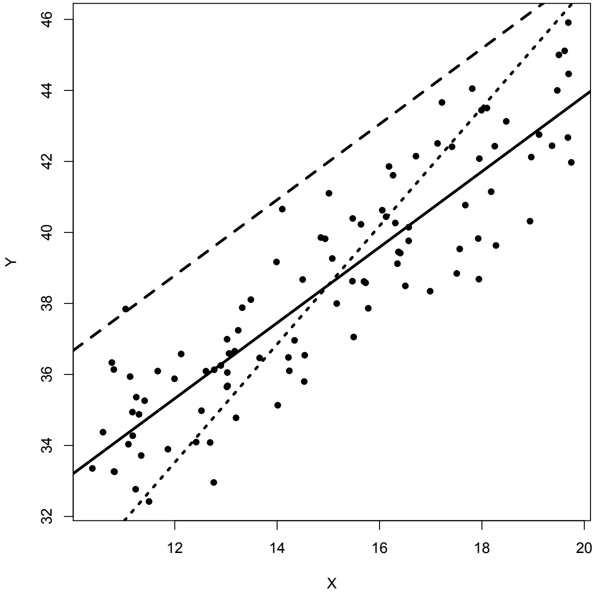
$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \tag{3}$$

and

$$\begin{aligned} \hat{u}_i &= Y_i - \hat{Y}_i \\ &= Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i \end{aligned} \tag{4}$$

Q: How to estimate $\hat{\beta}_0$ and $\hat{\beta}_1$?

Scatterplot: X and Y (with regression lines)



Choose $\hat{\beta}_0$ and $\hat{\beta}_1$ to minimize $\hat{S} = \sum_{i=1}^{N} \hat{u}_i^2$.

$$
\begin{aligned}
\hat{S} &= \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 \\
&= \sum_{i=1}^{N} (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2 \\
&= \sum_{i=1}^{N} (Y_i^2 - 2Y_i\hat{\beta}_0 - 2Y_i\hat{\beta}_1 X_i + \hat{\beta}_0^2 + 2\hat{\beta}_0\hat{\beta}_1 X_i + \hat{\beta}_1^2 X_i^2)
\end{aligned}
$$

Differentiate:

$$
\begin{aligned}
\frac{\partial \hat{S}}{\partial \hat{\beta}_0} &= \sum_{i=1}^{N}(-2Y_i + 2\hat{\beta}_0 + 2\hat{\beta}_1 X_i) \\
&= -2\sum_{i=1}^{N}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) \\
&= -2\sum_{i=1}^{N}\hat{u}_i
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \hat{S}}{\partial \hat{\beta}_1} &= \sum_{i=1}^{N}(-2Y_i X_i + 2\hat{\beta}_0 X_i + 2\hat{\beta}_1 X_i^2) \\
&= -2\sum_{i=1}^{N}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)X_i \\
&= -2\sum_{i=1}^{N}\hat{u}_i X_i
\end{aligned}
$$

OLS (continued)

Yields:

$$\sum_{i=1}^{N} Y_i = N\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^{N} X_i$$

and

$$\sum_{i=1}^{N} Y_i X_i = \hat{\beta}_0 \sum_{i=1}^{N} X_i + \hat{\beta}_1 \sum_{i=1}^{N} X_i^2$$

OLS (continued)

Solving yields:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{N}(X_i - \bar{X})^2} = \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X} \tag{5}$$

and

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \tag{6}$$

Variation in Y

$$\begin{aligned}\operatorname{Var}(Y) &= \operatorname{Var}(\hat{Y}+\hat{u}) \\ &= \operatorname{Var}(\hat{Y})+\operatorname{Var}(\hat{u})+2\operatorname{Cov}(\hat{Y},\hat{u}) \\ &= \underbrace{\operatorname{Var}(\hat{Y})}_{\text{“Systematic”}} + \underbrace{\operatorname{Var}(\hat{u})}_{\text{“Stochastic”}}\end{aligned}$$

TSS	$=$	**MSS**	$+$	**RSS**
(“Total”)		(“Estimated,” or “Model”)		(“Residual”)

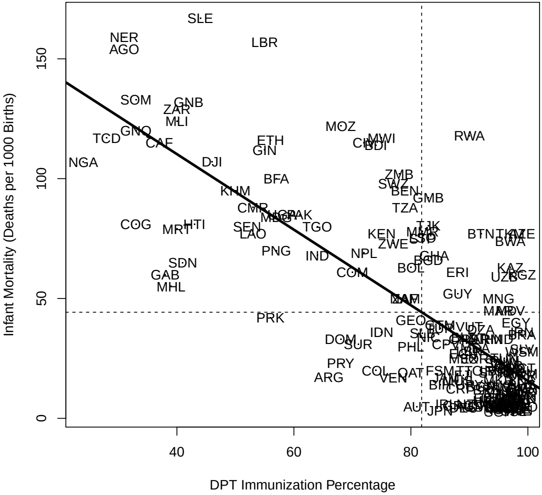
Running Example: Infant Mortality

```
> url <- getURL("https://raw.githubusercontent.com/PrisonRodeo/
    PLSC503-2025-git/master/Data/IMDPT25.csv")
> Data <- read.csv(text = url) # read the "countries" data
> rm(url)
>
> # Summary statistics
>
> # install.packages("psych") <- Install psych package, if necessary
> library(psych)

> with(Data, describe(InfantMortalityPerK))
   vars   n  mean    sd median trimmed   mad min max range skew kurtosis   se
X1    1 177 44.26 40.41   29.3   38.85 34.69 2.9 167 164.1 0.99     0.03 3.04

> with(Data, describe(DPTpct))
   vars   n  mean    sd median trimmed   mad min max range skew kurtosis   se
X1    1 177 81.85 19.57     90   85.22 11.86  24  99    75 -1.3     0.59 1.47
```

OLS Regression

```
> IMDPT<-lm(InfantMortalityPerK~DPTpct,data=Data,na.action=na.exclude)
> summary.lm(IMDPT)

Call:
lm(formula = InfantMortalityPerK ~ DPTpct, data = Data, na.action = na.exclude)

Residuals:
   Min     1Q Median     3Q    Max 
 -56.8  -16.3   -5.1   11.8   86.6 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  173.277      8.489    20.4   <2e-16 ***
DPTpct        -1.576      0.101   -15.6   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 26.2 on 175 degrees of freedom
Multiple R-squared:  0.582, Adjusted R-squared:  0.58
F-statistic:  244 on 1 and 175 DF,  p-value: <2e-16
```

Scatterplot: Infant Mortality and DPT Immunization Rates



```
> anova(IMDPT)

Analysis of Variance Table

Response: InfantMortalityPerK
           Df Sum Sq Mean Sq F value Pr(>F)
DPTpct      1 167423  167423     244 <2e-16 ***
Residuals 175 120033     686
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$\begin{aligned} TSS &= \text{total variability in } Y \text{ around its mean} \\ &= \sum (Y_i - \bar{Y})^2 \\ &= 167423 + 120033 \\ &= \mathbf{287456} \end{aligned}$$

$$\begin{aligned} MSS(\equiv \texttt{DPTpct}) &= \text{model ("explained" or "regression") sum of squares} \\ &= \sum (\hat{Y}_i - \bar{Y})^2 \\ &= \mathbf{167423} \end{aligned}$$

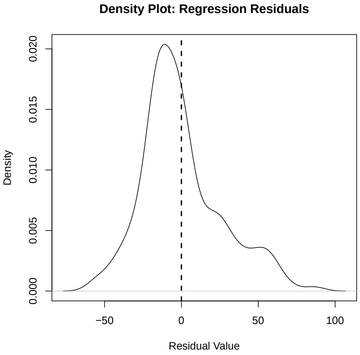
$$\begin{aligned} RSS(\equiv \texttt{Residuals}) &= \text{residual ("unexplained" or "error") sum of squares} \\ &= \sum \hat{u}_i^2 \\ &= \mathbf{120033} \end{aligned}$$

$$\begin{aligned} \hat{\sigma}^2 &= \frac{RSS}{N-k} \\ &= \frac{\sum \hat{u}_i^2}{N-2} \\ &= \frac{120033}{175} \\ &= \mathbf{686} \end{aligned}$$

$$\begin{aligned} \hat{\sigma} &= \text{"SEE" (the standard error of the estimate, a/k/a the } \texttt{Residual standard error}) \\ &= \sqrt{\hat{\sigma}^2} \\ &= \sqrt{686} \\ &= \mathbf{26.2} \end{aligned}$$

Fitted Values, Residuals, etc.

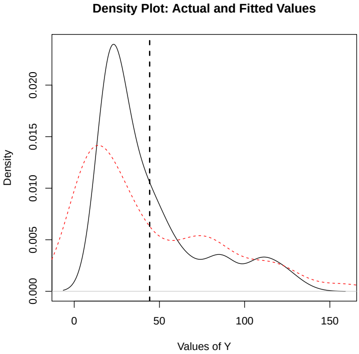
```
> # Residuals (u):
> Data$IMDPTres <- with(Data, residuals(IMDPT))
> describe(Data$IMDPTres)

  var   n mean    sd median   mad   min   max range skew kurtosis   se
1   1 177    0 26.12   -5.1 19.42 -56.8 86.59 143.4 0.75     0.44 1.96
```



Fitted Values

```
> # Fitted Values:
> Data$IMDPThat<-fitted.values(IMDPT)
> describe(Data$IMDPThat)

  var   n  mean    sd median  mad   min   max range skew kurtosis   se
1   1 177 44.26 30.84  31.41 18.7 17.22 135.4 118.2  1.3     0.59 2.32
```

Figure: Density Plot: Actual (Y) and Fitted Values ($\hat{Y}$)



Some Correlations

$\underline{\text{Corr}(Y, X)}$:

```
> with(Data, cor(InfantMortalityPerK,DPTpct,use="complete.obs"))
[1] -0.7632
```

$\underline{\text{Corr}(\hat{u}, Y)}$:

```
> with(Data, cor(IMDPTres,InfantMortalityPerK,use="complete.obs"))
[1] 0.6462
```

$\underline{\text{Corr}(\hat{u}, X)}$:

```
> with(Data, cor(IMDPTres,DPTpct,use="complete.obs"))
[1] 9.573e-17
```

$\underline{\text{Corr}(\hat{Y}, Y)}$:

```
> with(Data, cor(IMDPThat,InfantMortalityPerK,use="complete.obs"))
[1] 0.7632
```

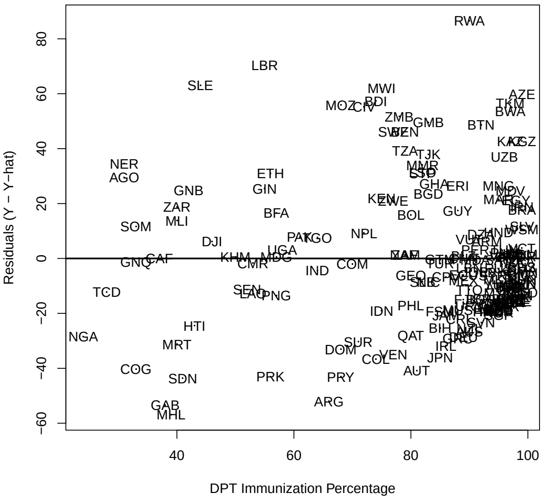
$\underline{\text{Corr}(\hat{Y}, X)}$:

```
> with(Data, cor(IMDPThat,DPTpct,use="complete.obs"))
[1] -1
```

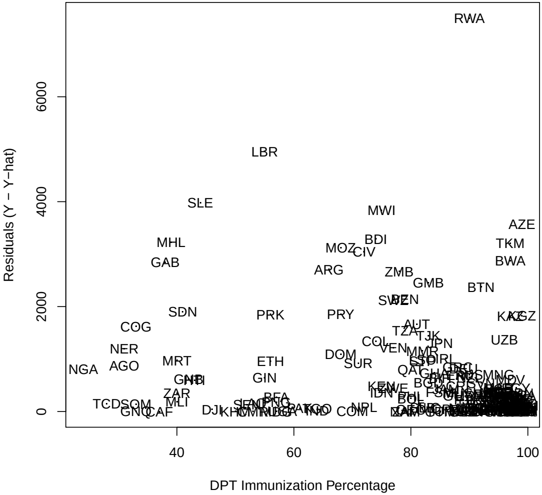
$\underline{\text{Corr}(\hat{u}, \hat{Y})}$:

```
> with(Data, cor(IMDPTres,IMDPThat,use="complete.obs"))
[1] 6.335e-17
```

Regression Residuals ($\hat{u}$) vs. DPT Percentage



Squared Residuals vs. DPT Percentage



Inference and Model Fit

The key point:

The estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are random variables.

Due to (*inter alia*):

- **Sampling variability**: Random samples from a population $\rightarrow$ slightly different $\hat{\beta}_0$s and $\hat{\beta}_1$s.
- **Random variability in X**: In cases where X is also a random variable...
- **Intrinsic variability in Y**: Because $Y_i = \mu + u_i$.

Start by assuming:

$$u_i \sim \text{i.i.d. } N(0, \sigma^2)$$

which means that:

$$\mathrm{Var}(Y|X, \beta) = \sigma^2$$

so:

$$\begin{aligned}
\mathrm{Var}(\hat{\beta}_1) &= \mathrm{Var}\left[\frac{\sum_{i=1}^{N}(X_i - \bar{X})Y_i}{\sum_{i=1}^{N}(X_i - \bar{X})^2}\right] \\
&= \left[\frac{1}{\sum(X_i - \bar{X})^2}\right]^2 \sum(X_i - \bar{X})^2 \,\mathrm{Var}(Y) \\
&= \left[\frac{1}{\sum(X_i - \bar{X})^2}\right]^2 \sum(X_i - \bar{X})^2 \,\sigma^2 \\
&= \frac{\sigma^2}{\sum(X_i - \bar{X})^2}.
\end{aligned}$$

$\text{Var}(\hat{\beta}_0)$ and $\text{Cov}(\hat{\beta}_0, \hat{\beta}_1)$

Similarly:

$$\text{Var}(\hat{\beta}_0) = \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \sigma^2$$

and :

$$\text{Cov}(\hat{\beta}_0, \hat{\beta}_1) = \frac{-\bar{X}}{\sum (X_i - \bar{X})^2} \sigma^2$$

Note that:

- $\text{Var}(\hat{\beta}_0)$ and $\text{Var}(\hat{\beta}_1) \propto \sigma^2$
- $\text{Var}(\hat{\beta}_0)$ and $\text{Var}(\hat{\beta}_1) \propto -\sum(X_i - \bar{X})^2$
- $\text{Var}(\hat{\beta}_0)$ and $\text{Var}(\hat{\beta}_1) \propto -N$
- $\text{sign}[\text{Cov}(\hat{\beta}_0, \hat{\beta}_1)] = -\text{sign}(\bar{X})$

For:

$$\hat{\beta}_1 = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2}$$

Rewrite:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(X_i - \bar{X})Y_i}{\sum_{i=1}^{N}(X_i - \bar{X})^2} = \left[\frac{\sum_{i=1}^{N}(X_i - \bar{X})}{\sum_{i=1}^{N}(X_i - \bar{X})^2}\right] Y_i.$$

Define "weights" k:

$$\hat{\beta}_1 = \sum k_i Y_i$$

with $k_i = \frac{X_i - \bar{X}}{\sum(X_i - \bar{X})^2}$.

Gauss-Markov (continued)

Alternative (non-LS) estimator:

$$\tilde{\beta}_1 = \sum w_i Y_i$$

Unbiasedness requires:

$$\begin{aligned}
\mathrm{E}(\tilde{\beta}_1) &= \sum w_i \mathrm{E}(Y_i) \\
&= \sum w_i (\beta_0 + \beta_1 X_i) \\
&= \beta_0 \sum w_i + \beta_1 \sum w_i X_i
\end{aligned}$$

Variance:

$$
\begin{aligned}
\mathrm{Var}(\tilde{\beta}_1) &= \mathrm{Var}\left(\sum w_i Y_i\right) \\
&= \sigma^2 \sum w_i^2 \\
&= \sigma^2 \sum \left[w_i - \frac{X_i - \bar{X}}{\sum (X_i - \bar{X})^2} + \frac{X_i - \bar{X}}{\sum (X_i - \bar{X})^2} \right]^2 \\
&= \sigma^2 \sum \left[w_i - \frac{X_i - \bar{X}}{\sum (X_i - \bar{X})^2} \right]^2 + \sigma^2 \left[\frac{1}{\sum (X_i - \bar{X})^2} \right]
\end{aligned}
$$

Because $\sigma^2 \left[\frac{1}{\sum(X_i - \bar{X})^2}\right]$ is a constant, $\min[\text{Var}(\tilde{\beta}_1)]$ minimizes

$$\sum \left[w_i - \frac{X_i - \bar{X}}{\sum(X_i - \bar{X})^2}\right]^2.$$

Minimized at:

$$w_i = \frac{X_i - \bar{X}}{\sum(X_i - \bar{X})^2}.$$

implying:

$$\begin{aligned} \text{Var}(\tilde{\beta}_1) &= \frac{\sigma^2}{\sum(X_i - \bar{X})^2} \\ &= \text{Var}(\hat{\beta}_1) \end{aligned}$$

If $u_i \sim N(0, \sigma^2)$, then:

$$\hat{\beta}_0 \sim N[\beta_0, \text{Var}(\hat{\beta}_0)]$$

and

$$\hat{\beta}_1 \sim N[\beta_1, \text{Var}(\hat{\beta}_1)]$$

Which means:

$$\begin{aligned} z_{\hat{\beta}_1} &= \frac{(\hat{\beta}_1 - \beta_1)}{\sqrt{\text{Var}(\hat{\beta}_1)}} \\ &= \frac{(\hat{\beta}_1 - \beta_1)}{\text{s.e.}(\hat{\beta}_1)} \\ &= \sim N(0, 1) \end{aligned}$$

A Small Problem...

$\sigma^2 =$???

Solution: use

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{N - k}$$

Yields:

$$\widehat{\text{Var}(\hat{\beta}_1)} = \frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2},$$

and

$$\widehat{\text{Var}(\hat{\beta}_0)} = \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \hat{\sigma}^2$$

Inference (continued)

$$\begin{aligned}
\widehat{\text{s.e.}(\hat{\beta}_1)} &= \sqrt{\widehat{\text{Var}(\hat{\beta}_1)}} \\
&= \sqrt{\frac{\hat{\sigma}^2}{\sum(X_i - \bar{X})^2}} \\
&= \frac{\hat{\sigma}}{\sqrt{\sum(X_i - \bar{X})^2}}
\end{aligned}$$

implies:

$$\begin{aligned}
t_{\hat{\beta}_1} \equiv \frac{(\hat{\beta}_1 - \beta_1)}{\widehat{\text{s.e.}(\hat{\beta}_1)}} &= \frac{(\hat{\beta}_1 - \beta_1)}{\frac{\hat{\sigma}}{\sqrt{\sum(X_i - \bar{X})^2}}} \\
&= \frac{(\hat{\beta}_1 - \beta_1)\sqrt{\sum(X_i - \bar{X})^2}}{\hat{\sigma}} \\
&\sim t_{N-k}
\end{aligned}$$

Point prediction:

$$\hat{Y}_k = \hat{\beta}_0 + \hat{\beta}_1 X_k$$

$\hat{Y}_k$ is an unbiased estimate of Y_k:

$$\begin{aligned}
\mathrm{E}(\hat{Y}_k) &= \mathrm{E}(\hat{\beta}_0 + \hat{\beta}_1 X_k) \\
&= \mathrm{E}(\hat{\beta}_0) + X_k \mathrm{E}(\hat{\beta}_1) \\
&= \beta_0 + \beta_1 X_k \\
&= \mathrm{E}(Y_k)
\end{aligned}$$

Variability:

$$\begin{aligned}
\mathrm{Var}(\hat{Y}_k) &= \mathrm{Var}(\hat{\beta}_0 + \hat{\beta}_1 X_k) \\
&= \frac{\sum X_i^2}{N \sum (X_i - \bar{X})^2} \sigma^2 + \left[\frac{\sigma^2}{\sum (X_i - \bar{X})^2} \right] X_k^2 + 2 \left[\frac{-\bar{X}}{\sum (X_i - \bar{X})^2} \sigma^2 \right] X_k \\
&= \sigma^2 \left[\frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right]
\end{aligned}$$

Variability of Predictions

$$\text{Var}(\hat{Y}_k) = \sigma^2 \left[\frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right]$$

means that $\text{Var}(\hat{Y}_k)$:

- Decreases in N
- Decreases in $\text{Var}(X)$
- Increases in $|X - \bar{X}|$

Estimated standard error of the prediction:

$$\widehat{\text{s.e.}(\hat{Y}_k)} = \sqrt{\hat{\sigma}^2 \left[\frac{1}{N} + \frac{(X_k - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]}$$

$\rightarrow$ (e.g.) confidence intervals:

$$95\%\,\text{c.i.}(\hat{Y}_k) = \hat{Y}_k \pm [1.96 \times \widehat{\text{s.e.}(\hat{Y}_k)}]$$

```
> summary(IMDPT)

Call:
lm(formula = InfantMortalityPerK ~ DPTpct, data = Data)

Residuals:
   Min     1Q Median     3Q    Max
 -56.8  -16.3   -5.1   11.8   86.6

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  173.277      8.489    20.4   <2e-16 ***
DPTpct        -1.576      0.101   -15.6   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 26.2 on 175 degrees of freedom
Multiple R-squared:  0.582, Adjusted R-squared:  0.58
F-statistic:  244 on 1 and 175 DF,  p-value: <2e-16
```

Var($\hat{\beta}$):

```
> vcov(IMDPT)

            (Intercept)   DPTpct
(Intercept)     72.0677 -0.83317
DPTpct          -0.8332  0.01018
```

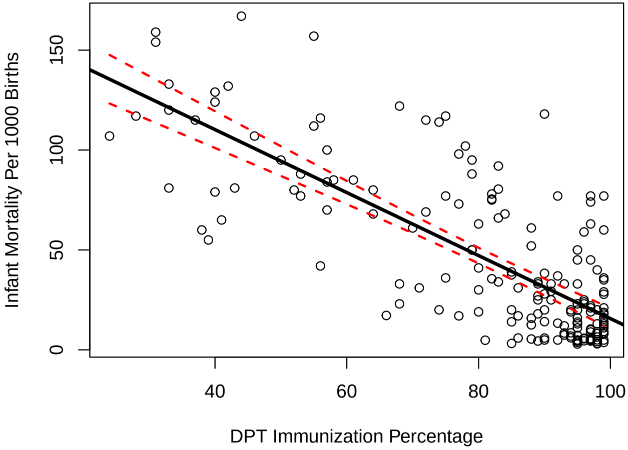
95 percent c.i.s:

```
> confint(IMDPT)

              2.5 %  97.5 %
(Intercept) 156.523 190.032
DPTpct       -1.775  -1.377
```

```
> SEs<-predict(IMDPT,interval="confidence")
> SEs
       fit    lwr    upr
1    25.10  20.53  29.68
2    17.22  12.05  22.40
3    23.53  18.84  28.21
.
.
<rows omitted>
.
.
175  21.95  17.15  26.75
176  39.29  35.36  43.23
177  17.22  12.05  22.40
```

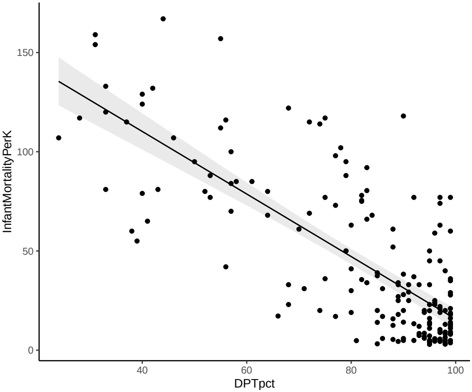
A Plot, With Confidence Intervals

Scatterplot of Infant Mortality and DPT Immunizations, along with
Least-Squares Line and 95% Prediction Confidence Intervals



Same Plot, But Easier

```
> library(marginaleffects)
> plot_predictions(IMDPT,condition="DPTpct",points=1) +
>  theme_classic()
```



General ideas:

- The closeness of the mapping between model-based values of Y and actual values of Y...
- Can be *in-sample* or *out-of-sample* ($\rightarrow$ "overfitting")
- Is (in part) a function of *model specification* (choice of predictors, functional form, interactions, etc.)
- Related (but not identical) to prediction / predictive ability

R^2 Introduced

$$
\begin{aligned}
R^2 &= \frac{\text{MSS}}{\text{TSS}} \\
&= \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} \\
&= 1 - \frac{\text{RSS}}{\text{TSS}} \\
&= 1 - \frac{\sum \hat{u}_i^2}{\sum(Y_i - \bar{Y})^2}
\end{aligned}
$$

R^2 Is...

R-squared:

- is "the proportion of variance explained"
- $\in [0, 1]$
  - $R^2 = 1.0 \equiv$ a "perfect (linear) fit"'
  - $R^2 = 0 \equiv$ no (linear) $X - Y$ association

For a single X,

$$\begin{aligned} R^2 &= \hat{\beta}_1^2 \frac{\sum (X_i - \bar{X})^2}{\sum (Y_i - \bar{Y})^2} \\ &= r_{XY}^2 \end{aligned}$$

Adjusted R^2 is:

$$R^2_{adj.} = 1 - \frac{(1 - R^2)(N - c)}{(N - k)}$$

where $c = 1$ if there is a constant in the model and $c = 0$ otherwise.

$R^2_{adj.}$:

- $R^2_{adj.} \to R^2$ as $N \to \infty$
- $R^2_{adj.}$ can be > 1, or < 0...
- $R^2_{adj.}$ increases with model "fit," but
- The extent of that increase is discounted by a factor proportional to the number of covariates.

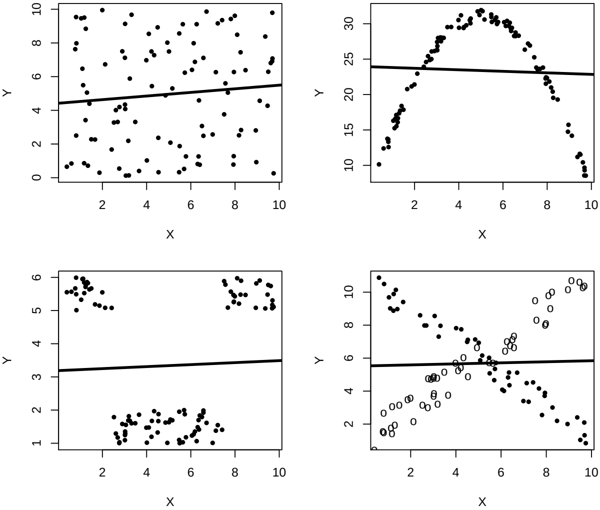
R^2 Alternatives

- Standard Error of the Estimate:

$$\text{SEE} = \sqrt{\frac{\text{RSS}}{N-k}}$$

- F-tests (later...)
- ROC / AUC
- Graphical methods

Caution: Different Ways to get $R^2 \approx 0$



The results:

```
> summary(IMDPT)

Call:
lm(formula = InfantMortalityPerK ~ DPTpct, data = Data)

Residuals:
   Min     1Q Median     3Q    Max 
 -56.8  -16.3   -5.1   11.8   86.6 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  173.277      8.489    20.4   <2e-16 ***
DPTpct        -1.576      0.101   -15.6   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 26.2 on 175 degrees of freedom
Multiple R-squared:  0.582, Adjusted R-squared:  0.58 
F-statistic:  244 on 1 and 175 DF,  p-value: <2e-16
```

A Better Table (using default-y `stargazer`)

Table: OLS Regression Model of Infant Mortality Rates, 2000

	Model I
(Constant)	173.30***
	(8.49)
DPT Immunization Rate	−1.58***
	(0.10)
Observations	177
R^2	0.58
Adjusted R^2	0.58
Residual Std. Error	26.19 (df = 175)
F Statistic	244.10*** (df = 1; 175)
Note:	*p<0.1; **p<0.05; ***p<0.01

Some Guidelines ("Rules"?)

Tables:

- *Use column headings descriptively.*
- *Use multiple rows / columns rather than multiple tables.*
- *Learn about significant digits, and don't report more than 4-5 of them.*
- *Use a figure to replace a table when you can.*
- *Be aware of norms about *s.*

Figures:

- *Report the scale of axes, and label them.*
- *Use as much "space" as you need, but no more.*
- *Use color sparingly.*