

CS & DA

PROBABILITY AND STATISTICS

DPP: 1

Statistics-1(Continuous Random Variable)

Q1 If X has the prob. density fx^n

$$f(x) = \begin{cases} ke^{-3x}; & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find k and $P(0.5 \leq X \leq 1)$

Q2 Find the distribution function of the random variable X whose probability density is given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Q3 If X is a random variable with density

$$f(x) = \frac{1}{2}e^{-\frac{|x|}{2}}, -\infty < x < \infty. \text{ Then } E(|X|) \text{ ---}$$

Q4 Let X be a random variable with a continuous uniform distribution on the interval (1, a), Where $\alpha > 1$. If $E[X] = 6\text{Var}[X]$, then $\alpha =$.

- (A) 2 (B) 3
(C) 4 (D) 7

Q5 Suppose the probability density function of a continuous random variable x is $= 3x^2$: $0 < x < 1$. Find 'a' and 'b' satisfying the following condition

- (A) $P[x \leq a] = P[x \geq a]$
(B) $P[x > b] = 0.05$

Q6 Find whether the following function is a probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Also obtain $P(0 < X \leq 1)$

Q7 The probability density function f (x) of a continuous random variable x is defined by

$$f(x) = \begin{cases} \frac{A}{x^3}, & 5 \leq x \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

then the value of A is

- (A) $\frac{100}{3}$
(C) $\frac{50}{3}$

- (B) $\frac{200}{3}$
(D) $\frac{3}{200}$

Q8 Let X be a random variable denoting the hours of life in electric light bulb. Suppose X is distributed with density function

$$f(x) = \frac{1}{1000}e^{-x/1000} \text{ for } x > 0$$

Find the expected life time of such a bulb.

Q9 If X has exponential distribution with mean 1/2, then is $P(X < 1 | X < 2)$.

- (A) $\frac{(1-e^{-2})}{(1-e^{-4})}$
(B) $\frac{(1-e^{-0.5})}{(1-e^{-1})}$
(C) $\frac{(1-e^{-1})}{(1-e^{-2})}$
(D) $\frac{(1-e^{-0.5})}{(1-e^{-4})}$

Q10 Customers arrive randomly and independently at a service window, and the time between arrivals has an exponential distribution with a mean of 12 minutes. Let X equal the number of arrivals per hour. What is $P[X = 10]$?

- (A) $\frac{10e^{-12}}{10!}$
(B) $\frac{10^{-12}e^{-10}}{10!}$
(C) $\frac{12^{-10}e^{-10}}{10!}$
(D) $\frac{5^{10}e^{-5}}{10!}$

Q11 The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. What is the probability that the repair time exceeds 2 h ?

Q12 Students arrive at a local bar and restaurant according to an approximate Poisson process at a mean rate of 30 students per hour. What is the probability that the bouncer has to wait more than 3 minutes to card the next student ?



- Q13** Let X be uniformly distributed over the interval $[a, b]$, where $0 < a < b$.
If $E(X) = 2V(X) = \frac{4}{3}$ the $P[X < 1]$ is-
(A) $\frac{3}{4}$ (B) 1
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$
- Q14** Suppose that a large conference room for a certain company can be reserved for no more than 4 hours. However the use of the conference room is such that both long and short conferences occur quite often. In fact it can be assumed that the duration X of a conference has a uniform distribution on the interval $[0, 4]$. What is the probability that any given conference lasts atleast 3 hours ?
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{4}$ (D) $\frac{3}{4}$
- Q15** X is uniformly distributed random variable that take values between 0 and 1. The value of $E(X^3)$ will be :
(A) 0 (B) $\frac{1}{8}$
(C) $\frac{1}{4}$ (D) $\frac{1}{2}$
- Q16** Buses arrive at a specified stop at 15 min intervals starting at 7 A.M., that is, they arrive at 7:00, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 A.M. find the probability that he waits ?
(a) less than 5 min for a bus and
(b) at least 12 min for a bus
- Q17** The mean height of 500 student is 151cm and the standard deviation is 15cm . Assuming that the heights are normally distributed, find how many students have heights between 120 and 155 cm?
Given $A(z=0 \text{ to } 2.07)=0.4808$
and $A(z=0 \text{ to } 0.27)=0.1084$
- Q18** A manufacturer knows from experience that the resistance of resistors he produces is normal with mean $\mu = 100$ ohms and standard deviation $\sigma = 2$ ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms?

- Q19** If the height of 300 student are normally distributed with mean 64.5 inches and standard deviation 3.3 inches, find the height below which 99% of the students lie.
Given $A(z=0 \text{ to } 2.327)=49\%$
- Q20** The life of army shoe is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months?
Given $F(2)=0.9772$
- Q21** The mean of a normal distribution is 50, its mode will be.
(A) 25 (B) 40
(C) 50 (D) 100
- Q22** For the standard normal variate, the mean and variance are.
(A) 1,0 (B) 0,0
(C) 0,1 (D) 1,1
- Q23** The mode of a normal distribution is 80 with SD 10. Then, its median will be.
(A) 8 (B) 800
(C) 80 (D) None of these
- Q24** If $\log_{10} X$ is normally distributed with mean 4 and variance 4, find the probability that X lies between 1.202 and 83180000.
Given that $\log_{10} 1202 = 3.08$ and $\log_{10} 8318 = 3.92$ and $A(|z| < 1.96) = 95\%$
(A) 1.05 (B) 0.95
(C) 0.78 (D) None of these
- Q25** In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal then, how many students score between 12 and 15 ? Given : Area from $Z = 0$ to 0.4 is 0.1554 and from $Z = 0$ to 0.8 is 0.2881.
(A) 0.443 (B) 44.3
(C) 444 (D) 82
- Q26**



If the actual amount of instant coffee which a filling machine puts into '6-ounce' jars is a RV having a normal distribution with $SD = 0.05$ ounce and if only 3% of the jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars? Given that area under $f(z)$ from 0 to 1.808 is 47%

- Q27** If the two regression lines are known, then $r = \dots\dots$
- (A) A.M of the two regression coefficients
 - (B) G.M of the two regression coefficients
 - (C) H.M of the two regression coefficients

(D) product of the two regression coefficients

- Q28** If the two lines of regression are perpendicular then the correlation coefficient $r = \dots\dots\dots$.
- Q29** If the two regression co-efficients are 0.8 and 0.2, what would be the value of co - efficient of correlation.
- Q30** Two random variables have the regression lines with equations $3x + 2y = 26$ and $6x + y = 31$.Find the mean values and the correlation coefficient between x and y .



Answer Key

Q1	k=3 $e^{-1.5} - e^{-3}$	Q14	(C)
Q2	$F(x) = \begin{cases} 0, & \text{for } x \leq 0 \\ \frac{x^2}{2}, & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1, & 1 \leq x \leq 2 \\ 1, & x > 2 \end{cases}$	Q15	(C)
Q3	4	Q16	$\frac{1}{3}$ and $\frac{1}{5}$
Q4	(B)	Q17	294
Q5	$2^{-\frac{1}{3}}, 0.9298$	Q18	68.26%
Q6	$\frac{1}{9}$	Q19	6ft 0.18 inches
Q7	(B)	Q20	4886
Q8	2000	Q21	(C)
Q9	(A)	Q22	(C)
Q10	(D)	Q23	(C)
Q11	0.3679	Q24	(B)
Q12	0.223	Q25	(C)
Q13	(D)	Q26	$\mu = 6.094$ ounces.
		Q27	(B)
		Q28	$r = 0.$
		Q29	$r = 0.4$
		Q30	7



Hints & Solutions

Q1 Text Solution:

$$\int_0^\infty f(x) dx = 1$$

$$\int_0^\infty ke^{-3x} dx = 1$$

Putting limits we get –

$$\frac{-k}{3} \times (e^{-\infty} - e^{-0}) = 1$$

$$\frac{-k}{3} \left[\frac{1}{\infty} - 1 \right] = 1$$

$$k = 3$$

Now solving for the second part-

$$P(0.5 \leq X \leq 1)$$

$$\int_{0.5}^1 ke^{-3x} dx$$

$$\int_{0.5}^1 3e^{-3x} dx$$

$$\frac{3}{-3} e^{-3x} \left(\text{limits from } 0.5 \text{ to } 1 \right)$$

$$e^{-1.5} - e^{-3}$$

Q2 Text Solution:

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{For } x \leq 0, F(x) = \int_{-\infty}^x f(x) dx = \int_{-\infty}^x 0 dx = 0$$

$$\text{For } 0 < x < 1, F(x) = \int_{-\infty}^x f(x) dx = 0$$

$$+ \int_0^x x dx = \frac{x^2}{2}$$

$$\text{For } 1 \leq x \leq 2, F(x) = \int_{-\infty}^x f(x) dx$$

$$= \int_{-\infty}^0 0 dx + \int_0^1 x dx + \int_1^x (2 - x) dx$$

$$\int_{-\infty}^x f(x) dx = \int_{-\infty}^0 (0) dx + \int_0^1 x dx +$$

$$\int_1^x (2 - x) dx$$

$$0 + \frac{1}{2} + \left(2x - \frac{x^2}{2} \right)_1^x = 2x - \frac{x^2}{2} - 1$$

$$\text{For } x > 2, F(x) = 1$$

Hence required distribution $F(x)$ is

$$F(x) = \begin{cases} 0, & \text{for } x \leq 0 \\ \frac{x^2}{2}, & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1, & 1 \leq x \leq 2 \\ 1, & x > 2 \end{cases}$$

Q3 Text Solution:

$$f(x) = \frac{1}{2} e^{-\frac{|x|}{2}}, -\infty < x < \infty.$$

$$E(|X|) = \int_{-\infty}^{\infty} |x| \times \frac{1}{2} e^{-\frac{|x|}{2}} dx$$

As it is an even function –

$$2 \int_0^{\infty} |x| \times \frac{1}{2} e^{-\frac{|x|}{2}} dx$$

$$\int_0^{\infty} x e^{-\frac{x}{2}} dx$$

$$\frac{x}{2} = t, dx = 2dt$$

$$2 \int_0^{\infty} (2t)^{2-1} e^{-t} dt$$

$$4 \times 1 = 4$$

Q4 Text Solution:

$$E(X) = \frac{b+a}{2} \text{ for continuous uniform distribution}$$

$$V(X) = \frac{(b-a)^2}{12}$$

$$E(X) = 6V(X)$$

$$\frac{\alpha+1}{2} = 6 \frac{(\alpha-1)^2}{12}$$

$$\alpha + 1 = \alpha^2 + 1 - 2\alpha$$

$$\alpha^2 - 3\alpha = 0$$

$$\alpha = 0, 3$$

Considering $\alpha = 3$

Q5 Text Solution:

Approach 1:

It is given that, $P[x \leq a] = P[x \geq a]$

$$\int_0^a f(x) dx = \int_a^1 f(x) dx \Rightarrow \int_0^a 3x^2 dx =$$

$$\int_a^1 3x^2 dx$$

$$\Rightarrow a^3 = 1 - a^3$$

$$2a^3 = 1$$



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$$a = \frac{1}{\sqrt[3]{2}} = 2^{-\frac{1}{3}}$$

(B) From the condition B

It is given as, $P[x > b] = 0.05 \Rightarrow$

$$\int_b^1 3x^2 dx = 0.05 \Rightarrow 3 \cdot \frac{(1-b^3)}{3} = 0.05$$

$$\Rightarrow b^3 = 0.95 \Rightarrow b = \left(\frac{19}{20}\right)^{\frac{1}{3}} = 0.9298$$

Q6 Text Solution:

$$\text{Here, } \int_{-\infty}^{\infty} f(x) dx = \int_{-1}^2 \frac{x^2}{3} dx$$

$$= \frac{x^3}{9} \Big|_{-1}^2 = \frac{8}{9} + \frac{1}{9} = 1$$

Hence, $f(x)$ is a probability density function,

Now,

$$P(0 < X \leq 1) = \int_0^1 \frac{x^2}{3} dx = \frac{x^3}{9} \Big|_0^1 = \frac{1}{9}$$

Q7 Text Solution:

Here,

$$f(x) = \frac{A}{x^3} \quad (5 \leq x \leq 10)$$

$\therefore f(x)$ is probability density function, so

$$\int_5^{10} \frac{A}{x^3} dx = 1$$

$$\Rightarrow \left[\frac{-A}{2x^2} \right]_5^{10} = 1$$

$$\Rightarrow \frac{3A}{200} = 1$$

$$\Rightarrow A = \frac{200}{3}$$

Q8 Text Solution:

$$f(x) = \frac{1}{1000} e^{-\frac{x}{1000}}$$

here $x > 0$

$$E(X) = \int_0^{\infty} x f(x) dx$$

$$\int_0^{\infty} x \frac{1}{1000} e^{-\frac{x}{1000}} dx$$

Now using –

$$\int_0^{\infty} t^{n-1} e^{-t} dt = n!$$

$$\text{Thus here } t = \frac{x}{1000}, dx = 1000 dt$$

$$1000 \int_0^{\infty} t^{2-1} e^{-t} dt = (2-1)! \times 1000$$

$$= 1000$$

Q9 Text Solution:

$$P(X < 1 | X < 2)$$

$$\frac{P(X < 1)}{P(X < 2)}$$

$$f\left(\frac{x}{\mu}\right) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

$$\frac{\int_0^1 f(x) dx}{\int_0^2 f(x) dx}$$

$$\frac{\int_0^1 2e^{-2x} dx}{\int_0^2 2e^{-2x} dx}$$

$$\frac{\int_0^1 2e^{-2x} dx}{\int_0^2 2e^{-2x} dx}$$

evaluating numerator in between from 0 to 1 and denominator from 0 to 2 –

$$\frac{\frac{e^{-2x}}{2}}{\frac{e^{-2x}}{2}} = \frac{(1-e^{-2})}{(1-e^{-4})}$$

Q10 Text Solution:

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

Using the direct formula –

$$\text{Here } \lambda = 5 \text{ as } \frac{60}{12} = 5$$

$$P(X = 10) = \frac{5^{10} e^{-5}}{10!}$$

Q11 Text Solution:

If X represents the time to repair the machine, the density function of X is given by :

$$f(x) = \lambda e^{-\lambda x} = \frac{1}{2} e^{-\frac{x}{2}}, x > 0$$

$$P(X > 2) = \int_2^{\infty} \frac{1}{2} e^{-\frac{x}{2}} dx$$

$$= \left[-e^{-\frac{x}{2}} \right]_2^{\infty} = e^{-1} = 0.3679$$

Q12 Text Solution:

X { Number of students per minute }

$$\lambda = 30 \text{ students / hr} = 30 \text{ students / 60 mins} = \frac{1}{2} \text{ students / min}$$

i.e, in every 2 min, one student is coming.

i.e Inter Arrival time of two successive students = 2 mins

and we know that inter arrival time follows Exponential Distribution.

So Average waiting time for Bounce = $\frac{1}{\mu} = 2 \text{ mins.}$



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where $t = \{\text{waiting time}\}$ and p. d. f is $f(t) =$

$$\begin{cases} \mu e^{-\mu t} & t \geq 0 \\ \text{otherwise} & t < 0 \end{cases}$$

 So, $P(t > 3) = \int_3^{\infty} f(t) dt = \int_3^{\infty} \mu e^{-\mu t} \cdot dt$

$$= \mu \left(\frac{e^{-\mu t}}{-\mu} \right)_3^{\infty} = -1 [e^{-\infty} - e^{-3\mu}] = \frac{1}{e^{3\mu}} = \frac{1}{e^{3/2}} = 0.223$$

Q13 Text Solution:

$$E(X)=2, V(X)=$$

$$\frac{4}{3} \text{ over } [a, b]$$

We have to find $P(X < 1)$ so,

using the formula for expectation

$$\frac{a+b}{2} = 2$$

$$a + b = 4$$

using the formula for variance

$$\frac{(b-a)^2}{12} = \frac{4}{3}$$

$$b - a = 4, b - a = -4$$

solving the equations

$$a + b = 4$$

$$b - a = 4$$

we get $a = 0, b = 4$

solving the equations

$$a + b = 4$$

$$b - a = -4$$

we get $a = 4, b = 0$

Considering $a = 0, b = 4$

Now $P(X < 1)$ will be equal to –

$$\int_0^1 \frac{1}{b-a} dx$$

$$\int_0^1 \frac{1}{4} dx$$

$$1/4$$

Q14 Text Solution:

As it is mentioned that it is an uniform distribution thus considering the values

$$f(x) = \frac{1}{4-0} = \frac{1}{4}$$

$$\begin{aligned} \text{We require } P(x \geq 3) &= \int_3^4 f(x) dx \\ &= \int_3^4 \frac{1}{4} dx = \frac{1}{4} [x]_3^4 = \frac{1}{4} \end{aligned}$$

Q15 Text Solution:

$$E(x^3) = \int_{-\infty}^{\infty} x^3 f(x) dx = \int_0^1 x^3 (1) dx$$

As it is mentioned that the variable will take the values between 0 and 1 only thus we are taking the limit in between 0 and 1.

$$= \left(\frac{x^4}{4} \right)_0^1 = \frac{1}{4}$$

Q16 Text Solution:

Let X denote the time in minute past 7 A.M when the passenger arrives at the stop.

Then X is uniformly distributed over $(0, 30)$

i.e., $f(x) = \frac{1}{30}$, $0 < x < 30$.

(a) The passenger will have to wait less than 5 min if he arrives at the stop between 7:10 and 7:15 or 7:25 and 7:30.

∴ Required Probability

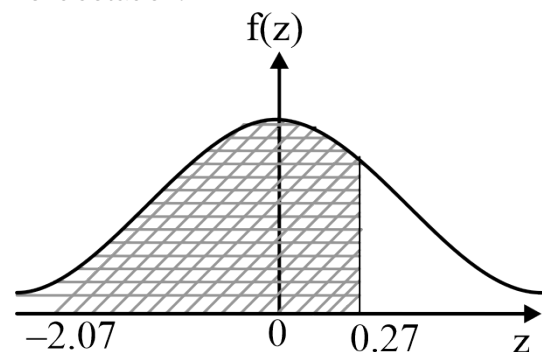
$$= P(10 < x < 15) + P(25 < x < 30)$$

$$= \int_{10}^{15} \frac{1}{30} dx + \int_{25}^{30} \frac{1}{30} dx = \frac{1}{3}$$

(b) The passenger will have to wait at least 12 min if he arrives at the stop between 7:00 and 7:03 or 7:15 and 7:18.

∴ Required probability = $P(0 < x < 3) + P(15 < x < 18)$

$$\int_0^3 \frac{1}{30} dx + \int_{15}^{18} \frac{1}{30} dx = \frac{1}{5}$$

Q17 Text Solution:

No. of student = 500

$\therefore N = 500$; Mean, $\mu = 151\text{cm}$

When $x_1 = 120\text{cm}$

Standard normal variable

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{120 - 151}{15} = \frac{-31}{15} = -2.07$$

When $x_2 = 155\text{ cm}$

Standard normal variable

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{155 - 151}{15} = 0.27$$

$$\therefore P(120 < x < 155) = P(-2.07 < z < 0.27) \\ = P(-2.07 \leq z \leq 0) + P(0 \leq z \leq 0.27)$$

$z \leq 0.27$)

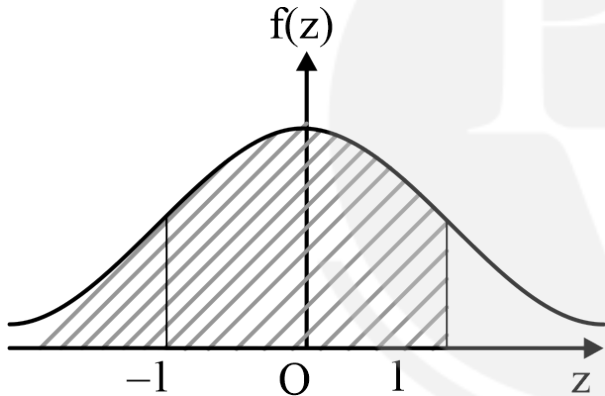
As the graph is symmetric thus-

$$= P(0 \leq z \leq 2.07) + P(0 \leq z \leq 0.27) \\ = 0.4808 + 0.1084 =$$

0.5892

\therefore The required number of student = $0.5892 \times 500 = 294$ (Appx.)

Q18 Text Solution:



$$\mu = 100\Omega, \sigma = 2\Omega$$

$$x_1 = 98\Omega, x_2 = 102\Omega$$

$$\therefore z_1 = \frac{x_1 - \mu}{\sigma} = \frac{98 - 100}{2} = -1$$

$$\text{and } z_2 = \frac{x_2 - \mu}{\sigma} = \frac{102 - 100}{2} = 1$$

$$\text{Now, } P(98 < x < 102) = P(-1 < z < 1) \\ = P(-1 \leq z \leq 0) + P(0 \leq z \leq 1) \\ = P(0 \leq z \leq 1) + P(0 \leq z \leq 1) \\ = 0.3413 + 0.3413 = 0.6826$$

\therefore Percentage of resistors having resistance between 98 ohms and 102ohms = 68.26%

Q19 Text Solution:

Mean $\mu = 64.5$ inches, S.D. $\sigma = 3.3$ inches

Area between 0 and

$$\frac{x - 64.5}{3.3} = 0.99 - 0.5 = 0.49$$

From the table, for the area 0.49, $z = 2.327$

The corresponding value of x is given by $\frac{x - 64.5}{3.3} = 2.327$

$$\Rightarrow x - 64.5 =$$

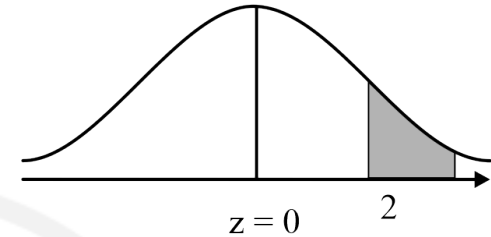
7.68

$$\Rightarrow x = 7.68 + 64.5 =$$

72.18 inches

Hence 99% student are of height less than 6ft 0.18 inches.

Q20 Text Solution:



Mean (μ) = 8

Standard deviation (σ) = 2

Number of pairs of shoes = 5000

Total months (x) = 12

$$\text{When } z = \frac{x - \mu}{\sigma} = \frac{12 - 8}{2} = 2$$

$$\text{Area when } (z \geq 2) = 1 - P(Z < 2) = 1 - 0.9772 = 0.0228$$

Number of pairs whose life is more than 12 months ($z > 2$) = $5000 \times 0.0228 = 114$

Replacement after 12 months = $5000 - 114 = 4886$ pairs of shoes

Q21 Text Solution:

The mean of a normal distribution is 50, its mode will be 50 as both are same in this case.

Q22 Text Solution:

The standard normal distribution is the normal distribution with mean 0 and variance 1

Q23 Text Solution:

The mode of a normal distribution is 80, then the median will also be 80, as in this type of distribution they all are same.

Q24 Text Solution:

As log X is a non-decreasing function of X, we have $P(1.202 < X < 83180000)$

$$= P(\log_{10} 1.202 < \log_{10} X < \log_{10} 83180000)$$

$$= P(0.08 < \log_{10} X < 7.92)$$

$$= P(0.8 < Y < 7.92)$$



Given,

$$Y = \log_{10} X \sim N(4, 4)$$

$$\text{Now, when } Y = 0.08, Z = \frac{0.08-4}{2} = -1.96$$

$$\text{and when } Y = 7.92, Z = \frac{7.92-4}{2} = 1.96$$

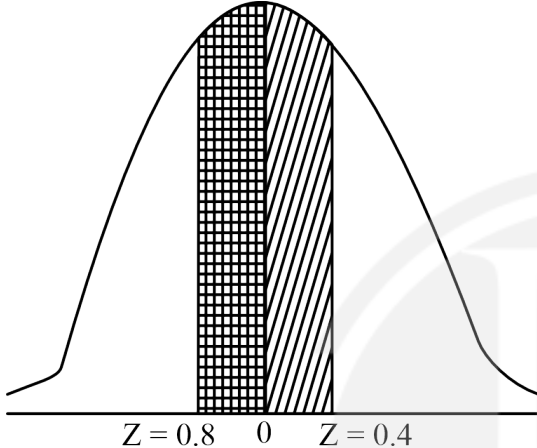
$$\text{Required probability} = 2P(-1.96 < Z < 1.96) \\ = 2 \times 0.475 = 0.95$$

Q25 Text Solution:

$$n = 1000, \mu = 14, \sigma = 2.5$$

$$\text{Here, } z_1 = \frac{x_1 - \mu}{\sigma} = \frac{12-14}{2.5} = -0.8$$

$$\text{and } z_2 = \frac{15-14}{2.5} = \frac{1}{2.5} = 0.4$$



Thus, the area lying between $z = -0.8$ to $z = 0.4$ \\ = [Area from ($z = 0$) to ($z = 0.8$)] + [Area ($z = 0$) to ($z = 0.4$)]

$$\text{Hence, the required number of students} \\ = 1000 \times 0.4435 = 443.5 = 444$$

Q26 Text Solution:

Let X be the actual amount of coffee put into the jars.

Then X follows $N(\mu, 0.05)$

$$P(X < 6) = 0.03$$

$$\therefore P\left\{-\infty < \frac{X-\mu}{0.05} < \frac{6-\mu}{0.05}\right\} = 0.03$$

$$\text{i.e. } \therefore P\left\{-\infty < Z < \frac{\mu-6}{0.05}\right\} = 0.47 \quad (\text{by symmetry})$$

From the table of areas, we have

$$P(0 < Z < 1.808) = 0.47$$

$$\therefore \frac{\mu-6}{0.05} = 1.808$$

$$\therefore \mu = 6.094 \text{ ounces.}$$

Q27 Text Solution:

Correlation coefficient ' r ' is the geometric mean (GM) of two regression coefficients.

$$r = \sqrt{b_{yx}b_{xy}}$$

Q28 Text Solution:

Angle between two regression lines

$$\tan \theta = \frac{1-r^2}{r} \left[\frac{\sigma_x \cdot \sigma_y}{\sigma_x^2 + \sigma_y^2} \right]$$

two regression lines are perpendicular

$$\theta = \frac{\pi}{2}; \text{ only when } r = 0.$$

Q29 Text Solution:

If the two regression co-efficients are positive correlation coefficients is also positive.

$$r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{0.8 \times 0.2} = \sqrt{0.16} \\ r = 0.4.$$

Q30 Text Solution:

Two regression equation are

$$3x + 2y = 26 \rightarrow (1) \text{ and } 6x + y = 31 \rightarrow (2)$$

Equation (1) is regression line of y on x and re-written as

$$y = \frac{-3x}{2} + 13 \rightarrow (3)$$

Equation (2) is regression line of x on y and re-written as

$$x = -\frac{1}{6}y + \frac{31}{6} \rightarrow (4)$$

$$\text{Hence } b_{yx} = -\frac{3}{2} \text{ and } b_{xy} = -\frac{1}{6}$$

$$r = -\sqrt{b_{yx} \times b_{xy}}$$

(\therefore both b_{yx} and b_{xy} are negative)

$$r = -\sqrt{\frac{3}{12}} = -\frac{1}{2}$$

$$r = -\frac{1}{2}$$

Both regression lines pass through (\bar{x}, \bar{y})

$$3\bar{x} + 2\bar{y} = 26$$

$$6\bar{x} + \bar{y} = 31 \text{ we get}$$

$$\bar{x} = 4 \text{ and } \bar{y} = 7$$

