DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE

& also for CS/IT

Permutations and Combinations

Lecture No. 04

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Recap of previous lecture

Topic

Permutation & Combination
(Part-3)

Topics to be Covered

Topic

"PERMUTATION & COMBINATION"

(Part- 4)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

Gmsj → Not Accepted.

COUNTING PRINCIPLE

RECAP

Fundamental Principle of Addition → If we have to perform only one of the job at a time out of n jobs then use this principle.

Keywords: "Either or, only one, Anyone"

Fundamental Principle of Multiplication → If we have to perform all the jobs at a time out of n jobs then use this principle.

Keywords: "AND, Both, All"

GAZAB KA Conclusion → (M. Imp Slide) RECAP

① if $n > r$ & RNA, then Multi Rule = Perm. Rule

② if $n = r$ & RNA, Then Multi Rule = Perm Rule = Factorial Rule

③ if RA, then only use Multi Rule.

ie the concept of ${}^nC_r$, ${}^nP_r$ & $r!$ is applicable only when RNA

Some Useful Information (Based on Experience) → RECAP

1. Always together / Not separated → Assume them as one unit with in Bracket.
2. All Never together / All do not come together → $\boxed{\text{Total} - \text{Always together.}}$
3. No two Girls are together → First arrange Boys.
4. Alternately (Linear case) < Two Cases will arise.
5. Alternately (Circular case) → only one Case will arise.
6. Particular / fix → No Need to select & No Need to arrange
7. At least one = Total − None.
8. At least = Go upto last Point (using common sense)
9. At Most = Include None also (if Possible)

CIRCULAR PERMUTATION →

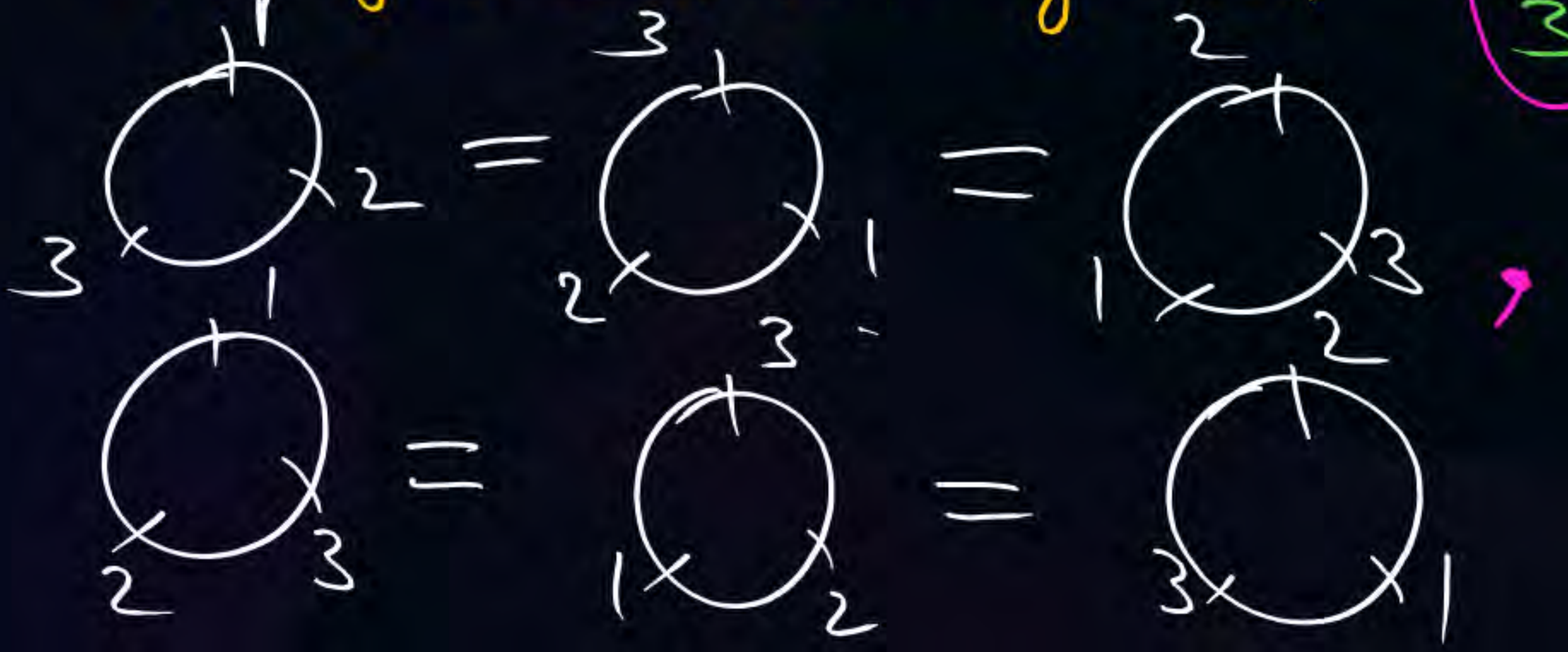
Number of Linear Arrangements of n different things $= n!$

Number of Circular Arrangements of n different things $= \frac{n!}{n} = \boxed{(n-1)!}$

Let $n=3$ then No. of linear arrangements $= 3! = 6$

eg $(1\,2\,3), (1\,3\,2), (2\,1\,3), (2\,3\,1), (3\,1\,2), (3\,2\,1)$

& No of Circular Arrangements $= \frac{3!}{3} = (3-1)! = 2! = 2$



There are n persons sitting in a row. Two of them are selected at random. Then how many selections are possible if, two selected persons are not together?

(a) ${}^{n}C_2$ (b) ${}^{n-1}C_1$

(c) ${}^{n}C_2 - {}^{n-1}C_1$ (d) ${}^{n-1}C_2$ (e) ${}^{n}C_2 - {}^{n-1}C_2$

Sol. (c)

Total possible selections of two persons out of n persons $= {}^{n}C_2$ ($\because$ RNA)

$P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, \ldots\ldots, P_{n-2}, P_{n-1}, P_n \longrightarrow$ i.e. we have n−1 pair

Two selected persons were together $= {}^{n-1}C_1$ so Req Ans = Total − Always together

$= {}^{n}C_2 - {}^{n-1}C_1$

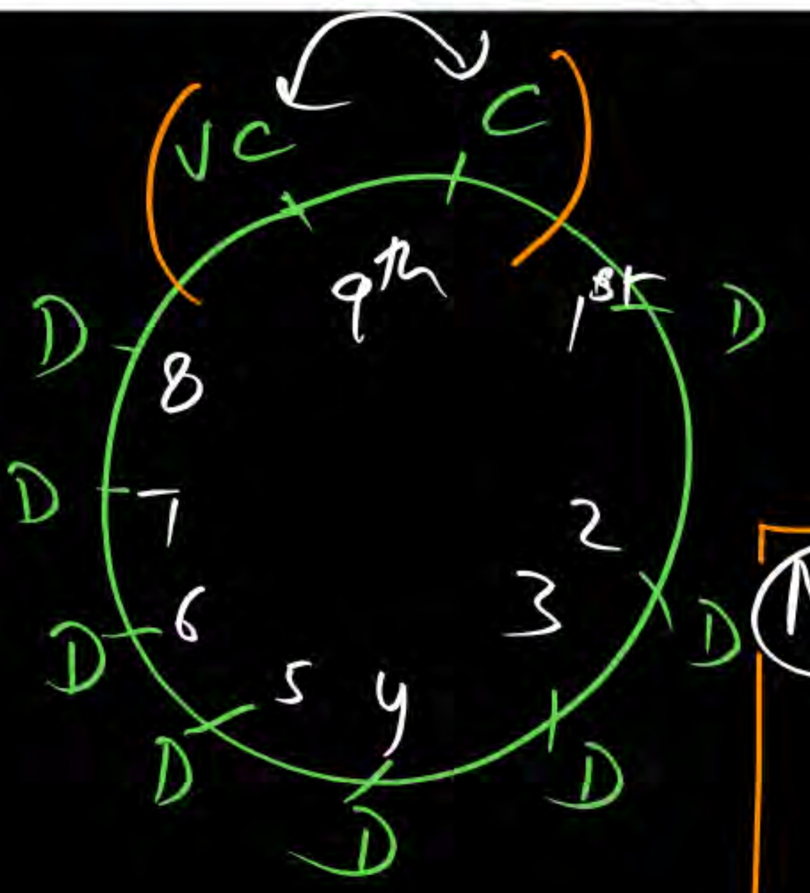
(ii) Also find the prob that two selected persons are not together ?

$$\text{Req Prob} = \frac{\text{fav}}{\text{Total}} = \frac{{}^nC_2 - {}^{n-1}C_1}{{}^nC_2} = 1 - \frac{{}^{n-1}C_1}{{}^nC_2} = 1 - \frac{(n-1)}{\frac{n(n-1)}{2}}$$

$$= 1 - \frac{2}{n} \quad \underline{\underline{\text{Ans}}}$$

In how many ways can 8 Directors, Vice-Chairman & Chairman of a firm be seated at a round table, if the Chairman has to sit between Vice-Chairman & Director

(a) $2 \times 9!$ (b) $2 \times 8!$ (c) $2 \times 7!$ (d) $3! \times 9!$



$= (9-1)! \times 2!$ Ans

(Person) (C & V.C)

M-II

$= ({}^{8}C_1 \times 1) \times (8-1)! \times 2!$

$= 8 \times 7! \times 2! = 8! \times 2!$

Dearrangements → when No one goes at right place assigned for him then such types of arrangements are called Dearrangements.

If there are n persons & n directed places. then

Total No. of arrangements $= n!$

All Correct " $= 1$

All wrong " $= n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\cdots\cdots+(-1)^n\frac{1}{n!}\right]$

this shortcut is applicable only when RNA.

for $n=2$, No. of Dearrangements $= 2!\left[1-\frac{1}{1!}+\frac{1}{2!}\right] = 2!\left[0+\frac{1}{2!}\right] = 1$

for $n=3$, " " $= 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right] = \frac{3!}{2!}-\frac{3!}{3!} = 3-1 = 2$

for $n=4$, " " $= 4!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right] = \frac{4!}{2!}-\frac{4!}{3!}+\frac{4!}{4!} = 12-4+1 = 9$

for $n=5$, " " $= 5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]$

$= \frac{5!}{2!}-\frac{5!}{3!}+\frac{5!}{4!}-\frac{5!}{5!} = 60-20+5-1 = 44$

for $n=6$, " " $= 6!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right] = 265$

Eg: If there are 3 Letter & 3 addressed envelopes then find Various arrangements?

Sol: Total arrangements (RNA) $= 3! = 6$

$\begin{pmatrix} E_1 & E_2 & E_3 \\ L_1 & L_2 & L_3 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_1 & L_3 & L_2 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_2 & L_1 & L_3 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_2 & L_3 & L_1 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_3 & L_1 & L_2 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_3 & L_2 & L_1 \end{pmatrix},$

All correct arrangements $= 1$.

All wrong arrangements $= 2$
(De-Arrangements)

MI for $n=3$, No. of De-Arrangements $= 3!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right]$

$= \frac{3!}{2!} - \frac{3!}{3!} = 3 - 1 = 2$

Q: There are six students appearing in a test in which there are 6 directed chairs. Then find the number of arrangements in which

(1) None of the student is seated on his/her chair.

(a) 1

(b) 720

(c) 265

(d) 120

For $n = 6$, Total arrangements $= 6! = 720$

All Correct " $= 1$ way

All wrong Arrangements (RMA) $= 265$

Q2: Each student is seated on his/her assigned chair?

All Correct arrangement $= 1$.

(3) There are six students appearing in a test in which there are 6 directed chairs. Then find the number of arrangements in which at least two students are seated on wrong chair?

(a) 1

(b) 720

(c) 719

(d) 265

(e) 120

$\boxed{\text{At least two}}$ students are seated on wrong chair

$= \boxed{\text{Total} - \text{None}}$ of the student is seated on wrong chair

$= \text{Total} - \text{All correct arrangement}$

$= 720 - 1$

$= 719$

Deep Analysis (PODCAST) → Various possibilities are as follows,

$$\text{(No wrong) or } \underbrace{\text{(two wrong) or (three wrong) or (four wrong) or (five wrong) or (six wrong)}}_{\text{At least two wrong}} = \text{Total}$$

Dearrangements

(All Correct) or (2W) or (3W) or (4W) or (5W) or (All 6 wrong) $= 6!$

$$^6C_0 \times 1 + {}^6C_2 \times 1 + {}^6C_3 \times 2 + {}^6C_4 \times 9 + {}^6C_5 \times 44 + {}^6C_6 \times 265 = 720$$

$$1 + (15 \times 1) + (20 \times 2) + (15 \times 9) + (6 \times 44) + (1 \times 265) = 720$$

④ No. of arrangements in which exactly 4 students are seated on wrong chair = ?

$$Ans = ({}^6C_4 \times 9) \times ({}^2C_2 \times 1) = {}^6C_4 \times 9 = 15 \times 9 = 135$$

⑤ No. of arrangements in which at least 4 students are seated on wrong chair = ?

$$Ans = ({}^6C_4 \times 9) \text{ or } ({}^6C_5 \times 44) + ({}^6C_6 \times 265)$$

$$= (15 \times 9) + (6 \times 44) + (1 \times 265) = ? = 664 \text{ arrangements}$$

⑥ No. of arrangements in which 4 particular students are seated on wrong chair?

$$Ans = ({}^4C_4 \times 9) \times ({}^2C_2 \times 1) = 9 \text{ ways.}$$

HWQ

Gate There are 5 Balls & 5 distinct cells. Then find the number of arrangements in which $\underbrace{\text{Ball } B_i \text{ is not placed in cell } C_i \ \forall i}_{\text{De-Arrangements.}}$ & $\boxed{\text{No cell Remains empty.}}$

(RNA)

(a) 44

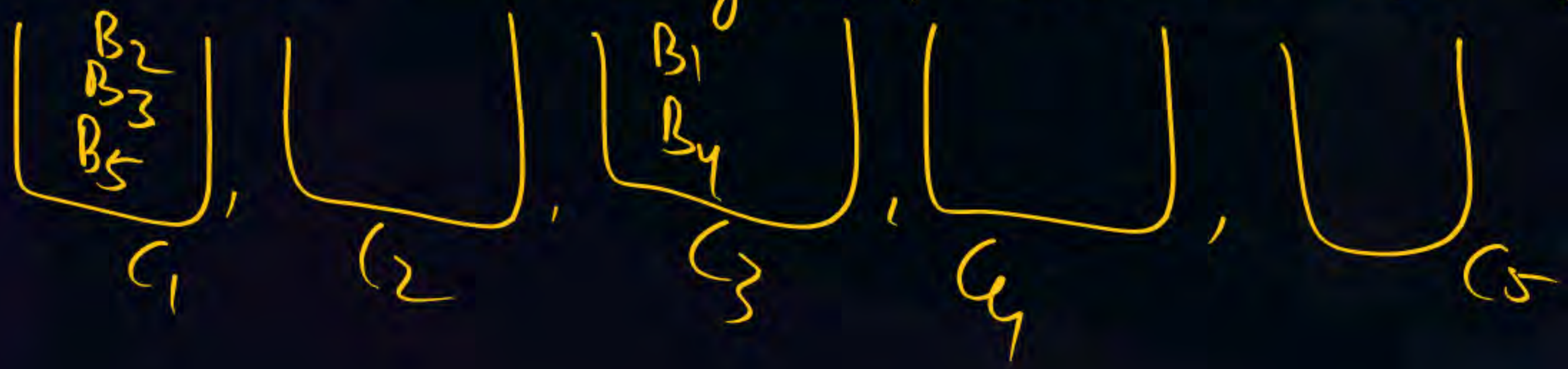
(b) 1

(c) 119

(d) 120

For $n=5$, All wrong arrangements $= 44$

Note:- Above 44 Dearrangements do not include following types of DeArrange...



Permutation of Alike items :→

(i) No. of linear arrangements of n different things $= n!$

(ii) No of Circular " " n different things $= (n-1)!$

(iii) No of Linear arrangement of n things in which p are alike, q are alike, r are alike & rest are different $= \dfrac{n!}{p!\,q!\,r!} = \dfrac{{}^{n}P_{n}}{p!\,q!\,r!}$

where $p+q+r+$ Rest items $=$ Total items (n)

Meaning of RA → If we have a liberty to Repeat any item, desired number of times then we will say that R.A.

Eg: How many 4 letter words can be formed using $\underbrace{a, a, a}_{\text{Alike}}, b$?

M-I Total 4 letter words = 4

(a a a b), (a a b a), (a b a a), (b a a a)

M-II Total wrong arrangements $= 4! = 24$

So Total correct arrangements $= \frac{4!}{3!} = 4$

Q: How many 4 digit Number Can be formed using 5, 5, 6, 6.

(M-I) Total 4 digit Nos are as follows; $= 6$ ✓ Ans

(5566), (5656), (5665), (6655), (6565), (6556)

(M-II) Total wrong arrangements $= 4! = 24$

Total Correct " $= \dfrac{4!}{2!\,2!} = 6$ ✓

Q How many 11 letter words can be formed using 'MISSISSIPPI'

$\underbrace{SSSS}\ \underbrace{IIII}\ \underbrace{PP}\ M$, Total letters $= 11$

(Toys)

So Total 11 letter words $= \dfrac{11!}{4!\,4!\,2!}$

(RNA)

Q. In how many ways 2 Maths Books, 3 Physics Books, 4 Chemistry Books 1 Hindi Book, 1 English Book & 1 Geography Book can be arranged in a shelf?

$$\underbrace{M, M}_{p=2}, \underbrace{P, P, P}_{q=3}, \underbrace{C, C, C, C}_{r=4}, \underbrace{H, E, G}_{\text{Rest items}} \Rightarrow n = 12$$

$$\text{Total arrangements} = \frac{12!}{2!\,3!\,4! \times 1! \times 1! \times 1!} = \frac{12!}{2!\,3!\,4!}$$

FQ 4 : How many 11 letter words can be formed using the letters of the words $\boxed{\text{MATHEMATICS}}$ if

① there is No Restriction = ? $= \dfrac{11!}{2!2!2! \times (1!)^5} = \dfrac{{}^{11}P_{11}}{2! \times 2! \times 2!}$ (Max Ans)

M, M, A, A, T, T, E, H, I, C, S , $n = 11$

TOYS (RNA)

② Vowels are not separated = ? = Vowels are always together

$\overset{1}{M}\ \overset{2}{H}\ \overset{3}{S}\ \overset{4^{th}}{(AAEI)}\ \overset{5}{C}\ \overset{6}{M}\ \overset{7}{T}\ \overset{8^{th}}{T}$

$= \dfrac{8!}{\underset{(M)}{2!}\ \underset{(T)}{2!}} \times \dfrac{4!}{\underset{(A)}{2!}}$

③ All vowels do not come together = ? = Total − Vowels always together $= \dfrac{11!}{2!2!2!} - \left(\dfrac{8!}{2!2!} \times \dfrac{4!}{2!}\right)$

$\underbrace{\text{do not come together}}_{\text{Never}}$

(4) No two Consonants are together = ? = first arrange vowels

✓ A ✓ A ✓ E ✓ I ✓

$= \frac{4!}{2!} \times$ (senseless 8.) $= 0$ way.

Vacant places = 5 But Consonants = 7

(5) No two vowels are together = ? = first arrange Consonants

✓ M ✓ M ✓ T ✓ T ✓ H ✓ C ✓ S ✓

Vacant places (free for Vowels) = 8

But Vowels are = 4

So they can be arranged by $= \frac{^8P_4}{2!}$

AA E I

$= \frac{^7P_7}{2!\,2!} \times \frac{^8P_4}{2!} =$ Ans

$= \frac{7!}{2!\,2!} \times \frac{^8P_4}{2!} =$ Ans

(Consonant) (Vowels)

Explanation: After arranging consonants, there are 8 far places for vowels (6 in b/n & 2 at the corners).

But vowels are only 4 so they can be arranged by ${}^8P_4$ ways.

Again it is wrong ∵ two 'A' are alike

So finally vowels can be arranged by $\frac{{}^8P_4}{2!}$ ways.

⟶ In this Ans, Most of the cases are of that types in which two or more consonants are together.

(6) Vowels & Consonants are alternately = ?

AMAMETITCNS → ✗

MAMATETICNS → ✗

= Senseless Quest

(7) Vowels occupy even places = ?

✓ _ ✓ _ ✓ _ ✓ _ ✓ _ ✓

2th 4th 6th 8th 10th

$$= \frac{^5P_4}{2!} \times \left(\frac{^7P_7}{2! \times 2!}\right)$$

(Vowels) Consonants

Vowels = 4, Even places = 5

Consonants = 7, Odd places = 6

But wait, we have one Even place also which is still vacant & this place can be filled by consonants. Now Total fair places for Consonants = 7

Qs In how many ways 7 (+) sign & 5 (−) sign are to be arranged in a Row so that No two (−) sign are together ?

Toys (RNA)

(a) $\frac{12!}{5!\,7!}$

(b) $12!$

(c) 56

(d) 0

= First arrange +ve sign in a Row. $= \frac{{}^7P_7}{7!} \times \frac{{}^8P_5}{5!}$

$\_ + \_ + \_ + \_ + \_ + \_ + \_ + \_$ $= 1 \times {}^8C_5 = {}^8C_3 = 56$

Vacant Places (few places for −ve sign) = 8 (6 in b/w & 2 at the corners)

−ve sign = 5 & they can be arranged by $\frac{{}^8C_5 \times 5!}{5!} = \frac{{}^8P_5}{5!}$

Note. Total arrangements (w/o any Restrictions) $= \frac{12!}{7!\,5!}$ So Prob $= \frac{56}{12!/7!5!}$

$2+3=5$

$2+3=5$

$2+3=5$

is +ve sign can be assumed as alike item.

Q: Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to number of heads is ?

(a) 20

(b) 120

(c) 9

(d) 40

(e) 720

Fav. Arrangements = {No. of heads = No of tails = 3 times each}

eg (H T H H T T) ⟶ $\frac{6!}{3!\,3!} = 20$

Deep Analysis :- (PODCAST) ⟶ Total ways = $\underset{P_1}{2} \times \underset{P_2}{2} \times \underset{P_3}{2} \times \underset{P_4}{2} \times \underset{P_5}{2} \times \underset{P_6}{2}$ = 64 ways.

(HHHHHH) or (HHHHHT) or (HHHHTT) or (HHHTTT) or (HHTTTT) or (HTTTTT) or TTTTTT = Total

$\frac{6!}{6!} + \frac{6!}{5!} + \frac{6!}{4!\,2!} + \frac{6!}{3!\,3!} + \frac{6!}{2!\,4!} + \frac{6!}{5!} + \frac{6!}{6!} = 2^6$

$({}^6C_6 = 1)$ $({}^6C_5 = 6)$ $({}^6C_4 = 15)$ $({}^6C_3 = 20)$ $({}^6C_2 = 15)$ $({}^6C_1 = 6)$ $({}^6C_0 = 1)$ = 64 ways

Thank you

Keep Hustling!