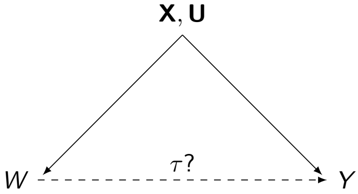
PLSC 504 – Fall 2022
Causal Inference with Observational Data

September 26, 2022

What We're On About



Potential Confounding

Here:

- Y is the outcome of interest,
- W is the primary predictor / covariate ("treatment") of interest,
- T_i is the "treatment indicator" for observation i,
- We're interested in estimating τ, the "treatment effect" of W on Y,
- **X** are <u>observed</u> confounders,
- **U** are <u>unobserved</u> confounders.

Things We Can Do

- **Randomize**

  (or...)

- Instrumental Variables Approaches
- Selection on Observables:
  - Regression / Weighting
  - Matching (propensity scores, multivariate/minimum-distance, genetic, etc.)
- Regression Discontinuity Designs ("RDD")
- Differences-In-Differences ("DiD")*
- Synthetic Controls*
- Others...

* We'll discuss these approaches in a couple weeks, as models for panel/time-series cross-sectional data.

Under Randomization

$$\begin{tikzcd} & \mathbf{U} \arrow[d] & \\ W \arrow[r] & Y & \mathbf{X} \arrow[l] \end{tikzcd}$$

$=$ no confounding!

Note:

- Randomized assignment of W "balances" covariate values – both observed and unobserved – *on average*...
- That is, under randomization of W:

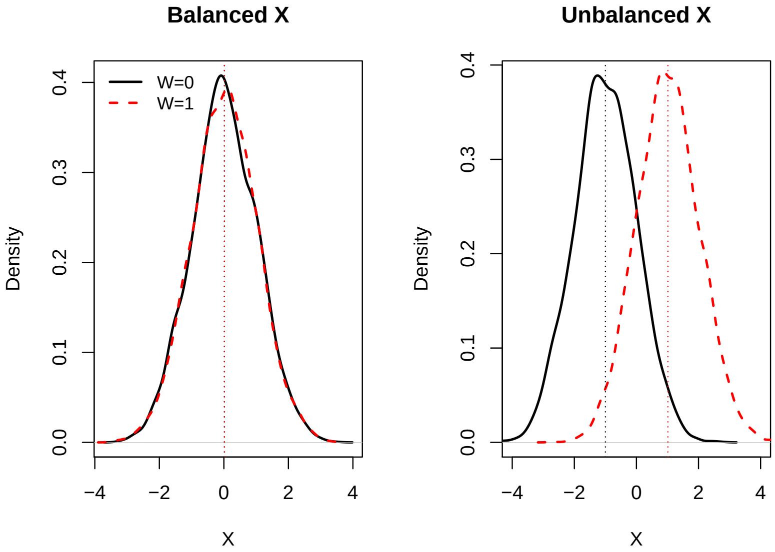
$$\mathrm{E}(\mathbf{X}_i, \mathbf{U}_i \mid W_i = 0) = \mathrm{E}(\mathbf{X}_i, \mathbf{U}_i \mid W_i = 1)$$

  or, more demandingly,

$$\mathrm{E}[f(\mathbf{X}, \mathbf{U}) \mid W_i = 0] = \mathrm{E}[f(\mathbf{X}, \mathbf{U}) \mid W_i = 1]$$

- Can yield imbalance by random chance...

Covariate Balance / Imbalance



Covariate Imbalance Under Randomization

Why seek balance when randomizing?

- More accurate estimates of treatment effects
- Higher statistical power

Possible Approaches:

1. Force balance by design:
   - Stratification / blocking
   - Matching / paired randomization (see below)
   - Rerandomization approaches (e.g., Morgan and Rubin 2012)
2. Post-randomization analysis:
   - Pre- vs. post-treatment Y values / "gain scores"
   - (Post-treatment) stratification by $\mathbf{X}$
   - (Pre-treatment) covariate adjustment via weighting / regression

Nonrandom Assignment of W_i

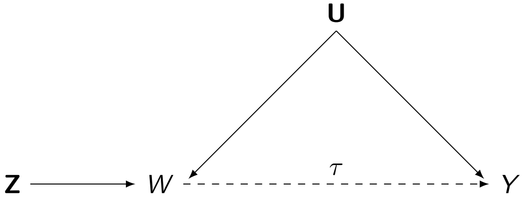
Valid causal inference requires $Y_{0i}, Y_{1i} \perp W_i | \mathbf{X}_i, \mathbf{U}_i$

- That is, treatment assignment W_i is *conditionally ignorable*

"What if I have unmeasured confounders?"

- In general, that's a bad thing.
- One approach: obtain *bounds* on possible values of τ
  - Assume you have one or more unmeasured confounders
  - Undertake one of the methods described below to get $\hat{\tau}$
  - Calculate the range of values for $\hat{\tau}$ that could occur, depending on the degree and direction of confounding bias
  - Or ask: How strong would the effect of the **U**s have to be to make $\hat{\tau} \to 0$?
- Some useful cites:
  - Rosenbaum and Rubin (1983)
  - Rosenbaum (2002)
  - DiPrete and Gangl (2004)
  - Liu et al. (2013)
  - Ding and VanderWeele (2016)

Digression: Instrumental Variables



Instrumental Variables

As in the more general regression case where we have $\text{Cov}(\mathbf{X}, \epsilon) \neq 0$, instrumental variables can be used to address confounding in causal analyses.

Instrumental Variables (continued)

Considerations:

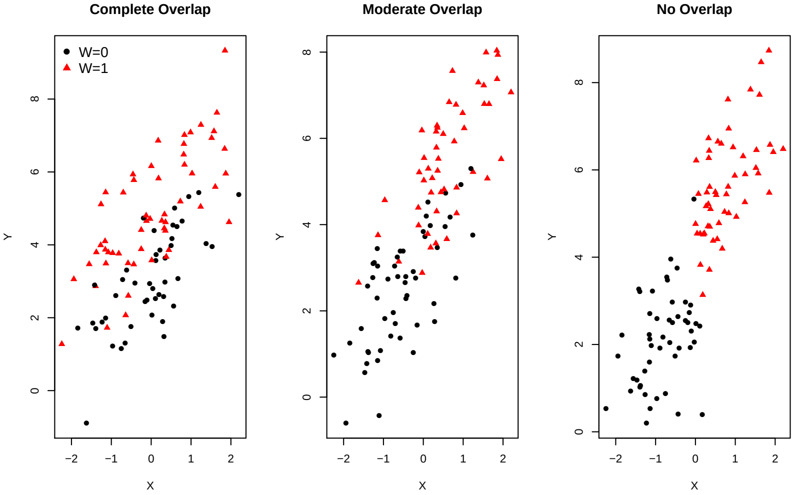
- Requires:
  1. Cov($\mathbf{Z}, W$) $\neq 0$
  2. $\mathbf{Z}$ has no independent effect on Y, except through W
  3. $\mathbf{Z}$ is exogenous [i.e., Cov($\mathbf{Z}, \epsilon$) $= 0$]
- Arguably most useful when treatment compliance is uncertain / driven by unmeasured factors ("intent to treat" analyses)
- Mostly, they're not that useful at all...
  - Bound et al. (1995): Weak instruments are worse than endogeneity bias
  - Young (2021): Inferences in published IV work (in economics) are wrong and terrible
  - Shalizi (2020, chapters 20-21): Gathers all the issues together, sometimes hilariously
- Other useful references:
  - Imbens et al. (1996) (the overly-cited one)
  - Hernan and Robins (2006) (making sense of things)
  - Lousdal (2018) (a good intuitive introduction)

Nonrandom Assignment of W_i (continued)

So...

- Causal inference with observational data typically requires that $\mathbf{U} = \varnothing$...
- This typically requires a strong theoretical motivation in order to assume that the observed $\mathbf{X}$ exhausts the list of possible confounders.

- **Even if** this assumption is reasonable, there are two (related) important concerns:
  - Lack of *covariate balance* (as above)
  - Lack of *overlap* among observations with $W_i = 0$ vs. $W_i = 1$
  - The latter is related to *positivity*, the requirement that each observation's probability of receiving (or not receiving) the treatment is greater than zero


Complete Overlap
Moderate Overlap
No Overlap
W=0
W=1
X
Y
−2
−1
0
1
2
0
2
4
6
8

In general:

- Ensuring overlap allows us to make counterfactual statements from observational data
  - Requires that we have comparable $W_i = 0$ and $W_i = 1$ units
  - It's *necessary* – no overlap means any counterfactual statements are based on assumption
  - Think of this as an aspect of *model identification* (Crump et al. 2009)
  - Most often handled via matching
- Ensuring covariate balance corrects potential bias in $\hat{\tau}$ due to (observed) confounding
  - This can be done a number of different ways: stratification, weighting, regression...
  - Key: Adjusting for (observable) differences across groups defined by values of W
- In general, we usually address overlap first, then balance...

Matching

Matching is a way of dealing with one of both of covariate overlap and (im)balance.

The process, generally:

1. Choose the $\mathbf{X}$ on which the observations will be matched, and the matching procedure;
2. Match the observations with $W_i = 0$ and $W_i = 1$;
3. Check for balance in $\mathbf{X}_i$; and
4. Estimate $\hat{\tau}$ using the matched pairs.

Variants / considerations:

- 1:1 vs. 1:k matching
- "Greedy" vs. "Optimal" matching (see Gu and Rosenbaum 1993)
- Distances, calipers, and "common support"
- Post-matching: Balance checking...

Flavors of Matching

- Simplest: Exact Matching
  - For each of the n observations i with $W = 1$, find a corresponding observation j with $W = 0$ that has identical values of $\mathbf{X}$
  - Calculate $\hat{\tau} = \frac{1}{n} \sum (Y_i - Y_j)$
  - Generally not practical, especially for high-dimensional $\mathbf{X}$
  - Variants: "coarsened" exact matching (e.g., Iacus et al. 2011)
- Multivariate Matching
  - Match each observation i which has $W = 1$ with a corresponding observation j with $W = 0$, and whose values on $\mathbf{X}_j$ are the most similar to $\mathbf{X}_i$
  - One example: Mahalanobis distance matching, based on the distance:

$$d_M(\mathbf{X}_i, \mathbf{X}_j) = \sqrt{(\mathbf{X}_i - \mathbf{X}_j)' \mathbf{S}^{-1} (\mathbf{X}_i - \mathbf{X}_j)}.$$

Flavors of Matching (continued)

- Propensity Score Matching
  - Match observation i which has $W = 1$ with observation j having $W = 0$ based on the closeness of their *propensity score*
  - The propensity score is, $\Pr(W_i = 1|\mathbf{X}_i)$, typically calculated as the predicted value of T_i (the treatment indicator) from a logistic (or other) regression of T on $\mathbf{X}$.
  - The assumptions about matching [that Y is orthogonal to $W|\mathbf{X}$ and that $\Pr(W_i = 1|\mathbf{X}_i) \in (0, 1)$] mean that $Y \perp W| \Pr(T|\mathbf{X})$.
  - In practice: read this...

- Other variants: Genetic matching (Diamond and Sekhon 2013), etc.[1]

[1] Shalizi (2016) notes that "(A)pproximate matching is implicitly doing nonparametric regression by a nearest-neighbor method," and that "(M)aybe it is easier to get doctors and economists to swallow "matching" than "nonparametric nearest neighbor regression"; this is not much of a reason to present the subject as though nonparametric smoothing did not exist, or had nothing to teach us about causal inference."

Matching Software

Interestingly, quite a few of the good matching programs written for R have been written by political scientists...

- the `Match` package (does propensity score, M-distance, and genetic matching, plus balance checking and other diagnostics)
- the `MatchIt` package (for pre-analysis matching; also has nice options for checking balance)
- the `optmatch` package (suite for 1:1 and 1:k matching via propensity scores, M-distance, and optimum balancing)
- `matching` (in the `arm` package)

Regression Discontinuity Designs

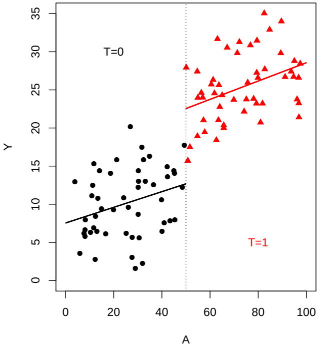
"RDD":

- Treatment changes abruptly [usually at some threshold(s)] according to the value(s) of some measured, continuous, pre-treatment variable(s)
    - This is known as the "assignment" or "forcing variable(s)," sometimes denoted **A**
    - Formally:

$$T_i = \begin{cases} 0 \text{ if } A_i \leq c \\ 1 \text{ if } A_i > c \end{cases}$$

- Intuition: Observations near but on either side of the threshold(s) are highly comparable, and can be used to (locally) identify τ
- This is because variation in T_i near the threshold is effectively random (a "local randomized experiment")
- E.g. Carpenter and Dobkin (2011) (on the relationship between the legal drinking age and public health outcomes like accidental deaths)

- Pluses:
  - Can be estimated straightforwardly, as:
    $$Y_i = \beta_0 + \beta_1 A_i + \tau T_i + \gamma A_i T_i + \epsilon_i$$
  - Generally requires fewer assumptions than IV or DiD (and those assumptions are easier to observe and test)
- Minuses:
  - Provides only an estimate of a local treatment effect
  - Fails if (say) subjects can manipulate A in the vicinity of c
- Lee and Lemieux (2010) is an excellent (if fanboi-ish) review
- R packages: `rddtools`, `rdd`, `rdrobust`, `rdpower`, `rdmulti`



Software Matters

- R
  - Packages for matching are listed above (`Matching`, `MatchIt`, etc.)
  - Similarly for RDD (`rddtools`, `rdd`, etc.)
  - IV regression: `ivreg` (in `AER`), `tsls` (in `sem`), others
  - See generally the *Econometrics* and *SocialSciences* CRAN Task Views
- Stata also has a large suite of routines for attempting causal inference with observational data
- And there's a pretty good NumPy/SciPy-dependent package for Python, called (creatively) Causalinference

Example: Sports and Grades in High School

Question: Does participation in high school varsity sports help or hinder academic achievement (i.e., grades)?

Data: "High School And Beyond" survey (1983 wave) ($N = 1375$)

Variables:

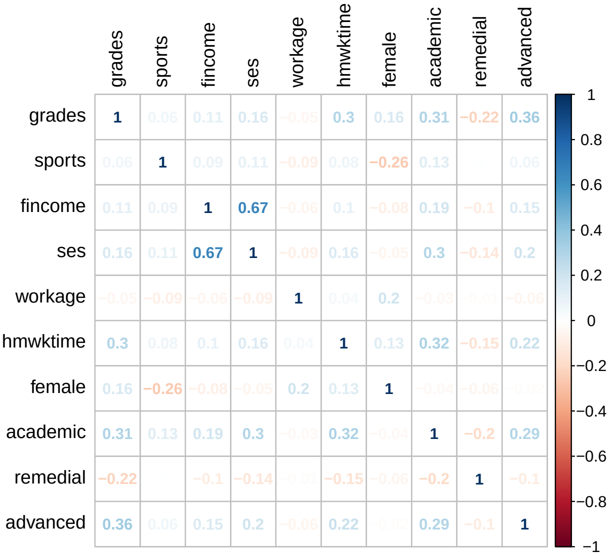
- `grades`: As=4, As & Bs=3.5, etc.
- `sports`: 1 if participated in varsity sports, 0 otherwise
- `fincome`: Family income (7-point scale)
- `ses`: Socioeconomic Status: 1=low, 2-middle, 3=high
- `workage`: Age at which started working
- `hmwktime`: Time spent on homework (7-point scale)*
- `female`: 1 = female student, 0 = male student
- `academic`: 1 if the student is on an academic track, 0 else
- `remedial`: 1 if the student took $\geq$1 remedial course
- `advanced`: 1 if the student took $\geq$1 advanced course

* Likely post-treatment, so we'll omit in the examples below.

Summary Statistics

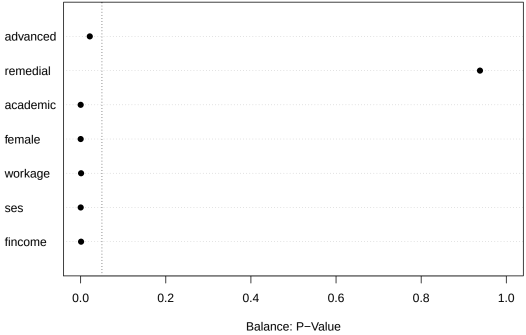
```
> summary(sports)
     grades          sports          fincome           ses       
 Min.   :0.0   Min.   :0.00   Min.   :1.0   Min.   :1.00  
 1st Qu.:2.5   1st Qu.:0.00   1st Qu.:3.0   1st Qu.:1.00  
 Median :3.0   Median :0.00   Median :5.0   Median :2.00  
 Mean   :2.9   Mean   :0.37   Mean   :4.4   Mean   :1.96  
 3rd Qu.:3.5   3rd Qu.:1.00   3rd Qu.:6.0   3rd Qu.:2.00  
 Max.   :4.0   Max.   :1.00   Max.   :7.0   Max.   :3.00  
    workage         hmwktime        female         academic   
 Min.   :11.0   Min.   :1.0   Min.   :0.00   Min.   :0.00  
 1st Qu.:13.0   1st Qu.:4.0   1st Qu.:0.00   1st Qu.:0.00  
 Median :15.0   Median :4.0   Median :1.00   Median :0.00  
 Mean   :14.6   Mean   :4.5   Mean   :0.52   Mean   :0.41  
 3rd Qu.:16.0   3rd Qu.:6.0   3rd Qu.:1.00   3rd Qu.:1.00  
 Max.   :21.0   Max.   :7.0   Max.   :1.00   Max.   :1.00  
    remedial        advanced    
 Min.   :0.00   Min.   :0.00  
 1st Qu.:0.00   1st Qu.:0.00  
 Median :0.00   Median :0.00  
 Mean   :0.36   Mean   :0.37  
 3rd Qu.:1.00   3rd Qu.:1.00  
 Max.   :1.00   Max.   :1.00  
```

Correlation Plot



```
> with(sports, t.test(grades~sports))

Welch Two Sample t-test

data:  grades by sports
t = -2, df = 1064, p-value = 0.02
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.183 -0.014
sample estimates:
mean in group 0 mean in group 1
            2.9             3.0


> summary(lm(Model,data=sports))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.71145    0.13397   20.24  < 2e-16 ***
sports       0.10119    0.03969    2.55    0.011 *
fincome      0.00435    0.01378    0.32    0.753
ses          0.02216    0.03487    0.64    0.525
workage     -0.01879    0.00794   -2.37    0.018 *
female       0.30062    0.03881    7.75  1.8e-14 ***
academic     0.29063    0.04099    7.09  2.1e-12 ***
remedial    -0.23215    0.03919   -5.92  4.0e-09 ***
advanced     0.44435    0.04004   11.10  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.68 on 1366 degrees of freedom
Multiple R-squared:  0.231,Adjusted R-squared:  0.226
F-statistic: 51.2 on 8 and 1366 DF,  p-value: <2e-16
```

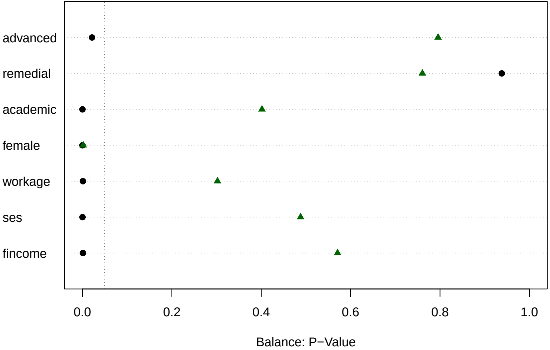
Balance Tests (Pre-Matching)



These are P-values associated with t-tests (for binary predictors) or Kolmogorov-Smirnov tests (for continuous predictors) for balance between `sports` $= 0$ and `sports` $= 1$.

```
> M.exact <- matchit(sports~fincome+ses+workage+female+academic+
+                    remedial+advanced,data=sports,method="exact")
> M.exact

Call:
matchit(formula = sports ~ fincome + ses + workage + female +
    academic + remedial + advanced, data = sports, method = "exact")

Exact Subclasses: 166

Sample sizes:
          Control Treated
All           864     511
Matched       287     239
Unmatched     577     272

> # Output matched data:

> sports.exact <- match.data(M.exact,group="all")

> dim(sports.exact)
[1] 526  12
```

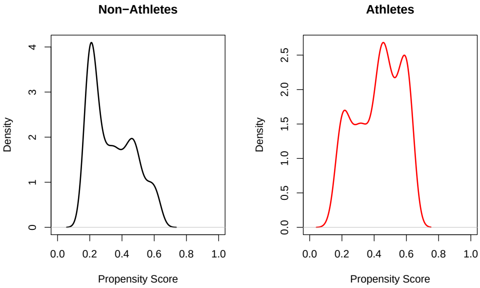
Exact Matching: Balance



These are P-values associated with t-tests (for binary predictors) or Kolmogorov-Smirnov tests (for continuous predictors) for balance between $\texttt{sports} = 0$ and $\texttt{sports} = 1$. Black dots are pre-matching; green triangles are after exact matching.

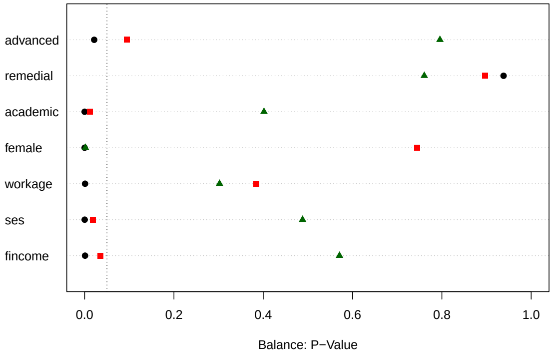
Propensity Score Matching

```
> PSfit <- glm(sports~fincome+ses+workage+female+academic+remedial+
+               advanced,data=sports,family=binomial(link="logit"))

> # Generate scores & check common support:

> PS.df <- data.frame(PS = predict(PSfit,type="response"),
+                     sports = PSfit$model$sports)
```



```
> M.prop<-matchit(sports~fincome+ses+workage+female+academic+
+                      remedial+advanced,data=sports,
+                      method="nearest")
> summary(M.prop)
.
.
.
Percent Balance Improvement:
         Mean Diff. eQQ Med eQQ Mean eQQ Max
distance         80      83       80      63
fincome          29       0       30       0
ses              34       0       35       0
workage          71       0       68      25
female           96       0       96       0
academic         41       0       41       0
remedial        -88       0     -100       0
advanced         19       0       19       0

Sample sizes:
          Control Treated
All           864     511
Matched       511     511
Unmatched     353       0
Discarded       0       0
```

Propensity Score Matching: Balance



These are P-values associated with t-tests (for binary predictors) or Kolmogorov-Smirnov tests (for continuous predictors) for balance between `sports` $= 0$ and `sports` $= 1$. Black dots are pre-matching; green triangles are after exact matching; red squares are after propensity score matching.

```
> with(sports, t.test(grades~sports))$statistic # No matching
     t
-2.286

> with(sports.exact, t.test(grades~sports))$statistic # Exact
     t
-1.395

> with(sports.prop, t.test(grades~sports,paired=TRUE))$statistic # PS
    t
-2.98

> with(sports.genetic, t.test(grades~sports))$statistic # Genetic
     t
-1.367
```

Regression Results

	No Matching	Exact	Propensity Score	Genetic
(Intercept)	2.71*	3.05*	2.84*	2.75*
	(0.13)	(0.23)	(0.16)	(0.17)
sports	0.10*	0.12*	0.09*	0.08
	(0.04)	(0.06)	(0.04)	(0.05)
fincome	0.00	0.05	−0.00	0.01
	(0.01)	(0.03)	(0.02)	(0.02)
ses	0.02	−0.14	0.05	0.03
	(0.03)	(0.07)	(0.04)	(0.05)
workage	−0.02*	−0.03*	−0.03*	−0.02*
	(0.01)	(0.01)	(0.01)	(0.01)
female	0.30*	0.34*	0.31*	0.29*
	(0.04)	(0.06)	(0.05)	(0.05)
academic	0.29*	0.24*	0.31*	0.31*
	(0.04)	(0.08)	(0.05)	(0.05)
remedial	−0.23*	−0.28*	−0.28*	−0.21*
	(0.04)	(0.06)	(0.05)	(0.05)
advanced	0.44*	0.51*	0.43*	0.40*
	(0.04)	(0.08)	(0.05)	(0.05)
R^2	0.23	0.29	0.26	0.22
Adj. R^2	0.23	0.28	0.25	0.21
N	1375	526	1022	939

$^*p < 0.05$

Some Questions...

- What – if anything – can the general robustness of our results tell us about the relationship between varsity athletics and grades?
- What can they tell us about our model?
- What mechanism(s) / circumstances might allow us to investigate the relationship between varsity athletic participation and grades using an RDD?
- What circumstances – if any – might allow us to investigate this relationship using instrumental variables?
- What sort(s) of experiments – natural or otherwise – might allow us to investigate this same relationship?

Resources

- Good references:
  - Freedman (2012)*
  - Shalizi (someday)*
  - Morgan and Winship (2014)
  - Pearl et al. (2016)
  - Peters et al. (2017)
- Courses / syllabi (a sampling):
  - Eggers (2019)
  - Frey (2019)
  - Hidalgo (2020)
  - Imai (2021)
  - Simpson (2019)
  - Xu (2018)
  - Yamamoto (2018)
- Other useful things:
  - The Causal Inference Book
  - Some useful notes

* I really like this one.