DATA SCIENCE & ARTIFICIAL INTELLIGENCE

& also for CS/IT

Permutations and Combinations

Lecture No. 01

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Topics to be Covered

Topic

"PERMUTATION & COMBINATION (Part 1)"

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

" If, what if, AGAR, YADI, TOH, ... "

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

COUNTING PRINCIPLE

Fundamental Principle of Addition → If we have to perform only one of the job at a time out of n jobs then use this principle.

Keywords: "Either or, only one, Anyone"

Fundamental Principle of Multiplication → If we have to perform all the jobs at a time out of n jobs then use this principle.

Keywords: "AND, Both, All"

Eg: There are 10 Boys & 8 Girls in a class then in how many ways we can Select

(i) Either a Boy or a Girl = ? = 10 ways + 8 ways = 18 ways

(ii) A Boy & a Girl = ? = 10 ways × 8 ways = 80 ways.

Eg (B_1, G_1) (B_1, G_2) (B_1, G_3) - - - - (B_1, G_8)
(B_2, G_1) (B_2, G_2) (B_2, G_3) - - - (B_2, G_8)
(B_3, G_1) (B_3, G_2) - - - - (B_3, G_8)
(B_4, G_1) - - - - - - (B_4, G_8)
⋮ - - - - - - ⋮
(B_{10}, G_1) - - - - - - (B_{10}, G_8)

} = 80 pairs

eg: there are 3 students appearing for Maths scholarship test, 4 for Physics test, & 5 for Chemistry scholarship test. then in how many ways ?

(i) these scholarships can be awarded = ? = 3 ways × 4 ways × 5 ways
(we want to perform all the jobs at a time) = 60 ways

(ii) one of these scholarships can be awarded = ? = 3 ways + 4 ways + 5 ways
(we have to perform only one of the job at a time) = 12 ways.

Note = Job 1 → awarding Maths scholarship.
Job 2 → " Physics "
Job 3 → " Chemistry "

Explanation:

$(M_1P_1C_1), (M_1P_2C_1), \ldots\ldots (M_1P_4C_1)$

$(M_1P_1C_2), (M_1P_2C_2), \ldots (M_1P_4C_2)$

$(M_1P_1C_3), (M_1P_2C_3), \ldots (M_1P_4C_3)$

$(M_1P_1C_4), (M_1P_2C_4), \ldots (M_1P_4C_4)$

$(M_1P_1C_5), (M_1P_2C_5), \ldots (M_1P_4C_5)$

$(M_2P_1C_1), (M_2P_2C_1), \ldots \vdots$

$(M_2P_1C_2), \ldots \vdots$

$(M_2P_1C_3), M_2P_2C_2), \ldots \vdots$

$(M_2P_1C_4), (M_2P_2C_3), \ldots \vdots$

$(M_2P_1C_5), (M_2P_2C_4), \ldots \vdots$

$(M_2P_2C_5), \ldots (M_3P_4C_5)$

} 60 Cases.

Eg In a Restaurant there are (8) Veg dishes & (5) Non Veg dishes then in how many ways you can order (a) dish ?

Sol: Total ways of ordering a dish = ? = we can order either Veg or Non Veg

$= 8 \text{ ways} + 5 \text{ ways} = 13 \text{ ways}$.

Eg If there are 15 NITS & 20 IITS in INDIA & you are selected in JEE then in how many ways student can choose a College ?

Sol: Total ways of opting College = Either we can opt NIT or IIT's.

$= 15 \text{ ways} + 20 \text{ ways} = 35 \text{ ways}$.

Q: If there are 20 IITS, each having 7 Branches then in how many ways toppers can take addmission?

Sol: No. of ways of taking addmisson = ?

(selecting a College) & (selecting a Branch)

= (20 ways) × (7 ways) = 140 ways.

Ex There are two friends sharing a Room in a hostel in IIT DELHI & they have Common wardrobe in which there are only 5 jeans & 4 shirts. If they want to prepare for a party then in how many ways they can dress up?

Sol: number of ways of dressing up $= \underset{f_1}{\left(\underset{J}{5} \times \underset{S}{4}\right)} \times \underset{f_2}{\left(\underset{J}{4} \times \underset{S}{3}\right)}$

$= 240$ ways.

(a) 20 ways (Senseless)

(b) 40 ways.

(c) 32 ways (Senseless)

(d) 240 ways.

Note: If only one friend can go to the party then Ans = ?

= Either 1st can go or 2nd can go.

$(5 \times 4) + (5 \times 4) = 40$ ways.

Q. There are 4 letters & 5 letter Boxes. In how many ways we can post these letters ?

Sol:

(a) 16 ways

(b) 24 ways

(c) 20 ways

(d) 1024 ways

(e) 625 ways

Total ways of posting letters = $\underset{L_1}{5 \text{ ways}} \times \underset{L_2}{5 \text{ ways}} \times \underset{L_3}{5 \text{ ways}} \times \underset{L_4}{5 \text{ ways}}$

(RA)

$= 5^4 \text{ ways} = 625 \text{ ways}$

WRONG APP:- Total ways = $\underset{B_1}{4} \times \underset{B_2}{4} \times \underset{B_3}{4} \times \underset{B_4}{4} \times \underset{B_5}{4} = 4^5$ choices

Here Letter Boxes are moving (which is Not possible)

Eg: there are (3) Rings & 4 fingers then in how many ways we can WORN these Rings if

(1) there is No Restriction = ? (RA)

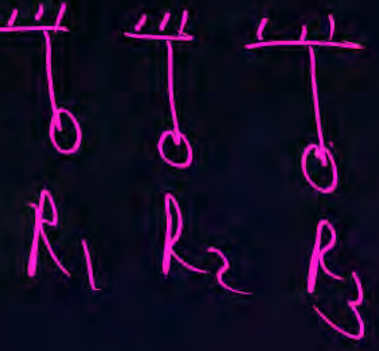
$$\underset{R_1}{4 \text{ ways}} \times \underset{R_2}{4 \text{ ways}} \times \underset{R_3}{4 \text{ ways}} = 4^3 = 64 \text{ ways}$$

(2) with at most one R in a single f = ? (RNA)

$$\underset{R_1}{4 \text{ ways}} \times \underset{R_2}{3 \text{ ways}} \times \underset{R_3}{2 \text{ ways}} = 24 \text{ ways}.$$

WRONG APP :- Req ways $= \underset{f_1}{3} \times \underset{f_2}{3} \times \underset{f_3}{3} \times \underset{f_4}{3} = 3^4$ ways.

Here we are Cracking fingers (Not possible)



Q: How many (3) letter words (with or W/o meaning) can be formed using vowels if

(i) there is no Restriction = ? (RA)

$$\underset{P_1}{5 \text{ ways}} \times \underset{P_2}{5 \text{ ways}} \times \underset{P_3}{5 \text{ ways}} = 125 \text{ ways}$$

(ii) A letter can come at most once = ? (RNA)

Vowels = a, e, i, o, u

$$\underset{P_1}{5 \text{ ways}} \times \underset{P_2}{4 \text{ ways}} \times \underset{P_3}{3 \text{ ways}} = 60 \text{ ways}$$

(X) ∵ we have to fill all 3 places at a time so we will use Multi. Rule

Eg: How many (4) digit Number can be formed using the digits, (1,2,3,4,5,6,7)

(i) RA = ? $\underset{P_1}{7\text{ways}} \times \underset{P_2}{7\text{ways}} \times \underset{P_3}{7\text{ways}} \times \underset{P_4}{7\text{ways}} = 7^4 = 2401\text{ways}$

(ii) RNA = ? $\underset{P_1}{7\text{ways}} \times \underset{P_2}{6\text{ways}} \times \underset{P_3}{5\text{ways}} \times \underset{P_4}{4\text{ways}} = 7\times6\times5\times4$

$= 42\times20$

$= 840\text{ ways}$

In a test there are eight questions, in which four have three possible answers, three has two possible answers each and one question has five possible answers. The total number of possible answers will be?

(a) 2880 (b) 78 (c) 94 (d) 3240

Total possible answers = ?

$$= \underbrace{3 \times 3 \times 3 \times 3}_{4} \times \underbrace{2 \times 2 \times 2}_{3} \times 5 = 3^4 \times 2^3 \times 5$$

$$= 3240$$

Combination → (When Counting is based on Selection only then use this Rule)

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

&

$$^nC_r = {}^nC_{n-r}$$

eg $^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}$

eg $^{22}C_4 = \frac{22\times21\times20\times19}{4\times3\times2\times1}$

eg $^7C_2 = \frac{7\times6}{2\times1}$ & so on....

eg $^{22}C_{19} = ? = {}^{22}C_{22-19} = {}^{22}C_3 = \frac{22\times21\times20}{3\times2\times1}$

Sp. Results: $^nC_3 = \frac{n(n-1)(n-2)}{3\times2\times1}$

$^nC_0 = 1 = {}^nC_n$ eg $^6C_0 = 1 = {}^6C_6$

$^nC_1 = n = {}^nC_{n-1}$ eg $^8C_1 = 8 = {}^8C_7$

$^nC_2 = \frac{n(n-1)}{2} = {}^nC_{n-2}$ eg $^5C_2 = 10 = {}^5C_3 = \frac{5\times4\times3}{3\times2\times1} = 10$

Q. In a party there are 5 persons then find the number of Handshakes in that party ?

(M-I) Using Manual Calculation :-

Various Handshakes are as follows ;

(P_1P_1), (P_1P_2), (P_1P_3), (P_1P_4), (P_1P_5)

(P_2P_1), (P_2P_2), (P_2P_3), (P_2P_4), (P_2P_5)

(P_3P_1), (P_3P_2), (P_3P_3), (P_3P_4), (P_3P_5)

(P_4P_1), (P_4P_2), (P_4P_3), (P_4P_4), (P_4P_5)

(P_5P_1), (P_5P_2), (P_5P_3), (P_5P_4), (P_5P_5)

Total ways = 10 Handshakes.

(M-II) Total Handshakes = $^5C_2 = 10$ ways.

Q. In a party there are 66 Handshakes then find No. of persons in that party?

(a) 66

(b) 10

(c) 11

(d) 12

(e) 2145

Let Total Persons = n then No of Handshakes = ${}^{n}C_2$

then A.T.Q

$${}^{n}C_2 = 66$$

$$\frac{n(n-1)}{2} = 66$$

$$n(n-1) = 132$$

$$n(n-1) = 12 \times 11$$

$$n(n-1) = 12(12-1)$$

$$n = 12$$

(a) $n = 66 \Rightarrow \frac{66 \times 65}{2} \neq 66$

(b) $n = 10 \Rightarrow \frac{10 \times 9}{2} = 45 \neq 66$

(c) $n = 11 \Rightarrow \frac{11 \times 10}{2} = 55 \neq 66$

(d) $n = 12 \Rightarrow \frac{12 \times 11}{2} = 66$ ✓

(e) $n = 2145 \Rightarrow$ No Need to check

Blunder Method - e

No. of Persons = ${}^{66}C_2 = 2145$

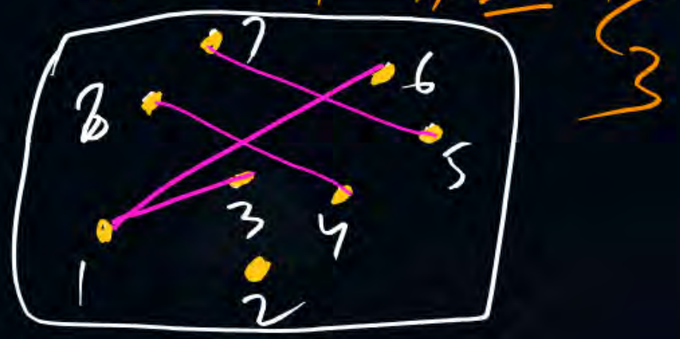
Q: How many straight lines & Δ's can be formed using 8 Non Collinear Points

Sol: No. of straight lines that can be formed using n Non Collinear Points $= {}^nC_2$

& " of Δ's that " " " " " " " " $= {}^nC_3$

(i) No. of st. lines $= {}^8C_2 = \frac{8\times 7}{2} = 28$

(ii) No of Δ's $= {}^8C_3 = \frac{8\times 7\times 6}{3\times 2} = 56$



CARDS (52)

RED CARDS (26)

Diamond (13)

Heart (13)

BLACK CARDS (26)

Club (13)

Spade (13)

1. Cards → 4 suits (D or H or C or S)
2. FACE CARDS (12) (eg K, Q, J)
3. HONOUR CARDS (16) (eg K, Q, J, A)

Q. In how many ways we can select (4) cards out of (52) cards if

(1) There is No Restriction = ? = $^{52}C_4$

(2) Four Cards are of same suits = $\underbrace{{}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4}_{\text{M-I}} = \underbrace{{}^{4}C_1 \times {}^{13}C_4}_{\text{M-II}} = 4 \times {}^{13}C_4$

(3) " " " " different suits = $^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = (^{13}C_1)^4$

(4) " " " " Face Cards = $^{12}C_4$

(5) " " " " Honour Cards = $^{16}C_4$

(6) Two are Black & Two are Red = $^{26}C_2 \times {}^{26}C_2$

(7) 4 Cards are of Same Colour = Either all R or all B

$= {}^{26}C_4 + {}^{26}C_4 = 2 \times {}^{26}C_4$

eg: A person has 6 friends. In how many ways he/she can organise B'day party.?

Sol: (In-I) Number of ways to deal with each friend = 2 ways (either I or R)

Total No. of ways to deal with all friends = $\underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} = 2^6$

$f_1\ f_2\ f_3\ f_4\ f_5\ f_6$ = 64

So Req. ways $= 2^6 - 1 = 64 - 1 = 63$ ways

Analysis: (SSSSSS) or (SSSSSR) or (SSSSRR) or (SSSRRR) or (SSRRRR) or (SRRRRR) or (RRRRRR)

$= 1 = {}^6C_6$ $= 6 = {}^6C_5$ $= 15 = {}^6C_4$ $= 20 = {}^6C_3$ $= 15 = {}^6C_2$ $= 6 = {}^6C_1$ $= 1 = {}^6C_0$

To organise party, All Rejections are not possible

M-II He/She can organise B'day party either by

= Selecting (1f) or (2f) or (3f) or (4f) or (5f) or (6f)

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= 6 + 15 + 20 + 15 + 6 + 1 = 63$ ways.

Q2 By selecting exactly 4f at a time = ? $= {}^6C_4 = 15$ ways

Q3 " " at least 4f at a time = ? ${}^6C_4 + {}^6C_5 + {}^6C_6 = 15 + 6 + 1 = 22$

Q4 " " at least one f at a time = ? = Same as part (1) = 63 ways.

M-II At least one f = Total − None = Total − No f $= 2^6 - {}^6C_6 = 64 - 1 = 63$

PODCAST Various possibilities are;

(0f) or (1f) or (2f) or (3f) or (4f) or (5f) or 6f = Total (64)

(0f) = None

(1f) or (2f) or (3f) or (4f) or (5f) or 6f = At least one f

At least one f = At least 4f

(X) At Most 2f = (1f or 2f) = $^6C_1 + {}^6C_2 = 6 + 15 = 21$

Thank you

Keep Hustling!