Probability distribution

→ Probability mass function

→ Probability density function.

* **Uniform distribution** → A uniform distribution refers to a type of probability distribution in which outcomes are equally likely.

Uniform distribution

→ Discrete Uniform Distribution

→ Continuous Uniform distribution.

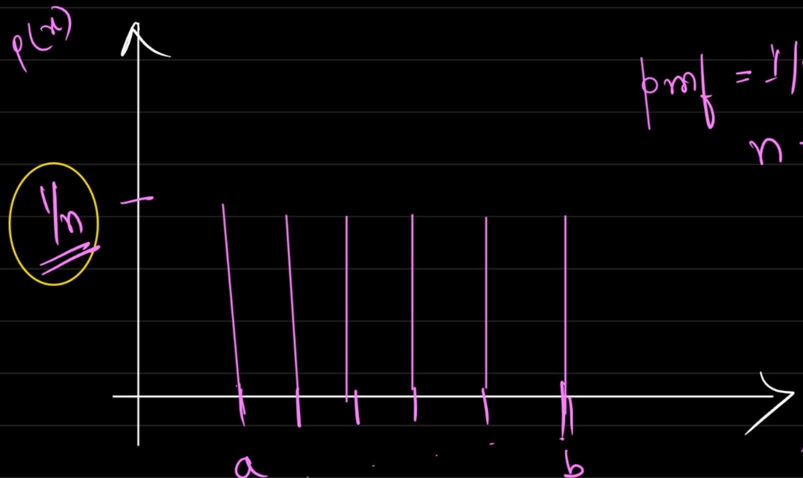
→ In a discrete Uniform distribution, the outcomes are discrete and have the same probability.

eg. rolling a dice

eg. tossig a coin.

eg. Picking up a card from well shuffled deck.

Notation of Uniform distribution: $U(a,b)$ | $\text{Unif}\{a,b\}$



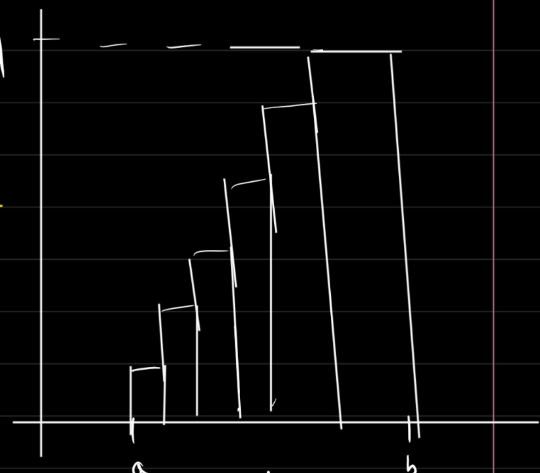
pmf of discrete UD

$\text{pmf} = 1/n$

$n = b - a + 1$

$= 6 - 1 + 1$

$\Rightarrow 6$



cdf of discrete UD

Q What is probability of getting 3 when you throw a dice?

$\rightarrow \quad p(x=3) \Rightarrow \frac{1}{6}$

* Mean of discrete Uniform distribution $= \frac{a+b}{2}$
* Variance of discrete Uniform distribution $= \frac{n^2-1}{12}$

dice → 1, 2, 3, 4, 5, 6

mean $= \frac{1+2+3+4+5+6}{6} = 3.5$

Expected value → In the long run average value of repeatations of experiment it represent.

→ long term average value of a random variable

$EV = \sum_{i=1}^{6} x_i \cdot p(x_i)$

$p(x_i)$ → probability of random value

x_i → random value

x	prob	$x \cdot p$
1	$1/6 = 0.167$	0.17
2	0.167	0.33
3	0.167	0.50
4	0.167	0.67
5	0.165	0.83
6	0.167	1

Sum$(x p) = 3.50$

* mean is used for frequency distribution
* Expected value is used for probability distribution

$E(x) \mid \mu = \sum_{i=1}^{N} x_i \, p(x_i)$

$Var(x) = E(x^2) - (E(x))^2$

$E(x) = \sum_{i=1}^{N} x_i \, p(x_i) = \sum_{i=1}^{N} x_i \frac{1}{N} = \frac{1}{N}(1+2+\dots N)$

$= \frac{1}{\cancel{N}}\left(\frac{\cancel{N}(N+1)}{2}\right)$

$= \frac{N+1}{2}$

→ Sum of first 'N' no $= \frac{N(N+1)}{2}$

1, 2, 3, ... 10

$\Rightarrow \frac{10(10+1)}{2}$

→ Sum of First N Squares $\Rightarrow \sum_{i=1}^{N} x^2 = \frac{N(N+1)(2N+1)}{6}$

$1^2+2^2+ \dots 10^2 \Rightarrow \frac{10(10+1)(2\times10+1)}{6}$

→ $(a+b)^2 = a^2+b^2+2ab$

$$E(x^2) = \sum_{i=1}^{N} x^2 p(x) = \frac{1}{N}\sum_{i=1}^{N} x^2 \Rightarrow \frac{1}{N}(1^2+2^2+ \cdots N^2)$$

$$\Rightarrow \frac{1}{\cancel{N}}\left(\frac{\cancel{N}(N+1)(2N+1)}{6}\right)$$

$$\Rightarrow \frac{(N+1)(2N+1)}{6}$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= \frac{(N+1)(2N+1)}{6} - \left(\frac{N+1}{2}\right)^2$$

$$= \frac{N(2N+1)+1(2N+1)}{6} - \frac{N^2+2N+1}{4}$$

$$Var(x) = \frac{N^2-1}{12}$$

$$\sigma = \sqrt{\frac{N^2-1}{12}}$$