Probability distribution

↙ ↘

Pmf Pdf

* Uniform distribution refers to a type of probability distribution in which outcomes are likely.

Uniform distⁿ

↙ ↘

Discrete Uniform distⁿ (pmf)

Continuous Uniform distribution (pdf)

* A Continuous uniform probability distribution is a distribution that has infinite no of values defined in a specified range / bound.

→ random variable is continuous.

→ rectangular distribution.

Example → A perfect random no generator.
eg. OTP.

→ Probability of guessing exact time at any moment

→ waiting time at a bus stop
→ Bus arrival is continuous and consistent
Bus is coming every 30 mins at a stop.

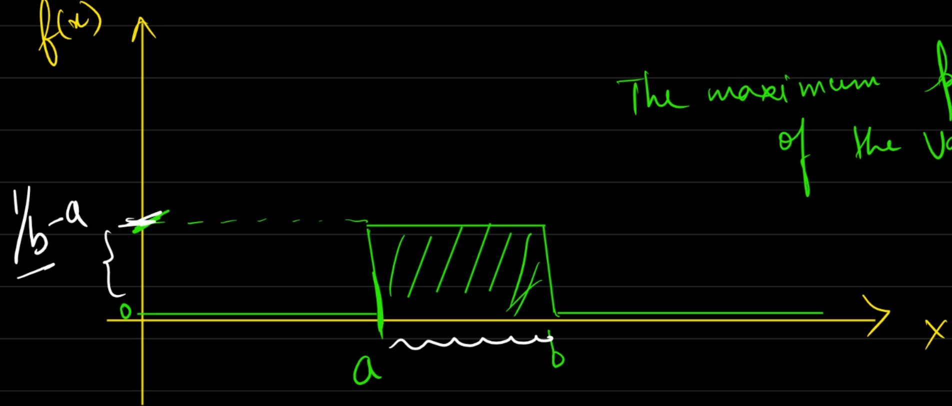
→ Temp variation in a day
(If a temp is fluctuating between minimum & maximum value)

Notation : $U(a, b)$

Parameter : $-\infty < a < b < \infty$

$b >> a$, a is min & b is max

The maximum probability of the variable x is 1



pdf of Continuous UD

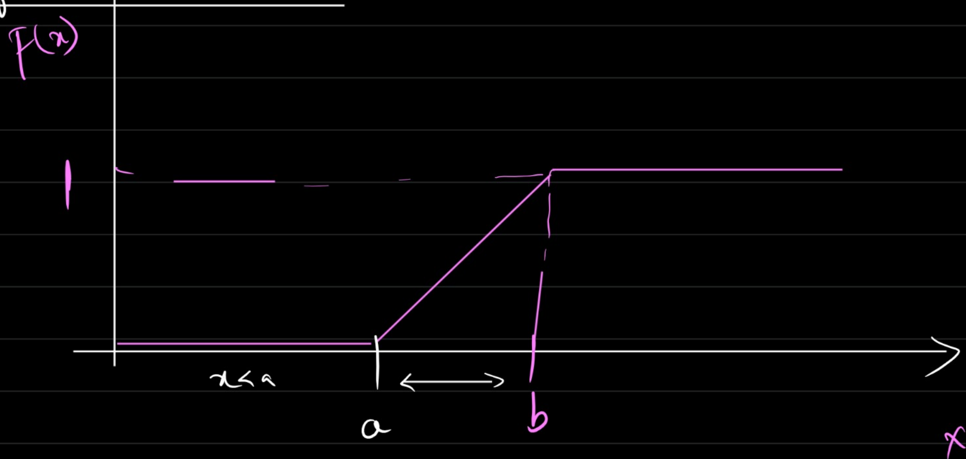
Prob = Area of rectangle = 1

$\Rightarrow$ base $\times$ height

$b-a \times f(x) = 1$

$$\boxed{f(x) = \frac{1}{b-a}}$$

Cdf of Uniform dist^n^



$$\text{cdf} = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} \text{ (line eq}^n\text{)} & x \in [a, b] \\ 1 & x > b \end{cases}$$

mean | median $= \frac{1}{2}(a+b)$

$\Downarrow$

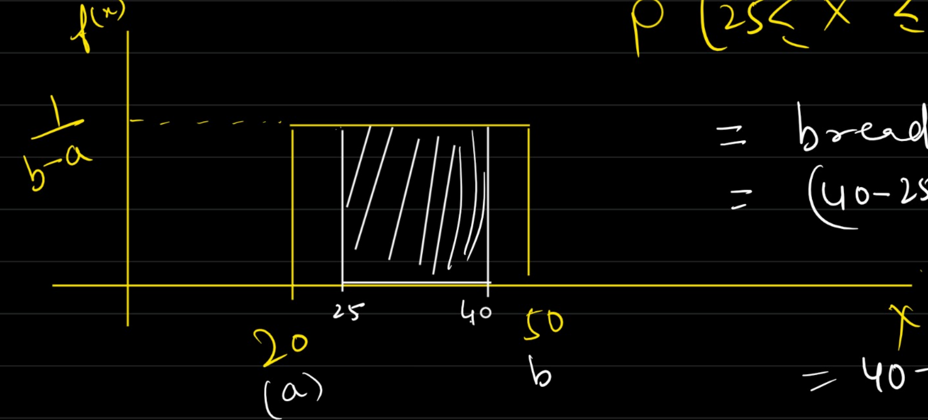
Avg of Pdf | Center of distribution.

Variance $= \frac{1}{12}(b-a)^2$

Q1. The number of items sold at a shop daily is Uniformly distributed with maximum minimum ited sold 50 & 20 respectively.

Probability of daily sales to fall between 25 & 40.

$\rightarrow$



$$P(25 \le X \le 40)$$

$= \text{breadth} \times \text{height}$

$= (40-25) \times \frac{1}{b-a}$

$= 40-25 \times \frac{1}{50-20}$

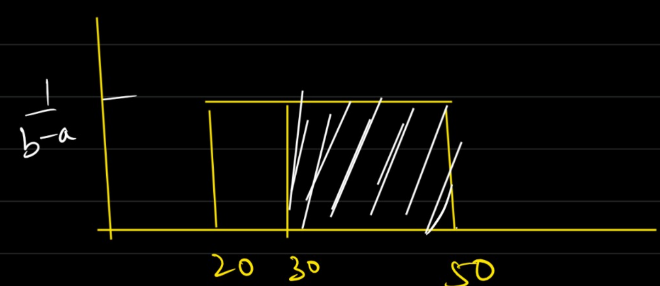
$= 15 \times \frac{1}{30} = \frac{1}{2} = 0.5$

50% chance that the number of item sold [25, 40].

* prob that sales ≥ 30

$P(X \geq 30) =$

$= (50-30) \times \frac{1}{(50-20)}$



$= 2\cancel{0} \times \frac{1}{3\cancel{0}} = 66.66$

Q. The amont of time for pizza delivery is Uniformly distributed b/w 15 & 60 mins. What is standard deviatim of the amount of time it takes for a pizza to be delivered.

$\Rightarrow$ $a = \min = 15$

$b = \max = 60$

✓ $Var(x) = \frac{1}{12}(b-a)^2 = \frac{1}{12}(60-15)^2 = \frac{45^2}{12} = 168.75$ min

✓ $\sigma = \sqrt{Var} = \sqrt{168.75} = 13$ mins for a continuous random variable with prob density function $f(x)$

$$E(x) = \int_{-\infty}^{\infty} x \cdot \underline{f(x)\,dx}$$

part of Expectation formulae

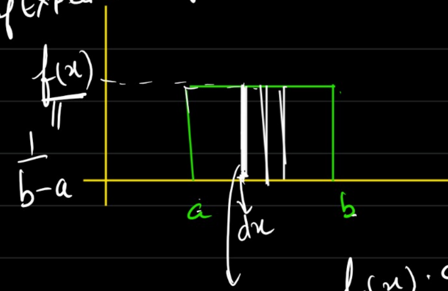
$\sum x f(x)$

for pmf

discrete $= \sum$

continuous $= \int$

$$E(x) = \int_a^b x\, f(x)\,dx$$



Area $= f(x) \cdot dx$

$$= \int_a^b x \cdot \frac{1}{b-a}\,dx$$

$$\int_a^b x\,dx$$

$$= \frac{1}{b-a} \int_a^b x\,dx$$

$$= \frac{x^{1+1}}{2}$$

$$= \left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{2(b-a)}(b^2-a^2)$$

$$= \frac{1}{2}\left[x^2\right]_a^b$$

$$= \frac{1}{2\cancel{(b-a)}}\cancel{(b-a)}(b+a)$$

$$= \frac{1}{2}(b^2-a^2)$$

$\Downarrow$

$b^2 - a^2 = (b-a)(b+a)$

$$\boxed{= \frac{b+a}{2}} \quad \text{if } X \sim U(a,b)$$

$$Var(x) = E[x^2] - \left(\underline{E[x]}\right)^2$$

$b^3 - a^3 = (b-a)(b^2 + ab + a^2)$

$\Downarrow$ $\qquad$ $\frac{a+b}{2}$

$$= \int_a^b x^2 f(x)\,dx = \frac{1}{b-a}\int_a^b x^2\,dx = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$\frac{1}{(b-a)}$

$$= \frac{1}{3(b-a)}(b^3-a^3)$$

$$= \frac{1}{3\cancel{(b-a)}}\cancel{(b-a)}(b^2+ab+a^2)$$

$$= \frac{b^2+ab+a^2}{3}$$

$$\mathrm{Var}(x) = \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^2+ab+a^2}{3} - \frac{a^2+b^2+2ab}{4}$$

$$= \frac{4b^2+4ab+4a^2-3a^2-6ab-3b^2}{12}$$

$(a-b)^2$
$= a^2-2ab+b^2$

$$= \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}$$

if $U(a,b)$, $\mathrm{Var}(x) = \frac{(b-a)^2}{12}$