DS & AI
CS & IT

Probability & Statistics - I

Probability
Lecture - 07

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

BASICS of PROBABILITY (Part-5)

(Baye's Theorem)

Topics to be Covered

Topic

BASICS of PROBABILITY (Part 6)

(Miscellaneous Topics)

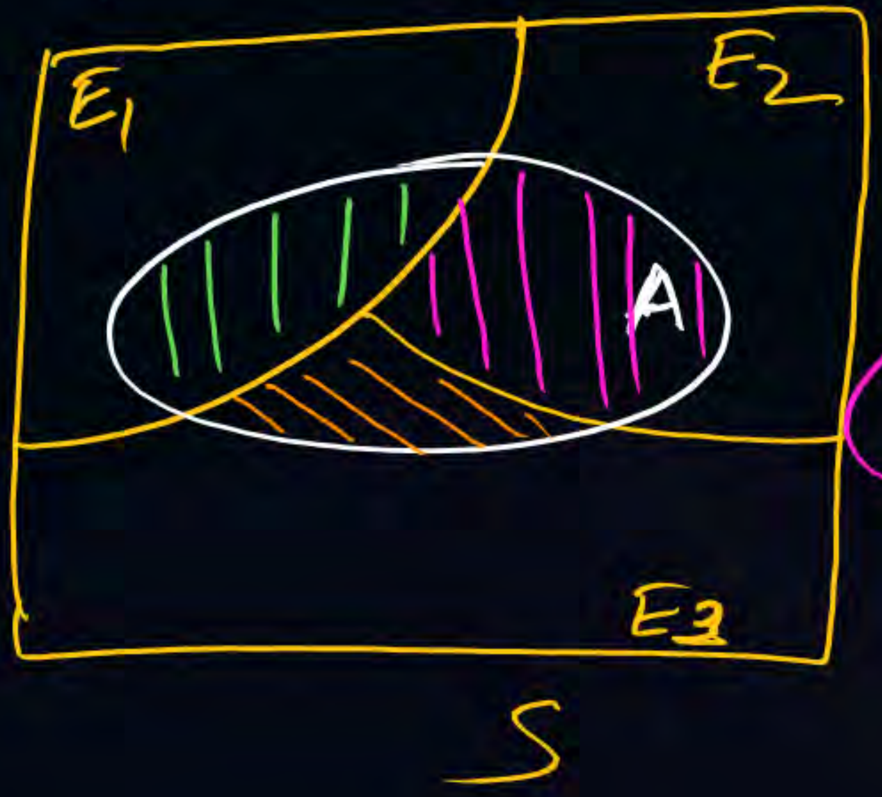
Thumb Rule of this Chapter → Try to avoid making Question by using following words;

" If, what if, AGAR, YADI, TOH, ... "

OR

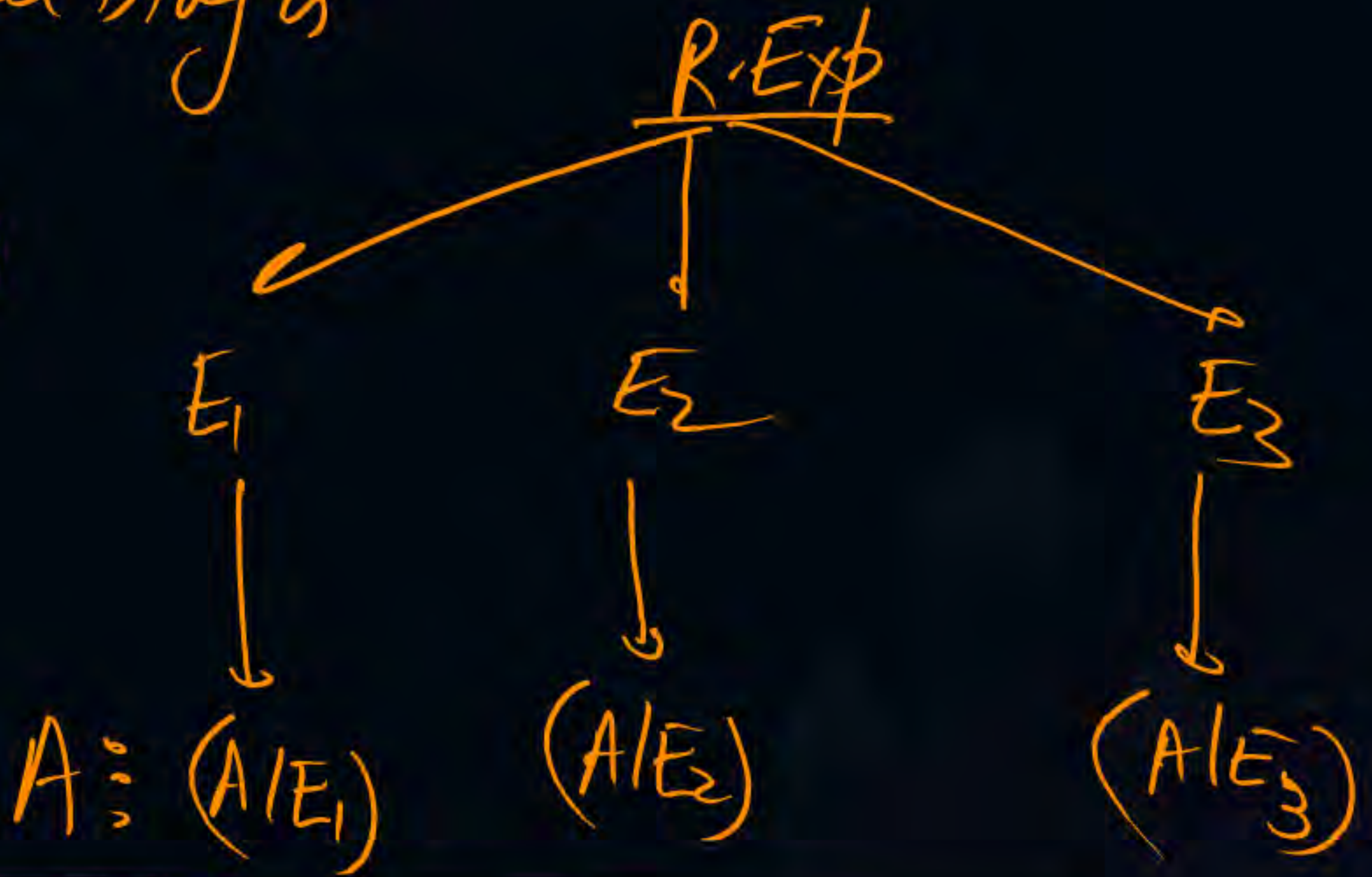
Don't Try to develop Question (by your $\boxed{\text{little mind}}$ until you have a complete understanding of the chapter) & try to solve the Quest.

Law of Total Prob → Let E_1, E_2, E_3 are ME & Exhaustive events associated with S.Space S & A is an Event which can occur with all E_1, E_2, E_3 i.e



& it's Tree Diag is

RECAP



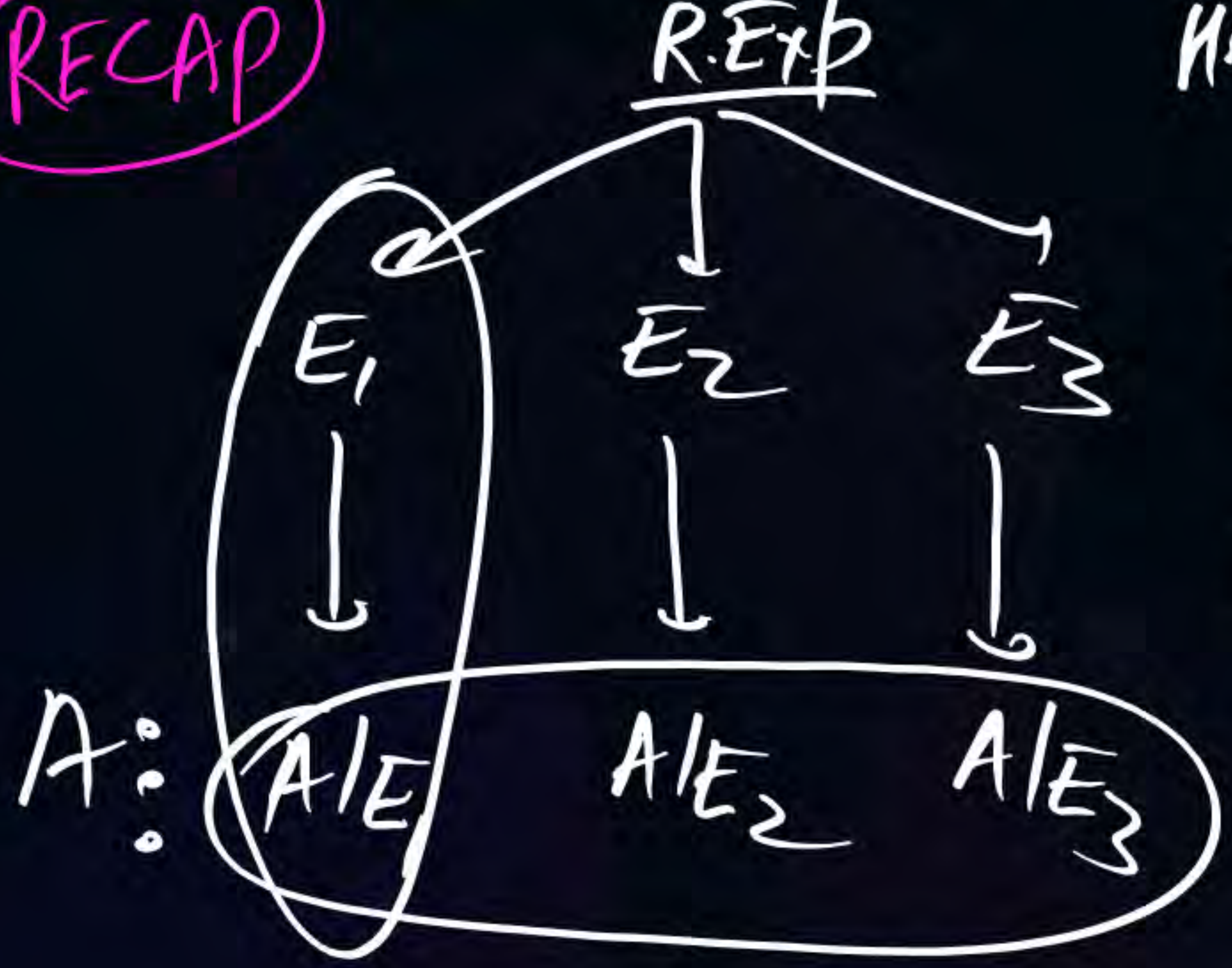
then $$\boxed{P(A) = P(E_1)\cdot P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

Baye's Theorem (Inverse prob theorem) $\longrightarrow$.

(this Theorem is useful to solve complex (= tough) Questions of Conditional Probability.)

Theory Same as above

RECAP



Here $\boxed{P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$ — (I)

$$\left.\begin{aligned} P(E_1/A) &= \frac{P(E_1)\cdot P(A/E_1)}{P(A)} \\ P(E_2/A) &= \frac{P(E_2)P(A/E_2)}{P(A)} \\ P(E_3/A) &= \frac{P(E_3)P(A/E_3)}{P(A)} \end{aligned}\right\} \text{Baye's Th.}$$

Important Points :→ RECAP

1. Necessary condition for the existence of Law of Total Prob & Baye's Th is
Associated events must be ME & Exhaustive.

2. In Law of Total Prob : A = { Assume that event as A which is Required }

3. In Baye's Th : A = { Assume that event as A, which is given as condition }

4. If in a Question, there is a feeling of $\boxed{\text{CROSS Check}}$ the given condition we can use Baye's Th.
& if we have No condition in a Question (or No feeling of CROSS Check) then use Law of Total Prob.

Q: A Person is known to speak Truth 3 out of 4 times.

HW

$\underbrace{\text{(He throw a die)}}_{\text{R Exp}}$ & $\underbrace{\text{Reports that it is six}}_{\text{Condition}}$ then Find the prob that it is Actually six?

$A = \{\text{Man Reports that it is six}\}$, $P(6) = \frac{1}{6}$, $P(\bar{6}) = \frac{5}{6}$

$P(\text{Telling Truth}) = \frac{3}{4}$ & $P(\text{Telling lie}) = \frac{1}{4}$

M-I

Die

$\xrightarrow{1/6}$ Six occurs (E_1) $\rightarrow$ $A/E_1 = \frac{3}{4}$

$\xrightarrow{5/6}$ Six Not occurs (E_2) $\rightarrow$ $A/E_2 = \frac{1}{4}$

Man Reports that it is six A:

$P(A/E_1) = P\left(\frac{\text{Reports that it is six}}{\text{six occurs}}\right) = P(T.T) = \frac{3}{4}$

$P(A/E_2) = P\left(\frac{\text{Reports that it is six}}{\text{six Not occurs}}\right) = P(T.L) = \frac{1}{4}$

$P(\text{actually six}) = P(E_1/A) = \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$

M-II

Die

$\xrightarrow{3/4}$ Telling Truth (E_1) $\longrightarrow$ $A/E_1 = \frac{1}{6}$

$\xrightarrow{1/4}$ Telling Lie (E_2) $\longrightarrow$ $A/E_2 = \frac{5}{6}$

Man Reports that it is six A:

$P(A/E_1) = P\left(\frac{\text{Reports that it is six}}{\text{Telling Truth}}\right) = P(6 \text{ occurs}) = \frac{1}{6}$

$P(A/E_2) = P\left(\frac{\text{Reports that it is six}}{\text{Telling lie}}\right) = P(6 \text{ Not occurs}) = \frac{5}{6}$

$P(A) = \frac{3}{4}\times\frac{1}{6} + \frac{1}{4}\times\frac{5}{6} = \frac{8}{24}$

$P(\text{actually six}) = P(E_1/A) = \frac{\frac{3}{4}\times\frac{1}{6}}{(8/24)}$

$= 3/8$.

$P(\text{Man Reports that it is six}) = 8/24$.

out of 24 Reports given by him, 8 are representing that 'It is six'

& out of 8 Reports representing that 'It is six' only 3 are correct.

PLAYING CARDS: →

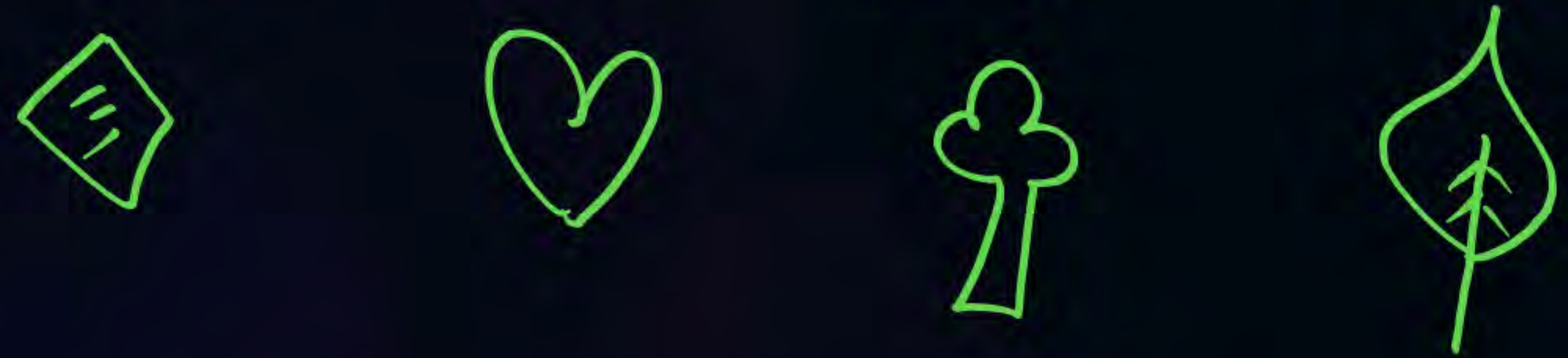
there are 4 suits eg (D, H, C, S)

FACE CARDS = 12 (K, Q, J)

HONOUR CARDS = 16 (K, Q, J, A)

52 Cards

- 26 RED
  - 13 Diamond
  - 13 Heart
- 26 Black
  - 13 Club
  - 13 Spade



Q: From a pack of 52 cards, while shuffling, four cards are accidently dropped then Find the prob that missing cards will be one from each suits ?

Sol: Elements of S-Space are in the form of ordered Quadruples.
So Not easy to write S-Space in terms of set.

App II: Total ways of loosing 4 cards (accidently / simultaneously) $= {}^{52}C_4$ ways

fav ways " " " $= \{\text{one from each suit}\}$

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = ({}^{13}C_1)^4$ ways.

So Req. Prob $= \frac{\text{fav}}{\text{Total}} = \frac{({}^{13}C_1)^4}{{}^{52}C_4} = ? = 0.1054$

Q:- From a pack of 52 Cards, while shuffeling one Card is lost & then two Cards are drawn at Random then find the prob that both the selected Cards are of Spade?

(M-I) To Perform this R-Exp we have to Complete two jobs
- job 1 = one Card is lost → E_1, E_2
- job 2 = Now two Cards are drawn (A)

Now for job 1
- Case I → Spade is lost (E_1) $\Rightarrow P(E_1) = \dfrac{{}^{13}C_1}{{}^{52}C_1} = \dfrac{1}{4}$
- Case II → Spade is Not lost (E_2) $\Rightarrow P(E_2) = \dfrac{{}^{39}C_1}{{}^{52}C_1} = \dfrac{3}{4}$

Now for job 2
- for Case I → (12 Sp & 39 oth) $\Rightarrow$ P(Both are Spade) $= \dfrac{{}^{12}C_2}{{}^{51}C_2}$
- for Case II → (13 Sp & 38 oth) $\Rightarrow$ P(Both are Spade) $= \dfrac{{}^{13}C_2}{{}^{51}C_2}$

Req Prob $= P(\text{using Case I}) + P(\text{using Case II})$

$= P(\text{spade is lost}) \times P(\text{two cards drawn are of spade})$

$+ P(\text{spade not lost}) \times P(\text{two cards drawn are of spade})$

$$= \frac{1}{4} \times \frac{{}^{12}C_2}{{}^{51}C_2} + \frac{3}{4} \times \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{1}{17}$$

(M-II) (Using L.T.P) → $A = \{$ Both the selected cards are of spade $\}$

Shuffling of Cards (52)

$\frac{13}{52} = \frac{1}{4}$ $\qquad$ $\frac{39}{52} = \frac{3}{4}$

Spade is Lost (E_1) $\qquad$ Spade Not Lost (E_2)

(12 Sp & 39 oth) $\qquad$ (13 Sp & 38 oth)

A :

$$A/E_1 = \frac{{}^{12}C_2}{{}^{51}C_2} \qquad A/E_2 = \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$P(A) = \frac{1}{4} \times \frac{{}^{12}C_2}{{}^{51}C_2} + \frac{3}{4} \times \frac{{}^{13}C_2}{{}^{51}C_2}$$

$$= \frac{1}{17}$$ Ans

Concept of with or w/o Replacement →

Eg: 3 Cards are drawn from a pack of 52 Cards then find the number of ways if Cards are drawn

(1) Simultaneously (at Random) $= {}^{52}C_3 = \frac{52\times51\times50}{3\times2\times1}$

(2) one by one with Replacement $= {}^{52}C_1 \times {}^{52}C_1 \times {}^{52}C_1$

(3) one by one w/o Replacement $= {}^{52}C_1 \times {}^{51}C_1 \times {}^{50}C_1$

Q: From a pack of Regular playing Cards, two Cards are drawn then find the Prob that both will be kings if 1st Card is not Replaced ?

$\underbrace{\text{1st Card is not Replaced}}_{\text{w/o Replacement}}$

Sol:

$$\text{Req Prob} = P(K \cap K) = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^3C_1}{{}^{51}C_1}$$

$$= \frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

(ii) Also find the ans if Cards are drawn one by one with Replacement ?

Sol:

$$\text{Req Prob} = P(K \cap K)$$

$$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

Q: A Box Contains 3R & 4B Marbles and 3 Marbles are drawn one by one w/o Replacement then find the prob of drawing 1R & 2B Marbles?

M-I (By Making Various Cases) →

Req Prob = P[(RBB) or (BRB) or (BBR)]

$$= \left(\frac{3}{7}\cdot\frac{4}{6}\cdot\frac{3}{5}\right) + \left(\frac{4}{7}\cdot\frac{3}{6}\cdot\frac{3}{5}\right) + \left(\frac{4}{7}\cdot\frac{3}{6}\cdot\frac{3}{5}\right)$$

$$= \left(\frac{3}{7}\times\frac{4}{6}\times\frac{3}{5}\right)\times 3$$

M-III R. Prob $= P(RBB)\times\frac{3!}{2!}$

$$= \left(\frac{3}{7}\times\frac{4}{6}\times\frac{3}{5}\right)\times 3$$

M-II (Using Hypergeometric Dist) →

7 Marble

3R   3M   4B

1R   2B

Req Prob $= \frac{\text{fav}}{\text{Total}} = \frac{{}^3C_1 \times {}^4C_2}{{}^7C_3} = \frac{3\times\left(\frac{4\times 3}{2}\right)}{\left(\frac{7\times 6\times 5}{3\times 2\times 1}\right)} = \left(\frac{3\times 4\times 3}{7\times 6\times 5}\right)\times 3$

(3R & 4B)

(ii) Also find the Prob, if Marbles are drawn one by one with Replacement.

M-I (By Making Cases) →

Req Prob $= P[(RBB) \text{ or } (BRB) \text{ or } (BBR)]$

$$= \left(\frac{3}{7}\times\frac{4}{7}\times\frac{4}{7}\right) + \left(\frac{4}{7}\times\frac{3}{7}\times\frac{4}{7}\right) + \left(\frac{4}{7}\times\frac{4}{7}\times\frac{3}{7}\right)$$

$$= \left(\frac{3}{7}\times\frac{4}{7}\times\frac{4}{7}\right)\times 3$$

M-II Using Binomial Distribution

(Have patience)

Q8) There are 10 Markers on a Table in which 6 are Defective & 4 Non Defective. If 3 Markers are drawn one by one w/o Replacements then find the prob that there will be exactly one Defective?

M-I (By Making Cases) →

Req Prob $= P[(DNN) \text{ or } (NDN) \text{ or } (NND)]$

$$= \left(\frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}\right)+\left(\frac{4}{10}\times\frac{6}{9}\times\frac{3}{8}\right)+\left(\frac{4}{10}\times\frac{3}{9}\times\frac{6}{8}\right)$$

$$= \left(\frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}\right)\times 3$$

$$= \frac{3}{10}$$

M-II using Hypergeometric Dist

10 Markers → 6 Def, 3 Markers, 4 N·D

6 Def → 1 Def

4 N·D → 2 N·D

$$\text{Req Prob} = \frac{\text{fav}}{\text{Total}} = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3} = \frac{6\times\left(\frac{4\times3}{2\times1}\right)}{\left(\frac{10\times9\times8}{3\times2\times1}\right)} = \left(\frac{6\times4\times3}{10\times9\times8}\right)\times 3 = \frac{3}{10}$$

6D & 4 N.D

(ii) Also find the Ans if Markers are drawn one by one with Replacement

M-I By Making Cases :→

Req Prob = P[(DNN) or (NDN) or (NND)]

$$= \left(\frac{6}{10}\times\frac{4}{10}\times\frac{4}{10}\right)+\left(\frac{4}{10}\times\frac{6}{10}\times\frac{4}{10}\right)+\left(\frac{4}{10}\times\frac{4}{10}\times\frac{6}{10}\right)$$

$$= \left(\frac{6}{10}\times\frac{4}{10}\times\frac{4}{10}\right)\times 3$$

M-III Req Prob = $P(DNN)\times\frac{3!}{2!} = \left(\frac{6}{10}\times\frac{4}{10}\times\frac{4}{10}\right)\times 3$

M-II Using Binomial Dist.

Have patience

A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

(a) $\frac{1}{5}$ (b) $\frac{1}{25}$

(c) $\frac{20}{99}$ (d) $\frac{19}{495}$

M-II

100 items

20 Def — 2 items — 80 N.Def

2 Def — 0 N.Def

Req Prob $= \frac{F}{T} = \frac{{}^{20}C_2 \times {}^{80}C_0}{{}^{100}C_2} = \frac{19}{495}$ (d)

M-I Req Prob $= P(D \cap D) \times \frac{2!}{2!}$

$= \frac{20}{100} \times \frac{19}{99} \times 1$

$=$ (d)

Q: Also find P(one is Def & other is N.D) = ?

Req Prob $= P(ND \text{ or } DN) = \left(\frac{80}{100} \times \frac{20}{99}\right) + \left(\frac{20}{100} \times \frac{80}{99}\right) = \frac{32}{99}$

M-II R.Prob $= P(ND) \times 2! = \frac{32}{99}$

A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its colour recorded and it is put back in the bag. This process is repeated $\boxed{\text{3 times.}}$ The probability that no two of the marbles drawn have the same colour is

ie all three are of different colours.

10 Blue
20 Black
30 Red

with Replacement

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

Various Cases are as follows:-

$(R B B_l), (R B_l B), (B_l R B), (B_l B R), (B B_l R), (B R B_l) = 6$ Cases

Fav. Cases $= \{(\text{Blue}, \text{Black}, \text{Red})\} \times 3!$ Cases

$= \left(\frac{10}{60} \times \frac{20}{60} \times \frac{30}{60}\right) \times 6 = \frac{1}{6}$

Questions Based on Special Type of Tree Diagram $\longrightarrow$

Equally likely Events $\rightarrow$ E_1, E_2, E_3 are called equally likely if $P(E_1) = P(E_2) = P(E_3)$

for eg $S_D = \{1, 2, 3, 4, 5, 6\} \Rightarrow$ each outcome is equally likely

$\therefore P(E_1) = P(E_2) = \cdots = P(E_6) = \frac{1}{6}$

for eg if Die is thrown twice then $S = \left\{ \begin{matrix} (11)\ (12)\ (13) \ldots (16) \\ (21)\ (22) \ldots (26) \end{matrix} \right\} = 36$ pair

so $P(E_1) = P(E_2) = P(E_3) = \cdots = P(E_{36}) = \frac{1}{36}$ each is equally likely.

(*) the concept of $\frac{\text{fav}}{\text{Total}}$ is $\boxed{\text{Valid only}}$ in that types of Questions whose individual elements of S. Space are equally likely.

Qs. GATE A coin is tossed until Head appears then find the prob that required No. of tosses to end such type of Game will be odd?

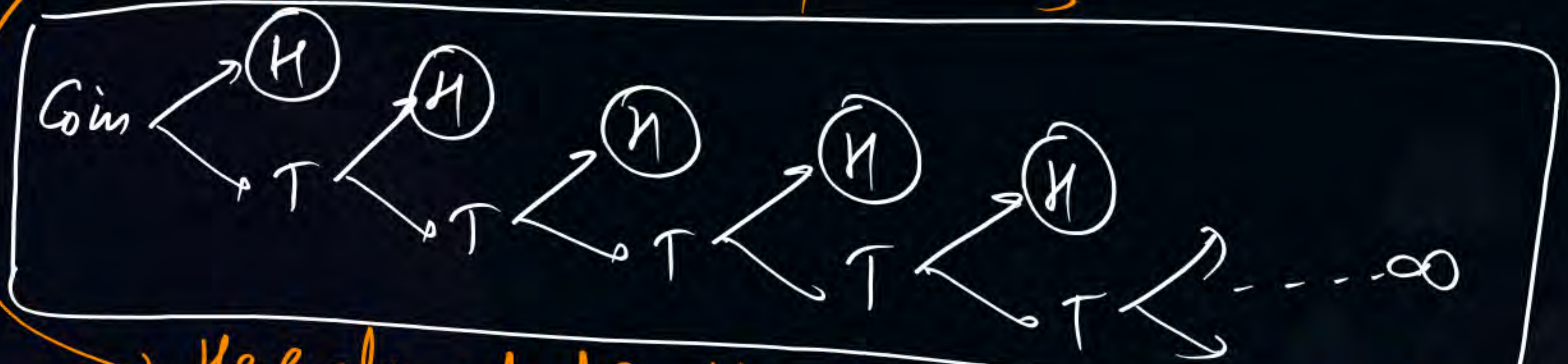
(a) 6/11

(b) 2/3

(c) 1/3

(d) 1/2

$$S = \{\underbrace{H}_{1},\ \underbrace{TH}_{2},\ \underbrace{TTH}_{3},\ \underbrace{TTTH}_{4},\ \underbrace{TTTTH}_{5},\ \ldots\ldots\}$$



Here elements of Sample Space are not equally likely

So $\cancel{P(\text{odd tosses}) = \dfrac{\text{Half are odd tosses}}{\text{Total tosses}} = \dfrac{1}{2}}$

<u>Correct App</u> :- Req Prob $= P[\text{odd tosses}]$

$= P[(H) \text{ or } (TTH) \text{ or } (TTTTH) \text{ or } (TTTTTTH) \text{ or } ----]$

$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^7 + ------$

$\because a = \frac{1}{2}$

$r = \left(\frac{1}{2}\right)^2$

$= \dfrac{1/2}{1-\left(1/2\right)^2} = \dfrac{1/2}{3/4} = \dfrac{2}{3}$

<u>Note</u>: (1) $S_n = a + ar + ar^2 + \ldots\ldots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}$ or $\dfrac{a(r^n-1)}{r-1}$

(2) $S_\infty = a + ar + ar^2 + ar^3 + \ldots\ldots \infty = \dfrac{a}{1-r}$, $-1 < r < 1$

MSQ

Q. Two persons A & B play a game of Dice alternately, in which any one can win if six appears 1st time then find their resp chances of winning if A starts the game?

≈ Prob

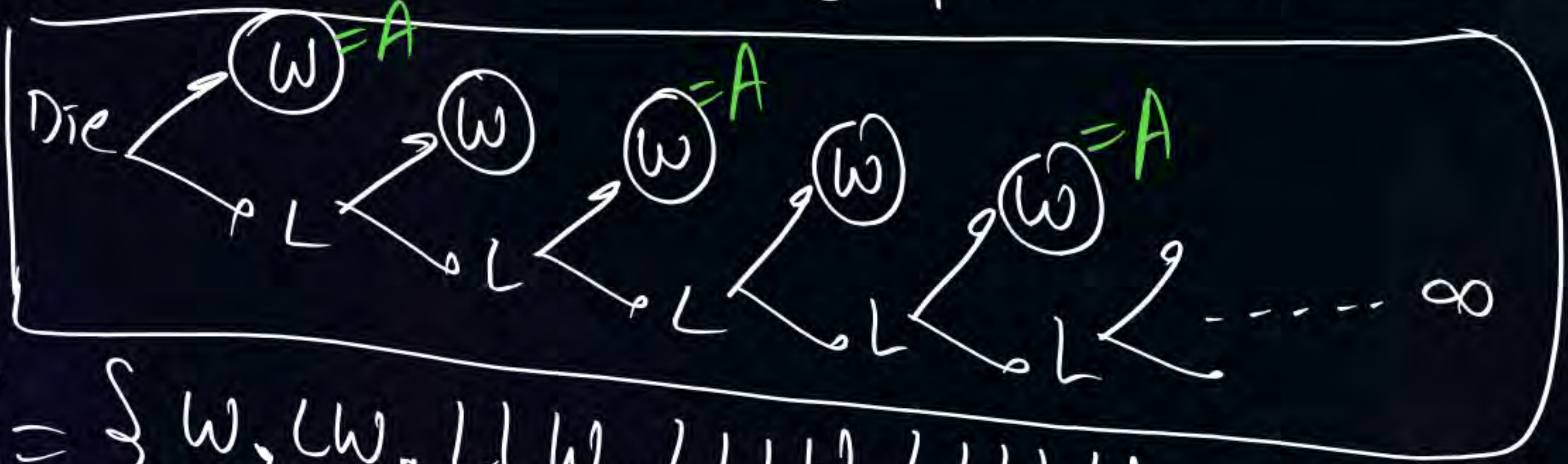
(a) $2/3$

(b) $6/11$ ✓

(c) $5/11$ ✓

(d) $\frac{1}{6}$

$P(W) = P(\text{six occurs}) = \frac{1}{6}$ & $P(L) = P(\text{six Not occurs}) = \frac{5}{6}$



$S = \{W, LW, LLW, LLLW, LLLLW, \ldots\}$

fav cases for A to win $= \{W$ or LLW or $LLLLW$ or $\ldots\}$

Req Prob $= P[W$ or LLW or $LLLLW$ or $\ldots]$

$$= \left(\frac{1}{6}\right) + \left(\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}\right) + \left(\left(\frac{5}{6}\right)^4 \frac{1}{6}\right) + \ldots\ldots$$

$$P(A\text{ win}) = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots \infty\right] = \frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^2}\right] = \frac{6}{11}$$ so (b) ✓

& $P(B\text{ win}) = 1 - P(A\text{ win}) = 1 - \frac{6}{11} = \frac{5}{11}$

HW Q. Three persons A, B, C play a game of Dice, $\boxed{\text{one after another}}$ in which anyone can win if 6 appears 1st time then find their respective chances of winning? if A starts.

in succession

Ans: $\left(\frac{36}{91}, \frac{30}{91}, \frac{25}{91}\right)$

A = B = C =

$(A \to B \to C) \longrightarrow (A \to B \to C) \longrightarrow (A \to B \to C) \to \ldots \ldots \infty$

How Free Diag Questions Based on equally likely events →

See Next Slide.

Q: A Basket contains 20 Apples & 10 oranges in which 5 Apples & 3 oranges are Rotten. If 2 fruits are choosen at Random then find the prob that either both are Apples or both are good?

Sol: $S = \{$ elements are in the form of ordered pair $\} = \begin{Bmatrix} (A_1, O_1)(A_1, O_2) \ldots (A_1, O_{10}) \\ (A_2 O_1), (A_2 O_2) \ldots \\ \ldots ? \end{Bmatrix}$ = Not easy to write

App II

Total ways of drawing 2 fruits $= {}^{30}C_2$

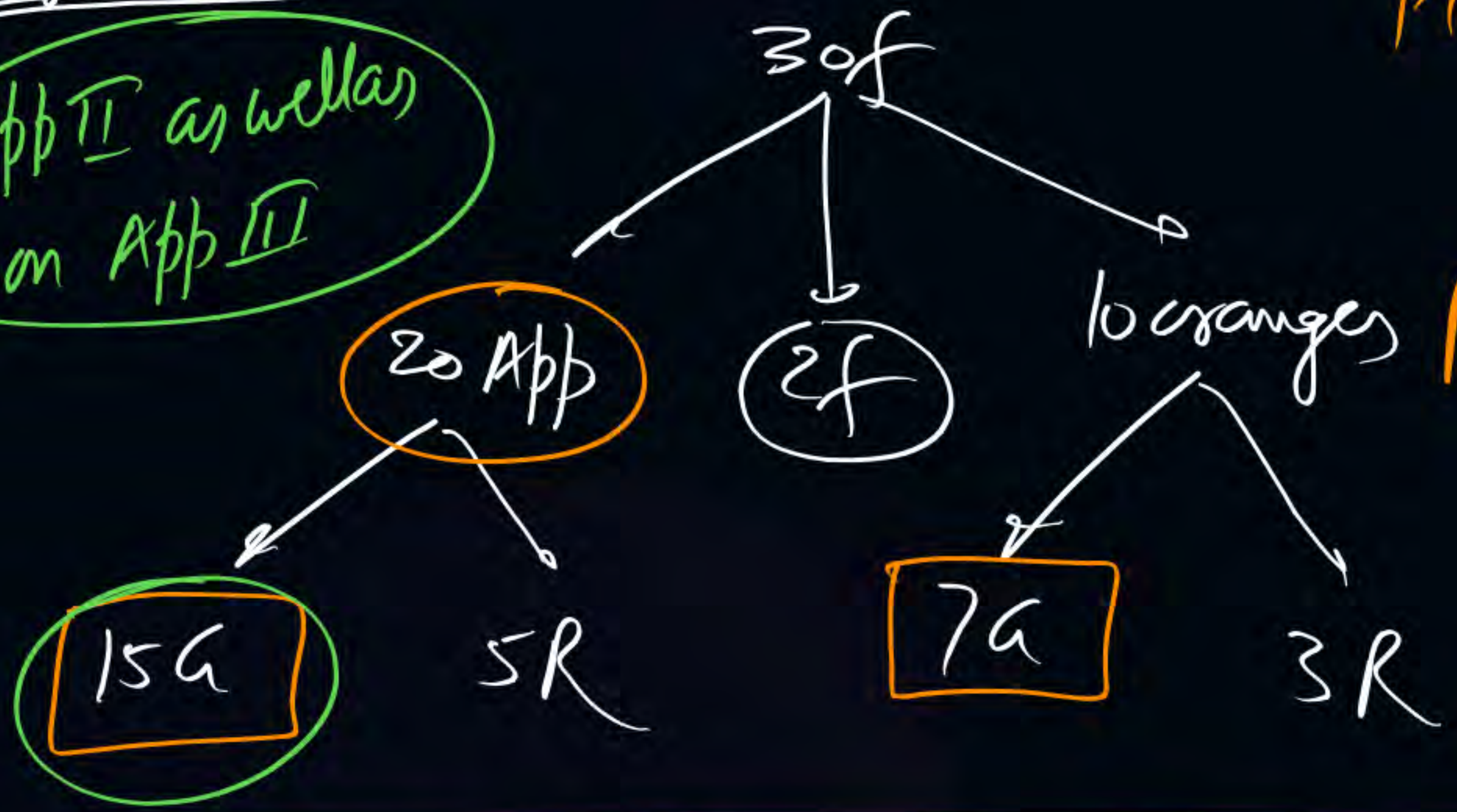
$A = \{$Both the fruits are Apples$\} \Rightarrow P(A) = \dfrac{{}^{20}C_2}{{}^{30}C_2}$

$G = \{$ .. .. .. are Good$\} \Rightarrow P(G) = \dfrac{{}^{22}C_2}{{}^{30}C_2}$

$A \cap G = \{$Both the fruits are Good Apples$\} \Rightarrow P(A \cap G) = \dfrac{{}^{15}C_2}{{}^{30}C_2}$

Info. Tree

App II as well as on App III



$$P(A) = \frac{{}^{20}C_2}{{}^{30}C_2}$$

$$P(G) = \frac{{}^{22}C_2}{{}^{30}C_2}$$

$$P(A \cap G) = \frac{{}^{15}C_2}{{}^{30}C_2}$$

$$\boxed{P(A \cup G) = ? = P(A) + P(G) - P(A \cap G)}$$

$$= \frac{{}^{20}C_2 + {}^{22}C_2 - {}^{15}C_2}{{}^{30}C_2} = \frac{316}{435}$$

Q. Aishwarya studies either CS or Maths on each day.
If she studies CS on a day then the prob of studying M on next day is 0.6 &
If " " M " " " " " CS " " is 0.4.
Given that Aish studies CS on Monday, then find the prob that she will also study CS on wednesday. (Ans = 0.4 or 0.6
(?) (?))

INFO TREE

NIRMAN
2025

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Thank
YOU