

PLSC 503 – Spring 2024

Binary Response Models, II

April 1, 2024

Running Example: House Vote on NAFTA (1993)

Response / Outcome

- `vote` – Whether (`=1`) or not (`=0`) the House member in question voted in favor of NAFTA.

Predictors

- `PropHisp` – The proportion of the House member's district who are of Latino/hispanic origin.
- `Democrat` – Whether the House member in question is a Democrat (`=1`) or a Republican (`=0`).
- `COPE` – The 1993 AFL-CIO (COPE) voting score of the member in question; the original variable ranges from 0 to 100, with higher scores indicating more pro-labor positions. Rescaled to range from 0 to 1.
- `DemXCOPE` – The multiplicative interaction of `Democrat` and `COPE`.

$$\Pr(\text{vote}_i = 1) = f[\beta_0 + \beta_1(\text{PropHisp}_i) + \beta_2(\text{Democrat}_i) + \beta_3(\text{COPE}_i) + \beta_4(\text{Democrat}_i \times \text{COPE}_i) + u_i]$$

```
> summary(NAFTA)
```

Vote		PropHisp		Democrat		COPE		DemXCOPE	
Min.	:0.000	Min.	:0.000	Min.	:0.000	Min.	:0.000	Min.	:0.000
1st Qu.:	0.000	1st Qu.:	0.010	1st Qu.:	0.000	1st Qu.:	0.170	1st Qu.:	0.000
Median	:1.000	Median	:0.030	Median	:1.000	Median	:0.810	Median	:0.750
Mean	:0.539	Mean	:0.088	Mean	:0.585	Mean	:0.602	Mean	:0.516
3rd Qu.:	1.000	3rd Qu.:	0.100	3rd Qu.:	1.000	3rd Qu.:	1.000	3rd Qu.:	1.000
Max.	:1.000	Max.	:0.830	Max.	:1.000	Max.	:1.000	Max.	:1.000

Logit:

$$\Pr(Y_i = 1) = \frac{\exp(\mathbf{X}_i\beta)}{1 + \exp(\mathbf{X}_i\beta)}$$

or probit:

$$\Pr(Y_i = 1) = \Phi(\mathbf{X}_i\beta)$$

Probit Estimates

```
> NAFTA.probit<-glm(Vote~PropHisp+Democrat+COPE+DemXCOPE,  
                    NAFTA,family=binomial(link="probit"))  
> summary(NAFTA.probit)
```

Call:

```
glm(formula = Vote ~ PropHisp + Democrat + COPE + DemXCOPE,  
     family = binomial(link = "probit"), data = NAFTA)
```

Deviance Residuals:

	Min	1Q	Median	3Q	Max
	-3.173	-0.677	0.362	0.764	1.817

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.078	0.153	7.03	2.1e-12 ***
PropHisp	1.279	0.467	2.74	0.0062 **
Democrat	3.034	0.739	4.11	4.0e-05 ***
COPE	-2.201	0.440	-5.00	5.8e-07 ***
DemXCOPE	-2.888	0.903	-3.20	0.0014 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 598.99 on 433 degrees of freedom
Residual deviance: 441.06 on 429 degrees of freedom
AIC: 451.1

Number of Fisher Scoring iterations: 8

Logit Estimates

```
> NAFTA.fit<-glm(Vote~PropHisp+Democrat+COPE+DemXCOPE,  
                  NAFTA,family=binomial)  
> summary(NAFTA.fit)
```

Call:

```
glm(formula = Vote ~ PropHisp + Democrat + COPE + DemXCOPE,  
     family = binomial, data = NAFTA)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-3.264	-0.650	0.310	0.728	1.818

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.792	0.275	6.50	7.8e-11 ***
PropHisp	2.091	0.794	2.63	0.00846 **
Democrat	6.866	1.547	4.44	9.1e-06 ***
COPE	-3.650	0.760	-4.80	1.6e-06 ***
DemXCOPE	-6.705	1.820	-3.68	0.00023 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 598.99 on 433 degrees of freedom
Residual deviance: 436.83 on 429 degrees of freedom
AIC: 446.8

Number of Fisher Scoring iterations: 5

```
> # Equivalent to:
```

```
>
```

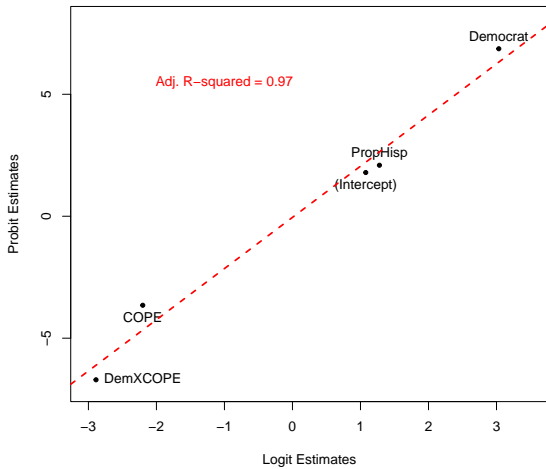
```
> fit<-glm(Vote~PropHisp+Democrat*COPE,NAFTA,family=binomial)
```

Models (table via `modelsummary`)

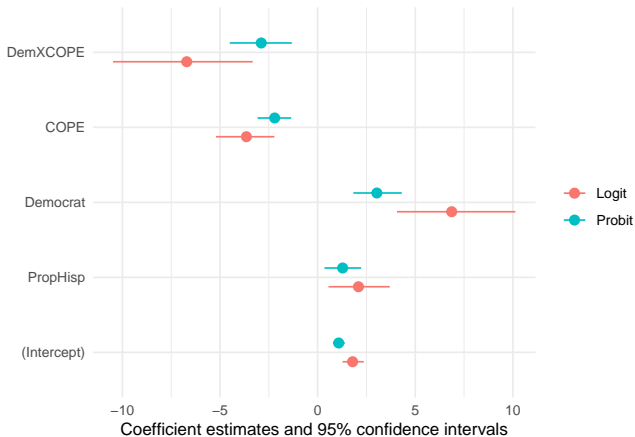
Table: Logits and Probits

	Logit	Probit
(Intercept)	1.792 (0.275)	1.078 (0.153)
PropHisp	2.091 (0.794)	1.279 (0.467)
Democrat	6.866 (1.547)	3.034 (0.739)
COPE	-3.650 (0.760)	-2.201 (0.440)
DemXCOPE	-6.705 (1.820)	-2.888 (0.903)
Num.Obs.	434	434
AIC	446.8	451.1
BIC	467.2	471.4
Log.Lik.	-218.414	-220.532
F	26.622	30.723
RMSE	0.40	0.41

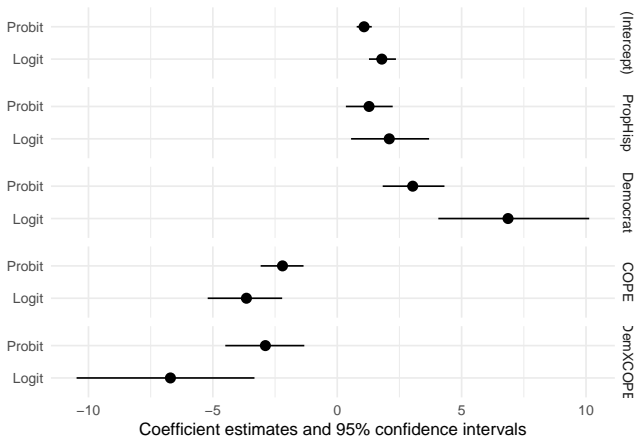
$\hat{\beta}_{\text{probit}}$ VS. $\hat{\beta}_{\text{logit}}$



Coefficient Plots (via `modelplot`)



Faceted Plot (also via `modelplot`)



Log-Likelihoods, “Deviance,” etc.

- R / glm reports “deviances”:
 - “Residual” deviance = $2(\ln L_S - \ln L_M)$
 - “Null” deviance = $2(\ln L_S - \ln L_N)$
 - stored in `object$deviance` and `object$null.deviance`
- So:

$$\begin{aligned} LR_{\beta=0} &= 2(\ln L_M - \ln L_N) \\ &= \text{“Null” deviance} - \text{“Residual” deviance} \end{aligned}$$

Example:

```
> LLR<-NAFTA.fit$null.deviance - NAFTA.fit$deviance  
  
> LLR  
[1] 162  
  
> pchisq(LLR,4,lower.tail=FALSE)  
[1] 5.04e-34
```

Interpretation: “Signs-n-Significance”

For both logit and probit:

- $\hat{\beta}_k > 0 \leftrightarrow \frac{\partial \Pr(Y=1)}{\partial X_k} > 0$
- $\hat{\beta}_k < 0 \leftrightarrow \frac{\partial \Pr(Y=1)}{\partial X_k} < 0$
- $\frac{\hat{\beta}_k}{\hat{\sigma}_k} \sim N(0, 1)$

Interactions:

$$\hat{\beta}_{\text{COPE}|\text{Democrat}=1} \equiv \hat{\phi}_{\text{COPE}} = \hat{\beta}_3 + \hat{\beta}_4$$

$$\text{s.e.}(\hat{\beta}_{\text{COPE}|\text{Democrat}=1}) = \sqrt{\text{Var}(\hat{\beta}_3) + (\text{Democrat})^2 \text{Var}(\hat{\beta}_4) + 2(\text{Democrat}) \text{Cov}(\hat{\beta}_3, \hat{\beta}_4)}$$

```
> NAFTA.fit$coeff[4]+NAFTA.fit$coeff[5]

COPE
-10.4

> # z-statistic:
>
> (NAFTA.fit$coeff[4]+NAFTA.fit$coeff[5]) /
+   (sqrt(vcov(NAFTA.fit)[4,4] +
+   (1)^2*vcov(NAFTA.fit)[5,5] +
+   2*1*vcov(NAFTA.fit)[4,5]))
COPE
-6.25

> # Square that, and it's a chi-square statistic:
>
> ((NAFTA.fit$coeff[4]+NAFTA.fit$coeff[5]) /
+   (sqrt(vcov(NAFTA.fit)[4,4] +
+   (1)^2*vcov(NAFTA.fit)[5,5] +
+   2*1*vcov(NAFTA.fit)[4,5]))))^2
COPE
39
```

(Or use car...)

```
> library(car)
> linearHypothesis(NAFTA.fit,"COPE+DemXCOPE=0")
Linear hypothesis test
```

```
Hypothesis:
COPE + DemXCOPE = 0
```

```
Model 1: restricted model
```

```
Model 2: Vote ~ Democrat + PropHisp + COPE + DemXCOPE
```

	Res.Df	Df	Chisq	Pr(>Chisq)
1	430			
2	429	1	39	0.000000000042 ***

```
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1
```

The *marginal effect* is:

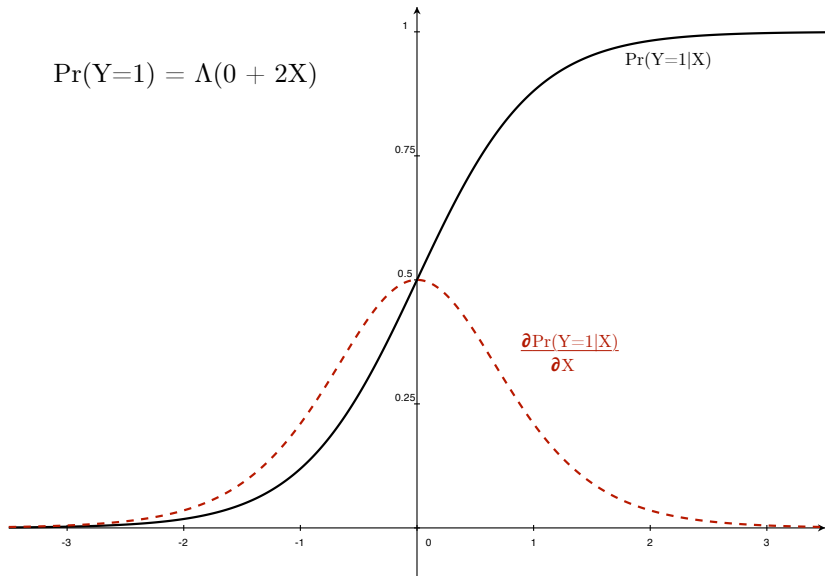
$$\begin{aligned}\frac{\partial \Pr(\hat{Y}_i = 1)}{\partial X_k} &= \frac{\partial F(\mathbf{X}_i \hat{\beta})}{\partial X_k} \\ &= f(\mathbf{X}_i \hat{\beta}) \hat{\beta}_k \\ &= \Lambda(\mathbf{X}_i \hat{\beta}) [1 - \Lambda(\mathbf{X}_i \hat{\beta})] \hat{\beta}_k \quad (\text{for logit}) \text{ or} \\ &= \phi(\mathbf{X}_i \hat{\beta}) \hat{\beta}_k \quad (\text{for probit})\end{aligned}$$

Note that these depend on $\mathbf{X}_i \hat{\beta}$, which means we either have to:

1. ...hold $\mathbf{X}_i \hat{\beta}$ constant at some value(s), or
2. ...average over the actual values of $\mathbf{X}_i \hat{\beta}$ observed in the data.

Marginal Effects Illustrated

$$\Pr(Y=1) = \Lambda(0 + 2X)$$

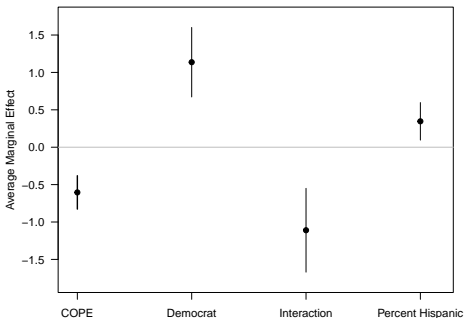


Marginal Effects In Action

```
> summary(margins(NAFTA.fit))
```

factor	AME	SE	z	p	lower	upper
COPE	-0.6043	0.1139	-5.3048	0.0000	-0.8275	-0.3810
Democrat	1.1366	0.2370	4.7953	0.0000	0.6720	1.6011
DemXCOPE	-1.1101	0.2858	-3.8836	0.0001	-1.6703	-0.5498
PropHisp	0.3462	0.1280	2.7054	0.0068	0.0954	0.5970

Plotted:



Log-Odds of $Y = 1$ are linear in \mathbf{X} :

$$\ln \Omega(\mathbf{X}) = \ln \left[\frac{\frac{\exp(\mathbf{X}\beta)}{1+\exp(\mathbf{X}\beta)}}{1 - \frac{\exp(\mathbf{X}\beta)}{1+\exp(\mathbf{X}\beta)}} \right] = \mathbf{X}\beta$$

That implies that:

$$\frac{\partial \ln \Omega}{\partial \mathbf{X}} = \beta$$

OR for a one-unit change in X_k :

$$\frac{\Omega(X_k = \ell + 1)}{\Omega(X_k = \ell)} = \exp(\hat{\beta}_k)$$

OR for a δ -unit change in X_k :

$$\frac{\Omega(X_k = \ell + \delta)}{\Omega(X_k = \ell)} = \exp(\hat{\beta}_k \delta)$$

Also:

$$\text{Percentage Change in the Odds} = 100[\exp(\hat{\beta}_k \delta) - 1]$$

Odds Ratios Implemented

```
> P<-qnorm(0.975)

> lreg.or <- function(model)
+ {
+   coeffs <- coef(summary(model))
+   lowerCI <- exp(coeffs[,1] - P * coeffs[,2])
+   OR <- exp(coeffs[,1])
+   upperCI <- exp(coeffs[,1] + P * coeffs[,2])
+   lreg.or <- cbind(OR,lowerCI,upperCI)
+   lreg.or
+ }

> lreg.or(NAFTA.fit)
      OR      lowerCI      upperCI
(Intercept)  5.99928  3.4965990  10.2933
PropHisp      8.09352  1.7068838   38.3770
Democrat    958.67832 46.1969511 19894.4757
COPE         0.02599  0.0058625   0.1152
DemXCOPE      0.00122  0.0000345   0.0434

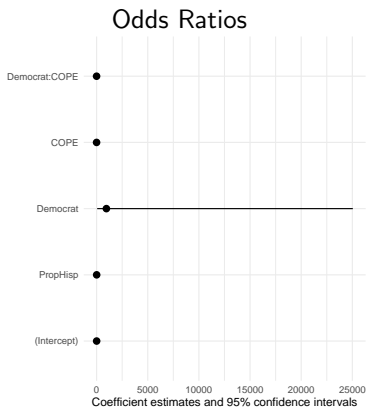
> Or via -confint-...

> exp(cbind(OR=coef(NAFTA.fit),confint.default(NAFTA.fit)))
      OR      2.5 %      97.5 %
(Intercept)  5.99928  3.4965990  10.2933
PropHisp      8.09352  1.7068838   38.3770
Democrat    958.67832 46.1969511 19894.4757
COPE         0.02599  0.0058625   0.1152
DemXCOPE      0.00122  0.0000345   0.0434
```

Odds Ratios via modelsummary / modelplot

Table: Odds Ratios

	(1)
(Intercept)	5.999 (1.652)
PropHisp	8.094 (6.427)
Democrat	958.678 (1483.358)
COPE	0.026 (0.020)
Democrat \times COPE	0.001 (0.002)
Num.Obs.	434
AIC	446.8
BIC	467.2
Log.Lik.	-218.414
F	26.622
RMSE	0.40



What Does This *Mean*?

```
> NAFTA.fit

Coefficients:
(Intercept)    PropHisp    Democrat      COPE    DemXCOPE
      1.79         2.09         6.87     -3.65     -6.71

> exp(cbind(OR=coef(NAFTA.fit), confint.default(NAFTA.fit))))
              OR      2.5 %      97.5 %
(Intercept)  5.99928  3.4965990  10.2933
PropHisp      8.09352  1.7068838  38.3770
Democrat    958.67832  46.1969511 19894.4757
COPE         0.02599  0.0058625   0.1152
DemXCOPE      0.00122  0.0000345   0.0434
```

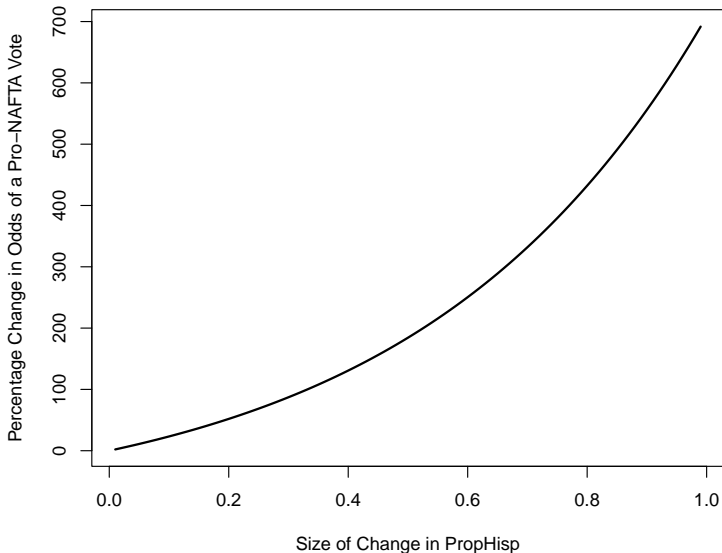
Consider PropHispc:

- A one-unit change (from 0 percent to 100 percent) in PropHispc corresponds to a $[(8.094 - 1) \times 100] = 709$ percent expected increase in the odds of a member of Congress voting in favor of NAFTA.
- A change of 0.10 (that is, a ten percentage-point increase) in the proportion of a member's district who is Hispanic corresponds to an odds ratio of:

$$\begin{aligned}\exp(2.09 \times 0.10) &= \exp(0.209) \\ &= 1.232\end{aligned}$$

- This means that an increase of 0.10 in PropHispc corresponds to a $[(1.232 - 1) \times 100] = 23.2$ percent expected increase in the odds that a member of Congress would have voted in favor of NAFTA.
- For an increase of 0.20 (that is, 20 percentage points), the corresponding odds ratio and percent increase are 1.519 and 51.9 percent, respectively.

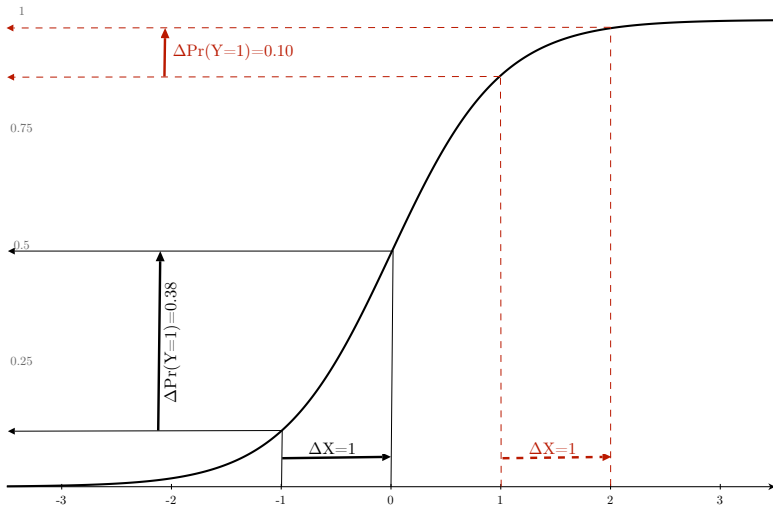
Percentage Change in Odds, by $\Delta\text{PropHisp}$



Predicted probabilities:

$$\begin{aligned}\widehat{\Pr(Y_i = 1)} &= F(\mathbf{X}_i \hat{\boldsymbol{\beta}}) \\ &= \frac{\exp(\mathbf{X}_i \hat{\boldsymbol{\beta}})}{1 + \exp(\mathbf{X}_i \hat{\boldsymbol{\beta}})} \text{ for logit,} \\ &= \Phi(\mathbf{X}_i \hat{\boldsymbol{\beta}}) \text{ for probit.}\end{aligned}$$

Predicted Probabilities Illustrated



Predicted Probabilities: Standard Errors

$$\begin{aligned}\text{Var}[\widehat{\text{Pr}(Y_i = 1)}] &= \left[\frac{\partial F(\mathbf{X}_i \hat{\beta})}{\partial \hat{\beta}} \right]' \hat{\mathbf{V}} \left[\frac{\partial F(\mathbf{X}_i \hat{\beta})}{\partial \hat{\beta}} \right] \\ &= [f(\mathbf{X}_i \hat{\beta})]^2 \mathbf{X}_i' \hat{\mathbf{V}} \mathbf{X}_i\end{aligned}$$

So,

$$\text{s.e.}[\widehat{\text{Pr}(Y_i = 1)}] = \sqrt{[f(\mathbf{X}_i \hat{\beta})]^2 \mathbf{X}_i' \hat{\mathbf{V}} \mathbf{X}_i}$$

Changes in $\Pr(\widehat{Y} = 1)$:

$$\Delta \Pr(\widehat{Y} = 1)_{\mathbf{x}_A \rightarrow \mathbf{x}_B} = \frac{\exp(\mathbf{x}_B \hat{\beta})}{1 + \exp(\mathbf{x}_B \hat{\beta})} - \frac{\exp(\mathbf{x}_A \hat{\beta})}{1 + \exp(\mathbf{x}_A \hat{\beta})}$$

or

$$= \Phi(\mathbf{x}_B \hat{\beta}) - \Phi(\mathbf{x}_A \hat{\beta})$$

Standard errors obtainable via delta method, bootstrap, etc...

In-Sample Predictions

```
> preds<-NAFTA.fit$fitted.values

> hats<-predict(NAFTA.fit,se.fit=TRUE)
> hats
$fit
      1      2      3      4 ...
9.01267619 7.25223902 6.11013844 5.57444635 ...
...
$se.fit
      1      2      3      4 ...
1.5331506 1.2531475 1.1106989 0.9894208 ...

> XBUB<-hats$fit + (1.96*hats$se.fit)
> XBLB<-hats$fit - (1.96*hats$se.fit)
> plotdata<-cbind(as.data.frame(hats),XBUB,XBLB)
> plotdata<-data.frame(lapply(plotdata,binomial(link="logit")$linkinv))
```

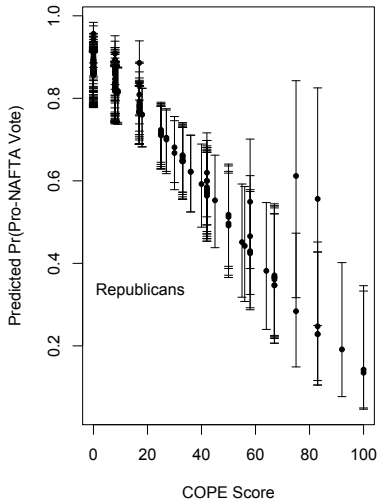
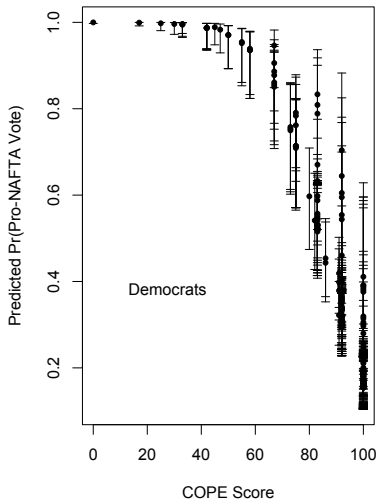
...

```
> par(mfrow=c(1,2))
> library(plotrix)

> with(NAFTA,
+   plotCI(COPE[Democrat==1],plotdata$fit[Democrat==1],ui=plotdata$XBUB[Democrat==1],
+         li=plotdata$XBLB[Democrat==1],pch=20,xlab="COPE Score",ylab="Predicted
+         Pr(Pro-NAFTA Vote)")

> with(NAFTA,
+   plotCI(COPE[Democrat==0],plotdata$fit[Democrat==0],ui=plotdata$XBUB[Democrat==0],
+         li=plotdata$XBLB[Democrat==0],pch=20,xlab="COPE Score",ylab="Predicted
+         Pr(Pro-NAFTA Vote)")
```

In-Sample Predictions



Out-of-Sample Predictions

“Fake” data:

```
> sim.data<-data.frame(PropHisp=mean(NAFTA$PropHisp),Democrat=rep(0:1,101),  
                        COPE=seq(from=0,to=1,length.out=101))  
> sim.data$DemXCOPE<-sim.data$Democrat*sim.data$COPE
```

Generate predictions:

```
> OutHats<-predict(NAFTA.fit,se.fit=TRUE,newdata=sim.data)  
> OutHatsUB<-OutHats$fit+(1.96*OutHats$se.fit)  
> OutHatsLB<-OutHats$fit-(1.96*OutHats$se.fit)  
> OutHats<-cbind(as.data.frame(OutHats),OutHatsUB,OutHatsLB)  
> OutHats<-data.frame(lapply(OutHats,binomial(link="logit")$linkinv))
```

Plot:

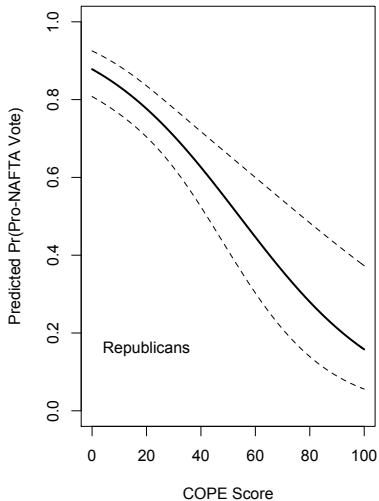
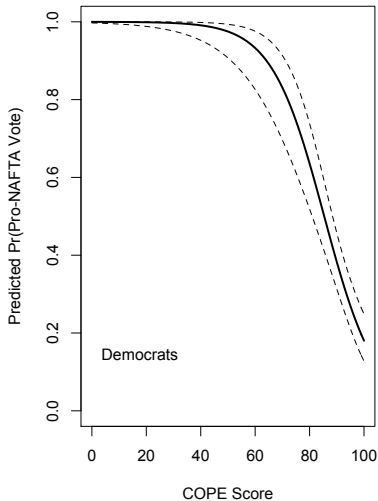
```
> both<-cbind(sim.data,OutHats)
> both<-both[order(both$COPE,both$Democrat),]
> bothD<-both[both$Democrat==1,]
> bothR<-both[both$Democrat==0,]

> par(mfrow=c(1,2))

> plot(bothD$COPE,bothD$fit,t="l",lwd=2,ylim=c(0,1),
      xlab="COPE Score",ylab="Predicted Pr(Pro-NAFTA Vote)")
> lines(bothD$COPE,bothD$OutHatsUB,lty=2)
> lines(bothD$COPE,bothD$OutHatsLB,lty=2)
> text(0.3,0.2,label="Democrats")

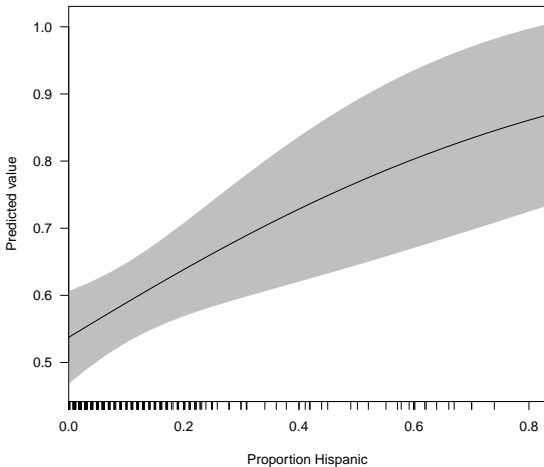
> plot(bothR$COPE,bothR$fit,t="l",lwd=2,ylim=c(0,1),
      xlab="COPE Score",ylab="Predicted Pr(Pro-NAFTA Vote)")
> lines(bothR$COPE,bothR$OutHatsUB,lty=2)
> lines(bothR$COPE,bothR$OutHatsLB,lty=2)
> text(0.7,0.9,label="Republicans")
```


Out-of-Sample Predictions



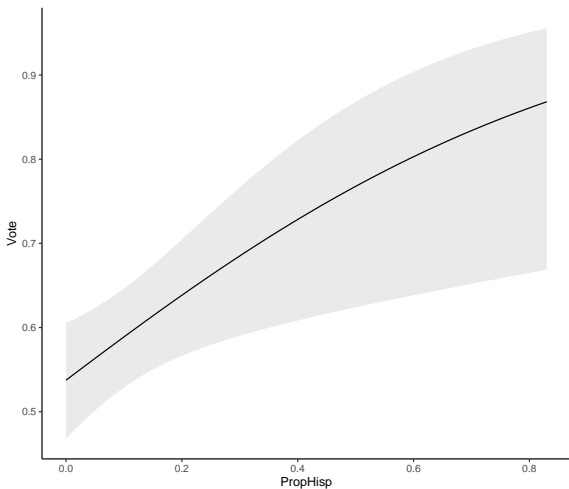
Single-Variable Example (using cplot)

```
> cplot(NAFTA.fit,"PropHisp",xlab="Proportion Hispanic")
```



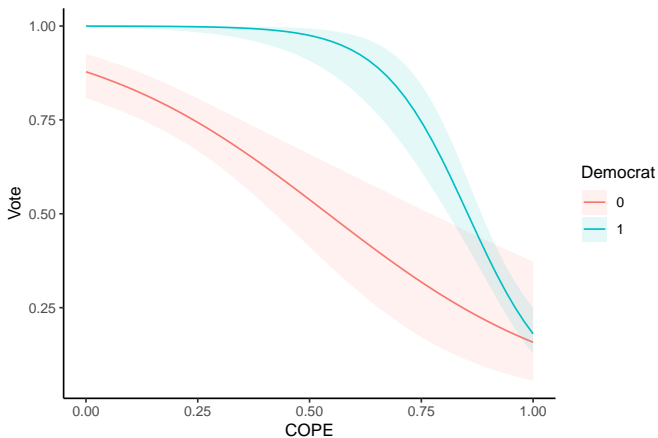
Same, using `marginalEffects::plot_prediction`

```
> plot_predictions(NAFTA.fit, condition="PropHisp") + theme_classic()
```



Interactive part, using plot_prediction

```
> plot_predictions(fit,condition=c("COPE","Democrat")) + theme_classic()
```



Goodness of Fit

Some alternatives....

- Pseudo- R^2 (no!)
- Proportional reduction in error (PRE) – a/k/a “accuracy”
- ROC curves.

Model Fit: Predictions

Suppose we assign:

$$\begin{aligned}\hat{Y}_i &= 0 & \text{if } \Pr(\widehat{Y_i = 1}) \leq \tau \\ \hat{Y}_i &= 1 & \text{if } \Pr(\widehat{Y_i = 1}) > \tau\end{aligned}$$

This would then give us a “confusion matrix”:

Actual Y_i	Predicted \hat{Y}_i	
	$\hat{Y}_i = 0$	$\hat{Y}_i = 1$
$Y_i = 0$	True Negative (“TN”)	False Positive (“FP”)
$Y_i = 1$	False Negative (“FN”)	True Positive (“TP”)

This means we have:

- Total actual negatives = TN + FP
- Total actual positives = TP + FN
- Number correctly predicted = TP + TN

Proportional Reduction in Error (PRE):

$$\text{PRE} = \frac{N_{MC} - N_{NC}}{N - N_{NC}}$$

- N_{NC} = number correctly predicted under the “null model,”
- N_{MC} = number correctly predicted under the estimated model,
- N = total number of observations.

PRE tells us how much (proportionally) better our model does at predicting Y in-sample than would a model that only contained an intercept.


```
> Assume tau = 0.5...
>
> table(NAFTA.fit$fitted.values>0.5,nafta$vote==1)
```

	FALSE	TRUE
FALSE	148	49
TRUE	52	185

$$\begin{aligned}
 \text{PRE} &= \frac{N_{MC} - N_{NC}}{N - N_{NC}} \\
 &= \frac{(148 + 185) - 234}{434 - 234} \\
 &= \frac{99}{200} \\
 &= \mathbf{0.495}
 \end{aligned}$$

Chi-Square test:

```
> chisq.test(NAFTA.fit$fitted.values>0.5,NAFTA$Vote==1)
```

Pearson's Chi-squared test with Yates' continuity correction

```
data:  NAFTA.fit$fitted.values > 0.5 and NAFTA$Vote == 1
X-squared = 120, df = 1, p-value <2e-16
```

Concepts:

- *Sensitivity* (or “true positive rate”)
 - The proportion of all actual positives that were predicted correctly
 - $\text{Sensitivity} = \frac{\text{TP}}{\text{TP} + \text{FN}}$
- *Specificity* (or “true negative rate”)
 - The proportion of all actual negatives that were predicted correctly
 - $\text{Specificity} = \frac{\text{TN}}{\text{TN} + \text{FP}}$
- False positive rate = $1 - \text{Specificity}$
- False negative rate = $1 - \text{Sensitivity}$

Suppose we set $\tau = 0.00001$. Then:

- We would essentially *always* predict $\hat{Y}_i = 1$, which means
- ...we would always correctly predict all the actual positives (maximize TPs), but
- ...we'd also always get every actual negative wrong (maximize FPs).

Similarly, if we set $\tau = 0.99999$. Then:

- We would essentially *always* predict $\hat{Y}_i = 0$, which means
- ...we would always correctly predict all the actual negatives (maximize TNs), but
- ...also always get every actual positive wrong (maximize FNs).

Values of τ between the extremes trade off true positives for false positives; as τ increases, we have fewer of the former and more of the latter.

NAFTA Examples

```
> # Tau = 0.2:
```

```
> Hats02<-ifelse(NAFTA.fit$fitted.values>0.2,1,0)
> CrossTable(NAFTA$Vote,Hats02,prop.r=FALSE,prop.c=FALSE,
  prop.t=FALSE,prop.chisq=FALSE)
```

NAFTA\$Vote	Hats02		Row Total
	0	1	
0	96	104	200
1	1	233	234
Column Total	97	337	434

TPR = $233/234 = 0.996$

FPR = $104/200 = 0.520$

```
> # Tau = 0.8:
```

```
> Hats08<-ifelse(NAFTA.fit$fitted.values>0.8,1,0)
> CrossTable(NAFTA$Vote,Hats08,prop.r=FALSE,prop.c=FALSE,
  prop.t=FALSE,prop.chisq=FALSE)
```

NAFTA\$Vote	Hats08		Row Total
	0	1	
0	178	22	200
1	123	111	234
Column Total	301	133	434

TPR = $111/234 = 0.474$

FPR = $178/200 = 0.890$

“Receiver Operating Characteristic” (ROC) Curves

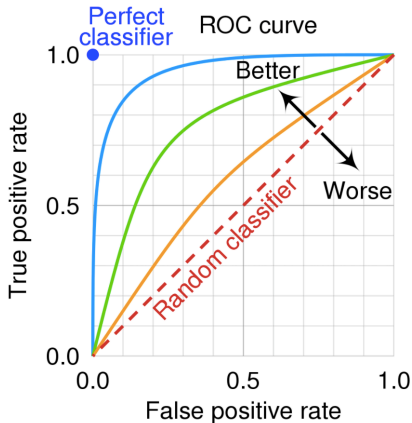
Now, imagine:

1. Fit a model
2. Choose a value of τ very near 0
3. Generate \hat{Y}_i s
4. Calculate and save the TPR and FPR for that value of τ
5. Increase τ by a very small amount
6. Go to (3), and repeat until τ is very close to 1.0

We could then plot the true positive rate vs. false positive rate (i.e., *Specificity* vs. $1 - \textit{Sensitivity}$)

ROC Curves (continued)

- If the model fits perfectly, it will have a 1.0 true positive rate, and a 0.0 false negative rate
- If the model fits no better than random chance, the curve defined by those points will be a diagonal line.
- (Intuition: If each prediction is no better than a (weighted) coin flip, the rate of true positives and false positives will increase together.)
- In between these extremes, better-fitting models will have curves that are closer to the upper-left corner



(Source)

“AUROC”: Area under the ROC curve
→ assessment of model fit

ROC Curves Implemented

```
> library(ROCR)

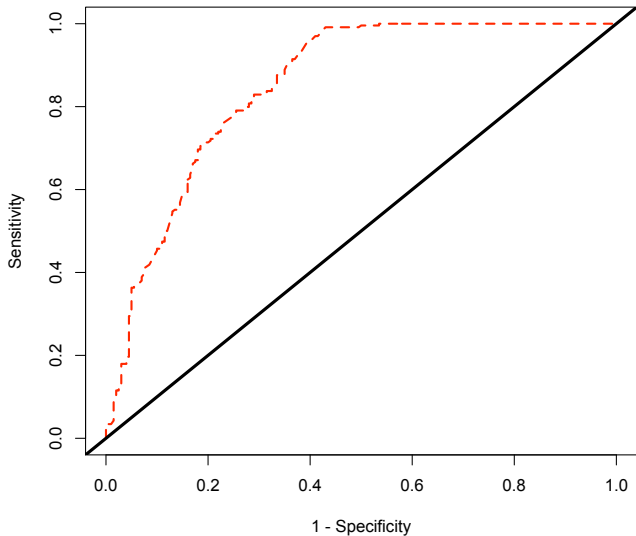
> NAFTA.hats<-predict(NAFTA.fit,type="response")

> preds<-ROCR::prediction(NAFTA.hats,NAFTA$Vote)

> plot(performance(preds,"tpr","fpr"),lwd=2,lty=2,
       col="red",xlab="1 - Specificity",ylab="Sensitivity")

> abline(a=0,b=1,lwd=3)
```

ROC Curve: Example



Interpreting AUROC Curves

- Area under ROC = 0.90-1.00 → Excellent (A)
- Area under ROC = 0.80-0.90 → Good (B)
- Area under ROC = 0.70-0.80 → Fair (C)
- Area under ROC = 0.60-0.70 → Poor (D)
- Area under ROC = 0.50-0.60 → Total Failure (F)

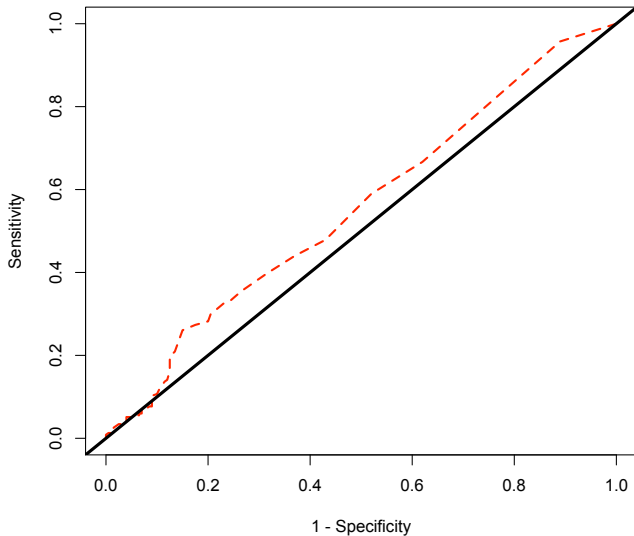
ROC Curve: A Poorly-Fitting Model

Model is:

$$\Pr(\text{vote}_i = 1) = f[\beta_0 + \beta_1(\text{PropHisp}_i) + u_i]$$

```
> NAFTA.bad<-with(NAFTA,  
  glm(Vote~PropHisp,family=binomial(link="logit")))  
> NAFTA.bad.hats<-predict(NAFTA.bad,type="response")  
> bad.preds<-ROCR::prediction(NAFTA.bad.hats,NAFTA$Vote)  
  
> plot(performance(bad.preds,"tpr","fpr"),lwd=2,lty=2,  
  col="red",xlab="1 - Specificity",ylab="Sensitivity")  
> abline(a=0,b=1,lwd=3)
```

Bad ROC!



Comparing ROCs

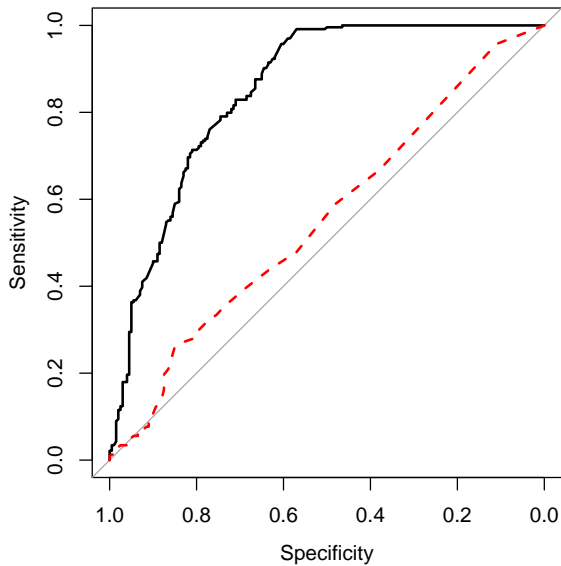
```
> install.packages("pROC")
> library(pROC)

> GoodROC<-roc(NAFTA$Vote,NAFTA.hats,ci=TRUE)
> GoodAUC<-auc(GoodROC)
> BadROC<-roc(NAFTA$Vote,NAFTA.bad.hats,ci=TRUE)
> BadAUC<-auc(BadROC)

> GoodAUC
Area under the curve: 0.85

> BadAUC
Area under the curve: 0.556
```

Combined Plot



Model Fitting, etc.:

- `glm` (in base stats)
 - `Binary responses = family(binomial)`
 - Links: `logit`, `probit`, `cloglog`, `log`, `cauchit` (Cauchy)
- Some `easystats` packages:
 - `datawizard` (standardizing variables, etc.)
 - `correlation` (what the name says...)

Model Interpretation + Visualization:

- `modelsummary` (tables and plots of estimates, ORs, etc.)
- `marginaleffects` (generate and plot of predictions, etc.)
- `margins` (marginal effects)
- `ROCR`, `pROC` (generate / plot ROC curves, calculate AUROC)
- `easystats` packages:
 - `report + parameters` (tables, output, etc.)
 - `modelbased + effectsize` (substantive interpretation of models)
 - `performance` (model fit: R^2 , AUROC, etc.)

Other Binary-Response Topics

Things we'll probably talk about later:

- Rare events and “separation”
- Binary responses in panel / longitudinal data
- Multilevel / hierarchical models for binary responses
- Models with (binary) sample selection
- Measurement models for binary outcomes (e.g., item response models)
- Semi- and non-parametric models (see, e.g., Horowitz and Savin 2001)
- “Heteroscedastic” models (where $\sigma_i^2 \neq \sigma^2 \forall i$) (see, e.g., Alvarez and Brehm 1995, 1997; Tutz 2018)
- “Bivariate” probit models, where

$$\{Y_{1i}, Y_{2i}\} \sim BVN(0, 0, 1, 1, \rho)$$