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DATA SCIENCE AND AI

ENGINEERING MATHEMATICS

LINEAR ALGEBRA

DPP 03

Q1 The eigen values of the matrix $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$ are :

(A) (a + 1), 0
(B) a, 0
(C) (a − 1), 0
(D) 0 , 0

Q2 Consider $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

The eigenvalues of M are

(A) 0, 1, 2
(B) 0, 0, 3
(C) 1, 1, 1
(D) −1, 1, 3

Q3 Consider the matrix $M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$

The eigenvalues of M are

(A) −5, −2, 7
(B) −7, 0, 7
(C) −4i, 2i, 2i
(D) 2, 3, 6

Q4 The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are

(A) +1 and +1
(B) Zero and +1
(C) Zero and +2
(D) −1 and +1

Q5 The eigenvalues of (A^4 + 3A -2I), where A is $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$, are

(A) 2, 20, 88
(B) 1, 2, 3
(C) 2, 20, 3
(D) 1, 20, 88

Q6 Eigen value of matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are

(A) −2, −1, 1, 2
(B) −1, 1, 0, 2
(C) 1, 0, 2, 3
(D) −1, 1, 0, 3

Q7 The eigenvalues of a matrix are i, −2i and 3i. The matrix is

(A) Unitary
(B) Anti-Unitary
(C) Hermitian
(D) Anti-Hermitian

Q8 The eigenvalue of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are

(A) Real and Distinct
(B) Complex and Distinct
(C) Complex and Coinciding
(D) Real and Coinciding

Q9 The eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

(A) 5, 2, −2
(B) −5, −1, 1
(C) 5, 1, −1
(D) −5, 1, 1

Q10 A 3 × 3 matrix has element such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is:

(A) 18
(B) 12
(C) 9
(D) 6

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The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(A) 0, 1,1 (B) $0, -\sqrt{2}, \sqrt{2}$

(C) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (D) $\sqrt{2}, \sqrt{2}, 0$

Q12 The trace of a 2 × 2 matrix is 4 and its determinant is 8. If one of the eigenvalue is 2(1 + *i*), the other eigenvalue is

(A) 2(1 – i) (B) 2(1 + i)

(C) (1 + 2i) (D) (1 – 2i)

Q13 The eigenvalues of the matrix representing the following pair of linear equations

x + iy = 0

ix + y = 0

are

(A) 1 + ***i***, 1 + ***i***

(B) 1 – i, 1 – i

(C) 1, i

(D) 1 + ***i***, 1 – ***i***

Q14 If the matrix $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then the eigen values of $A^2 + 5A + 8I$, are :

(A) $-1,\ 27, -8$

(B) $1,\ 3, -2$

(C) $2,\ 32, 4$

(D) 2,50, 10

Q15 Two of the eigen values of a 3×3 matrix, whose determinant equals 4 , are -1 and +2 the third eigen value of the matrix is equal to :

(A) -2 (B) -1

(C) 1 (D) 2

Q16 If A is a singular hermitian matrix, then the least eigen value of A^2 is :

(A) 0 (B) 1

(C) 2 (D) None of these.

Q17 If λ is an eigen value of matrix 'M' then for the matrix $(M - \lambda I)$,

Which of the following statement (s) is/ are correct ?

(A) Skew symmetric.

(B) Non singular.

(C) Singular.

(D) None of these.

Q18 Let A be a matrix whose characteristic roots are 3, 2,-1 . If $B = A^2 - A$ then |B| = ________.

(A) 24 (B) -2

(C) 12 (D) -12

Q19 The Eigen vector correseponding to the largest Eigen value of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is______

(A) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ (D) None.

Q20 The Eigen vector of the matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ are :

(A) (1, 0)

(B) (0, 1)

(C) (1, 1)

(D) (1, -1)

Q21 The vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then corresponding eigen values of A is :

(A) 1 (B) 2

(C) 5 (D) -1

Q22 The column vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is a simultaneous eigenvector of A = $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and B =

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$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

(A) b = 0 or a = 0

(B) b = 0 or a=-c

(C) b = 2a or b = −a

(D) b = a/2 or b = −a/2

Q23 A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 & + & x_2 \\ x_2 & - & x_3 \end{pmatrix}$, transform a vector $\overrightarrow{x}$ three dimensional space to a two-dimensional real space. The transformation matrix T is

(A) $\begin{pmatrix} 1 & 1 & 0 & \\ & 0 & 1 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & & 1 & 1 \\ -1 & & 1 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Q24 The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} are$

(A) 6,1and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(B) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(C) 6, 2 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(D) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Q25 (i) The eigen vectors X of a matrix A, is not

a) $\mathrm{X'}_1\ \mathrm{X}_2 = 0$

(ii) Two eigen vectors X_1 and X_2 are called orthogonal if

b) $\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$

(iii) Normalised form of vectors $\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$ is obtained on dividing each element by.

c)unique

(iv) Every square matrix satisfies its own.

d)Characteristic equation

(A) (i)-c
(ii)-a
(iii)-b
(iv)-d

(B) (i)-a
(ii)-c
(iii)-b
(iv)-d

(C) (i)-c
(ii)-a
(iii)-d
(iv)-b

(D) (i)-c
(ii)-d
(iii)-b
(iv)-a

Q26 Calculate the matrix A^2 ,by the use of Cayley – Hamilton theorem (or) otherwise is :

$$A = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & -i \end{vmatrix}$$

Q27 The matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ is given and the eigen values of $4\mathbf{A}^{-1} + 3\mathbf{A} + 2I$ are.

(A) 6,15 (B) 9,12

(C) 9,15 (D) 7.15

Q28 In matrix $\mathbf{A} = \begin{bmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{bmatrix}$ if a + d = ad- bc = 1 then A^3 = ________.

(A) A − I (B) A + I

(C) − I (D) 0

Q29 Let A be an $\mathbf{n} \times \mathbf{n}$ complex matrix whose characteristic polynomial is : $\mathrm{f}(\mathrm{t}) = \mathrm{t}^{\mathrm{n}} + \mathrm{c}_{\mathrm{n}-1}\mathrm{t}^{\mathrm{n}-1} + \ldots + \mathrm{c}_1\mathrm{t} + \mathrm{c}_0$ then __________.

(A) $\det\ A = C_{n-1}$

(B) $\det\ A = C_0$
(C) $\det\ A = (-1)^{n-1} C_{n-1}$
(D) $\det\ A = (-1)^n C_0$

Q30 The constant term of the characteristic polynomial of the matrix.

$$\begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix}$$ is_______.

Q31 Two matrices A and B are said to be similar if B = $P^{-1}AP$ for some invertible matrix P. Which of the following statements is NOT TRUE?

(A) Det A = Det B

(B) Trace of A = Trace of B

(C) A and B have the same eigenvectors

(D) A and B have the same eigenvalues

Q32 If A and P be square matrices of the same type and if P is invertible, then the matrices A and P^{-1} AP have .... characteristic roots.

Q33 If $\mathrm{A}_{2\times 2}$ s. t $\mathrm{A}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$ & $\mathrm{A}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$

then find A = ?

(A) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0\\2 & 2\end{bmatrix}$

(C) $\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}$ (D) $\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}$

Q34 If $\mathrm{A}_{3\times 3}$ then number of L. I Eigen vectors of A to have diagonalisation possible will be ?

(A) 1 (B) 2

(C) 3 (D) less than 3.

Answer Key

Q1	(A)
Q2	(B)
Q3	(B)
Q4	(C)
Q5	(A)
Q6	(A)
Q7	(D)
Q8	(B)
Q9	(C)
Q10	(B)
Q11	(B)
Q12	(A)
Q13	(D)
Q14	(C)
Q15	(A)
Q16	(A)
Q17	(C)
Q18	(A)
Q19	(B)
Q20	(C)
Q21	(C)
Q22	(B)
Q23	(A)
Q24	(A)
Q25	(A)
Q26	IDENTITY MATRIX
Q27	(C)
Q28	(C)
Q29	(D)
Q30	(0)
Q31	(C)
Q32	same
Q33	(C)
Q34	(C)

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Hints & Solutions

Q1 Text Solution:

Matrix A = $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$

Characteristic equation of matrix A,

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} a-\lambda & 1 \\ a & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - (a+1)\lambda + a - a = 0$

$\Rightarrow \lambda = 0, \ a+1.$

Alternative Solution :

sum of eigen values = Trace of matrix

Hence option A is correct.

Q2 Text Solution:

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$The\ eigen\ values\ are\ given\ by -$

$|M - \lambda I| = 0$

$$Det\ of \begin{pmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{pmatrix} = 0$$

$$\left(1-\lambda\right)\left((1-\lambda)^2 - 1\right) - 1\left(1-\lambda-1\right)$$

$$+1\left(1-\left(1-\lambda\right)\right) = 0$$

$$\left(1-\lambda\right)\left(-\lambda\left(2-\lambda\right)\right) + \lambda + \lambda = 0$$

$\lambda = 0$

$$\left(1-\lambda\right)\left(\lambda-2\right) + 2 = 0$$

$\lambda - 2 - \lambda^2 + 2\lambda + 2 = 0$

$3\lambda - \lambda^2 = 0$

$\lambda = 0, 3$

Q3 Text Solution:

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

$|M - \lambda I| = 0$

$$\begin{vmatrix} 0-\lambda & 2i & 3i \\ -2i & -\lambda & 6i \\ -3i & -6i & -\lambda \end{vmatrix} = 0$$

$-\lambda\left(\lambda^2 + 36i^2\right) - 2i\ \left(2i\lambda + 8i^2\right) + 3i$

$\left(12\, i^2 - 3i\lambda\right) = 0$

$-\lambda^3 + 36i^2\ (-\lambda) - 4i^2\ \lambda - 36i^3\ + 36i^3$

$-\ 9i^2\lambda = 0$

$-\lambda^3\ +\ 36\lambda\ +\ 4\lambda\ -36\,i + 36i - 9\lambda = 0$

$\lambda^3 -\ 36\lambda -\ 4\lambda\ - 9\lambda = 0$

$\lambda^3 - 49\lambda = 0$

$\lambda^3 - \left(\lambda^2 - 49\lambda\right) = 0$

$\lambda = 0,\ 7,\ \ -7$

(B) is Correct.

Q4 Text Solution:

$M = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

$|M - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & i \\ -i & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)^2 + i^2 = 0$

$1 + \lambda^2 - 2\lambda - 1 = 0$

$\lambda^2 - 2\lambda = 0$

$\lambda - (\lambda - 2) = 0$

$\lambda = 0, 2$

Option C is correct

Q5 Text Solution:

Eigen value of $A^4 + 3A - 2I$ where.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{vmatrix}$$

$(3-\lambda)\ (2-\lambda)\ (1-\lambda) = 0$

$(\lambda - 1)\ (\lambda - 2)\ (\lambda - 3) = 0$

$\lambda = 1, \ 2, \ 3$

For λ = 1, eigen value of A^4 = 1 , 3A = 3, 2I = 2

thus, eigen values of A^4 + 3A − 2I = 1 + 3 − 2 = 2

For λ = 2, eigen value of A^4 = 16 , 3 A = 6, 2 I = 2

thus, eigen values of

$A^4 + 3A - 2I = 16 + 6 - 2$

= 16 + 4 = 20.

For λ = 3, eigen value of A^4 = 81 , 3 A = 9, 2 I = 2

Thus the eigen values of-

$A^4 + 3A - 2I = 81 + 9 - 2 = 88$

Thus (A) is correct options.

Q6 Text Solution:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 0-\lambda & 1 & 0 & 0 \\ 1 & 0-\lambda & 0 & 0 \\ 0 & 0 & 0-\lambda & -2i \\ 0 & 0 & 2i & 0-\lambda \end{vmatrix} = 0$$

$$(-\lambda)\begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & -2i \\ 0 & 2i & -\lambda \end{vmatrix} - 1\begin{vmatrix} 1 & 0 & 0 \\ 0 & -\lambda & -2i \\ 0 & 2i & -\lambda \end{vmatrix}$$

$(-\lambda)\left((-\lambda)\left(\lambda^2 + 4i^2\right)\right) - 1\left(\lambda^2 + 4i^2\right) = 0$

$(-\lambda)\left(-\lambda^3 + 4\lambda\right) - \lambda^2 + 4 = 0$

$\lambda^4 - 4\lambda^2 - \lambda^2 + 4$

$\lambda^4 - 5\lambda^2 + 4 = 0$

$\left(\lambda^2 - 1\right)\left(\lambda^2 - 4\right) = 0$

$\lambda^2 - 1 = 0 \ \rightarrow \ \lambda = \pm 1$

$\lambda^2 - 4 = 0 \ \rightarrow \ \lambda = \pm 2$

thus, $\lambda = +1, -1, +2, -2$

Option (A) is correct.

Q7 Text Solution:

The eigen values are $i, -2i, \ 3i$ All the eigen values are imaginary,

thus according to the property of anti - hermitian all the eigen values are either 0 or imaginary .

thus option (D) is the correct option.

Q8 Text Solution:

$$A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 0-\lambda & i \\ i & 0-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} -\lambda & i \\ i & -\lambda \end{vmatrix} = 0$$

$\lambda^2 - i^2 = 0$

$\lambda^2 - 1 = 0$

$\lambda = \pm i$

thus option (B) is correct.

Q9 Text Solution:

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)\left((2-\lambda)^2 - 9\right) = 0$

$(1-\lambda)\left(4 + \lambda^2 - 4\lambda - 9\right) = 0$

$(1-\lambda)\left(+\lambda^2 - 4\lambda - 5\right) = 0$

$(1-\lambda)\left(\lambda^2 - 4\lambda - 5\right) = 0$

$(\lambda - 1)\left((\lambda - 5)(\lambda + 1)\right) = 0$

$\lambda = 1, 5, -1$

option (c) is correct.

Q10 Text Solution:

A matrix is of order $3 \ \times \ 3.$

Trace = 11

$\{\lambda i = 11\}$

$\lambda_1 + \lambda_2 + \lambda_3 = 11$

Determinant = 36

$\lambda_1 \ \lambda_2 \ \lambda_3 = 36$

thus option (d) will be the correct option.

as - 6, 3, 2

6 + 3 + 2 = 11 ⟶ sum of eigen values.

$6 \ \times \ 3 \ \times \ 2 = 36$ →product of eigen values.

Q11 Text Solution:

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$$M = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$|M - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & 0-\lambda & 1 \\ 0 & 1 & 0-\lambda \end{vmatrix} = 0$$

$$-\lambda \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} - 1 \begin{pmatrix} 1 & 1 \\ 0 & -\lambda \end{pmatrix} = 0$$

$$-\lambda(\lambda^2 - 1) - 1(-\lambda) = 0$$

$$-\lambda^3 + \lambda + \lambda = 0$$

$$2\lambda - \lambda^3 = 0$$

$$\lambda^3 - 2\lambda = 0$$

$$\lambda(\lambda^2 - 2) = 0$$

$$\lambda = 0, +\sqrt{2}, -\sqrt{2}$$

Option B is correct.

Q12 Text Solution:

The trace of a 2×2 matrix is 4, thus $\lambda_1 + \lambda_2 = 4$, 2 the determinant is product of eigen values thus $\lambda_1 \,.\, \lambda_2 = 8$

$$\lambda_1 + \lambda_2 = 4$$

$$\lambda_1 + 2 + 2i = 4$$

$$\lambda_1 + 2 + 2i = 2(1 - i)$$

Thus option A is correct.

Q13 Text Solution:

x + iy = 0

ix + y = 0

$$\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

thus the matrix $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & i \\ i & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - i^2 = 0$$

$$1 + \lambda^2 - 2\lambda + 1 = 0$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$(\lambda - (1+i))(\lambda - (1-i)) = 0$$

$$\lambda = 1 + i, i - i.$$

Q14 Text Solution:

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -1-\lambda & 2 & 3 \\ 0 & 3-\lambda & 5 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)(3-\lambda)(-1-\lambda) = 0$$

$$\lambda = -2$$

$$\lambda = 3$$

$$\lambda = -1$$

eigen value of A^2 will be , 4, 9, 1

5 A $\quad -10,\ 15,\ -5$

8 I $\quad 8,\ 8,\ 8$

Now eigen value of A^2 + 5 A + 8 I will be

2, 32, 4

thus (c) is the correct option.

Q15 Text Solution:

Now $\lambda_1 + \lambda_2 + \lambda_3 = -1 + 2 + \lambda_3$

$$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 4$$

$$-1 \cdot 2 \cdot \lambda_3 = 4$$

$$\lambda_3 = -2$$

thus (A) is the correct option.

Q16 Text Solution:

A is a singular hermition matrix.

Thus all the eigen values of a hermition matrix are always real.

And as the matrix is singular thus the determinant that is the product of eigen values are zero.

thus , the least eigen value of A^2 is 0.

as all the eigen values of A^2 will be greater than or equal to 0.

Q17 Text Solution:

If λ is an eigen value of M , then $|M - \lambda . I| = 0$, that is the matrix $B = M - \lambda . I$ is singular.

Q18 Text Solution:

Therefore for A Eigen values are : 3 ,2,-1

for A^2 the eigen values will be: 9,4,1

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Eigen values for A^2=9 ,4 , 1

Eigen values for A = 3 ,2, -1

Eigen values for B(i.e $A^2 - A$) =Eigen values for

A^2– Eigen values for A

=(9-3),(4-2),(1-(-1))

=6,2,2

Determinant value of B = Product of Eigen values

= (6*2*2)

=24

Q19 Text Solution:

$$Det\ of\ A - \lambda I = Det\ of$$

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$\left(1-\lambda\right)\left((2-\lambda)^2 - 1\right) = 0$$

$$\left(1-\lambda\right)\left(1-\lambda\right)\left(3-\lambda\right) = 0$$

$$\lambda = 3, 1, 1$$

$$The\ largest\ is\ 3$$

$$AX = 3X$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 3\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$2a + b + c = 3a$$

$$a + 2b + c = 3b$$

$$c = 3c$$

$$Thus\ c = 0$$

$$putting\ c = 0,$$

$$b = a$$

$$Thus\ the\ eigen\ vector,$$

$$\begin{bmatrix} a \\ a \\ 0 \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$Let\ k = 1\ thus\ the\ eigen\ vector\ is$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Q20 Text Solution:

$$\mathrm{A} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

For eigen values :

$$|\mathrm{A} - \lambda \mathrm{I}| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = 0$$

$$1 + \lambda^2 - 2\lambda - 1 = 0$$

$$\lambda = 0,\ 2$$

For $\lambda = 0$

A x = O

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \mathrm{x} \\ \mathrm{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x - y = 0$$

$$y - x = 0$$

$$\Rightarrow \mathrm{y} = \mathrm{x}$$

Let x = k $\Rightarrow$ y = k

So, $\mathrm{X} = \begin{bmatrix} \mathrm{k} \\ \mathrm{k} \end{bmatrix} = \mathrm{k}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathrm{k} \in \mathrm{R}$

Hence option c is correct.

Q21 Text Solution:

If λ_1 be the eigen value of matrix A and X_1 the corresponding eigen vector then.

$$\mathrm{AX_1} = \lambda_1\ \mathrm{X_1}$$

$$\Rightarrow \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ -5 \end{bmatrix}$$

$$= 5 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\Rightarrow \lambda_1$ = 5 is the eigen value of A corresponding to the eigen vector $[12-1]^T$

Q22 Text Solution:

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$Given\ that\ X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}\ is\ an\ eigen\ vector\ of$

$A.$

$AX = BX$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{bmatrix}$$

$$\begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} b+c \\ a+c \\ a+b \end{bmatrix}$$

$b = 0, a = -c$

Q23 Text Solution:

$Lrt\ us\ consider\ the\ matrix\ A$

$$= \begin{pmatrix} 1 & 1 & 0 & \\ & 0 & 1 & -1 \end{pmatrix}$$

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 & + & x_2 \\ x_2 & - & x_3 \end{pmatrix}$$

$$TX = \begin{pmatrix} 1 & 1 & 0 & \\ & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= \begin{pmatrix} x_1 & + & x_2 \\ x_2 & - & x_3 \end{pmatrix}$$

$Thus\ A\ is\ the\ correct\ option.$

Q24 Text Solution:

$The\ eigen\ value\ of\ the\ matrix\ A$

$$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

$are -$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$\left(5-\lambda\right)\left(2-\lambda\right) - 4 = 0$

$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 7\lambda + 6 = 0$

$\left(\lambda - 6\right)\left(\lambda - 1\right) = 0$

$6, 1\ \ are\ the\ eigen\ values.$

$Let\ X = \begin{bmatrix} a \\ b \end{bmatrix}\ be\ the\ eigen\ value$

$corresponding\ to\ eigen\ value\ 1.$

$AX = 1.X$

$AX = X$

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$5a + 4b = a$

$4a + 4b = 0$

$a = -b$

$a + 2b = b$

$a = -b$

$Thus\ the\ eigen\ vector\ is\ \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$Let\ X = \begin{bmatrix} a \\ b \end{bmatrix}\ be\ the\ eigen\ value$

$corresponding\ to\ eigen\ value\ 6.$

$AX = 6.X$

$AX = X$

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 6a \\ 6b \end{bmatrix}$$

$5a + 4b = 6a$

$4b = a$

$a + 2b = 6b$

$a = 4b$

$Thus\ the\ eigen\ vector\ is\ \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

Q25 Text Solution:

The correct solution will be

(i)-c
(ii)-a
(iii)-b
(iv)-d

Q26 Text Solution:

Given,

$$A = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & -i \end{vmatrix}$$

Since it is U.T.M So eigen values of A are $\lambda = 1, \ -1, \ i, \ -i$

so C Eq. of A can be taken as.

$(\lambda - 1)\ (\lambda + 1)\ (\lambda - i)\ (\lambda + i)\ = 0$

$\Rightarrow \ \ (\lambda^2 - 1)\ (\lambda^2 + 1) = 0$

$\Rightarrow \ \lambda^2 - 1 = 0$

By Cayley Harmilton theorem , are can replace λ by A .

So,

$A^2 - I = 0$

A^2 = I.

Q27 Text Solution:

$$\mathrm{A} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$$

$The\ eigen\ value\ of\ A\ is\ -$

$$\begin{vmatrix} 1-\lambda & 0 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

$$\left(1-\lambda\right)\left(4-\lambda\right) = 0$$

$\lambda = 1, 4$

$Thus\ the\ eigen\ value\ of\ A^{-1} = 1, 1/4, 3A$
$= 3, 12\ AND\ 2I = 2, 2$

$4\mathrm{A}^{-1} + 3\mathrm{A} + 2I$
$First\ eigen\ value\ is\ -$
$4 + 3 + 2 = 9$
$second\ eigen\ value\ will\ be\ -$
$1 + 12 + 2 = 15$

Q28 Text Solution:

$We\ will\ from\ the\ characteristic\ equation\ thus\ -$

$\lambda^2 - \lambda + 1 = 0$

$According\ to\ the\ cayley\ -\ Hamilton\ theorem\ every\ sqaure\ matrix\ satisfies\ it\ own\ characteristic\ equation\ thus\ -$

$A^2 - A + I = 0$
$A^3 - A^2 + A = 0$
$A^3 = A^2 - A$
$A^3 = A - I - A$
$= -I$

Q29 Text Solution:

Let A be an n x n complex matrix whose characteristic polynomial is :

$\mathrm{f}\left(\mathrm{t}\right) = \mathrm{t}^{\mathrm{n}} + \mathrm{c}_{\mathrm{n}-1}\mathrm{t}^{\mathrm{n}-1} + \ldots + \mathrm{c}_1\mathrm{t} + \mathrm{c}_0$

$\mathrm{f}\left(\mathrm{t}\right) = \mathrm{t}^{\mathrm{n}} - \mathrm{trace}\ \mathrm{A}\ \mathrm{t}^{\mathrm{n}-1} + \ldots + \mathrm{c}_1\mathrm{t}$
$+ (-1)^n\,|A|$

$Thus\ \det\ A = (-1)^n C_0$

Q30 Text Solution:

The constant term of a polynomial P(x) is **its value when x = 0**. By definition of the characteristic polynomial, its value when x = 0 is the determinant of the matrix. Answer: The determinant of the matrix.

Now cal the determinant value of the matrix we get-

$$\begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix}$$

$R_1\ and\ R_3\ are\ equal\ thus\ the\ det\ is\ 0$

Q31 Text Solution:

GATE

$Let\ P^{-1}AP = B$
$|B - \lambda I| = |P^{-1}AP - \lambda I|$
$= |P^{-1}AP - P^{-1}\lambda P|$
$= |P^{-1}(A - \lambda I)P|$
$= |(A - \lambda I)|$
$They\ have\ the\ same\ eigen\ value.$
$Now, P^{-1}AP = B$
$|P^{-1}AP| = |B|$
$|P^{-1}||A||P| = |B|$
$|A| = |B|$
$Tr(P^{-1}AP) = TrB$
$Tr(APP^{-1}) = TrB$
$Tr(AI) = TrB$
$TrA = TrB$

Q32 Text Solution:

$Let\ P^{-1}AP = B$
$|B - \lambda I| = |P^{-1}AP - \lambda I|$
$= |P^{-1}AP - P^{-1}\lambda P|$
$= |P^{-1}(A - \lambda I)P|$
$= |(A - \lambda I)|$

Q33 Text Solution:

A $\left[\frac{1}{0}\right] = \left[\frac{1}{0}\right]$ & A $\left[\frac{2}{1}\right] = \left[\frac{4}{2}\right]$

$Let\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$Now, \begin{bmatrix} a & b \\ c & d \end{bmatrix}\left[\frac{1}{0}\right] = \left[\frac{1}{0}\right]\ and\ \begin{bmatrix} a & b \\ c & d \end{bmatrix}\left[\frac{2}{1}\right]$
$= \left[\frac{4}{2}\right]$

$Solving\ them\ we\ get -$
$a = 1, c = 0, 2a + b = 4, 2c + d = 2$
$Thus\ b = 2, d = 2$

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} = A$$

Q34 Text Solution:

An n × n matrix A is diagonalizable if and only if A has n linearly independent eigenvectors.Thus as the order of matrix is 3 , thus the total number of LI eigen vectors will be 3.