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Unit 1

Method 1

Q
H 5

⟹ • Graph: A set of vertices connected by edges

• Connected graph: A graph where there is a path between every pair of vertices

• Degree of vertex: The number of edges incident to the vertex

• Circuit: A closed path with no repeated edges or vertices except the starting and ending vertex.

• tree: A connected graph with no circuits

• Path tree: A tree where intermidiate vertices have 2 and end vertices have degree 1

• Star ~~graph~~ tree: A tree with one central vertex of degree n-1 and all other vertices (leaves) of degree 1.

Q
H 6

⟹ • Family trees: Representing ancestry and descendants

• File systems: Organizing files and folders hierarchically.

• Decision Trees: Modeling decisions and their possible Consequences.

Method 2.

Q H5

⟹ Distance : Number of edges in shortest path between two vertices.

- Eccentricity : Maximum distance from vertex to any other vertex
- Radius : Minimum eccentricity among all vertices.
- Center : Vertex or vertices with eccentricity equal to radius.

Q H6

⟹

let a tree be with 3 vertices (path tree)


$t$ $\begin{bmatrix} E(1) = 2 \\ E(2) = 1 \\ E(3) = 2 \end{bmatrix}$

We know that

Diameter of $t$ = 2

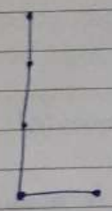
Radius of $t$ = 1 [2 is the center of $t$]

Hence Diameter = 2 × Radius

Hence Proved

Q H7

⟹



let $G_1$ be tree with vertices 5

$G_1$

And the Given tree $G$ is

→ The degree sequences of both Graphs are given below

$G_1 = (1, 1, 2, 2, 2)$

$G = (1, 1, 2, 2, 2)$

We know that

$\sum d(v)_{t_5} = 8$

for $G_1$ $\sum d(v) = 8$

for $G$ $\sum d(v) = 8$.

∴ $G_1$ & $G$ are isomorphic Graph/tree

Q H7

⟹ Tree with more leaves push some vertices farther from "the farthest leaf", raising those vertices' eccentricity; however, the center(s) remains the vertices of minimum eccentricity.

~~Total degree sum is fixed at 2(n-1)~~

Q H8

(i) $d(a,b) = 1$ $\qquad$ $d(j,k) = 1$

$d(a,c) = 2$

$d(a,d) = 3$

$d(a,e) = 4$

$d(a,f) = 5$ . . .

(ii)

| | | |
|---|---|---|
| $E(a) = 6$ | $E(f) = 5$ | $E(k) = 6$ |
| $E(b) = 5$ | $E(g) = 6$ | $E(l) = 6$ |
| $E(c) = 4$ | $E(h) = 6$ | $E(m) = 6$ |
| $E(d) = 3$ | $E(i) = 5$ | $E(n) = 6$ |
| $E(e) = 4$ | $E(j) = 5$ | |

(iii) Radius = 3

(iv) Diameter = 6

(v) Center = d

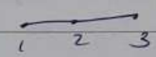
H9

⟹ Minimum = 2. Every tree with $n \geq 2$

$\because$ Sum of degree is $2(n-1)$;

Distribute this over n positive degrees leaves two 1's.

Ex:-> A path tree with 3 v.

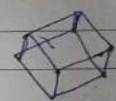


Method 3

H5

⟹ Given graphs

$G_1$



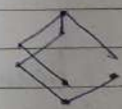
$G_2$

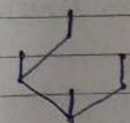
for $G_1$

1) 

2) 

for $G_2$

1) 

2) 

H6

$\Longrightarrow$ In $G = P_n$, then $G$ is connected and has no circuits.

$\longrightarrow$ The $P_n$ contains $n$ vertices with $(n-1)$ edges

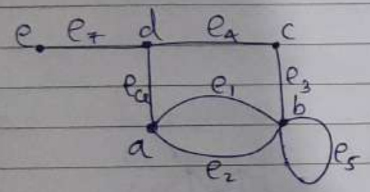
$\longrightarrow$ It is minimally connected.

$\therefore$ $P_n$ is a ~~tree~~ spanning tree on its own

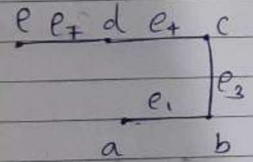
H7

$\Longrightarrow$ Given

G.




spanning tree from G =



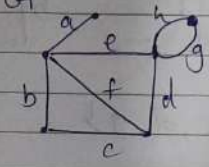

branch $\Rightarrow \{e_1, e_3, e_4, e_7\}$

chord $\Rightarrow \{e_2, e_5, e_6\}$

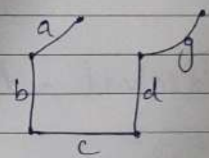
Method 4

H3

$\Longrightarrow$ Given G




Spanning tree from G.




branch = $\{a, b, c, d, g\}$

chord = $\{e, f, h\}$

fundamental circuits :→

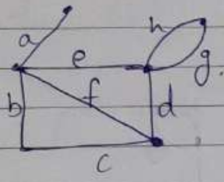
1. $\{e, b, c, d\}$

2. $\{f, b, c\}$

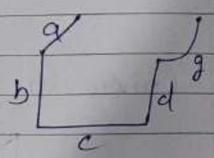
3. $\{h, g\}$

H4

⟹ Given G



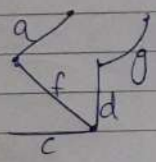

tree



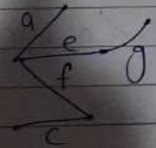

branch = {a, b, c, d, g}

chord = {e, f, h}

1) add 1 chord and remove 1 branch.




1



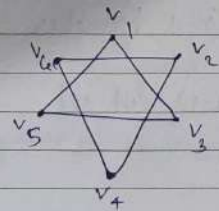

2

## Unit 2

Method 1

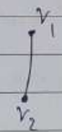
H5

⟹ for $G_1$



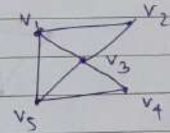

cut set ⇒ {($v_1$, $v_5$), ($v_1$, $v_3$)}

for $G_2$




cut-set = {($v_1$, $v_2$)}

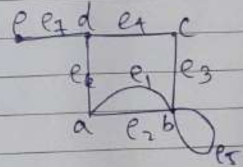
for $G_3$




cut-set = {($v_1$, $v_2$), ($v_1$, $v_3$), ($v_1$, $v_5$)}

H6

⟹ Given G




a) {$e_4$, $e_1$, $e_2$} = cut-set

b) {$e_1$, $e_6$, $e_4$} = not cut-set

c) {$e_1$, $e_2$} = not cut-set

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H7
⟹ A tree contains $(n-1)$ edges.

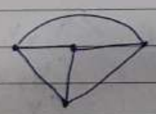
∴ A tree have $(n-1)$ branches

∴ A tree has $(n-1)$ cut-set

Method 2

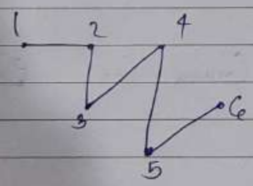
H3
⟹ Given

G1



G2

Spanning tree



fundamental cut-set ⇒ $\{(1,3), (1,2), (1,4)\}$

$\{(1,2), (1,3)\}$

No. of f.C.S ⇒ 3 [no. of branches]

5 [No. of branches]

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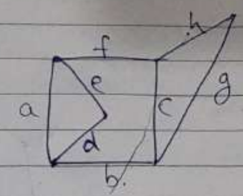
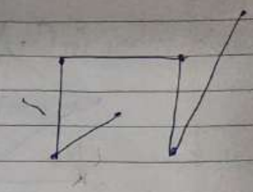
H4
⟹ Given G.





Given s.t = $\{d, a, f, c, g\}$

f.C.S ⇒ $\{h, g\}$.

There are 4 other fundamental cutset

$\{h, c, b\}, \{f, b\}, \{e, d\}, \{a, e, b\}$

H5
⟹ Given that

The graph is made by removing an edge from minimally connected graph

And given its Rank = 9.

We know that

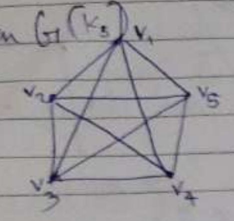
Rank = $n - k$ [Here k is component]

$9 = n - 2$ [it's a disconnected graph]

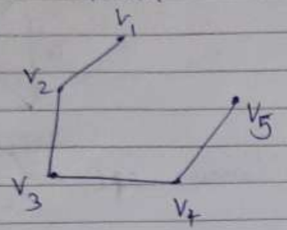
$\boxed{\therefore n = 11}$

H6

$\Longrightarrow$ Given $G(K_5)$




Sp. tree




* fundamental Circuits.

$\Rightarrow \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_1, v_3, v_4\},$

$\{v_1, v_3, v_5\}, \{v_1, v_4, v_5\}$

where.

$\{v_2, v_3, v_4, v_5\} \therefore \{v_2 v_3, v_2 v_4, v_2 v_5, v_3 v_4, v_3 v_5, v_4 v_5 \ldots\}$

* fundamental cutset

$\{(v_1 v_5), (v_1 v_4), (v_1 v_3) (v_1 v_2)\},$

$\{(v_1 v_5), (v_1 v_4), (v_1 v_3), (v_2 v_5), (v_2 v_4) (v_2 v_3)\}$

$\{(v_1 v_5) (v_1 v_4) (v_2 v_5) (v_3 v_5) (v_2 v_4) (v_3 v_4)\}$

$\{(v_1 v_5) (v_2 v_5) (v_3 v_5) (v_4 v_5)\}$

H7

$\Longrightarrow$ ~~A tree on n vertices has exactly n-1 cut-set~~

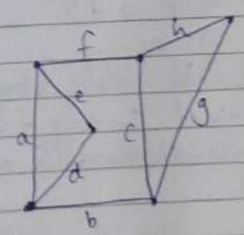
Let S be a cut-set of G. By definition, G-S is disconnected. The Complement of S are the edges remaining after deleting S. Since G-S is disconnected no spanning tree can be contained entirely in $E(G) \setminus S$. Therefore the Complement of cut-set Cannot contain a Spanning tree.

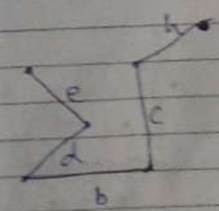
$\longrightarrow$ Let T be a spanning tree of G. The complement $E(G) \setminus T$ are the chords. By the notes, every cut-set in a connected graph must Contain at least one Branch of every spanning tree of G; Since $E(G) \setminus T$ contains no branch of T, it cannot Contain a cut-set. Therefore the complement of a spanning tree does not contain a cut-set.

Method 3

H3

$\Longrightarrow$ Given G.




t

Here,
Branch $= \{e, d, b, c, h\}$
chords $= \{a, f, g\}$

→ With respect to a spanning tree T, a chord $C_i$ that determines a fundamental circuit C occurs in every fundamental cut-set associated with the branches in that circuit and in no other fundamental cut-set.

for example.

for chord a

which creates a fundamental circuit

$\Rightarrow \{a, e, d\}$

∴ chord will be present in the fundamental cut-set of all branches of that circuits.

for fundamental cut-sets.

$\Rightarrow \{b, a, e\}$

$\Rightarrow \{f, a, d\}$

- Dually, a branch $b_i$ that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S and in no other fundamental circuit.

for example.

for branch e.

fundamental cut-set $= \{a, e, f\}$

∴ fundamental circuits containing e ⇒

$\{a, e, d\}$, $\{b, e, d, b, c\}$

Method 4

H5

⟹ Given : Tree T and a vertex v with $d(v) > 1$

To prove : v is a cut-vertex.

→ A tree has no cycles and is minimally connected.

If $d(v) \geq 2$, then there are at least two distinct neighbors u & w of v.

→ In a tree, the unique path between u to w must contain v

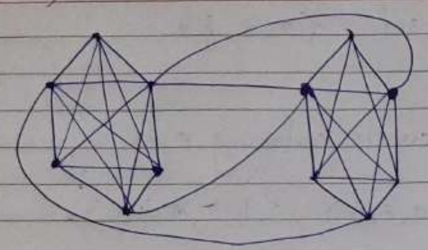
→ By removing vertex v, you are removing the distinct path between u & w

which makes them a two different components.

∴ The graph becomes disconnected.

∴ The vertex v is a cut-vertex.

H6 ⟹



H7 ⟹ Edge Connectivity :→ ($\lambda(G)$): The ~~minimum~~ minimum number of edges whose removal disconnects G.

Example :→

In a tree $\lambda = 1$ because trees are minimally connected.

Vertex Connectivity :→ the minimum no. of vertices whose removal disconnects G.

for Example

$K_{1n}$ [Here $k = 1$ Because all n vertices are only connected to 1 virtex]

H8 ⟹ The vertex Connectivity of $K_{1n} = 1$

The Edge Connectivity of $K_{1n} = 1$.

H9 ⟹ A cut-edge is an edge whose removal increases the no. of connected components of the graph.

→ Yes, In a tree, every edge is a bridge. Thus every tree with $n \geq 2$ has at least one cut-edge

H10 ⟹ Given, degree 2 Regular Graph with $n \geq 3$

The vertex & Edge connectivity of this graph is 2

Unit 3

H5

⟹ Given: place a minimum no. of queens on an 8X8 board so every square is controlled.

→ The domination no. of for queens is 5.

let Horizontal scalling be (a, b, c, d, e, f, g, h)

& Vertical Scalling be (1, 2, 3, 4, 5, 6, 7, 8)

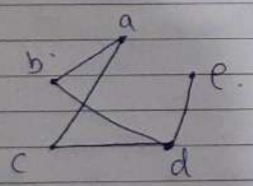
Then one of the Solution is

⇒ (a1, d1, g2, b6, b8)

H6

⟹ Given: $v = \{a, b, c, d, e\}$

$E = \{(a,b), (a,c), (b,d), (c,d), (d,e)\}$.




$d_1 = \{a, d\}$

$d_2 = \{b, d\}$

H7

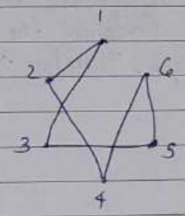
⟹ Given: $V = \{1, 2, 3, 4, 5, 6\}$

$E = \{(1,2), (1,3), (2,4), (3,5), (4,6), (5,6)\}$

1) $\{2, 5, 6\}$ → yes, This set can cover all the vertex.

2) $\{1, 3, 4\}$ → yes, ,,

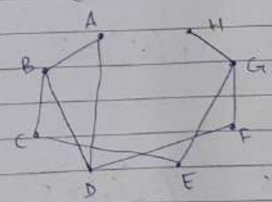
3) $\{2, 5\}$ → yes. ,,




H8

⟹ Given: $V = \{A, B, C, D, E, F, G, H\}$

$E = \{(AB), (AD), (B,C), (BD), (CE), (DF), (EG), (FG), (GH)\}$



for the Given Graph

→ Minimum Number of Cameras = 2.

{B, G}.

H9

⟹ Given $S_n = K_{1,n-1}$

→ The center vertex has degree $(n-1)$, so its closed neighborhood is the entire vertex set. Thus the singleton set Contains the center dominates the whole graph. Therefore $\gamma(S_n) = 1$

## Method 2

H7

⟹ Find: $\gamma(P_n) = ?$,

$\gamma(C_n) = ?$, for $6 \le n \le 9$

for $n = 6$.

$\gamma(P_6) = 3$, $\gamma(C_n) = 3$.

for $n = 9$

$\gamma(P_9) = 5$, $\gamma(C_9) = 5$

$\therefore \gamma(P_n) = \gamma(C_n)$ [for $6 \le n \le 9$] [3, 4, 4, 5]

H8

⟹ 1) $K_6$:

$\gamma(K_6) = 1$ & $\gamma_t(K_6) = 2$

2) $P_6$ : $\gamma(P_6) = 2$ & $\gamma_t(P_6) = 3$.

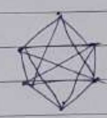
3) $C_6$ : $\gamma(C_6) = 2$ & $\gamma_t(C_6) = 3$

4) $K_{4,3}$ : $\gamma(K_{4,3}) = 3$ & $\gamma_t(K_{4,3}) = 4$.

H9

⟹ Given : $\gamma(G) = 2 \times \gamma_t(G)$

⟹ $K_6$

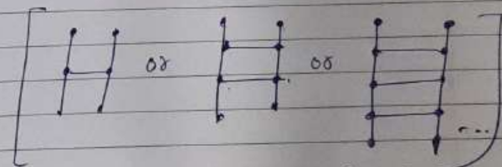


H10

⟹ Given $\gamma(G) = \gamma_t(G) = k$ ; $k \ge 2$.

for Graph such as ladder graph.

Like



$k$ is $\ge 2$. & $\gamma(G) = \gamma_t(G)$

H12

$\Longrightarrow$ Given $G_1$

1) The domination number of $G_1$ = 3.

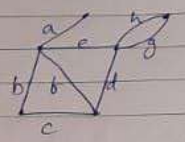
2) The total domination ~~number~~ number of $G_1$ = 4.

# Method 3

H4

$\Longrightarrow$ Given:

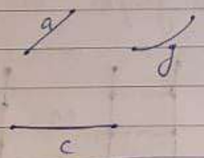
$G_1$



$G_2$

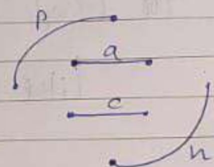
Maximum matching of $G_1$

$\Rightarrow$



$\boxed{\therefore \text{Matching no. of } G_1 = 3}$

Maximum matching of $G_2$

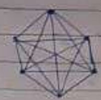


$\boxed{\therefore \text{Matching no of } G_2 = 4}$

H5

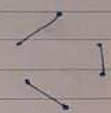
$\Longrightarrow$ Given $G_1 = K_6$

$K_6$ =



for maximum matching pairs = $\frac{n}{2} = \frac{6}{2} = 3$

$\boxed{\therefore \nu(K_6) = 3}$ $\longrightarrow$



H6

$\Longrightarrow$ Given $G_1 = C_6$

$C_6$ =

$\boxed{\nu(C_6) = 3}$ $\longrightarrow$

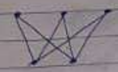
H7

$\Longrightarrow$ Given $G_1 = P_6$

$P_6$ =

$\boxed{\nu(P_6) = 3}$, $\longrightarrow$

H8
$\Longrightarrow$ Given $G = K_{2,3}$

$K_{2,3} \longrightarrow$ 

$\boxed{\nu(K_{2,3}) = 2} \longrightarrow$ 

H9
$\Longrightarrow$ Given $G = S_n$

$S_n =$ 

$\boxed{\nu(S_n) = 1} \rightarrow$ Because there is only one non-pendent vertex & which has $n-1$ degree

H10
$\Longrightarrow$ Prove = maximum is always maximal but maximal is not necessarily maximum.

We know that,

Maximal matching :-> maximum no. of edges with no adjecent edges.

Maximum Matching :-> It is a maximal matching which contains the highest no. of edges.

$\because$ Maximum is sub-set of Maximal

Hence every Maximum is Maximal but not converse is not true.

# Method 4

H5
$\Longrightarrow$ Given = Connected graph of order 4 that has no perfect Maching

let Graph be $K_{1,3}$

$K_{13} =$ 

In above graph, which has order 4 not have perfect matching.

H6
$\Longrightarrow$

let $G$ be $K_{BG}$ [ B = set of Boys, G = set of Girls ]

By Hall's theorem,

if for every subset $X \subseteq B$

$|N(X)| \geq |X|$ , $\therefore \boxed{|N(B)| \geq |B|}$

H7

→ Given $U = \{v, w, x, y, z\}$, $W = \{a, b, c, d, e\}$

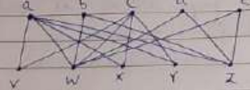
$G_1$ =

for $G_1$, for every $X \subseteq U$

$|N(X)| \geq |X|$

$\therefore$ U can be matched to W for $G_1$,

$G_2$ =



for $G_2$, let $X$ be $= \{b, d, e\}$

$\therefore N(X) = \{w, z\}$

Here, $|X| > |N(X)|$

→ it doesn't follows the Hall's theorem

$|N(X)| \geq |X|$

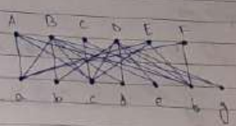
$\boxed{\therefore \text{ U can not be Matched in } G_2}$

H8 → Given

Applicants $= \{A, B, C, D, E, F\}$

positions $= \{a, b, c, d, e, f, g\}$

$G_1$ =



or

$A = a, c, f$

$B = a, b, c, d, e, g$

$C = c, f$

$D = b, c, d, e, f, g$

$E = a, c, f$

$F = a, f$

for $G_1$,

$X \subseteq$ Applicants,

$|N(X)| \ngeq |X|$

$\therefore$ There, a matching doesn't exist

for $X = \{A, C, E, F\}$

$N(X) = \{a, c, f\}$ $\therefore |N(X)| < |X|$

H9 ⟹

1) for every nonempty $X \subseteq X$-partite set.

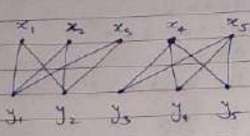
Compute neighbourhood $N(X)$. If any $X$ has $|N(X)| < |X|$, Hall fails and no perfect matching exists. Otherwise perfect matching exists.

for example ⇝

let $X = \{x_1, x_2, x_3, x_4, x_5\}$,

$Y = \{y_1, y_2, y_3, y_4, y_5\}$

And




Here, for $X' = \{x_1, x_2, x_3\}$

$N(X') = \{y_1, y_2\}$

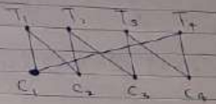
so, $|N(X')| = 2 < 3 = |X'|$.

Thus Halls fails.

∴ No perfect Matching exists.

H10 ⟹ Given $T = \{T_1, T_2, T_3, T_4\}, C = \{C_1, C_2, C_3, C_4\}$

$T_1 : C_1, C_2$
$T_2 : C_2, C_3$
$T_3 : C_3, C_4$
$T_4 : C_1, C_4$



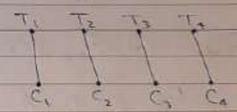

[ By, Hall's theorem $|N(X)| \leq |X|$ ]

for $\forall X \subseteq T$

$|N(X)| \leq |X|$ or $|N(T)| \leq |T|$

∴ There is a perfect matching for T & C.

Ex ⇒ $T_1 : C_1$
$T_2 : C_2$
$T_3 : C_3$
$T_4 : C_4$

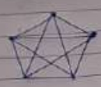



H11 ⟹ Same as $\boxed{H8}$.

Unit 4

Method 1

H5 $\Longrightarrow$ Given $K_5$, $d(v_i) = 4$,



We know $L = D - A$

for $K_n$: $D = (n-1)I$, $A = J - I$ (where $J$ is the all-ones matrix)

$$\therefore L(K_5) = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

H7 $\Longrightarrow$ Given $G = C_5$, $d(v_i) = 2$.

By $L = D - A$

$D = [2, 2, 2, 2, 2]$

$$\therefore L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

H8 $\Longrightarrow$ Given $G = K_{4,4}$, $d(v_i) = 4$

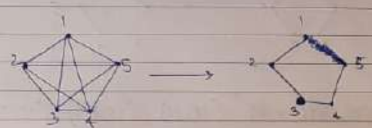
$D = \mathrm{diag}(4,4,4,4,4,4,4,4)$ or $4\,\mathrm{diag}(I_8)$.

$$A = \begin{bmatrix} O_{4\times4} & J_{4\times4} \\ J_{4\times4} & O_{4\times4} \end{bmatrix}$$ (where $J$ is all 1's matrix)

$$\therefore L(K_{4\times4}) = D - A = \begin{bmatrix} 4I_{4\times4} & -J_{4\times4} \\ -J_{4\times4} & 4I_{4\times4} \end{bmatrix}$$

Method 2

H5 $\Longrightarrow$ Given $G = K_5$



$n = 5$, $e = 10$, $c = e - n + 1 = 6$, $B = 4$.

f - cut-sets = {(13, 14, 15), (13, 14, 15, 24, 25), (14, 15, 24, 25, 35), (15, 25, 35)}

[chords only]

$$\therefore C_f(K_5) = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cccccc|cccc} 13 & 14 & 15 & 24 & 25 & 35 & 12 & 23 & 34 & 45 \end{array}$$

$$\left[\begin{array}{cccccc|cccc} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

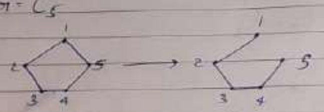
H6

$\Longrightarrow$ Given $G = S_5$, $n = 5$, $B = 4$, $C = 0$ [$S_5$ is tree]

$$\therefore C_f(S_5) = I_5 \text{ or } \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

H7

$\Longrightarrow$ Given $G = C_5$




$n = 5$, $B = 4$
$C = 1$

f-cut sets = {(15, 12), (15, 23), (15, 34), (15, 45)}

$$\therefore C_f(C_5) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

H8

$\Longrightarrow$ Given $G =$


f-cut sets = {(e, f), (e, h, d), (c, i, d), (c, g, b), (a, b)}

$$\therefore C_f(G) = \begin{array}{c|cccc|ccccc} & e & c & d & b & f & h & i & g & a \\ \hline 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

## Method 3

H4

$\Longrightarrow$ Given, $A = \begin{bmatrix} -5 & 4 & 34 \\ 0 & 0 & 4 \\ 0 & 0 & 4 \end{bmatrix}$.

for triangular Matrices, eigenvalues are diagonal entries.

$\therefore \lambda \in (-5, 0, 4)$.

$|A| = \lambda_1 \times \lambda_2 \times \lambda_3 = -5 \times 0 \times 4 = 0$.

$\therefore$ A is not investible

H5 $\Longrightarrow$ Given $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\because$ characteristic polynomial is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3$$

$S_1 = 0 + 0 + 0 = 0$ [trace of A]

$S_2 = (0-1) + (0-1) + (0-1) = -3$

[sum of principal 2x2 minors]

$S_3 = |A| = 2$

$\therefore \lambda^3 - 3\lambda - 2 = 0$

$(\lambda+1)(\lambda+1)(\lambda-2) = 0.$

$$\boxed{\therefore \begin{cases} \lambda_1 = -1 \\ \lambda_2 = -1 \\ \lambda_3 = 2 \end{cases}}$$

H6 $\Longrightarrow$ Given $G = K_4$

We know that

$L = nI - J$

$L = 4I - J$

$\therefore L = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$

& we know that

for $K_n \Rightarrow \lambda \in (0^1, n^{n-1})$

$$\boxed{\therefore \lambda \in \langle 0, 4, 4, 4 \rangle}$$

for $L = \begin{bmatrix} 3-\lambda & -1 & -1 & -1 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ -1 & -1 & -1 & 3-\lambda \end{bmatrix}$

let $a = 3-\lambda$ & $d = 4-\lambda$

after Row operations $L = \begin{bmatrix} a & -1 & -1 & -1 \\ -d & d & 0 & 0 \\ -d & 0 & d & 0 \\ -d & 0 & 0 & d \end{bmatrix}$

By Solving eq

we get $-\lambda(1-\lambda)^3 = 0$

$\therefore \lambda \in (0, 1, 1, 1)$

H7

$\Longrightarrow$ for $A_{P_4}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 1 & -\lambda & 1 & 0 \\ 0 & 1 & -\lambda & 1 \\ 0 & 0 & 1 & -\lambda \end{bmatrix}$$

$$|A - \lambda I| = \lambda^4 - 3\lambda^2 + 1 = 0.$$

let $x = \lambda^2$; then

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$\therefore \lambda = \pm\sqrt{\frac{3 \pm \sqrt{5}}{2}}$$

$$\therefore \lambda = \pm 1.6180, \pm 0.6180.$$

for $L_{P_4}$

$$(L - \lambda I) = \begin{bmatrix} 1-\lambda & -1 & 0 & 0 \\ -1 & 2-\lambda & -1 & 0 \\ 0 & -1 & 2-\lambda & -1 \\ 0 & 0 & -1 & 1-\lambda \end{bmatrix}$$

after solving $|L - \lambda I|$

$$= \lambda^4 - 6\lambda^3 + 10\lambda^2 - 4\lambda = 0$$

$$\lambda(\lambda^3 - 6\lambda^2 + 10\lambda - 4) = 0$$

$$\lambda(\lambda - 2)(\lambda^2 - 4\lambda + 2) = 0$$

$$\boxed{\therefore \lambda = 0, 2, 2 \pm \sqrt{2}}$$

for $S_4$

$A_{S_4}$

$$(A - \lambda I) = \begin{bmatrix} -\lambda & 1 & 1 & 1 \\ 1 & -\lambda & 0 & 0 \\ 1 & 0 & -\lambda & 0 \\ 1 & 0 & 0 & -\lambda \end{bmatrix}$$

$$|A - \lambda I| = \lambda^4 - 3\lambda^2 = \lambda^2(\lambda^2 - 3) = 0.$$

$$\boxed{\therefore \lambda = 0, 0, \pm\sqrt{3}}$$

for $L_e$,

$$(L-\lambda I) = \begin{bmatrix} 3-\lambda & -1 & -1 & -1 \\ -1 & 1-\lambda & 0 & 0 \\ -1 & 0 & 1-\lambda & 0 \\ -1 & 0 & 0 & 1-\lambda \end{bmatrix}$$

$$|L-\lambda I| = \lambda(\lambda-4)(\lambda-1)^2 = 0.$$

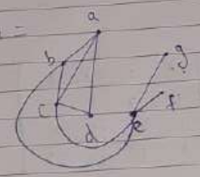
$$\boxed{\therefore \lambda = 0, 4, 1, 1}$$

## Method 4

H2

⟹ Given.

$G =$




Here.

→ The graph contains two dense triads $\{a,b,c,d\}$ & $\{e,f,g\}$ with bridging be, ce.

→ Spectral bisection separates these as:

$S=\{a,b,c,d\}$, $T=\{e,f,g\}$.