Bernoulli distribution

→ Binary ⟶ two outcomes

A discrete prob distribution of a r.v. which takes only two possible outcomes, typically labelled as success (coded as 1) and failure (coded as 0) which a fixed prob of success and failure, i.e sum of prob = 1

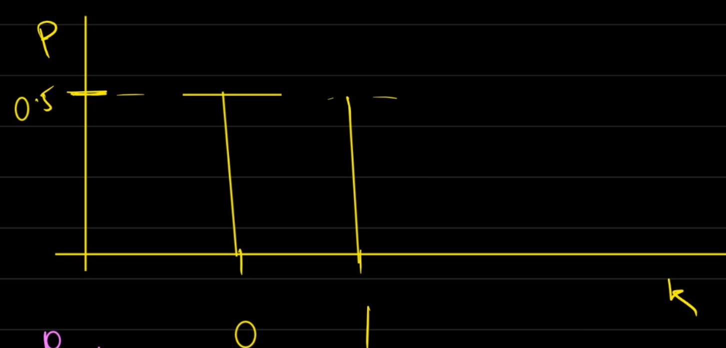
* To model any experiment with only two possible outcomes.

Eg. Tossing a coin.

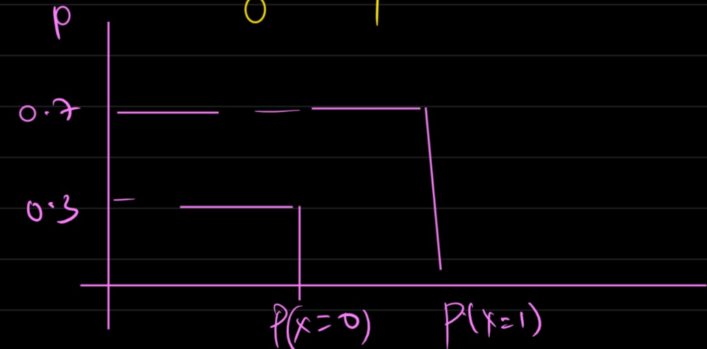
Head or tail $(k \in \{1, 0\})$

$$P(X=H) = P(X=1) = \frac{1}{2} = 0.5 = p$$

$$P(X=T) = P(X=0) = 1 - \frac{1}{2} = 1 - 0.5 = 0.5 = 1 - p = q$$



$$P(X=0) + P(X=1) = 1$$



pmf ⇒

$$P(X=k) = \begin{cases} p & \text{if } k=1 \\ 1-p & \text{if } k=0 \end{cases}$$

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Combining together

$$\boxed{P(X=K) = p^K(1-p)^{1-K}}$$

① if $K=1$

$$P(X=1) = p^1(1-p)^{1-1} = p\underbrace{(1-p)^0}_{1} = p$$

② if $K=0$ $\quad P(X=0) = \underbrace{p^0}_{1}(1-p)^{1-0} = 1-p$

$$\boxed{p + 1 - p = 1}$$

* Conditions of Bernoulli Distribution

① No of trial should be finite
② Each trial should be Independent.
③ Only two possible outcome — Pass / Fail.
④ Prob of each output should be same in every trial.

1	2	3	4
$P(H)=0.5$	$p(H)=0.5$	$p(H)=0.5$	$p(H)=0.5$

Examples

→ Tossing a coin
→ Someone has committed a fraud or not
→ getting a six or not
→ pass | fail
→ win / not win
→ Customer churn | convert

→ rain or not.

Q Bumrah bowls 6 balls at wicket with prob of 0.6 at hitting the stump with each ball. What is prob of not hitting a wicket?

→ Bernoulli dist (as only two outcomes hit the wicket or not)

$p(\text{hitting a wicket}) = 0.6$

$p(\text{hitting not a wicket}) = 1 - 0.6 = \underline{0.4}$

* Mean & Variance of Bernoulli distribution

Mean ⇓ p

Variance ↓ $p(1-p)$

① Mean

$$E(x) = \sum_{i=1}^{n} x\, p(x)$$

X can have two values 0 or 1

$= x=1 + x=0$

$= 1 \times 0.5 + 0 \times 0.4$

$= \underline{\underline{0.5}}$

assume a random Experiment

$p(x=1) = 0.6 = p$

$p(x=0) = 1-p = 0.4 = q$

$E(x) = 1 \times 0.6 + 0 \times 0.4$

$= \underline{\underline{0.6}}$

$\underline{\underline{E(x) = p}}$

② $Var(x) \Rightarrow E(x^2) - (\underbrace{E(x)}_{p})^2$

$$E(x^2) = \sum x^2 p(x)$$

$x=1 \quad x=0$

$= 1^2 \times p + 0 \times (1-p)$

$= \underline{p}$

$Var(x) = p - p^2$

$= p(1-p)$ — proved.