GATE

DS AND AI

PROBABILITY AND STATISTICS

DPP: 01

Statistics -1 (Discrete Random Variable)

Q1 A class consists of 50 students, out of which 30 are girls. The mean of marks scored by girls in a test is 73 (out of 100), and that of boys is 71. Determine the mean score of the whole class.

Q2 Find the median for the data 8, 5,7,10,15, 21.

Q3 For a moderately skewed distribution, mean = 12 and mode = 6. Using these values, find the value of the median.

Q4 The marks scored by a student in different subjects are 45, 91, 62,71, 55. Find the median of the given data using the median formula.

Q5 For any given data, the mean is 45.5, and the median is 43. Find the modal value.

Q6 Let X be random variable with mean μ_X and variance $\sigma_x^2 > 0$, then Var (aX +b) is

(A) $a\sigma_x^2$ (B) $a^2\sigma_x^2$

(C) $a\sigma_x^2 + b$ (D) $a^2\sigma_x^2 + b$

Q7 If X and Y are tow random variables with respective expected values E(X) and E(Y), then Cov (X,Y)is-

(A) E(XY) – E (X) E(Y)

(B) E(X/Y) = E (XY)

(C) E(XY) – E (X/Y)

(D) E(XY) – E (X) /E(Y)

Q8 The covariance of two independent variate is equal to-

(A) zero

(B) unity

(C) sum of their expectation

(D) The product of their expectations

Q9 If X_1 and X_2 are independent, then $\mathrm{V}\left(\mathrm{X}_1-\mathrm{X}_2\right)$ is equal to :

(A) $\mathrm{V}\left(\mathrm{X}_1\right)+\mathrm{V}\left(\mathrm{X}_2\right)$

(B) $\mathrm{V}\left(\mathrm{X}_1\right)-\mathrm{V}\left(\mathrm{X}_2\right)-2\ \operatorname{Cov}\left(\mathrm{X}_1, \mathrm{X}_2\right)$

(C) $\mathrm{V}\left(\mathrm{X}_1\right)-\mathrm{V}\left(\mathrm{X}_2\right)$

(D) $\mathrm{V}\left(\mathrm{X}_1\right)+\mathrm{V}\left(\mathrm{X}_2\right)-2\ \operatorname{Cov}\left(\mathrm{X}_1, \mathrm{X}_2\right)$

Q10 A random variable X has the following probabaility function:

x	0	1	2	3	4	5	6
$f(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

(i) Find k,

(ii) Find P ($\geq$ 5), P(3 < X $\leq$6), P(X < 4)

Q11 If X is the number of points rolled with a balanced die, find the expected value of g(X) = $2X^2 + 1$

Q12 A discrete random variable X has the following probability distribution:

X	1	2	3	4	5
P(X = x)	0.05	0.15	0.35	0.4	0.05

Calcuate:

(i) F(3)

(ii) E[X]

(iii) var [X]

Q13 If random variable X assumes only positive integral values, with the probability $P(X=x)=\frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x=1,2,3, \ldots \ldots,$ then E(X) is

(A) 2/9 (B) 2/3

(C) 1 (D) 3/2

Q14 A discrete random variable X has the following probability function

x	: 10	20	30	40	50
y	: 0.1	0.1	0.4	0.3	0.1

Denote by μ_x and σ_x the mean and the standard deviation of X. Find $P(|X-\mu_x| \leq \sigma_x)$

(A) 1 (B) 0.8
(C) 0.7 (D) 0.5

Q15 An unbiased coin is tossed until a head is obtained. If N denotes the number of tosses required, what is P(N > 1)?
(A) 1/2 (B) 1
(C) 1/4 (D) 1/8

Q16 Probability mass function of a discrete random variables are given below.

X_i =	0	1	2	3	4	5	6
$P(X_i)$ =	k	3k	5k	7k	9k	11k	13k

(A) Find K
(B) Evalate P (X < 4)
(C) Evaluate $P(3 < x \leq 6)$

Q17 Which one of the following is not possible for a binomail distribution ?
(A) Mean = 2, variance = 3/2
(B) Mean = 5, variance = 9
(C) Mean = 10, variance = 5
(D) Mean = 4, variance = 8/3

Q18 A coin is twice as likely to turn up tails as heads. If the coin is tossed independently, what is the probability that the third head occurs on the fifth toss?
(A) $\frac{8}{81}$ (B) $\frac{40}{243}$
(C) $\frac{16}{81}$ (D) $\frac{80}{243}$

Q19 If X follows bionmial distribution with parameters n and p, the variance of X/n is-
(A) $\frac{p(1-p)}{n}$ (B) np (1–p)
(C) p(1–p) (D) $\frac{p(1-p)}{n^2}$

Q20 In a class of Pathology, the professor decided to conduct a test of swine flu on all the students. Test result says that one student in every ten is having swine flue. What is the probability that out of 5 students expected to attend a class, at least 4 will not have swine flu?

Q21 If the probability of a defective item is 0.1, find (i) the mean (ii) standard deviation for the distribution of items in a total of 400.

Q22 The mean and variance of a binomial distribution are 2.5 and 1.875 respectively. Obtain the binomial probability distribution.

Q23 100 dice are thrown. How many are expected to fall 6. What is the variance in the number of 6's ?

Q24 Let X be a poison random variable and P(X = 1) + 2 P(X = 0) = 12 P(X = 2). Which one of the following statements is TRUE ?
(A) $0.40 < P(X = 0) \leq 0.45$
(B) $0.40 < P(X = 0) \leq 0.50$
(C) $0.50 < P(X = 0) \leq 0.55$
(D) $0.55 < P(X = 0) \leq 0.60$

Q25 In a Poisson distribution, the probability of observing 3 is 2/3 times that of observing 4. The mean of the distribution is
(A) 5 (B) 6
(C) 7 (D) 8

Q26 The number of misprints per page of a book (X) follows the poisson distribution such that (X =1) = P(X =2). If the book contains 500 pages, the expected number of pages containing at most one misprint is-
(A) $5005e^{-2}$
(B) $10005e^{-2}$
(C) $15005e^{-2}$
(D) $500(1 - 3e^{-2})$

Q27 A manufacturing company supplies condensers with 1% defective pieces. Condensers are pack in boxes of 100. Find the probability that a box picked at random will have four or more faculty condensers.

Q28 One fifth percent of the blades produced by a manufacturing factory turn out to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective , one defective and two defective blades respectively in a consignment of 1,00,000 packets. Given $e^{-0.02} = 0.9802$

Q29 A certain company sells tractors which fail at a rate of 1 out of 1000. If 500 tractors are purchased form this company what is the probability of 2 of them failing with in first year ?

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Answer Key

Q1	72.2
Q2	9
Q3	10
Q4	62
Q5	38
Q6	(A)
Q7	(A)
Q8	(A)
Q9	(D)
Q10	16/49.
Q11	?
Q12	= 0.88
Q13	(A)
Q14	(A)
Q15	(A)
Q16	33/49
Q17	(A)
Q18	(A)
Q19	(A)
Q20	$\left(\frac{9}{10}\right)^4 \times \frac{7}{5}$
Q21	6
Q22	10
Q23	= 13.9
Q24	(A)
Q25	(A)
Q26	(A)
Q27	0.019
Q28	$19.6 \cong 20.$
Q29	0.07582

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Hints & Solutions

Q1 Text Solution:

Given,

Total number of students in a class = 50

Number of girls in the class - 30

Number of boys in the class = 50 - 30 = 20

Mean marks scored by girls = 73

Mean marks scored by boys = 71

Thus, the total marks scored by girls = 73 × 30 = 2190

Also, the total marks scored by boys = 71 × 20 = 1420

Mean score of the class = (Total marks scored by girls and boys)/Total number of students

= (2190 +1420)/50

= 3610/50

= 72.2

Q2 Text Solution:

Arranging the given data in ascending order, we get, 5, 7,8,10,15, 21.

Here, the number of observations is 6, which is even.

Hence, Median = [(n/2)th term + ((n/2) + l)th term]/2

Median = [(6/2)th term + ((6/2) + l)th term]/2

Median = (3rd term + 4 term)/2

Here, 3rd term = 8 and 4th term = 10

Therefore, median = (8+10)/2 = 18/2 = 9

Hence, the median of the given data is 9.

Q3 Text Solution:

Given that, mean = 12 and mode = 6

We know that, 3 Median = 2 Mean + Mode

Now, substitute the values in the formula, we get

3 Median = 2(12) + 6

3 Median = 24 + 6

3 Median = 30

Median = 30/3 = 10.

Hence, the value of median is 10.

Q4 Text Solution:

Given data: 45, 91, 62,71, 55

Arranging the given set in ascending order, we get 45, 55, 62, 71, 91.

Here, we have an odd number of observations, (i.e.) 5.

If "n" is odd, the median formula is [(n +l)/2]th term

= [(5 +l)/2]th term

= [(6)/2]th term = 3rd term = 62.

Hence, the median of the given data is 62.

Q5 Text Solution:

We know that,

Mode = 3 Median - 2 Mean

∴Mode = 3 × 43 - 2 x 45.5

= 129 - 91 = 38.

Mode = 38.

Q6 Text Solution:

Mean = μ_x = variance is $(6x)^2 > 0$

Var (ax + b) = Var (ax) + Var (b)

= a^2 Var (x)

= $a^2 . 6x^2$

Q9 Text Solution:

Var (ax + by) = a^2 Var (x) + b^2 Var (y) + 2ab Cov (x, y)

$\Rightarrow \quad \mathrm{Var}\ (\mathrm{x} - \mathrm{y}) = 1.\mathrm{Var}\ (\mathrm{x}) + 1\ \mathrm{Var}\ (\mathrm{y}) + 0$

Q10 Text Solution:

(i) $\sum f(x) = 1$

K + 3K + 5K + 7K + 9K + 11K + 13K = 1

$$\mathrm{K}\left(\cancel{1} + \cancel{3} + 5 + \cancel{7} + \cancel{9} + 11 + 13\right) = 1$$

K(10 + 10+ 29) = 1

$\mathrm{K} \times 49 = 1$

K = $\frac{1}{49}$

(ii) $\mathrm{P}(\mathrm{X} \geq 5) = \mathrm{P}(\mathrm{X} = 5) + \mathrm{P}(\mathrm{X} = 6)$

= 11K + 13K = 24K = 24/49

$\mathrm{P}(3 < \mathrm{X} \leq 6)$ = P(X = 4) + P(X = 5) + P(X = 6)

= 9K + 11K + 13K

= 20K + 13K

= 33K = 33/49

P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

= K + 3K + 5K + 7K
= 16K = 16/49.

Q11 Text Solution:

g(X) = $2X^2$ +1

X =	0	1	2	3	4	5	6
P(X) =	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

g(X) = $2X^2$ +1

E(g(x)) = E($2X^2$ +1)

= E($2x^2$) + E(1)

= 2E(x^2) + (1)

$= 2\left[\sum x_i^2\ P\left(x_i\right)\right] + 1$

$= 2$

$\left[\left(0 \times \frac{1}{6}\right) + \left(1 \times \frac{1}{6}\right) + \left(4 \times \frac{1}{6}\right) + \left(9 \times \frac{1}{6}\right)\right.$

$\left. + \left(16 \times \frac{1}{6}\right) + \left(25 \times \frac{1}{6}\right) + \left(36 \times \frac{1}{6}\right)\right] + 1$

$= 2\left[\left(\frac{1}{6}\right)\left(0 + 1 + 4 + 9 + 16 + 25 + 36\right)\right]$

$+ 1$

= 27 + 1

= 28.

Q12 Text Solution:

(i) f(3) = f(x) = P(X $\leq$ n)

$= P\left(X \leq 3\right)$

= P(X = 1) + P(X = 2) + P(X = 3)

= 0.05 + 0.15 + 0.35

= 0.05 + 0.50 = 0.55

(ii) E(x) = $\sum xi\ \ P\ \ (xi)$

=

$1\ \times\ 0.05\ +\ 2\ \times\ 0.15\ +\ 3\ \times\ 0.35 +\ 4\ \times$
$0.40\ +\ 5\ \times\ 0.05$

= 3.25

(iii). V(x) = E(x^2) – E($x)^2$

$\Rightarrow\ \ E\left(x^2\right) = \sum\left(xi\right)^2 P\left(xi\right)$

= 0.05 + 0.6 + 3.15 + 6.4+ 1.25

= 11.45

V(X) = 11.45 – $(3.25)^2$

= 0.88

Q13 Text Solution:

$E\left(X\right) = \sum_{i=1}^{x} xi, \left(xi\right)$

$= 1 \times \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{1-1} + 2 \times \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{2-1} + 3$

$\times \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{3-1}$

$= \frac{2}{3}$

$\left[1 \times \left(\frac{1}{3}\right)^0 + 2 \times \left(\frac{1}{3}\right)^1 + 3 \times \left(\frac{1}{3}\right)^2 + 4\right.$

$\left. \times \left(\frac{1}{3}\right)^3\right]$

$= \frac{2}{3}\left[\sum_{x=1}^{x} x\left(\frac{1}{3}\right)^{n-1}\right]$

$S = \left(\frac{1}{3}\right)^0 + 2 \times \left(\frac{1}{3}\right)^1 + 3 \times \left(\frac{1}{3}\right)^2$

Q14 Text Solution:

$P\left(\left|X - \mu_x\right| \leq 6_x\right)$

Here Mean = $\sum xi\left|, xi\right.$

$= 10\ \times\ 0.1\ +\ 20\ \times\ 0.1\ +\ 30\ \times\ 0.4\ +$
$40\ \times\ 0.3\ +\ 50\ \times\ 0.1$

$=\ 1\ +\ 2\ + 12 + 12 +\ 5$

= 24 + 8 = 32.

Variance = E(X^2) – $E(X)^2$

= E(X^2) = $\sum\left(xi\right)^2\left|, \left(xi\right)\right.$

$= 100 \times 0.1 + 400 \times 0.1 + 900 \times 0.4$
$+ 1600 \times 0.3 + 2500 \times 0.1$

= 1140

Var (x) = 1140 – 1024 = 116

= $P\ (1X - 32| \leq 116)$

= $116 \leq x - 32 \leq 116$

= $-84 \leq x \leq 148$

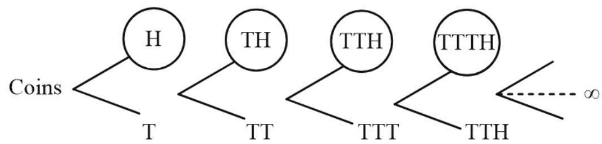
$P\left(0 \leq x \leq 148\right)$

$0.1 + 0.1 + 0.4 + 0.3 + 0.1 = 1$

Q15 Text Solution:

N = {Number of tosses required}

= {1, 2, 3, 4, 5, ...}



S = {H, TH. TTH, TTTH, ...}

N :	1	2	3	4	5	6
P(N) :	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$

$P\left(N > 1\right) = 1 - P\left(N = 1\right) = 1 - \frac{1}{2} = \frac{1}{2}$ i.e

Q16 Text Solution:

(A) We know that sum of PMF is one.

$$\sum_i P\left(x_i\right) = 1$$

k + 3k + 5k + 7k + 9k + 11k + 13k = 1

49 k = 1

$k = \frac{1}{49}$

(B) x < 4 ⇒x = 0 or x = 1 or x = 2 or x =3

There fore, P[x < 4] = p [x = 0] + p(x = 1) + p(x = 2) + p(x = 3) = k + 3k + 5k + 7k = $\frac{16}{49}$

(C) 3 < x ≤ 6 – x = 4 or x = 5 or x = 6

Therefore, P[3 <x ≤6] = 9k + 11k + 13k = 33k = $\frac{33}{49}$

Q17 Text Solution:

Mean = np

Variance = npq = np $(1-\mathrm{p})$

(A) Mean = 2

Variance = 3/2

$\frac{\text{Variance}}{\text{Mean}} = \frac{3}{4} = \mathrm{q} \quad \mathrm{P} = \frac{1}{4}$

np = 2

$\mathrm{n} \times \frac{1}{4} = 2$

n = 8, thus possible.

(B) Mean = 5

Variance = 9

q = 9/5

P = $1 - \frac{9}{5} = -\frac{4}{5}$ →Not possible.

(c) Mean = 10.

Variance = 5

$\mathrm{q} = \frac{5}{10} = \frac{1}{2}$

$\mathrm{P} = 1 - \frac{1}{2} = \frac{1}{2}$

$\mathrm{n} \times \frac{1}{2} = 10$

n = 20 Possible.

(D) np = 4

mpq = 8/3

q = 2/3

p = $\frac{1}{3}$

np = 4

n = 12

Thus possible.

Q18 Text Solution:

Let x = { Number of Heads Turning up}

& Let Head = Sucess then p = P(H) = $\frac{1}{3}$ & q = P(H) = $\frac{2}{3}$

P(3rd Head in 5th toss) = P (exactly 2H in 1st 4 Tosses)

× P(H is 5th toss)

= $\left[{}^{\mathrm{n}}\mathrm{C_r}\ \mathrm{P^r}\ \mathrm{q^{n-r}}\right] \times \frac{1}{3}$

= $\left[{}^{4}\mathrm{C}_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{4-2}\right] \times \frac{1}{3} = 6 \times \frac{2^2}{3^4} \times \frac{1}{3}$

= $\frac{8}{81}$ i.e

Q19 Text Solution:

X —B (n, p)

Mean = np

Variance = npq

Variance (x/n) = $\frac{1}{\mathrm{n}^2}$ Var (x)

$= \frac{\mathrm{npq}}{\mathrm{n}^2}$

$\frac{1}{\mathrm{n}}\mathrm{P}(1-\mathrm{p})$

Q20 Text Solution:

P (Student not having swing flu) = $\frac{9}{10}$

P(4 student safe) = ${}^{n}C_r p^r q^{n-r} = {}^{5}C_4\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^1 = 5 \times \frac{9^4}{10^5}$

P(5 student safe)= ${}^{5}C_5\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^0 = 1 \times \frac{9^5}{10^5}$

Total = P(4 student safe) + P (5 student safe) = $5 \times \frac{9^5}{10^5} + \frac{9^5}{10^5} = \frac{9^5 \times 14}{10^5} = \left(\frac{9}{10}\right)^4 \times \frac{7}{5}$

Q21 Text Solution:

p = 0.1, n = 400

Mean = np = 400 × 0.1 = 40

Standard deviation = $\sqrt{npq}$

= $\sqrt{400 \times 0.1 \times (0.9)}$

= $\sqrt{400 \times 0.09}$

= 20×0.3 = 6

Q22 Text Solution:

Mean of the binomial distribution = np = 2.5

Variance of the binomial distribution = npq = 1.875

np = 2.5......(i)

npq = 1.875.....(ii)

$\frac{npq}{np} = \frac{1.875}{2.500} = \frac{1875}{2500}$

⇒ q = 0.75

since p = 1 – q = 1 –0.75 = 0.25

Again since, np = 2.5 or n·(0.25) = 2.5 ⇒ $n = \frac{2.5}{0.25} = 10.$

Now, with n = 10, p = 0.25 and q = 0.75 , the binomial probability distribution is

$p(r) = {}^{10}C_r(0.25)^r(0.75)^{10-r}, r = 0, 1, 2,$ $.....10$

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Where r is the number of successes.

Q23 Text Solution:

E(x) = np = $100 \times 1/6 = 16.7 \approx 17$

So, 17 out of 100 are expected to fall 6.

V(x) =

$\mathrm{n\,p\ (1-p) = 100\ \times\ 1/6\ \times\ (1-1/6)}$

$= 13.9$

So, variance is number of 6's = 13.9.

Q24 Text Solution:

P(X = 1) + 2 P(X = 0) = 12 P (X = 2)

$\frac{\mathrm{e}^{-\lambda}\cdot\lambda^1}{1\mathrm{i}} + 2\frac{\mathrm{e}^{-\lambda}\cdot\lambda^0}{0\mathrm{i}} = 12\frac{\mathrm{e}^{-\lambda}\cdot\lambda^2}{2\mathrm{i}}$

$\lambda + 2 = 6\,\lambda^2$

= $6\lambda^2 - \lambda - 2 = 0$

$(3\lambda - 2)(2\lambda + 1) = 0$

$\lambda = \frac{2}{3}, \frac{-1}{2}$

But λ = Variance for Poisson Distribution So it can't be - ve

Hence λ = 2/3.

Now P (X = 0) = $\frac{\mathrm{e}^{-\lambda}\,\lambda^0}{0\mathrm{i}}$

= $\mathrm{e}^{-2/3}$

= 0.5/34

Hence option (c) is correct Answer.

Q25 Text Solution:

NA

Q26 Text Solution:

NA

Q27 Text Solution:

P = 1%, n = 100 , λ = np =1

$\mathrm{P(K)} = \frac{e^{-1}(1)^k}{k!} = \frac{e^{-1}}{k!}$

P(4 or more faulty) = P(4) + P(1) + ...+P(100)

= 1 − [P(0)+P(1) + P(2) + P(3)] = 1 − [e^{-1}] $\left[\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!}\right]$

= $= 1 - e^{-1}\left[1 + 1 + 0.5 + \frac{1}{6}\right] = 1 - \frac{8}{3e}$

$= 0.019$

Q28 Text Solution:

Mean of Poisson distribution λ =np

$n = 10, p\frac{1}{500}$

Then, $\lambda = 10 \times \frac{1}{500} = 0.02$

$P(x = r) = \frac{e^{-\lambda}\lambda^r}{\lfloor r}.\, r = 0, 1, 2....$

(i) Probability of numbers of defective balde.

$P(r = 0) = \frac{e^{-0.02}(0.02)^0}{\lfloor 0}.\ e^{-0.02} = 0.9802$

Then number of packets with numbers of defective blade = 1,00,000 ×0.9802 = 98020

(ii) Probability of one defective blad

$P(r = 1) = \frac{e^{-0.02}(0.02)^1}{\lfloor 1}.\ e^{0.02} = 0.9802$

=0.019604

∴ Number of packets with one defective blade = 1,00,000 ×0.019604 = 1960

(iii) Probability of two defective blads

$P(r = 2) = \frac{e^{-0.02}(0.02)^2}{\lfloor 2} = 0.000196$

∴ Number of two defective blades = 1,00,000 × 0.000196

= 19.6 ≅20.

Q29 Text Solution:

$\lambda = \mathrm{np} = 500 \times \frac{1}{1000} = \frac{1}{2}$

$\mathrm{P\,(X = 2)} = \frac{\mathrm{e}^{-\lambda}\lambda^2}{2\mathrm{i}} = \mathrm{e}^{0.5}\frac{(0.5)^2}{2} = 0.07582$