DS & AI
CS & IT

Probability & Statistics -I

Probability
Lecture : 03

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Recap of previous lecture

Topic

BASICS OF PROBABILITY

(Part-1)

Topics to be Covered

Topic

BASICS of PROBABILITY

(Part-2)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

① Addition Theorem of Prob. : → $\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

② Multiplication Theorem of Prob → $\boxed{P(A\cap B) = P(A/B)\cdot P(B)}$

③ P(Neither A Nor B) = 1 − P(either A or B or Both)

$$\boxed{P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B)}$$

④ P(either A or B or Both) = 1 − P(Neither A Nor B)

$\boxed{P(\text{at least one of A or B}) = 1 - P(\text{None})}$

Mutually Exclusive Events →

If, two events can't occur simultaneously, then these are called M.E. Events

OR

If occurrence of one event prevents the occurrence of other event & vice versa than events are called ME Events. ie

If A & B are ME then only one can occur at a time

Mathematically: if E_1 & E_2 are ME events then $E_1 \cap E_2 = \phi$

Conclusion: if E_1 & E_2 are ME then

→ $P(E_1 \cap E_2) = 0$

→ $P(E_1 \cup E_2) = P(E_1) + P(E_2) - 0$

eg $S_D = \{1, 2, 3, 4, 5, 6\}$ & let us consider following events

$E_1 = \{1, 3, 5\}$ $\because E_1 \cap E_2 = \phi \Rightarrow E_1$ & E_2 are M.E & $P(E_1 \cap E_2) = 0$

$E_2 = \{2, 4, 6\}$ $\because E_2 \cap E_3 \neq \phi \Rightarrow E_1$ & E_2 are Not M.E

$E_3 = \{1, 2, 3, 4\}$ $\because E_1 \cap E_3 \neq \phi \Rightarrow E_1$ & E_3 are Not ME

$E_4 = \{2, 4\}$, $\because E_1 \cap E_4 = \phi \Rightarrow E_1$ & E_4 are also ME But $E_1 \cup E_4 \neq S$

i.e it is not Necessary that, in Case of ME Events, you will get their union as S. Space

$E_4 = \{x : 1 < x < 5$ & x is divisible by $2\}$

Independent Events. $\rightarrow$ If occurrence or Non occurrence of one event does not alter the occurrence or Non occurrence of other Event then Events are Called Independent events

$\Longleftrightarrow$

Mathematically: If A & B are Ind Events then $\Longleftrightarrow$ $\boxed{P(A \cap B) = P(A) \cdot P(B)}$

eg; $S_{Coin} = \{H, T\}$, $S_{Die} = \{1, 2, 3, 4, 5, 6\}$

$A = \{H\} \Rightarrow P(A) = \frac{1}{2}$ $\quad B = \{1, 2, 3, 4\} \Rightarrow P(B) = \frac{4}{6}$

then $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\because \text{ A \& B are Ind. Events.}} = \frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$

PODCAST :- (Deep SEEK)

① ME events are associated with SAME sample space
while Ind " " " " DIFFERENT sample spaces

② In case of Ind events, we can observe that

$\boxed{A \cap B = \phi}$ then why not these are ME ??

∵ we are forcefully drawing a wrong conclusion.
as the concept of Intersection is applicable with in the same S.Sp.
→ that's why it is WRONG conclusion.

③ Same sample space
- A & B may be ME
- " " may not be ME

(4) Events formed by individual elements of s.space are ME (T)

eg $S_D = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1\}, E_2 = \{2\}, E_3 = \{3\}, E_4 = \{4\}, E_5 = \{5\}, E_6 = \{6\}$

$\because E_i \cap E_j = \phi \ \forall i \,\&\, j \Rightarrow E_i \,\&\, E_j$ are ME.

eg $S_{coin} = \{H, T\}, E_1 = \{H\}, E_2 = \{T\}$

$\because E_1 \cap E_2 = \phi \Rightarrow E_1 \,\&\, E_2$ are ME

(5) If two Events E_1 & E_2 are associated with different S space then question of their ME Nature doesn't arise.

Short RECAP :-

Operation	P&C	Prob	Formula	ME	Ind.
Either or	Plus	union	Addition Th	$P(A\cup B)=P(A)+P(B)$	*
AND	Multiply	Intersection	Multi Th	$P(A\cap B)=0$	$P(A\cap B)=P(A)\cdot P(B)$

Addition Th: $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

* for Independency: $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

for ME : $P(A\cup B)=P(A)+P(B)-0$

Qs Four Dice are thrown simultaneously then find the Prob that sum of the outcomes is 22 ?

Sol:

$$S = \left\{ \begin{matrix} (1111), (1112), (1113) \ldots (1116) \\ (2111), (2112), \ldots (2116) \\ (3111) \ldots (6666) \end{matrix} \right\} \Rightarrow n(S) = \underset{D_1}{6} \times \underset{D_2}{6} \times \underset{D_3}{6} \times \underset{D_4}{6}$$

$= 6^4 = 1296$ Quadruples.

$$\text{fav Quadruples} = \left\{ \begin{matrix} (6664), (6646), (6466), (4666) \\ (6655), (6565), (6556) \\ (5566), (5656), (5665) \end{matrix} \right\} = 10 \text{ Quadruples}$$

So $\text{Prob}(\text{Sum} = 22) = \frac{\text{fav}}{\text{Total}} = \frac{10}{1296}$

App II:

$$\text{fav Cases} = \left\{ \begin{matrix} (6664) \ldots \\ (6655) \ldots \end{matrix} \right\} = \begin{matrix} \frac{4!}{3!} = 4 \text{ Quad} \\ \frac{4!}{2!2!} = 6 \text{ Quad} \end{matrix} \Bigg\} = 10 \text{ Quad. So } P = \frac{10}{1296}$$

Q. 7 Surgical Strikes occurred in a week (= 7 days) from INDIA on PAKISTAN then find the prob that all will occur on a same day ?

Sol: (App II) Total ways of occurring S.S (RA) $= \underset{S_1}{7} \times \underset{S_2}{7} \times \underset{S_3}{7} \times \underset{S_4}{7} \times \underset{S_5}{7} \times \underset{S_6}{7} \times \underset{S_7}{7} = 7^7$ ways

fav ways of occurring S.S = {All will occur on a same day}

$= \underset{S_1}{7\text{ ways}} \times \underset{S_2}{1} \times \underset{S_3}{1} \times \underset{S_4}{1} \times \underset{S_5}{1} \times \underset{S_6}{1} \times \underset{S_7}{1} = 7$ ways $= {}^7C_1$

Req Prob $= \dfrac{\text{fav ways}}{\text{Total ways}} = \dfrac{7}{7^7} = \dfrac{1}{7^6}$

Analysis:

$$\begin{array}{ccccccccc} & & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 \\ (1) & & (M & M & M & M & M & M & M) \\ & \text{or} & (T & T & T & T & T & T & T) \\ & \text{or} & (W & W & W & W & W & W & W) \\ & \text{or} & (Th & Th & Th & Th & Th & Th & Th) \\ & \text{or} & (F & F & F & F & F & F & F) \\ & \text{or} & (S & S & S & S & S & S & S) \\ & \text{or} & (Su & Su & Su & Su & Su & Su & Su) \end{array} \Bigg\} \text{ 7 choices}$$

So fav. choices $= {}^7C_1 = 7$

(2) Here all the 5 strikes are different.

What is the probability that a leap year selected at random will contain 53 Sundays?

Appr I

Leap year = 366 days = 364 days + 2 days

= 52 × 7 days + 2 days

= 52 weeks & 2 days.

i.e 100% we have 52 Sundays. Now for Remaining two days.

S = { (M T), (T W), (W Th), (Th, F), (F Sat), (Sat Sund), (Sund Mon) } = 7 pair

fav Pair = { (Sat Sund), (Sund Mon) } = 2 Pair

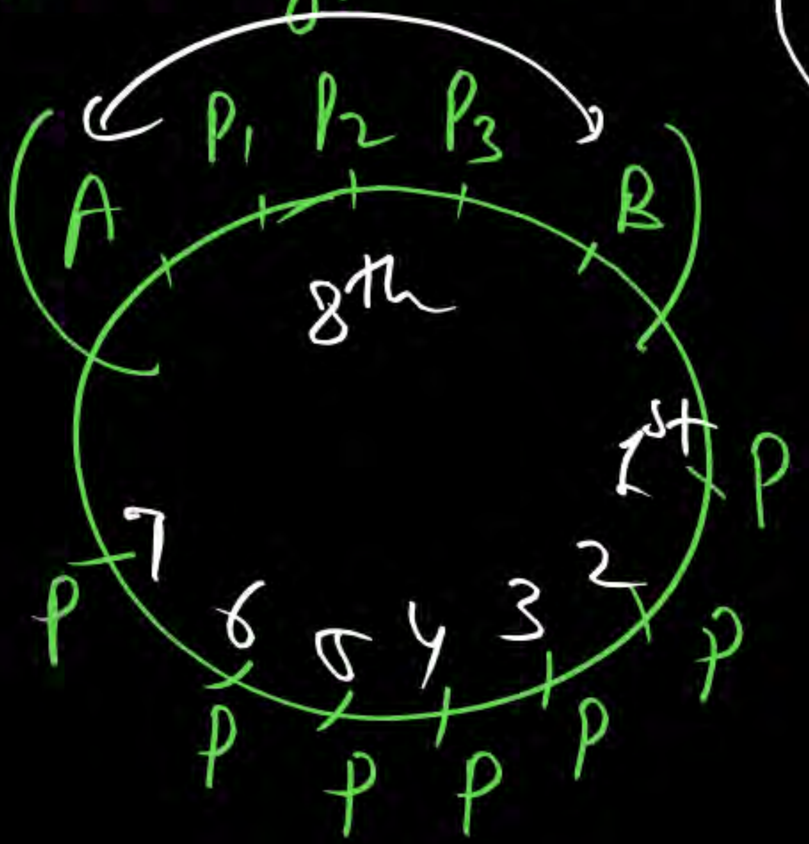
Req Prob = $\frac{\text{fav}}{\text{Total}} = \frac{2}{7}$

A and B stand in a ring with 10 other persons. If the arrangement of the 12 person is at random, find the chance that there are exactly 3 persons between A and B.

= Prob.

Total ways of arranging 12 persons in a circle $= (12-1)! = 11!$
(RNA)

fav ways $= \left({}^{10}C_3 \times 3!\right) \times (8-1)! \times (2!)$



So Req Prob $= \frac{\text{fav}}{\text{Total}} = \frac{{}^{10}C_3 \times 6 \times 7! \times 2}{11!}$

$= \frac{\cancel{10} \times \cancel{9} \times \cancel{8} \times 6 \times \cancel{7!} \times 2}{(3 \times 2 \times 1) \times 11 \times \cancel{10!}} = \frac{2}{11}$

Two dice each numbered from 1 to 6 are thrown together. Let A and B be two events given by

A : Even number on the first dice

B : Number on the second dice is greater than 4

(i) What is the value of $P(A \cap B)$ and $P(A \cup B)$ respectively?

(a) × 1/2, 1/6 (b) × 1/4, 2/3 (c) × 2/3, 1/6 (d) ✓ 1/6, 2/3

$S = \{(11)(12)(13) \ldots (16), (21), (22) \ldots, (65), (66)\} \Rightarrow n(S) = 36$ pair

App I ← App II

$A = \left\{ \begin{matrix} (21)(22) \ldots (26) \\ (41)(42) \ldots (46) \\ (61)(62) \ldots (66) \end{matrix} \right\} = 18$ pair

$B = \left\{ \begin{matrix} (15),(16),(25)(26)(35),(36) \\ (45),(46),(55),(56),(65),(66) \end{matrix} \right\} = 12$ pair

$A \cap B = \{(25),(26),(45),(46),(65),(66)\} = 6$ pair

$P(A) = \frac{18}{36} = \frac{1}{2}$, $P(B) = \frac{12}{36} = \frac{1}{3}$

① $P(A \cap B) = \frac{6}{36} = \frac{1}{6} = \frac{1}{2} \times \frac{1}{3} = P(A).P(B)$

② $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

Q. (3) Dice are thrown simultaneously then find the prob that at least one die show digit '4' ?

Sol: App II $S = \left\{ \begin{matrix} (111)(112) \ldots (116), (311) \ldots (316) \\ (211)(212) \ldots (216), (411) \ldots (666) \end{matrix} \right\} \Rightarrow n(S) = \underset{D_1}{\underline{6}} \times \underset{D_2}{\underline{6}} \times \underset{D_3}{\underline{6}} = 216$

i.e. 216 Triplets.

un.fav Triplets = { None die will show digit 4 } = $\underset{D_1}{\underline{5}} \times \underset{D_2}{\underline{5}} \times \underset{D_3}{\underline{5}}$ = 125 Triplets.

App III

P (None die will show digit 4) = $\frac{125}{216}$

Hence $\boxed{P(\text{at least one die show digit } 4) = 1 - P(\text{None})} = 1 - \frac{125}{216} = \frac{91}{216}$.

F.Q 2: If six coins are tossed simultaneously then write it's s. space?

$$S = \{(HHHHHH), (HHHHHT), (HHHHTT), (HHHTTT), (HHTTTT), (HTTTTT), (TTTTTT)\} = 64 \text{ tuples}$$

$$\frac{6!}{6!} = {}^6C_6 = 1, \quad \frac{6!}{(5!)(1!)} \; {}^6C_5 = 6, \quad \frac{6!}{4!\,2!} \; {}^6C_4 = 15, \quad \frac{6!}{3!\,3!} \; {}^6C_3 = 20, \quad \frac{6!}{2!\,4!} \; {}^6C_2 = 15, \quad \frac{6!}{(1!)(5!)} \; {}^6C_1 = 6, \quad \frac{6!}{6!} \; {}^6C_0 = 1.$$

$$\text{Total Cases} = \underset{c_1}{\underline{2}} \times \underset{c_2}{\underline{2}} \times \underset{c_3}{\underline{2}} \times \underset{c_4}{\underline{2}} \times \underset{c_5}{\underline{2}} \times \underset{c_6}{\underline{2}} = 2^6 = 64 \text{ tuples}$$

it means in our sample space we can directly see 64 M.E. events

& $P(H) = P(T) = \frac{1}{2}$ & All coins are Independent $\Longrightarrow$ Now we can multiply individual Probabilities if Required

H T	H T	H T	H T	H T	H T
c_1	c_2	c_3	c_4	c_5	c_6

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

$$e^{i(2\pi)} = \cos(2\pi) + i\sin(2\pi) = 1$$

$$e^{i\left(\frac{\pi}{2}\right)} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i \Rightarrow \boxed{i = e^{i\left(\frac{\pi}{2}\right)}}$$

Evaluate $i^i = ? = \left(e^{i\left(\frac{\pi}{2}\right)}\right)^i = e^{\frac{\pi}{2}(i^2)} = e^{-\frac{\pi}{2}}$ = Real & +ve No.

i.e i^i = Real & +ve No.

(1) Find the prob that all the outcomes are identical ?

App I: fav cases = {(HHHHHH), (TTTTTT)} = 2 Tuples.

So Req Prob = $\frac{fav}{Total} = \frac{2}{64} = \frac{1}{32}$

App II: Req Prob = P[(HHHHHH) or (TTTTTT)]

ME

$= P[(HHHHHH) \cup (TTTTTT)]$

$= \underbrace{\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)}_{\text{just bcoz of Ind}} + \underbrace{\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)}_{\text{just bcoz of Ind.}}$

$= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \frac{1}{32}$

(2) Find the prob that H & T appears alternately ?

App I: fav cases $= \{(HTHTHT), (THTHTH)\} = 2$

So Req Prob $= \dfrac{\text{fav}}{\text{Total}} = \dfrac{2}{64} = \dfrac{1}{32}$

ME

App III: Req Prob $= P\left[(HTHTHT) \text{ or } (THTHTH)\right]$

$= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6$ {Multi is bcoz of Ind Nature}

$= 2\left(\frac{1}{2}\right)^6 = \dfrac{2}{2^6} = \dfrac{1}{2^5} = \dfrac{1}{32}$

Analysis: Various cases in which we are getting exactly 3H & exactly 3T are as follows;

$^6C_3 = 20$ cases $=$ (HHHTTT), (HHTHTT), (HTHTHT), (HTTHHT) .... $= 10$

(TTTHHH), (TTHTHH), (THTHTT), (THHTTH) .... $= 10$

$\overline{20.}$

i.e. for getting H & T alternately we have only 2 cases.

(3) Find the prob that Both H & T appears at least once ?

App I unfav Cases = {(HHHHHH), (TTTTTT)} = 2 Cases

So fav Cases = 64 − 2 = 62 So Req Prob = $\frac{\text{fav}}{\text{Total}} = \frac{62}{64} = \frac{31}{32}$

(4) Find the prob that H appears at least once ?

App I unfav Cases = {(TTTTTT)} = 1 Cases.

So fav Cases = 64 − 1 = 63

Hence Prob = $\frac{\text{fav}}{\text{Total}} = \frac{63}{64}$

App II

Multi is bcoz of Ind Nature.

$P(\text{at least one } H) = 1 - P(\text{No Head})$

$= 1 - P(\text{all } T) = 1 - P(TTTTTT) = 1 - \left(\frac{1}{2}\right)^6$

(5) If 1^{st} three outcomes are H, H, H then find the prob of occurring Tail when coin is tossed again ? Condition/statement

Sol: App III (M-I) Req Prob $= P(T \text{ in } 4^{th} \text{ toss}) = ? = \frac{1}{2}$

App III (M-II) Req Prob $= P[\underline{H}\,\underline{H}\,\underline{H}\ \underline{T}\ \underline{S\,O}\,.\,\underline{S\,O}]$

given

(Multi is bcoz of Ind Nature)

$= 1 \times 1 \times 1 \times \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

App I: it is a Question Based on Conditional Prob so Have patience Later on it will be discussed.

(6) If $\underbrace{1^{st} \text{ three outcomes are } H, H, H}_{\text{condition}}$, then find the prob of occurring Tail in Remaining tosses ?

App III Req Prob $= P[HHH\ TTT]$

$= 1 \times 1 \times 1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

App I — Have patience

(7) Find the prob that only 1^{st} two tosses produces Head ?

App I fav Cases $= \{ (\underline{H}\,\underline{H}\,\underline{T}\,\underline{T}\,\underline{T}\,\underline{T}) \} = 1$ case.

Req Prob $= \dfrac{\text{fav}}{\text{Total}} = \dfrac{1}{64}$

App II Req Prob $= P[\text{only } 1^{st} \text{ two tosses are Head}] = P[H\,H\,\underline{T\,T\,T\,T}]$

$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

(8) Find the prob that, 1st two tosses produces Head ?

App I : fav Cases = { ($\underline{H}\ \underline{H}\ \underline{H/T}\ \underline{H/T}\ \underline{H/T}\ \underline{H/T}$) . . . . . . } = ? = Not easy to count.

App II : fav Cases = 1 way × 1 way × 2 ways × 2 ways × 2 ways × 2 ways

$= 1 \times 1 \times 2 \times 2 \times 2 \times 2 = 16$ Cases are fav

So Req Prob $= \frac{\text{fav}}{\text{Total}} = \frac{16}{64} = \frac{1}{4}$

App III : Req Prob $= P\left[\left(\underline{H}\ \underline{H}\ \underline{SO}\ \underline{SO}\ \underline{SO}\ \underline{SO}\right)\right] = \boxed{\frac{1}{2} \times \frac{1}{2} \times 1 \times 1 \times 1 \times 1} = \frac{1}{4}$

Analysis of Part (8) →

$$\text{Req Prob.} = \frac{\text{fav}}{\text{Total}} = \frac{16}{64} = \frac{(1\times1\times2\times2\times2\times2)}{2^6}$$

App II

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{2}{2}\times\frac{2}{2}\times\frac{2}{2}\times\frac{2}{2}$$

$$= \frac{1}{2}\times\frac{1}{2}\times \underbrace{1\times1\times1\times1}$$ → App III

$$= P(\underline{H}\ \underline{H}\ \underline{SO}\ \underline{SO}\ \underline{SO}\ \underline{SO})$$

$$= \frac{1}{4}$$

(9) Find the prob that exactly 2 H will occur ?

App II fav Cases = { eg (H H T T T T) .... } $= \frac{6!}{2!\,4!} = {}^6C_2 = 15$

fav eg = { (H H T T T T), (H T T T H T), (T T H T H T) ... (T T T T H H) }

$\underbrace{\qquad\qquad}_{15\ \text{Cases}}$

$$\text{Req Prob} = \frac{\text{fav}}{\text{Total}} = \frac{{}^6C_2}{2^6} = \frac{15}{64}$$

App III : we can use concept of Binomial Distribution (Have patience)

(10) Find the prob that exactly 3 H will occur?

App II Fav Cases = $\{\underbrace{(HHHTTT), (HTHHTT), (THTHTH), \ldots, (TTTHHH)}\}$

$$= \frac{6!}{3!\,3!} = {}^6C_3 = 20 \text{ tuples}$$

So Req Prob $= \frac{\text{Fav}}{\text{Total}} = \frac{{}^6C_3}{2^6} = \frac{20}{64}$ (we are not sure about the Location of H & T)

App III (Using Binomial Dist) $\longrightarrow$ Have Patience

(11) Find the prob that H & T appears equal number of times?

i.e. (exactly 3H & 3T)

same as part (10)

so Ans $= \frac{{}^6C_3}{2^6} = \frac{20}{64}$

Q. A coin is tossed 7 times then find the prob that

(1) H & T appears equal No. of times = ? = senseless Q i.e. Ans = 0

* (2) H & T appears alternately = ? $= \left(\frac{1}{2}\right)^7 + \left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

either (HTHTHTH) or (THTHTHT)

(Take care) → ME ✓

PODCAST

H & T are Alternately

(H T H T H T H)

or

(T H T H T H T)

Here R.A

4B & 3G.

B & G are Alternately

(B G B G B G B)

or

(G B G B G B B) → ~~X~~

Here RNA

Q8 A coin is tossed 10 times then find the prob that

(1) exactly 3 H will occur ?

(2) 4^{th} Head will occur in 9^{th} toss ? $= \frac{7}{64}$

HW

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2025

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Thank

you

Keep Hustling!