

Hypothesis testing and Statistical Analysis using z-test.

- ① Z test
 - ② t test
 - ③ Chi-square test
 - ④ ANOVA test
- Average
- Categorical data
- Variance of data.

Z test criteria

↓

→ S.S ≥ 30

→ σ population should be given.

CLT $\rightarrow \mu = \bar{X}, \sigma = \sigma/\sqrt{n}$

→ Sampling mean distribution \rightarrow Normal distn.

→ S.S ≥ 30

$$Z_{\text{score}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow n \text{ is S.S.}$$

Q. Suppose a child psychologist says that the average time working mother spend talking to their children is upto 11 mins. per day. To test the hypothesis you conducted an experiment with random sample of 100 working mothers and find that they spend 11.5 minutes per day talking with their children. Assume prior research suggests that the population standard deviation is 2.3 mins. conduct the test with 5% level of Significance ($\alpha = 0.05$)

Hypothesis →

Sample → $n = 100$
 $\bar{X} = 11.5$
 $\sigma = 2.3$

Z test.

→ $\mu = 11 \text{ mins}$ $\bar{X} = 11.5 \text{ min}$ $\sigma = 2.3 \text{ min}$ $n = 100$
 $\alpha = 0.05$

① frame the hypothesis.

upto 11 mins

LOS / MOE
Complete experiment

$$H_0: \mu \leq 11$$

$$H_A: \mu > 11$$

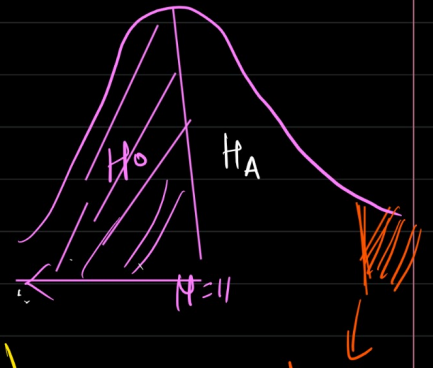
$$\mu \leq 11$$

$$\mu > 11$$

② $\alpha = 5\%$, one tail test

Type of test = Z test

rejection region
is at H_A side



③ Z score \bar{x} (test statistics)

\Downarrow
Z statistics

$$Z_{\text{score of } \bar{x}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{11.5 - 11}{2.3 / \sqrt{100}} = 2.17$$

$\mu \neq 11$ (two rejection region)
 \Downarrow
two tail test



One rejection region
at one side of distn
 \Downarrow
One tail test

* To identify two tail test.
 H_A has \neq

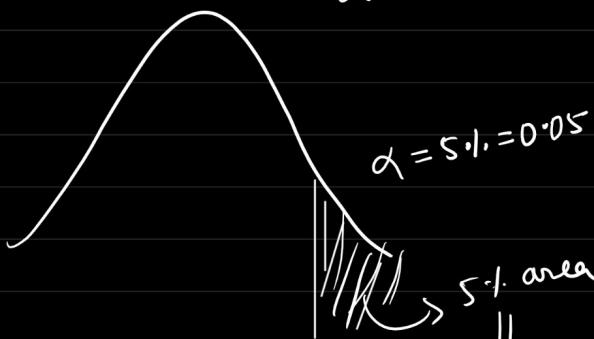
④

Z critical

\downarrow
 α to Z score

p-value

\downarrow
Z score of test to p-value



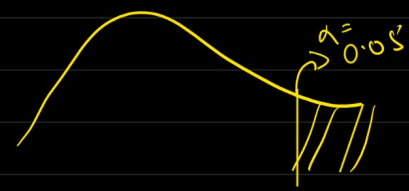
$$\alpha = 5\% = 0.05$$

5% area

To be converted
into Z score

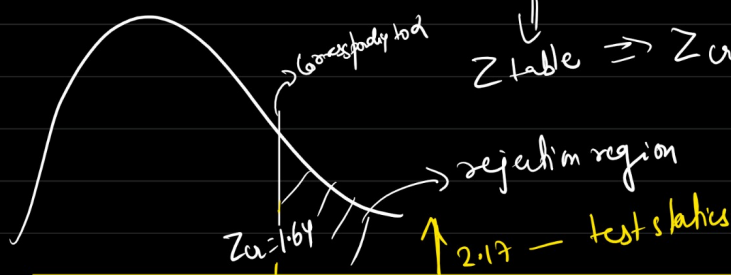
Z table \Rightarrow Z critical = 1.64

2.17 \rightarrow prob value



$$Z_{\text{score}} = 2.17$$

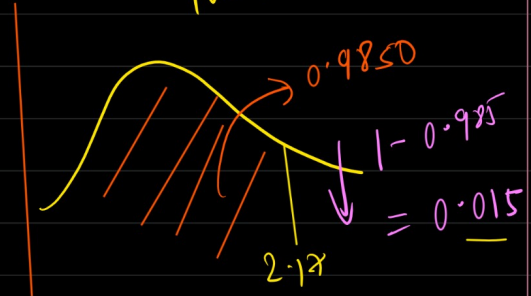
$$p\text{-value} = 0.98$$



$$Z_{\alpha} = 1.64$$

rejection region

\uparrow 2.17 — test statistics



$$2.17$$

$$0.9850$$

$$1 - 0.9850 = 0.0150$$

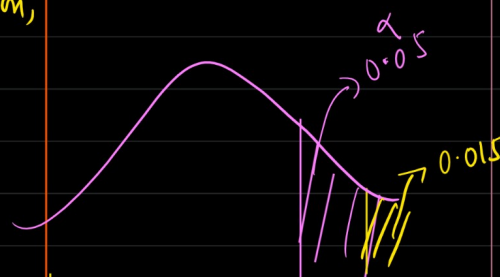
if Z statistics lies in Z_{α} region,
(2.17) you reject H_0

Step-5 Conclusion

$$Z_{\text{test}} > Z_{\alpha}$$

reject the H_0 .

meaning \rightarrow mothers spend more
than 11 mins.



$$p\text{-value} < 0.05$$

$$0.015 < 0.05$$

Since $p\text{-value} < 0.05$

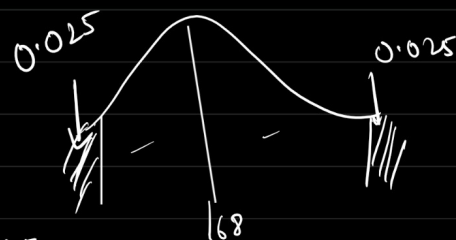
reject the H_0

Q. The average height of all residents in a city is 168cm with a σ of 3.9cm. One researcher believes that mean is different. He measured the height of 36 individuals and found that the average height is 169.5cm. Test the hypothesis at 95% confidence interval, $\alpha = 5\%$.

$$\mu_{\text{ht}} = 168\text{cm}, \sigma = 3.9\text{cm}, n = 36, \bar{x} = 169.5\text{cm}, \alpha = 5\%$$

Step-1: $H_0: \mu_{\text{ht}} = 168$

$$H_A: \mu_{\text{ht}} \neq 168$$



Step-2 $\alpha = 5\%$ but since two tail test

$$\alpha = 5/2 = 2.5\% \text{ or } 0.025$$

Z test

Step-3

$$Z_{\text{statistic}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{169.5 - 168}{3.9 / \sqrt{36}} = 2.31$$

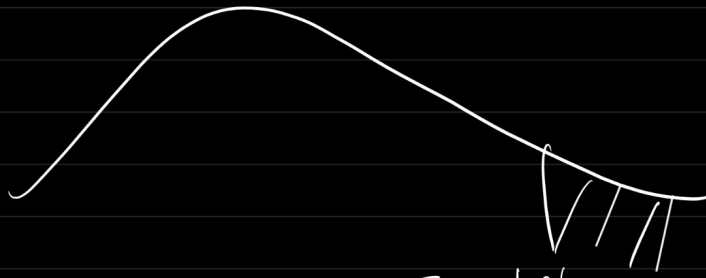
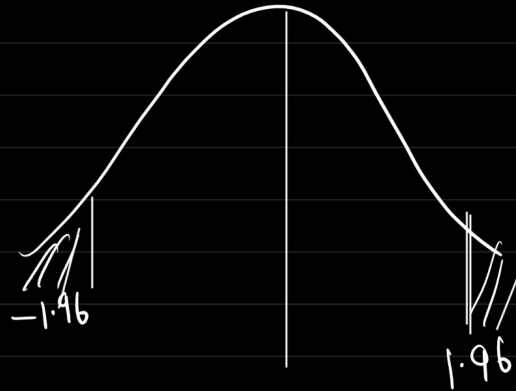
Step-4

$Z_{\text{critical}} \rightarrow$ Since it's a two tail test $= 0.025$

$$Z_{\alpha} = -1.96$$

$$Z_{\text{statistic}} = 2.31$$

$$Z_{\text{critical}} = 1.96$$



$$Z_{\text{stat}} = 2.31$$

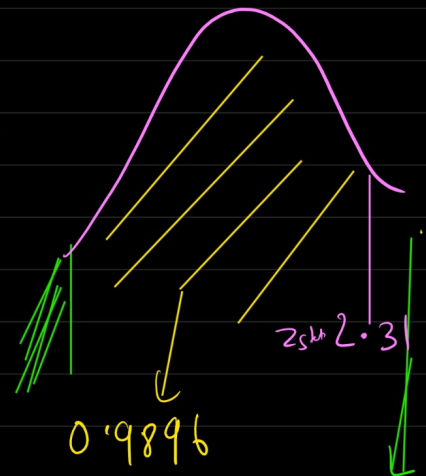
Step-5

$Z_{\text{stat}} > Z_{\alpha} = \text{reject the } H_0$

Conclusion = The avg height $\neq 168\text{cm}$

* 2nd approach

\rightarrow p-value approach

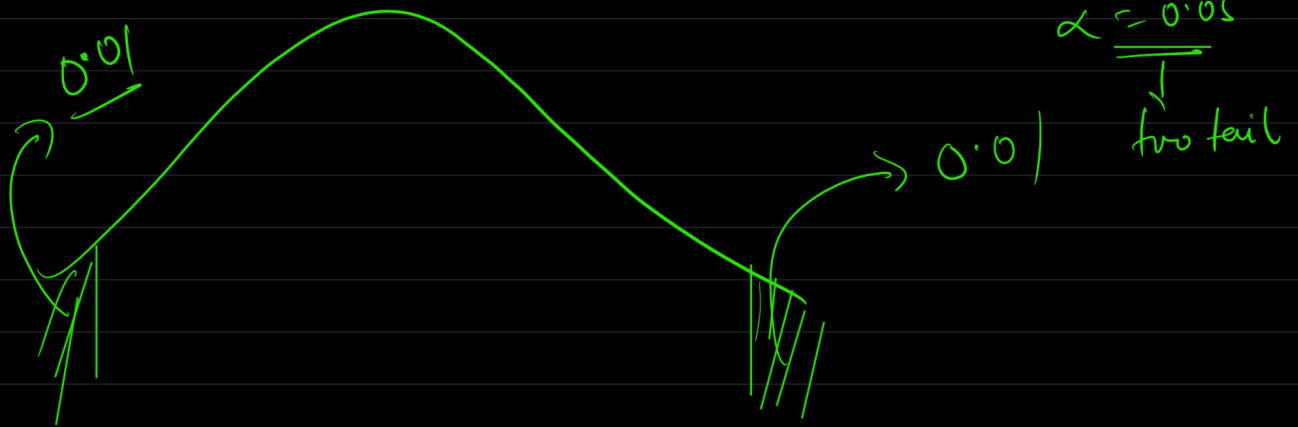


$$\Rightarrow 1 - 0.9896$$

$$\Rightarrow 0.01$$

two tail test

$$\begin{aligned} \text{total area} &= 0.01 + 0.01 \\ &= 0.02 \end{aligned}$$



$p\text{-value} < 0.05 \rightarrow \text{reject the } H_0$

$$0.02 < 0.05$$

\hookrightarrow
Conclusion $\mu_{\text{wt}} \neq 168\text{g}$