ANOVA test

Hypothesis testing in ANOVA

→ Partitioning of variance in ANOVA

H_0: $\mu_1 = \mu_2 = \mu_3 = \cdots \mu_k$

H_A: Atleast one of the sample mean is not equal.

$\mu_1 = \mu_2 = \mu_3 \cdots \neq \mu_k$

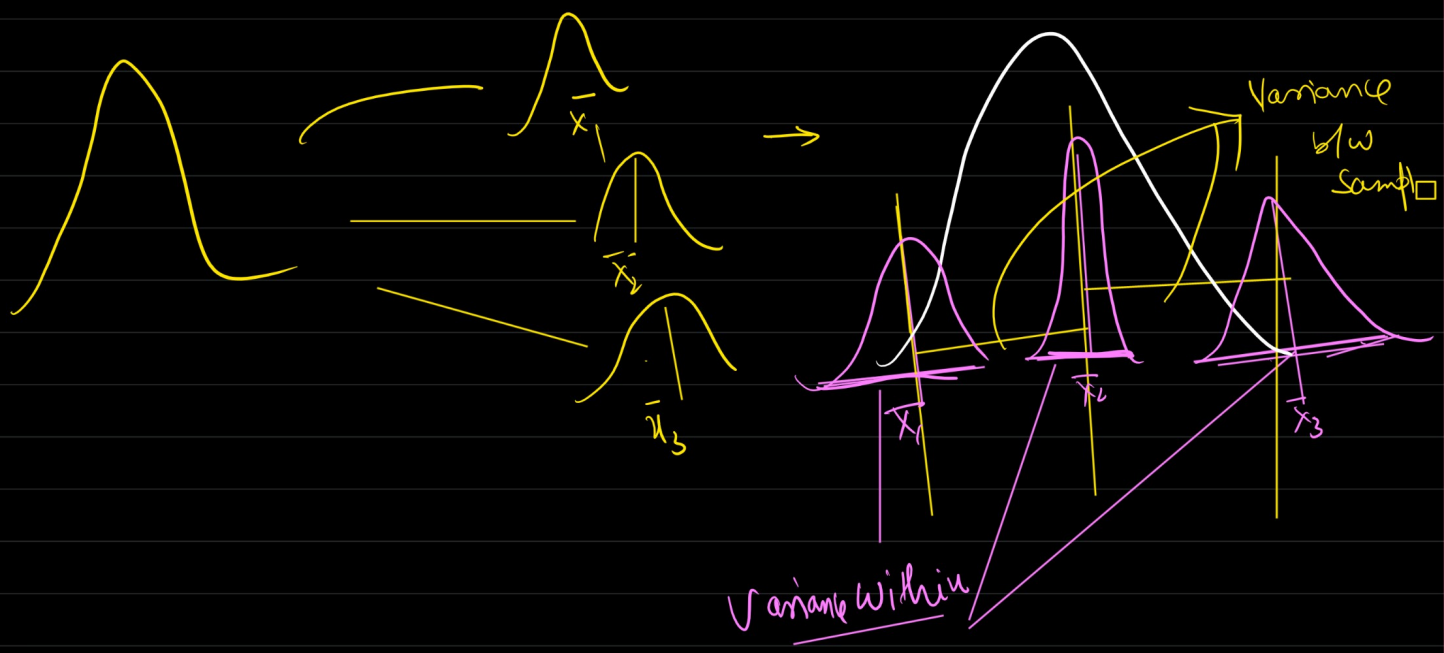
test statistics = $\frac{\text{Variance b/w Sample}}{\text{Variance within Sample}}$

var between the sample

x_1 x_2 x_3

Variance within Sample

x_1	x_2	x_3
1	5	9
2	6	10
3	7	11
4	8	12



One Way ANOVA

→ one factor with atleast two levels

Q. There are three dosage of a medicine given to three sample of the patients. The ratings if the headace is reduced (1-10). Are there differences in the three conditions ($\alpha = 0.05$)?

① H_0: $\mu_{0mg} = \mu_{10mg} = \mu_{100mg}$

H_A: Not all are equal

② $\alpha = 0.05$, one tail test

③ Calculate F statistics

$$\text{f statistics} = \frac{\text{Variance b/w Sample}}{\text{Variance within sample}}$$

0mg	10mg	100mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2
$\Sigma a_1 = 57$	$\Sigma a_2 = 47$	$\Sigma a_3 = 21$

	Sum of Squares (SS)	df	MS (SS/df)	F
✓ b/w the Sample				
✓ within sample				
Total.				

* Sum of squares b/w sample: $\sum \frac{(\sum a_i)^2}{n} - \frac{T^2}{N}$, where $n = 7$, $N = 21$

$0mg = \sum a_1 = 9+8+7+8+8+9+8 = 57$

$10mg = \sum a_2 = 7+6+6+7+8+7+6 = 47$

$100mg = \sum a_3 = 4+3+2+3+4+3+2 = 21$

$$= \frac{57^2 + 47^2 + 21^2}{7} - \frac{(57+47+21)^2}{21} \quad (57+47+21 \rightarrow 125)$$

$$= 98.67$$

* Sum of squares within the Sample

$$SS_{within} = \sum y^2 - \sum \frac{(\sum a_i)^2}{n}$$

$$= 9^2+9^2+7^2+8^2+ \dots\dots 7^2+6^2+6^2+ \dots\dots 4^2+3^2+ \dots 2^2$$

$$= 853$$

$$= 853 - \frac{(57+47+21)^2}{7} = 10.29$$

	Sum of Squares (ss)	df	MS (ss/df)	F
b/w the Sample	98.67	2	49.34	$\frac{49.34}{0.54} = 86.56$
within sample	10.29	18	0.54	
Total.	108.96	20		

df b/w the Sample: no of sample − 1 = 3 − 1 = 2

df within sample: N − a (no of groups) = 21 − 3 = 18

N − 1 = 21 − 1 = 20

$$F\text{statistics} = \frac{MS\ b/w}{MS\ within} \quad \left(\frac{\text{Var b/w Samples}}{\text{Var within Samples}}\right)$$

F statistics = 86.56

Steps

$F_{critical}$ for $\alpha = 0.05$

df b/w = 3 − 1 = 2 ← df_1

df within = 21 − 3 = 18 ← df_2

$F_{critical\ \alpha=0.05}(2, 18) = 3.55$

Steps:

$86.56 > 3.55$, reject the H_0.