

## Probability and Baye's theorem

Probability  $\rightarrow$  Share of success | Total no of possible outcomes.

eg if you toss a coin, what is the probability that you will get a head?

$$\rightarrow P(X=H) = 1/2 = 0.5$$

eg dice is rolled. What is the probability that the outcome is an even no?

$\rightarrow$  1, 2, 3, 4, 5, 6  
           $\uparrow$            $\uparrow$            $\uparrow$

$$P(\text{Even no}) = \frac{3}{6}$$

\* Probability rules:-

① For any even A  $\rightarrow 0 \leq P(A) \leq 1$

② The sum of all probabilities of all possible outcome is 1.

$P(H)$  or  $P(T)$

$$P(H) + P(T) = 1$$

$$\boxed{P(T) = 1 - P(H)}$$

$\leftarrow$  rule of subtraction.

③ Complement rule

$$P(\text{not } A) = 1 - P(A)$$

$$\rightarrow P(H) = \frac{1}{2}$$

$$P(\text{not } H) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\rightarrow P(3) = \frac{1}{6}$$

$$P(\text{not } 3) = 1 - \frac{1}{6} = \frac{5}{6}$$

# ④ General addition rule

$$P(\underline{A} \text{ or } \underline{B}) = P(A) + P(B) - P(A \text{ and } B)$$



$$P(A) + P(B) - P(A \text{ and } B)$$

↓  
 $A \cap B, B \cap A$

# ⑤ Multiplication rule



→ Tossing a coin  
→ Throwing a dice



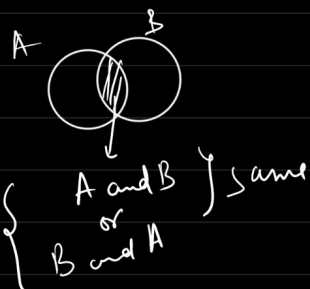
$P(Y) = \frac{2}{5}$  → one yellow ball is taken out  $P(Y) = \frac{1}{4}$

$P(R) = \frac{3}{5}$  → one yellow ball is taken out  $P(R) = \frac{3}{4}$

$$P(R \text{ and } Y) = P(R) \times P(Y|R)$$

$$= \frac{3}{5} \times \frac{3}{4}$$

→ The probability is changing based on the last event.  
These are called dependent events.



And if events are dependent

$$P(A \text{ and } B) = P(A) \times P(B/A)$$

Prob of A      ↓  
Probability of event B when A has already occurred.

$$P(A \text{ and } B) = P(A) * P(B/A) \text{ — ①}$$

$$P(B \text{ and } A) = P(B) * P(A/B) \text{ — ②}$$

equating eq<sup>n</sup> ① & ②

$$P(A) * P(B/A) = P(B) * P(A/B)$$

$$P(B/A) = \frac{P(B) * P(A/B)}{P(A)}$$

← Bayes' theorem

$$P(A/B) = \frac{P(A) * P(B/A)}{P(B)}$$

$P(A/B)$  = Prob of event A given B has already occurred

$P(B/A)$  = Prob of event B, given A has already occurred.

$P(A), P(B) \rightarrow$  Independent Prob of A and B

Q 10% of patients in a clinic have liver disease.  
Five percent of the clinical patients are alcoholics.  
Among these patients diagnosed with liver disease 7% are alcoholic.

What is prob of patients having liver disease given that he is an alcoholic?

$\rightarrow P(A) = \text{Prob of having liver disease} = 0.10$

$P(B) = \text{Prob of alcoholism} = 0.05$

$P(B/A) = 0.07$

$P(A/B) = ?$

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)} = \frac{0.07 \times 0.10}{0.05} = 0.14 \Rightarrow 14\%$$

\* Use of Bayes theorem

$\rightarrow$  Naive Bayes classifier  
ML model.

Bayesian statistics

$\downarrow$   
data analysis and parameter estimation based on bayes theorem

$x_1$ # of rooms	$x_2$ Area	$x_3$ locality	$y$ Price
-	-	-	-
-	-	-	-

$$P(y/x_1, x_2, x_3) = \frac{P(y) \cdot P(x_1, x_2, x_3/y)}{P(x_1, x_2, x_3)}$$

Bayes theorem.  $\rightarrow P(B)$