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*This article examines two alternative specifications for estimating event-count models in which the data-generating process results in a larger number of zero counts than would be expected under standard distributional assumptions. The author compares King's hurdle event count model and Greene's zero-inflated Poisson model, using data on congressional responses to Supreme Court decisions from 1979 to 1988. The author shows that each of these models is a special case of a more general dual regime data-generating process that results in extra-Poisson zero counts. Furthermore, because this data-generating process can produce overdispersion in its own right, these models are also shown to be related to variance function negative binomial specifications. The underlying correspondence between these models leads to similar results in estimating and interpreting them in practice.*

## An Analytic and Empirical Examination of Zero-Inflated and Hurdle Poisson Specifications

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**A**s social scientists have become increasingly aware of the importance of distributional assumptions in empirical analysis, they have increasingly adopted models that better reflect those assumptions. One place where this has been most apparent is in the use of event-count models in the social sciences. Long a staple in the natural sciences, models appropriate to event counts have recently come to find a place in the social scientist's tool kit. A wide range of applications of these models can be found in various areas of sociology (e.g., Hannan and Carroll 1992; Nagin and Land 1993), political science (e.g., Nixon 1991; Krause 1994), and economics (e.g., Hausman, Hall, and Griliches 1984). As the use of these models and their

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variations becomes more prevalent, a more complete understanding of their potential benefits and drawbacks becomes more important.

This article undertakes a comparison of two alternative models for use with event-count data. In practice, many social science event counts are characterized by a greater number of zero counts than would be expected under the Poisson or its variations. I explore some potential reasons for the presence of excess zeros in social science data, focusing on the case in which the data-generating process may be separated into two distinct phases: the first in which the count moves from zero to some discrete event-count distribution, and the second that generates the observed count. In so doing, I compare two alternative means for dealing with such mechanisms: the hurdle Poisson regression suggested by Mullahy (1986) and King (1989a) and the zero-inflated Poisson (ZIP) regression of Lambert (1992) and Greene (1994). These models are shown to be variants of a more general dual regime data-generating process. Furthermore, this process is itself shown to result in the appearance of under- or, more commonly, overdispersion, suggesting a link to variance function negative binomial models in which the dispersion parameter is allowed to vary as a function of independent variables.

I illustrate the general model and evaluate these alternative approaches to its estimation using data on the number of congressional responses to Supreme Court decisions of the Warren and Burger eras. In the example presented, the commonalities across the three models are shown to lead to similar results in practice, in terms of coefficient estimates, standard errors, and predicted values of both zeros and counts. The article concludes with suggestions for distinguishing among the different models in applied settings, as well as a discussion of some alternative applications and extensions of the models themselves.

#### *A GENERAL DUAL REGIME MODEL FOR EVENT-COUNT DATA*

Event-count data are precisely that; data composed of counts of the number of events occurring within a specific observation period.<sup>1</sup> These data take the form of nonnegative integers; for example, the

number of dissents per case on the U.S. Supreme Court (Nixon 1991). Under a few minimal assumptions, event counts can be shown to follow a Poisson distribution (see, e.g., King 1989b). This distribution is characterized by a mean level, usually denoted  $\lambda$ , which represents the unobserved expected rate of occurrence of events during the observation period. The probability density function (pdf) of the Poisson distribution is written

$$Prob(Y_i = y) = \frac{e^{-\lambda_i} \lambda_i^y}{y!}, \quad (1)$$

where  $y = 0, 1, 2, 3, \dots$  and  $\lambda_i > 0$ . Introduction of a vector of covariates  $X_i$  is typically accomplished by making them an exponential function of the Poisson parameter:

$$\lambda_i = \exp(X_i' \beta).$$

This specification ensures the nonnegativity of the endogenous variable.

Because event counts are bounded by zero, they are typically heteroscedastic; in particular, their variance increases with their expected value. In the case of a Poisson-distributed variable  $Y$ , this fact is seen in the restriction that the mean and variance of that variable are equal; that is,

$$E(Y) = \text{Var}(Y) = \lambda.$$

More generally, event counts where the variance exceeds the mean are commonly referred to as being *overdispersed*; where the mean is greater than the variance, the data are said to be *underdispersed*. The restrictive nature of the mean-variance equality assumption inherent to the Poisson has spawned the development of an avalanche of techniques both for detecting (e.g., Cameron and Trivedi 1990; Dean and Lawless 1989; Gurmu 1991; Lee 1986) and modeling (e.g., Cameron and Trivedi 1986; Hausman et al. 1984; King 1989b, 1989c; King and Signorino 1995) over- and (to a lesser extent) underdispersion.

In this article, I examine two variations on the Poisson regression model. Both are motivated by what I will refer to as a dual regime data-generating process.<sup>2</sup> The general form of the model presented here analyzes observations that undergo two separate stages in the determination of the final, observed event count. The first *transition* stage occurs when the observation moves from a state in which the event of interest does not (typically, cannot) occur to one in which events occur at some rate  $\lambda_i$ . The observed dependent variable  $Y_i$  thus takes on a value of zero for all observations that do not make the transition. The probability of observation  $i$  making such a transition is denoted  $p_i$ , and  $(1 - p_i)$ , or the probability that the observation will not make the transition to the other-than-zero state, is denoted  $q_i$ . If the observation does move into the state where events may occur (the *events* stage), the number of such events observed then assumes some variant of the Poisson distribution.<sup>3</sup>

A characteristic of this two-stage process is that we only observe the final count, which is only partially informative about the first stage transition probability. Following Greene (1994), I therefore write this general dual regime model as one of partial observability (e.g., Abowd and Farber 1982):

$$Y_i = p_i^* Y_i^*,$$

where  $p_i^*$  is a binary (0/1) realization of  $p_i$  and  $Y_i^*$  is distributed as Poisson with mean  $\lambda_i$ . In those instances where the probability of a transition is believed to vary according to a different set of independent variables than those that influence the subsequent event count, I denote that set of variables and their respective coefficient vector as  $Z_i'\gamma$  (I address the function relating  $p_i$  and  $Z_i$  below). Assuming  $Y_i^*$  and  $p_i^*$  are independent, we may write the probability of obtaining a zero count as:

$$\begin{aligned} \text{Prob}(Y_i = 0) &= \text{Prob}(p_i^* = 0) + \text{Prob}(p_i^* = 1, Y_i^* = 0) \\ &= q_i + p_i \text{Prob}(Y_i^* = 0) \\ &= q_i + p_i \exp(-\lambda_i). \end{aligned} \tag{2}$$

Similarly, the probability of obtaining a specific nonzero count  $y$  is

$$\begin{aligned}
 \text{Prob}(Y_i = y) &= \text{Prob}(p_i^* = 1) (\text{Prob}(Y_i^* = y)) \\
 &= p_i \frac{e^{-\lambda_i} \lambda_i^y}{y!}, y \in \{1, 2, 3, \dots\}.
 \end{aligned}
 \tag{3}$$

A number of substantive questions may motivate employment of a model such as the one examined here. Greene, for example, envisions the following situation:

Consider the response to the question how many trips have you taken to a certain sport fishing site? The answer to this question comes at two levels. There are individuals who would not visit a sport fishing site almost regardless of circumstances because this activity does not interest them, whereas there are others for whom the number of visits might follow more conventional patterns amenable to Poisson or negative binomial regression. (Greene 1994:12)

This general model thus encompasses a number of different possible variants, several of which have been discussed in the econometrics literature. In general, they may be classified according to two determining characteristics: the probability distribution assumed for  $p_i$  (or  $q_i$ ) and whether the second-stage distribution allows for zero counts; that is, is or is not truncated at zero. A characteristic of all such models, however, is that the number of observations having a value of zero (i.e., no events experienced) on the dependent variable will generally exceed that which would be expected under the Poisson.

#### *DUAL REGIME EVENT-COUNT MODELS: ZERO-INFLATED AND HURDLE POISSON SPECIFICATIONS*

##### *HURDLE POISSON MODELS OF EVENT COUNTS*

In his discussion of dual regime event-count models, Mullahy (1986) outlines what he calls a hurdle Poisson model. He states that

the idea underlying the hurdle formulations is that a binomial probability model governs the binary outcome of whether a count variate has

zero or a positive realization. If the realization is positive, the "hurdle" is crossed, and the conditional distribution of the positives is governed by a truncated-at-zero count data model. (P. 345)

Mullahy limits his discussion to the case in which both  $p_i^*$  and  $Y_i^*$  are Poisson variates, the latter truncated at zero (Johnson, Kotz, and Kemp 1992):<sup>4</sup>

$$Prob(Y_i = y | y > 0) = \frac{\lambda_i^y}{(\exp(\lambda_i) - 1) y!}, y \in \{1, 2, 3, \dots\}. \quad (4)$$

This is equivalent to splitting the distribution for the count and allowing  $\lambda$  to take on different values for the change from zero to one event than for other possible values. Mullahy then uses this model to examine survey data on beverage consumption.

King's (1989a) hurdle Poisson model closely mirrors Mullahy's in that it too assumes that the binomial hurdle probability is a Poisson variate, and adopts a truncated event-count distribution in the latter stage. King originates his model by assuming separate processes for zero and nonzero counts. The rates of occurrence for these two events (denoted  $\lambda_0$  and  $\lambda_+$ , respectively) are then specified as exponential functions of (potentially different) vectors of explanatory variables  $X_{0i}$  and  $X_{+i}$ . The Poisson assumption for the rate of transition  $\lambda_0$  means that the cumulative distribution function of the transition probability  $p_i$  is

$$p_i = 1 - \exp[-\exp(X_{0i}'\beta_0)]. \quad (5)$$

As King notes, this function is asymmetrical, meaning that the probability that the hurdle will be crossed as  $X_{0i}'\beta_0$  increases will approach one more quickly than it will go to zero as  $X_{0i}'\beta_0$  falls.

King's (1989a) rationales for these model properties are motivated by the characteristics of the problem he addresses: the impact of international alliances on the incidence and spread of war. He notes:

The substantive hypothesis is that the percent of alliances has a small or even negative effect on the probability of any additional nations

being involved in wars. Once the first additional nation commits to an international conflict, however, the existence of additional international alliances will drag new nations into the fray. (P. 131)

King (1989a) argues that, in the case of international conflict he examines, the asymmetry in the first-stage event probability "is quite plausible . . . since in the hurdle model the probability of crossing the hurdle might arbitrarily approach 1.0 when the expected number of nations initiating the conflict ( $\lambda_{+i}$ ) is high" (p. 134).

Both Mullahy's and King's hurdle models are a special case of the more general dual regime model presented above. As noted, the hurdle model assumes that the probability of making the transition from the zero state follows an asymmetric distribution, and the subsequent event count is truncated at zero, implying that some event (or events) is inevitable once the hurdle is crossed. The potential importance of these assumptions, versus their possible alternatives, may be considered more fully when analyzed in comparison to an alternative model with applicability to similar processes.

#### *THE ZERO-INFLATED DUAL REGIME POISSON REGRESSION MODEL*

A second model of this general form is Lambert's (1992) zero-inflated Poisson model. Lambert examines defects in manufacturing, noting that "when manufacturing equipment is properly aligned, defects may be nearly impossible. But when it is misaligned, defects may occur according to a Poisson distribution." She goes on to state that "one interpretation is that slight, unobserved changes in the environment cause the process to move randomly back and forth between a perfect state in which defects are extremely rare and an imperfect state in which defects are possible but not inevitable" (1992:1).<sup>5</sup> Because her focus is on manufacturing defects, Lambert is interested in the probability of remaining in the "perfect state" ( $q_i$ ), which she parameterizes as a logit. She examines two circumstances: one in which the probability of remaining in the perfect state is a simple multiplicative function of the variables that explain the event count



$$q_i = \frac{\exp(\tau X_i' \beta)}{1 + \exp(\tau X_i' \beta)} = \frac{\lambda^\tau}{1 + \lambda^\tau}, \quad (6)$$

and one in which the probability of remaining in the perfect state is allowed to vary independently of the variables that affect the event count

$$q_i = \frac{\exp(Z_i' \gamma)}{1 + \exp(Z_i' \gamma)}. \quad (7)$$

The former she labels the ZIP( $\tau$ ) model, and the latter the ZIP regression model. In addition, because defects are not a certainty in the imperfect state, the second-stage event-count model is not truncated at zero. Greene (1994) essentially adopts Lambert's model specification in his analysis of credit-reporting data and extends his discussion to both a probit specification of  $q_i$  and a negative binomial specification for the second-stage count process.

As a special case of the general model, the ZIP regression makes substantially different assumptions about the nature of the data-generating process than the hurdle model. Whether parameterized as a logit or a probit, the probability exiting the zero-only stage is assumed to follow a symmetric cumulative distribution. Likewise, even those cases that make the transition to the events stage may nevertheless have zero counts; crossing the hurdle in the ZIP model does not guarantee a positive realization of  $Y$ . Whether these differences in assumptions are important in practice is addressed below.

#### *EXCESS ZEROS, OVERDISPERSION, AND UNDERDISPERSION IN EVENT-COUNT DATA<sup>6</sup>*

As noted above, event counts in which the variance is greater than the mean are said to be overdispersed, whereas those with variance less than the mean are underdispersed. A commonly used alternative

for model estimation in the presence of overdispersion is to fit a negative binomial model to the data (Johnson, Kotz, and Kemp 1992):

$$Prob(Y_i = y) = \left( \frac{\Gamma(y + 1/\alpha)}{\Gamma(1/\alpha)y!} \right) \left( \frac{1/\alpha}{1/\alpha + \lambda_i} \right)^{1/\alpha} \left( \frac{\lambda_i}{1/\alpha + \lambda_i} \right)^y. \quad (8)$$

One interpretation of the negative binomial is as a mixture of Poisson variates with different mean parameters  $\lambda$  conditionally distributed according to a gamma distribution with unit mean and variance  $\alpha$  (see, e.g., Land, McCall, and Nagin 1996). Under this specification, the ratio of the variance to the expected value of  $Y$  varies as a linear function of the mean:

$$\frac{Var(Y_i)}{E(Y_i)} = 1 + \alpha\lambda_i. \quad (9)$$

The negative binomial model this reduces to the Poisson when  $\alpha = 0$ . Because the Poisson model is “nested” within the negative binomial,  $t$ -tests for  $\alpha = 0$ , or a likelihood ratio test, can be used to test for the presence of significant amounts of overdispersion. Generalization of this approach allows the variance parameter  $\alpha$  to be modeled as a (typically exponential) function of some set of explanatory variables as well (see, e.g., King 1989a).

A number of factors may result in over- or underdispersion in event-count data, including unobserved heterogeneity and contagion across events (see King 1989c for a fuller explication). The issues of positive and negative contagion are of unique interest here. Generally speaking, when contagion is present, events occurring within a single observation or period are nonindependent. The form of that nonindependence determines whether the contagion is of the positive or negative variety.

In the case of positive contagion,<sup>7</sup> the conditional probability of an additional event occurring in the period under observation is positively related to previous events during that period. That is, the occurrence of one event increases the probability of observing another

event during the same period. In this way, high values of the count variable are "pushed" even higher due to the positive dependence of individual events. Conversely, the absence of the occurrence of events in a positively contagious process will result in inertia, so that low values (particularly zeros) on these variables will tend to remain zeros. As a result, positive contagion leads to count data that are overdispersed; the presence of exceptionally high and low counts causes the variance to be large relative to the mean.

In contrast, negative contagion corresponds to underdispersed counts. Under negative contagion, the occurrence of one event during the period under observation decreases the probability of observing additional events during that same period. King (1989c), for example, considers the number of congressional attempts to override presidential vetoes per year. He notes that because such override attempts require the expenditure of substantial amounts of political capital, one such attempt in a session may reduce the probability that another will occur in the same session. This negative contagion suggests in turn that such annual counts will be underdispersed. Under conditions of negative contagion, observed counts will exhibit variance less than their mean levels because the effect of negative contagion is to restrict the extent of variation in the observed counts.

The importance of contagion and negative contagion and their effect on the dispersion of the data in the context of the dual regime event-count models described above is apparent. Given that these data-generating processes directly influence the occurrence of a particular count value (i.e., zero), they have important implications for the observed level of dispersion in the data. In general, a greater number of zeros than would otherwise be expected under the assumption of Poisson-distributed data will resemble positive contagion, and thus appear as overdispersion in the data. Conversely, observing fewer zeros than expected will appear to be the result of negative contagion, and thus be manifested as apparent underdispersion. As a result, data generated by a dual regime process will often appear to be over- or underdispersed, when in fact conditional on the transition stage they are neither. I will refer to over- or underdispersion caused by greater- or less-than-Poisson numbers of zero counts as being zero driven, to distinguish it from variation in the dispersion of the data due to true contagion, heterogeneity, or some other cause.

The presence and extent of under- or overdispersion that will be detected under these circumstances is directly related to one of the two signature assumptions of the models illustrated here. Specifically, dual regime models that do not assume that the events stage distribution is truncated at zero will *always* have a greater-than-Poisson number of zero counts, and thus will always appear to be, to some extent, overdispersed. In contrast, because the presence of zeros in the truncated models is entirely a function of the first (hurdle) stage, the number of zeros may be greater than, equal to, or even less than the expected quantity under Poisson assumptions, and thus may result in either overdispersion, underdispersion, or neither. This suggests that the assumptions one makes about the nature of the data-generating process can have important implications for the detection of over- and underdispersion in the observed count.

To see how this is the case, consider first a dual regime event-count model in which the second stage is not truncated at zero. Under these circumstances, zero counts may occur either as a result of a failure to move to the second stage or as actual zero observations following such a transition. Using equations (2) and (3), and assuming an *untruncated* Poisson model in the events stage, we may write the unconditional expected value of  $Y_i$  as

$$\begin{aligned} E(Y_i) &= E(p_i^*)E(Y_i^*) \\ &= p_i \lambda_i \end{aligned} \tag{10}$$

and the unconditional variance of  $Y_i$  as (see Greene 1994:15):

$$\begin{aligned} \text{Var}(Y_i) &= p_i \lambda_i + q_i (0 - p_i \lambda_i)^2 + p_i (\lambda_i - p_i \lambda_i)^2 \\ &= p_i \lambda_i + p_i \lambda_i^2 + p_i^2 \lambda_i^2 \\ &= p_i \lambda_i (1 + q_i \lambda_i). \end{aligned} \tag{11}$$

The mean-variance equality assumption of the Poisson is satisfied, and the standard Poisson model emerges, when  $q_i = 0$ . For nonzero  $q_i$ , note that by equation (10)

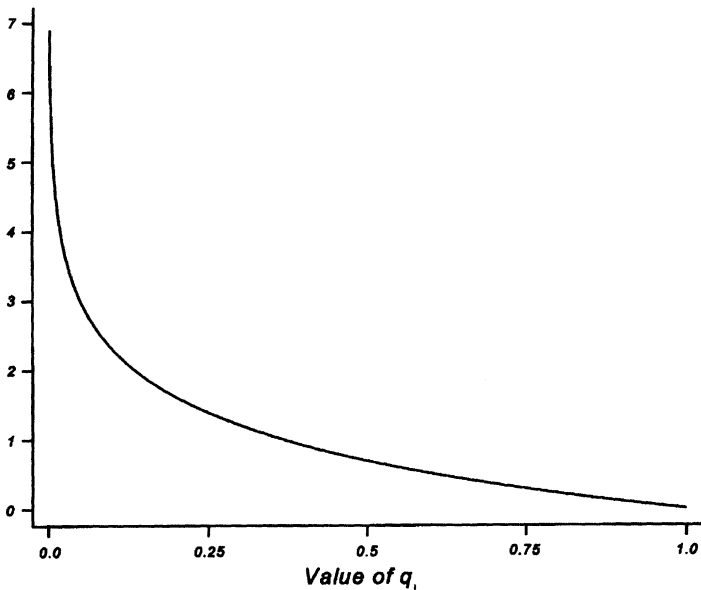
$$\begin{aligned}\frac{\text{Var}(Y_i)}{E(Y_i)} &= 1 + q_i \lambda_i \\ &= 1 + \frac{q_i}{p_i} E(Y_i).\end{aligned}\tag{12}$$

This shows how the dual regime nature of the model generates apparent overdispersion. In this context,  $q_i/p_i$  is analogous to the overdispersion parameter  $\alpha$  in equation (9);  $q_i = 0$  corresponds to  $\alpha = 0$ , and the extent of overdispersion increases as the probability of no transition to the event stage (and the subsequent number of zero counts) increases.<sup>8</sup>

These results thus show that analyses of data generated by nontruncated dual regime processes that fail to account for that process will tend to detect overdispersion when, conditional on the transition stage, none is present. Now consider the case in which the data-generating process in the events stage is truncated in nature, so that no zero counts are observed once the hurdle is crossed. Because the probability of a zero count, conditional on  $p_i^* = 1$ , is zero, equation (2) reduces to

$$\text{Prob}(Y_i = 0) = q_i.\tag{13}$$

The unconditional expected value and variance are as in equations (10) and (11), but with  $\lambda_i$  indicating the mean of a truncated-at-zero Poisson process. Under these circumstances, zero-driven over- or underdispersion will be observed when the probability of a zero count is other than what it would be under standard Poisson assumptions. That is, if the probability of crossing the hurdle ( $p_i$ ) is sufficiently small, there will be a large number of zeros in the data, and examinations of the data will tend to indicate overdispersion. In contrast, if the probability of transition into the count model regime is relatively large, fewer zero counts will be observed than would be expected under Poisson assumptions, and the data will appear to be underdispersed. Recalling that the probability of a zero count in a nontruncated Poisson is  $\exp(-\lambda)$ , no zero-driven over- or underdispersion will be apparent when  $q = \exp(-\lambda)$  (or, alternatively, when  $\lambda = -\ln(q)$ ), where



**Figure 1: Mean-Variance Equality in the Truncated Dual Regime Model**

NOTE: Graph is relationship between  $q_i$  and  $\lambda_i$  in the dual regime model with truncation. Parameter values on the line meet mean-variance equality restrictions.

$\lambda$  is the rate of occurrence in an untruncated Poisson distribution. The general form of this relationship is illustrated in Figure 1.

In Figure 1, the line represents the relationship between the probability of an observation remaining in the zero-count state and the expected value of the event count ( $\lambda$ ) in a model where the second-stage event count is truncated at zero. The  $x$ -axis represents the probability of remaining in the zero-only state. The  $y$ -axis is the value of  $\lambda$  for that data, ignoring the effects of the transition stage. Parameter values that fall along the line will appear to be Poisson-distributed in their number of zeros. That is, one will observe a number of zero counts equal to that which would be observed if the data were Poisson distributed; no zero-driven over- or underdispersion will be apparent. For example, in a model where no zeros are observed once the hurdle is crossed, if the probability of exiting the zero stage is 0.5, the number

of zero counts we would observe would be equal to that of an untruncated Poisson distribution with  $\lambda = -\ln(0.5) = 0.693$ .

In the case where data generated by such a process are examined under traditional Poisson assumptions, under- or overdispersion will be detected when the relationship between  $q$  and  $\lambda$  does not fall on the line. Specifically, data in which the values of  $q$  and  $\lambda$  lie below this line will exhibit underdispersion, as the number of zeros observed will be less than would be expected in a Poisson distribution with that expected value. Conversely, data with values of  $q$  and  $\lambda$  lying above the line will appear to be overdispersed and will have a greater number of zero counts than a Poisson distribution with equal mean.

Because covariates are typically included in one or both parts of the model, the actual relationship presented in Figure 1 is not always a simple one. It is unlikely that all of the observations in a given sample will fall on the line in Figure 1, nor is it likely that "Poisson-correct" numbers of zero counts will be observed. King's hurdle model, by assuming a Poisson distribution for  $1 - q_i$  (his  $\lambda_{0i}$ ), allows for this to occur when  $\lambda_0 = \lambda_+$  and  $\beta_0 = \beta_+$  (i.e., when explanatory variables and their coefficients are identical for both stages of the model). In that case, the hurdle Poisson specification reduces to the basic Poisson formulation. Other models that assume different distributions for  $p_i$  or  $q_i$  require different conditions for this to occur.

In sum, there is a distinct relationship between excess zero counts due to a dual regime data-generating process and under- and overdispersion in that data. This relationship is also seen in the commonalities between the types of models commonly used for estimating dual regime processes and techniques for including and parameterizing overdispersion in particular.<sup>9</sup> In the following sections, I examine these similarities using political science data, attempting to answer the question of whether and to what extent those similarities translate into equivalent results in practice.

#### *CONGRESS AND THE SUPREME COURT: AN EMPIRICAL APPLICATION*

Studies of Supreme Court reversals of congressional acts are counted among the classics of the study of American politics (e.g.,

Dahl 1957). Significantly less work has examined the inverse of this relationship: the extent to which Congress responds to the decisions of the Court. Previous analyses of Congress-Court interaction suggest that, as a general rule, Congress pays little attention to the decisions of the Court and that congressional attempts to modify or overturn the Court's decisions are relatively rare (see, e.g., Henschen 1983; Eskridge 1991). Whether due to institutional deference, agreement with case outcomes, or simple inattention, the typical Supreme Court decision is final: Congress rarely intervenes to modify or overturn the Court's ruling. As a result, the vast majority of Supreme Court cases are never addressed by Congress.

One possible explanation for the dearth of congressional attention to the decisions of the Court has its roots in the informational role of groups in the Congress. Drawing upon examinations of the importance of organized interests and lobbyists in providing members of Congress with specialized information (e.g., Hansen 1991), it has been suggested that, in the absence of pressure to do so, Congress will not address *any* Court decisions, and only in those cases where the affected parties bring the case to the attention of the members of Congress will any response be forthcoming. This characterization of interest group activity in this area suggests that the number of congressional responses to a decision of the Supreme Court during a given period depends on two critical factors: first, the probability that a group or groups will lobby Congress for redress, and second, conditional on that pressure, the number of actions taken by members of Congress in response to it. The dual nature of the process, and the resulting distribution of the count of such responses, makes this question ideal for an examination of the properties of the models discussed above.

For clarity of exposition, I will focus primarily on the influence of a single variable in the analyses done here. I choose to take an especially close look at the effect of a declaration of unconstitutionality on the incidence of congressional responses to Court decisions. Recent work in the positive political theory of institutions has modeled the decision of the Court to decide a case on constitutional or statutory grounds as endogenously determined by the positions of other actors in the political environment. Spiller and Spitzer's (1992) model of Supreme Court review of agency action, for example, allows the Court



to choose to review an agency rule on either constitutional or statutory grounds. Because the former are more restrictive (in the sense that they require a supermajority to reverse the decision via a constitutional amendment), it is more difficult for Congress to successfully challenge such decisions. Thus, in the presence of any costs to proposing legislation that does not pass, their model suggests that constitutional decisions will, *ceteris paribus*, have fewer challenges brought against them than nonconstitutional ones. In the analyses below, I give special attention to the effect of this variable, both for the substantive reasons given here and as an illustration of variable effects in each of the various models.

Several other characteristics, both relating to the cases themselves and to the political environment in which they are addressed, may exert influence on the probability of a response and the number of such responses. Here I focus on those factors peculiar to a particular Supreme Court decision and their influence at the various stages of the model. For example, because older decisions are seen as less relevant than more recent ones, it is generally believed that the age of the decision will be inversely related to the probability of its being addressed by Congress.<sup>10</sup> In addition, a number of characteristics of the decision may serve as cues for interest groups to target them in the legislature, including the presence of lower court conflict, the alteration of previously established precedent, and the unanimity of the decision. In general, both lower court disagreement and an alteration of precedent may be seen as giving a decision less institutional validity, thus increasing the opportunity for a successful challenge in the legislature. Unanimity has the opposite effect: Because of the weight given to unanimous decisions, I expect that they will be less likely to be challenged in Congress.

Finally, the political preferences of the actors in question undoubtedly have an impact on the actions of the legislature. Although a full elaboration of a model of institutional political preferences is beyond the scope of the present article, I do control for ideological effects in a rudimentary way: Inclusion of a dummy variable indicating a liberal Supreme Court decision will, to a limited extent, address issues of political preferences between Congress and the Court.<sup>11</sup> Table 1 presents summary statistics for the variables used in this analysis.

**TABLE 1: Descriptive Statistics for Dependent and Independent Variables**

<i>Variable</i>	<i>Mean</i>	<i>Standard Deviation</i>	<i>Minimum</i>	<i>Maximum</i>
Number of actions taken	0.11	0.64	0	11
ln(exposure)	2.04	0.55	0	2.30
Year of decision	1972.4	9.85	1953	1988
Liberal decision	0.52	0.50	0	1
Lower court disagreement	0.23	0.42	0	1
Alteration of precedent	0.02	0.15	0	1
Declaration of unconstitutionality	0.08	0.27	0	1
Unanimous vote	0.34	0.47	0	1

NOTE:  $N = 4,052$ . Data are all Supreme Court decisions handed down during the 1953-1987 terms and fall under the jurisdiction of House and Senate Judiciary Committees.

### *PRELIMINARY ESTIMATION RESULTS*

I examine congressional responses to Supreme Court decisions by analyzing the number of such responses to each Warren, Burger, or Rehnquist Court decision falling under Judiciary Committee jurisdiction during the 96th-101st Congresses. Data used in this analysis are drawn from the Supreme Court Judicial Database, 1953-1992 (ICPSR #9422), supplemented with data on actions taken against Supreme Court decisions by the House and Senate Judiciary Committees during the period from 1979 to 1988, as listed in Appendix B of Eskridge (1991).<sup>12</sup> Eskridge examines congressional responses to Supreme Court decisions, defined as either the introduction of a bill into either chamber or the holding of hearings, in response to the substantive result of one or more decisions of the Supreme Court.

The dependent variable of interest is the total number of actions taken in response to a decision of the Court in the House and Senate Judiciary Committees during the period 1979-1988. As is evident in Table 2, the vast majority of Supreme Court decisions received no congressional scrutiny. Of those that did, the total number of such actions ranged from 1 to 11, with a mean of 2.6. Although the event count nature of the variable suggests that the Poisson regression may be an appropriate technique with which to analyze these data, in fact the data contain significantly more zeros than would be predicted by a Poisson with a mean of 0.11. In nearly 96 percent of all cases analyzed here, no congressional response occurred during the 1979-1988 period. More specifically, under Poisson assumptions, we would

**TABLE 2: Frequencies: Numbers of House and Senate Actions Taken in Response to Supreme Court Decisions, 1979-1988**

<i>Number of Actions</i>	<i>Frequency</i>	<i>Percentage</i>
0	3,882	95.80
1	63	1.55
2	38	0.94
3	32	0.79
4	8	0.20
5	12	0.30
6	12	0.30
7	3	0.07
10	1	0.02
11	1	0.02
Total	4,052	100.0

expect  $4052(\exp(-0.109)) = 3634$  zeros in our data. The observed frequency of 3,882 (or about 107% of what is expected) gives us cause for concern that the Poisson is not, in fact, the optimal distribution for these data. Nonetheless, I proceed by estimating a Poisson regression, the results of which may serve as a baseline model for purposes of comparison.

Estimates of a Poisson regression model of congressional responses on the independent variables are presented in the first column of Table 3.<sup>13</sup> In general, the results are encouraging: all coefficients have the expected signs, and all save one are statistically significant at conventionally accepted levels. Turning to the coefficients on our primary variable of interest, we note that the impact of a decision being constitutional in nature is both statistically significant and substantively large. Holding all independent variables at their means, the value of  $\lambda$  is 0.034, suggesting that, as expected, congressional actions occur very infrequently (i.e., at a low rate).

Because we suspect that the large numbers of zero counts will result in apparent overdispersion in the data, the model was reestimated using a negative binomial specification; these results are presented in column 2 of Table 3. Results of the negative binomial model correspond to our expectations, albeit to a lesser extent than those obtained from the Poisson regression. Although all coefficients remain correctly signed, a number are now substantially smaller than their standard errors, reducing our confidence in their influence. The dis-

**TABLE 3: Results of Poisson, Negative Binomial, and Generalized Negative Binomial Model Estimates**

<i>Variable</i>	<i>Poisson</i>	<i>Negative Binomial</i>	<i>Generalized Negative Binomial</i>	
			<i>E(Y)</i>	$\alpha$
(Constant)	-160.125 (-9.91)	-134.411 (-4.93)	-150.723 (-4.26)	156.204 (4.53)
log(exposure)	0.544 (4.77)	0.178 (0.67)	0.604 (2.14)	-0.408 (-1.26)
Year of decision	0.079 (9.82)	0.067 (4.89)	0.075 (4.19)	-0.077 (-4.43)
Liberal decision	0.296 (3.02)	0.099 (0.45)	0.274 (1.16)	-0.022 (-0.09)
Lower court disagreement	-0.212 (-1.79)	-0.321 (-1.22)	-0.115 (-0.39)	0.052 (0.17)
Alteration of precedent	-0.254 (-0.67)	-0.102 (-0.13)	-0.233 (-0.35)	-0.683 (-0.71)
Declaration of unconstitutionality	-1.838 (-4.78)	-1.538 (-2.89)	-2.033 (-2.59)	1.331 (1.41)
Unanimous decision	-0.407 (-3.74)	-0.297 (-1.28)	-0.372 (-1.38)	0.554 (2.08)
( $\alpha$ )	—	32.233 (30.96)	—	
Log likelihood	-1,636.308	-989.542	-975.115	

NOTE: Numbers in parentheses are *t* ratios. *N* = 4,052. Likelihood-ratio tests of negative binomial versus Poisson equal 1,293.53 and 1,322.39 for single-parameter and generalized models, respectively (both significant at  $p < .01$ ). Likelihood-ratio test of generalized negative binomial versus single-parameter specification equals 28.85 ( $\chi^2(7)$ ,  $p < .01$ ).

persion parameter ( $\alpha$ ) confirms our expectations about the variance of the data; even holding constant the effect of the independent variables, the data are substantially overdispersed, with a variance-mean ratio of about 1.4 to 1. This fact is also reflected in the results of a likelihood-ratio test of the negative binomial versus the Poisson alternative. The negative binomial provides a substantially better fit to the data than the Poisson; with one degree of freedom, we can reject the null that the data are conditionally distributed as Poisson at the .001 level of significance. The value of  $\lambda$  holding all variables at their mean levels is 0.117, suggesting as did the Poisson results that the average rate of occurrence is very low.

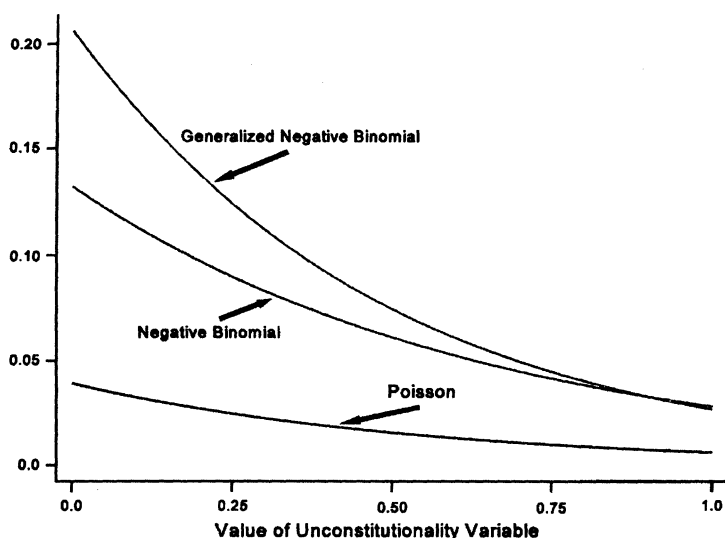
For purposes of comparison, I next estimated a generalized (or variance-function) negative binomial model in which the variance

parameter  $\alpha$  is allowed to vary as an exponential function of the same independent variables included in the model of the count. Our discussion above suggests that even if the observed overdispersion is the result of a dual regime data-generating process, a variance function negative binomial should yield similar results because of the relationship between  $q_i$  and the dispersion parameter  $\alpha$ .

Results of this generalized negative binomial model are presented in columns 3 and 4 of Table 3. In general, we would expect that the signs of the coefficients in the two parts of the model will be reversed, in that those factors causing greater counts (for example, more recent decisions) will also be expected to reduce the number of zero counts and thus mitigate the apparent overdispersion caused by the large numbers of zeros in the data. In fact, we find this predominantly to be the case. For example, year of decision has both a positive increase on the number of counts and a negative influence on the dispersion of the data. The impact of unanimity is the opposite: Unanimous decisions decrease the number of congressional responses while increasing the observed variance.

Comparing this model to the previous two, a likelihood-ratio test indicates that the model performs significantly better than both the Poisson regression and the single-parameter negative binomial. This confirms the conclusion we draw from the parameter estimates that the variance of the count experiences significant fluctuations as the values of a number of the independent variables change. With all variables at their mean levels, the dispersion parameter  $\alpha$  equals 43.84, again confirming the presence of significant overdispersion. Meanwhile, the value of  $\lambda$  at variable mean values is 0.175. Although this number remains somewhat low, it is important to note that as model fit improved, the mean-level value of  $\lambda$  increased. This fact reflects each successive model's ability to better account for the large number of zero counts present in the data.

To better illustrate this fact, as well as to provide a more intuitive presentation of variables' influence on the expected count in the different models, Figure 2 presents the change in the value of  $\lambda$  as the value of the variable indicating a declaration of unconstitutionality goes from zero to one. Although this variable is not, in fact, continuous, the graph does serve the useful purpose of outlining the general shape of the variable's effects on the observed count. Each line was



**Figure 2: Impact of Unconstitutionality: Poisson, Negative Binomial, and Generalized Negative Binomial Models**

NOTE: Lines represent values of  $\lambda$  corresponding to varying levels of the unconstitutionality variable for each of the different models, with all other variables held at their mean values.

calculated holding all other variables at their mean values. As would be expected from an examination of the coefficient estimates, this variable has its largest effect in the generalized negative binomial model. Despite this, however, the actual changes in the expected counts are relatively small for all three models. Furthermore, these estimates show that even extreme values of the most influential variables do not predict the actual numbers of counts well.

Thus, although successive generalization of the variance of a Poisson model showed substantial improvement in fitting our models to the data, these results remain less than optimal. In particular, the overall low levels of  $\lambda$  required to account for the large number of zero counts means that an adequate prediction of the dependent variable in cases where some action *was* observed cannot be made. I therefore turn to the hurdle and ZIP specifications, both to obtain more accurate results and to examine the properties of the models described above.

**TABLE 4: Results for Zero-Inflated and Hurdle Poisson Model Estimates**

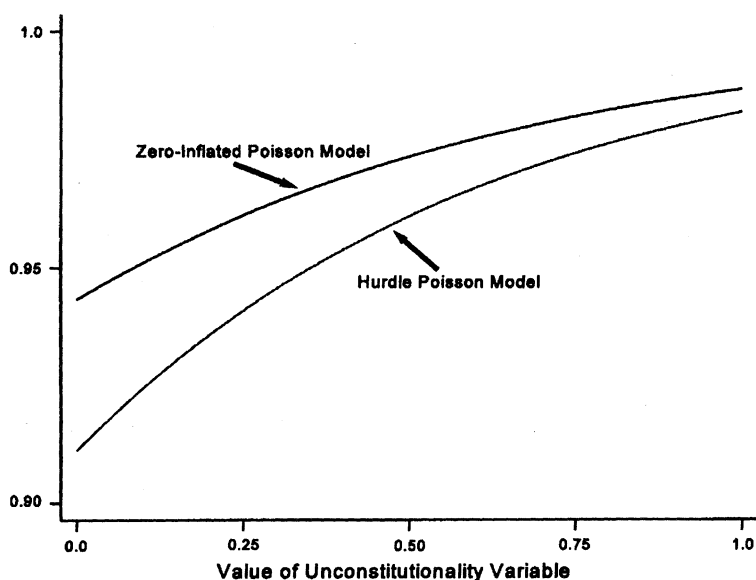
Variable	Zero-Inflated Poisson		Hurdle Poisson	
	Prob( $p^* = 1$ )	E( $Y p^* = 1$ )	Prob( $Y > 0$ )	E( $Y Y > 0$ )
(Constant)	153.580 (6.35)	-8.793 (-0.63)	-153.217 (-5.86)	-9.967 (-0.60)
log(exposure)	-0.487 (-2.64)	0.089 (0.65)	0.510 (2.76)	0.079 (0.62)
Year of decision	-0.076 (-6.24)	0.005 (0.68)	0.076 (5.77)	0.005 (0.64)
Liberal decision	-0.091 (-0.54)	0.190 (2.08)	0.139 (0.87)	0.192 (1.70)
Lower court disagreement	0.043 (0.22)	-0.138 (-1.30)	-0.079 (-0.43)	-0.147 (-1.01)
Alteration of precedent	-0.401 (-0.65)	-0.582 (-1.08)	0.171 (0.34)	-0.601 (-1.11)
Declaration of unconstitutionality	1.590 (2.42)	-0.421 (-0.69)	-1.696 (-2.88)	-0.367 (-0.78)
Unanimous decision	0.499 (2.58)	0.098 (0.96)	-0.460 (-2.59)	0.088 (0.70)
Log likelihood	-979.483		-975.111	

NOTE: Numbers in parentheses are  $t$  ratios.  $N = 4,052$ .

#### ZERO-INFLATED AND HURDLE POISSON ESTIMATION RESULTS

As noted previously, although the event count nature of the dependent variable implies the use of a Poisson model or its variants for the problem at hand, our theory also suggests that the process generating the event count is governed by a two-step structure similar to that described above. For purposes of comparison, I estimated both zero-inflated and hurdle Poisson regressions, again including all independent variables in both transition and event stages in both models. These results are presented in Table 4.

What is most striking about the results of these models is the remarkable similarity of the coefficient estimates, and their standard errors, across the two techniques. Examining first the models of the transition stages, we note that in all cases the estimates of the parameters are quite close, and in some cases nearly identical, between the ZIP and hurdle specifications. Note that the signs are reversed because the ZIP model estimates variable effects on  $q_i$ , whereas the hurdle



**Figure 3: Impact of Unconstitutionality on Transition Probabilities: ZIP and Hurdle Models**

NOTE: Lines represent changes in the probability of the observation not undergoing the transition to the events stage as the value of the unconstitutionality variable goes from zero to one.

model parameterizes  $p_i$ . Standard errors vary somewhat more but are still quite similar, and overall patterns and levels of significance correspond well. Minor differences in variable effects are likely due both to the different parameterization of the transition probability (logit vs. Poisson) and to truncation effects in the second stage. Substantively, however, variable effects are quite similar. Note also that because of the relationship outlined above with respect to overdispersion and transition probabilities, the results for both first stages are also quite similar to those for the variance in the generalized negative binomial model presented in Table 3, and the log likelihoods are also similar.<sup>14</sup>

Following my earlier presentation, Figure 3 illustrates the effect of a change from a nonconstitutional to a constitutional decision on the probability of a zero count in  $Y_i$  (i.e.,  $Prob(q_i = 1)$ ), again holding all other variables at their mean values. As is evident in Figure 3, the

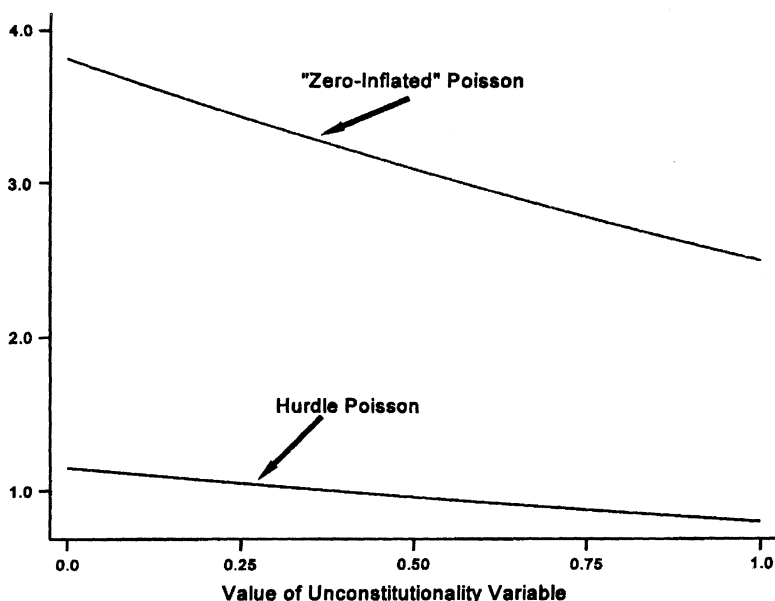


probabilities of no transition to the events stage are quite similar across the two models, as is the effect of an unconstitutional decision on that probability. Overall, the probabilities of transition are somewhat higher in the hurdle model, but the differences are slight.<sup>15</sup>

Turning to an examination of the event-count stages of the models, we again find that the estimated coefficients are remarkably similar across the two alternative specifications. Indeed, the estimates are even closer here than for the transition stage. This is likely due to the fact that, unlike in the transition stage, equivalent distributions are assumed to hold for the counts conditional on their respective transition stages. The sole difference in assumptions here is that the hurdle model's count distribution is assumed to be truncated at zero, whereas the ZIP specification count may take on zero values in the events stage. This difference, however, appears on the basis of these results to make little difference in the parameters estimated or on their standard errors.

As was the case in the models for the transition stage, coefficient estimates are also similar to those of the generalized negative binomial model. One significant difference here is the lack of important time effects in the dual regime models; once the transition stage is taken into effect, more recent decisions seem no more or less likely to receive congressional responses than older ones. This would imply that decision age, rather than having direct influence on the variance of the expected count, is instead influential in determining whether *any* action is taken against that decision, and that this, in turn, influences the count dispersion by its impact on the frequency of zero counts.

This also appears to be the case for the effect of a declaration of unconstitutionality. Although such a decision is significantly less likely to make the transition into the event-count stage, the influence of that variable once the second stage has been attained is small, and its standard errors are large by comparison. Predicted changes in the (conditional) expected count accompanying a change in this variable are presented in Figure 4. The difference in coefficient estimates is reflected in the different amount of change for each of the models; the unconstitutionality variable shows slightly greater influence in the ZIP specification than in the hurdle model. Overall expected values are higher in the ZIP model as well, due to the difference in the constant term between the two models.



**Figure 4: Impact of Unconstitutionality on Counts: ZIP and Hurdle Models**

NOTE: Lines represent values of  $\lambda$  corresponding to varying levels of the constitutionality variable for each of the two models, with all other variables held at their mean values.

The overall conclusion we may draw from this example is that in at least some circumstances, the performance of ZIP and hurdle Poisson models will be quite similar. This suggests that, as a practical matter and barring any strong theoretical considerations favoring one over the other, the choice between them may be made largely on the basis of convenience of estimation. Nonetheless, it would be advisable to investigate this similarity further, perhaps using Monte Carlo methods to assess under what circumstances results of the two may diverge. Speculation may lead us to believe that these divergences will be most likely when one form of the model provides a substantially better fit to the data-generating process than the other. For example, it is likely that the hurdle model presented here would have substantially different (and superior) results than the ZIP model in conditions where the transition probabilities were substantially asymmetrical.

## *CONCLUSIONS*

This article has examined zero-inflated and hurdle event-count models in some detail. I show that both models may be considered special cases of a more general type of model characterized by a dual regime data-generating process. Differences between these two model types are traceable to varying assumptions about the probability distribution governing the binomial transition process and about the possibility of zero counts occurring in the conditional (second stage) event count. The connection between excess zero counts and overdispersion (and thus between these types of models and variance function negative binomial models) was also explored, and a number of connections between the models were outlined. Most important, dual regime event-count models will often tend to indicate overdispersion in the data as a result of the large number of zero counts. This occurs because excess (i.e., extra-Poisson) zeros due to the split-model data-generating process are indistinguishable in practice from those due to contagion in the dependent variable. As a result, those excess zeros will tend to appear as overdispersion, and the estimated value of  $\alpha$  (the variance parameter) will be inflated. Furthermore, variables that in reality affect the zero-generating stage of the split model may appear to influence the count variance directly.

The empirical analysis of the article shows that, at least for the data given here, the two models yield strikingly similar results in practice: coefficient estimates, standard errors, and predicted values are all parallel across the two models. Although this may not always be the case, it does provide at least initial confirmation of the practical importance of the models' structural similarities. At the same time, this article also suggests a number of areas for future inquiry into the models themselves. First and foremost, work should be undertaken to better ascertain the statistical properties of the various estimators outlined here. Although the results here tentatively suggest that these models will yield similar results in application, the truth of that proposition remains to be proven beyond this special case. Likewise, it is important that we determine the robustness of these techniques to variations in the distribution of the dependent variable, model misspecification, and the host of other problems that all too frequently plague social science researchers. Finally, this line of research could

also extend to the derivation and analysis of the model types suggested by the general process but not present in the extant statistical literature; for example, hurdle negative binomial models.

More generally, the dual regime data-generating process investigated here is one common to many social science applications. To cite but one example, one potentially fruitful area of application is in the analysis of survey data, where oftentimes responses to survey items may reflect more than one aspect of the phenomenon of interest. For example, survey data on childbearing often take the form of a respondent's report of the number of children she or he has. If we have reason to believe that the factors that influence the decision to have a first child are in some way different than those that effect subsequent children, models such as those given here offer the prospect of extracting information to that effect from those data.

#### APPENDIX EXTRA-POISSON ZERO COUNTS AND THE PERFORMANCE OF REGRESSION-BASED TESTS FOR OVERDISPERSION

Economists and others have suggested a number of tests for determining if data exhibit overdispersion. These tests typically exploit the Poisson assumption of mean-variance equality, using the predicted values from a Poisson regression to test this assumption. They have come into wide use, primarily because estimating alternatives to the Poisson (e.g., the negative binomial and continuous parameter binomial) is both computationally costly and analytically complex. Three of the most commonly referenced regression-based tests are those of Cameron and Trivedi (1986, 1990) and Dean and Lawless (1989).

Cameron and Trivedi's tests make use of the fact that a Poisson variate has identical first and second moments; that is, data that are Poisson distributed will have variance equal to their mean. Their 1986 test analyzes the mean-variance equality assumption by regressing the squared residuals from the Poisson regression on the predicted values from that same regression:

$$(Y_i - \hat{Y}_i)^2 = \beta \hat{Y}_i + \epsilon_i. \quad (\text{A1})$$

Under the condition of mean-variance equality, we expect the estimated coefficient on  $\hat{Y}$  to equal one. The  $t$ -test for  $\beta = 1$  is therefore a means of testing for Poisson variation. In addition, the size of the coefficient will suggest the presence and magnitude of overdispersion in the data.

Similarly, Cameron and Trivedi's later (1990) test examines the condition of mean-variance equality by analyzing the regression:

$$((Y_i - \hat{Y}_i)^2 - \hat{Y}_i) = \beta[g(\hat{Y}_i)] + \epsilon_i. \quad (\text{A2})$$

In their example, Cameron and Trivedi use  $g(\hat{Y}_i) = \hat{Y}_i^2$ . Under the condition of mean-variance equality, the coefficient on  $\hat{Y}_i^2$  should be zero, in that  $E[\text{Var}(Y) - E(Y)] = 0$ . The  $t$ -test for  $\beta = 0$  is therefore another test of this restriction.

Finally, Dean and Lawless (1989) suggest a score test for overdispersion. Again relying on the mean-variance assumption, their  $T_1$  test is

$$T_1 = \frac{\sum_{i=1}^N (Y_i - \hat{Y}_i)^2 - Y_i}{\sqrt{2 \sum_{i=1}^N \hat{Y}_i^2}}. \quad (\text{A3})$$

This is a standardized version of Lee's (1986) score test for overdispersion.  $T_1$  is distributed as standard normal; high values reject the null of no overdispersion.<sup>16</sup>

Note that as the number of extra-Poisson zero counts approaches  $N$ , equations (A1) and (A2) become

$$\hat{Y}_i^2 = \beta \hat{Y}_i + \epsilon_i \quad (\text{A4})$$

and

$$(\hat{Y}_i^2 - \hat{Y}_i) = \beta \hat{Y}_i^2 + \epsilon_i. \quad (\text{A5})$$

Because the predicted values  $\hat{Y}_i$  will also be decreasing as the number of zeros approach  $N$ , it will become increasingly likely to reject the respective nulls.

Similarly, as the number of zero counts increases,

$$T_1 \rightarrow \frac{\sum_{i=1}^N \hat{Y}_i^2}{\sqrt{2 \sum_{i=1}^N \hat{Y}_i^2}} \quad (\text{A6})$$

which, for positive  $\hat{Y}_i$ , is strictly increasing in  $N$ . This suggests that in these circumstances, the result of the  $T_1$  test will depend not so much on the equality of mean and variance in the data but almost entirely on the sample size.

(continued)

## APPENDIX Continued

TABLE A1: Results of Regression-Based Tests for Overdispersion

<i>Test</i>	<i>Test Statistic</i>	<i>Distribution</i>	<i>Result</i>
Cameron and Trivedi (1986) <sup>a</sup>	6.74	$t$ , with $df = N - 1$	Reject null of no overdispersion at $p < .001$
Cameron and Trivedi (1990) <sup>b</sup>	7.08	$t$ , with $df = N - 1$	Reject null of no overdispersion at $p < .001$
Dean and Lawless $T_1$ test (1989) <sup>c</sup>	105.97	Standard normal	Reject null of no overdispersion at $p < .001$

a. Cameron and Trivedi's (1986) test is a  $t$ -test for  $\beta = 1$  on the estimated coefficient after regressing  $(Y - \hat{Y})^2$  on  $\hat{Y}$  without a constant term in the equation. See text for details.

b. Cameron and Trivedi's (1990) test is a  $t$ -test for  $\beta = 0$  on the estimated coefficient after regressing  $((Y - \hat{Y})^2) - \hat{Y}$  on  $\hat{Y}^2$  without a constant term in the equation. See text for details.

c. The Dean and Lawless (1989)  $T_1$  test is a standardized version of the score statistic also derived by Lee (1986) and Cameron and Trivedi (1986). See text for details.

In all three cases, each test is initiated by fitting a Poisson model to the data and generating predicted values. Violation of the mean-variance equality assumption thus forms the basis for the tests. In the case where the data-generating process is characterized by excessive zeros relative to those expected in the Poisson, however, the tests will tend to indicate the presence of overdispersion in the data. In one important respect, this is exactly what these tests *should* do; to the extent that dual regime models of the type discussed here are a particular kind of overdispersion, these tests will indicate the presence of that overdispersion.<sup>17</sup> One limitation of these tests, however, is that excess zeros caused by a two-part data-generating process "trick" these tests into indicating the presence of overdispersion when, controlling on the transition stage, little or none is present. That is, because they rely on the initial estimation of a (misspecified) Poisson model and fail to take account of the conditional nature of the count variable, these tests are of somewhat more limited value when the data-generating process takes a dual regime form.

For example, Table A1 presents results for each of these three tests on the data presented here. Each test statistic was calculated following estimation of the Poisson regression model in Table 3. In all three cases, because of the large number of zero counts, the tests reject the null of no overdispersion at high levels of significance.

## NOTES

1. For a good recent review of Poisson and negative binomial event count models, see Land et al. (1996).

2. This terminology should not be confused with the switching regime models of Goldfeld and Quandt (1975) and subsequent authors. Because of its two-stage nature, the general model here also bears a strong resemblance to the split-population duration model of Schmidt and Witte (1989).

3. Greene (1994) also examines the case in which the distribution of  $Y_i$  conditional on  $p_i^* = 1$  and  $X_i'\beta$  is a negative binomial; for purposes of brevity, I restrict my analysis to the Poisson and truncated Poisson cases here.

4. Mullahy (1986) also discusses a number of other models, including the with zeros (WZ) model, which augments or diminishes the probability of a zero count by a scalar factor. He also pays a good deal of attention to the geometric event-count model, limiting his empirical analyses to this latter distribution.

5. As Greene (1994:11) notes, Lambert's model in fact assumes that defects are impossible in the perfect state, since the only possible observed count is zero.

6. This section draws extensively on results presented by Greene (1994).

7. Positive contagion has also been referred to as *occurrence dependence* in relation to unemployment data (e.g., Heckman and Borjas 1980) and in models of "spells" in the biometric literature (e.g., Kekalaki 1983).

8. Furthermore, because  $q_i$  and  $p_i$  are constrained to be nonnegative, this equation also shows that underdispersion due to fewer-than-Poisson numbers of zero counts will not occur in the model with a nontruncated events stage. This result was first noted in Greene (1994).

9. It is also seen in the effect of extra-Poisson zero counts on regression-based tests for overdispersion. Because they are based on obtaining predicted values from a (misspecified) Poisson regression, these tests will generally be inaccurate in the presence of a dual regime data-generating process. See the appendix for details.

10. A similar theory, based on the interest group model of Landes and Posner (1975), would suggest an analogous result; older legislation has had a longer period in which the groups by whom it was "purchased" could reap its benefits. Thus, to the extent that the courts tend to address recently enacted legislative acts (Dahl 1957; Landes and Posner 1975), older decisions will generally be those relating to older legislation, and thus of little or no concern to the present Congress.

11. In some cases, the Court's decisions were handed down after 1979, whereas in other instances, Congress overturned the decision prior to 1988. In both such instances, those decisions were not available to have actions taken against them for the entire 10-year period. Following Maddala (1983:53), I control for this difference across cases by including a variable equal to the log of the number of years of exposure to congressional scrutiny during the 1979-1988 period.

12. Decisions are limited to those from the 1953 to 1986 terms, and to those touching on subject matter falling under the jurisdiction of the Judiciary Committees. The unit of analysis is the orally argued docket number. These criteria produce 4,065 cases for analysis, 13 of which contain missing data on one or more of the independent variables. Analyses are therefore conducted on 4,052 observations. See Eskridge (1991) for details on how data were selected.

13. Poisson, negative binomial, and generalized negative binomial models were estimated using Stata 3.1, the zero-inflated Poisson model using LIMDEP 7.0, and the hurdle Poisson model using COUNT 4.01. All estimations were done on a Dell 166-MHz Pentium running DOS 6.2. Convergence times varied considerably, with the Poisson taking the shortest time and the generalized negative binomial and hurdle models the longest. Data and estimation commands for replication of these analyses are available upon request from the author.

14. In the ZIP model, the log likelihood is not directly comparable to those of the Poisson and its variants. Recall that the standard Poisson model obtains here if  $q_i \rightarrow 0$ . Given the parameterization of  $q_i$ , this would require that some element of  $Z_i\gamma$  explode, a situation not



amenable to standard likelihood-based tests. Note, however, that Vuong's (1989) test for nonnested models rejects the Poisson in favor of the ZIP specification at the .001 level. In the hurdle case, the likelihood presented in Table 4 differs from that reported by King's COUNT program. The Poisson model is nested within the hurdle model, here obtaining if the two coefficient vectors are equal. Individual coefficient tests indicate that this is not, in fact, the case, and a traditional likelihood-ratio test of the hurdle model versus the Poisson favors the former at very high levels of significance. King's program omits from the likelihood terms that do not depend on the coefficient vectors. In particular, their program omits a term depending exclusively on  $Y$  from the likelihood that is maximized. To make the reported likelihood comparable, it is necessary to subtract from it a factor equal to  $\Sigma[\ln(Y_i!)]$ . In the example presented here, the value of  $-303.683$  was subtracted from the reported likelihood of  $-671.428$  to yield the comparable value presented.

15. One alternative reason for the similarity of these estimates also highlights an important implication of the choice between symmetric and asymmetric hurdle probabilities. That is, the direction of the asymmetry has implications for whether  $p_i$  or  $q_i$  is modeled. In the ZIP model, because the probability is symmetrical, the choice between estimating  $p_i$  or  $q_i$  is immaterial. Under the hurdle specification, however, the combination of the shape of the (asymmetric) probability distribution of the transition probability and the choice of  $p_i$  or  $q_i$  has substantive implications about the nature of the transition event.

In the models given here, the ZIP model estimates  $q_i$ , whereas the hurdle model examines  $p_i$ . Because, as noted by King (1989a), the Poisson and logistic distributions are similar on their lower tails, and because the transition probabilities  $p_i$  here are all quite small (and thus will fall toward the lower tail of either distribution), we might expect that our estimates would be similar, even in the presence of different distributional assumptions.

16. Dean and Lawless (1989) propose several other tests as well. They note that, compared to these alternatives, the  $T_1$  statistic is slow to approach normality as  $N \rightarrow \infty$ , but this fact is immaterial to the present discussion.

17. I thank an anonymous reviewer for pointing this out.

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