DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE

& also for CS/IT

Permutations and Combinations

Lecture No. 02

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

Permutation & Combination
(Part-1)

Topics to be Covered

Topic

"PERMUTATION & COMBINATION"

(Part-2)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

" If, what if, AGAR, YADI, TOH, ... "

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

COUNTING PRINCIPLE

Fundamental Principle of Addition → If we have to perform only one of the job at a time out of n jobs then use this principle.

Keywords : "Either or , only one , Anyone"

Fundamental Principle of Multiplication → If we have to perform all the jobs at a time out of n jobs then use this principle.

Keywords: "AND , Both , All"

Combination → (When Counting is based on Selection only then use this Rule).

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}} \quad \& \quad \boxed{{}^nC_r = {}^nC_{n-r}}$$

eg $^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}$

eg $^{22}C_4 = \frac{22\times21\times20\times19}{4\times3\times2\times1}$

eg $^{7}C_2 = \frac{7\times6}{2\times1}$ & so on....

eg $^{22}C_{19} = ? = {}^{22}C_{22-19} = {}^{22}C_3 = \frac{22\times21\times20}{3\times2\times1}$

Sp. Results: $\boxed{{}^nC_3 = \frac{n(n-1)(n-2)}{3\times2\times1}}$

$^nC_0 = 1 = {}^nC_n$ eg $^6C_0 = 1 = {}^6C_6$

$^nC_1 = n = {}^nC_{n-1}$ eg $^8C_1 = 8 = {}^8C_7$

$\boxed{{}^nC_2 = \frac{n(n-1)}{2}} = {}^nC_{n-2}$ eg $^5C_2 = 10 = {}^5C_3 = \frac{5\times4\times3}{3\times2\times1} = 10$

In a test there are three multiple choice questions having four choices each. Number of sequences in which a student can fail to get all answers correct is

(a) 11 (b) 15 (c) 80 (d) 63

Total possible sequences of Ans (RA) $= \frac{4\text{ way}}{Q_1} \times \frac{4\text{ way}}{Q_2} \times \frac{4\text{ way}}{Q_3} = 4^3 = 64$

But it includes one sequence in which student has given all Correct Ans

So Req way = Total seq − All correct seq.

$= 64 - 1 = 63$ (d)

F.Q 1: out of 6 Gents & 4 ladies, a committee of 5 persons is to be formed. then find the Number of Committees if

① there is No Restriction = ? (RNA) $= {}^{10}C_5 = 252$ Committees (Max Ans).

② Atleast 2L are there = ? = either (2L & 3G) or (3L & 2G) or (4L & 1G) or (5L ~~& 0G~~)

$= ({}^4C_2 \times {}^6C_3) + ({}^4C_3 \times {}^6C_2) + ({}^4C_4 \times {}^6C_1) = 186$ Committees.

M-II (WRONG APP) ~~Req Committees $= {}^4C_2 \times {}^8C_3 = 6 \times 56 = 336 >$ Max~~ (??)

Note No. of Committees in which there are exactly 2L = ? = (2L & 3G) = ${}^4C_2 \times {}^6C_3$

(3) At Most 2L are there = ? = (0L & 5G) or (1L & 4G) or (2L & 3G)

$= ({}^4C_0 \times {}^6C_5) + ({}^4C_1 \times {}^6C_4) + ({}^4C_2 \times {}^6C_3) = 186$ committees

(4) Gents are in Majority = ? = Same as part (3) = 186

(5) Ladies " " " = ? = (4L & 1G) or (3L & 2G)

$= ({}^4C_4 \times {}^6C_1) + ({}^4C_3 \times {}^6C_2) = 66$ Committees.

(6) there are exactly 3L in that Committee = ? = (3L & 2G) $= {}^4C_3 \times {}^6C_2 = 60$

(7) At least one L is there = ?

(M-I) Req ways = (1L & 4G) or (2L & 3G) or (3L & 2G) or (4L & 1G)

$$= ({}^4C_1 \times {}^6C_4) + ({}^4C_2 \times {}^6C_3) + ({}^4C_3 \times {}^6C_2) + ({}^4C_4 \times {}^6C_1) = 246$$

(M-II) At least one Lady = $\boxed{\text{Total} - \text{None}}$

= Total Committees − No Lady is there in a Committee

$$= \text{Total} - \text{All Gents} = {}^{10}C_5 - (0L \,\&\, 5G) = 252 - {}^6C_5 = 252 - 6$$

$$= 246$$

PODCAST: Various possibilities are as follows;

$$\underbrace{\underset{= \text{None}}{(0L\ \&\ 5G)} \text{ or } (1L\ \&\ 4G) \text{ or } \overbrace{(2L\ \&\ 3G) \text{ or } (3L\ \&\ 2G) \text{ or } (4L\ \&\ 1G)}^{\text{At least } 2L}}_{} = \text{Total Committees.}$$

$$({}^4C_0 \times {}^6C_5) + ({}^4C_1 \times {}^6C_4) + ({}^4C_2 \times {}^6C_3) + ({}^4C_3 \times {}^6C_2) + ({}^4C_4 \times {}^6C_1) = {}^{10}C_5$$

$$6 + \underbrace{60 + 120 + 60 + 6}_{\text{At least one } L} = 252$$

$$\underbrace{6 + 60 + 120}_{\text{At Most } 2L}$$

(*) P(in a Committee there are at least 2L) = ? $= \dfrac{\text{fav}}{\text{Total}} = \dfrac{120+60+6}{252} = \dfrac{186}{252}$

(*) P(" " " " exactly 2L) = ? $= \dfrac{\text{fav}}{\text{Total}} = \dfrac{120}{252}$

Factorial → $\boxed{n! = n(n-1)(n-2)(n-3)\ldots\ldots 3.2.1}$ ⇒ $\boxed{n! = n(n-1)!}$ Learn

eg $5! = 5\times4\times3\times2\times1 = 120$, eg $4! = 4\times3\times2\times1 = 24$, eg $1! = 1$ ......

$= 5\times4!$ $\quad$ $= 4\times3!$

eg $8! = 8\times7!$

Note: $\boxed{0! = 1}$ (defined)

eg: How many 5 digit Nos can be formed using odd digits (if RNA)?

Total 5 digit Nos (using 1, 3, 5, 7, 9) = ? = $\underset{P_1}{\underline{5\text{ ways}}} \times \underset{P_2}{\underline{4\text{ ways}}} \times \underset{P_3}{\underline{3\text{ ways}}} \times \underset{P_4}{\underline{2\text{ ways}}} \times \underset{P_5}{\underline{1\text{ way}}} = 5!$

(RNA)

eg How many 4 letter words can be formed using the letters "ROSE" if each letter is coming exactly once = ? = $\underset{P_1}{\underline{4}} \times \underset{P_2}{\underline{3}} \times \underset{P_3}{\underline{2}} \times \underset{P_4}{\underline{1}} = 4! = 24$

(RNA)

Analysis of $\boxed{0! = 1}$ $\rightarrow$ $\boxed{0! = ? = 1}$

$\frac{1!}{1}$ {MENTOS JINDAGI}

$5! = 5\times4\times3\times2\times1$

$2! = 2\times1$

$1! = 1\times1$

$0! = 0\times1 = 0$

??

$1! = 1\times1 = 1$

$2! = 2\times1 = 2$ $\frac{2!}{2}$

$3! = 3\times2\times1 = 6$ $\frac{3!}{3}$

$4! = 4\times3\times2\times1 = 24$ $\frac{4!}{4}$

$5! = 5\times4\times3\times2\times1 = 120$ $\frac{5!}{5}$

i.e fundamental Defn of Factorial is Not applicable in case of 0!

Eg: In how many ways 5 persons can be seated on 8 chairs ?

Total seating arrangements (RNA) $= \left(\underset{P_1}{\underline{8}} \times \underset{P_2}{\underline{7}} \times \underset{P_3}{\underline{6}} \times \underset{P_4}{\underline{5}} \times \underset{P_5}{\underline{4}}\right)$ choices

$$= \underset{(\text{Chairs})}{{}^{8}C_{5}} \times \underset{(\text{Arrangement})}{5!} = {}^{8}P_{5}$$

Eg: In how many ways 8 persons can be seated on 8 chairs ?

Total seating arrangements (RNA) $= \underset{P_1}{\underline{8}} \times \underset{P_2}{\underline{7}} \times \underset{P_3}{\underline{6}} \times \underset{P_4}{\underline{5}} \times \underset{P_5}{\underline{4}} \times \underset{P_6}{\underline{3}} \times \underset{P_7}{\underline{2}} \times \underset{P_8}{\underline{1}} = {}^{8}C_{8} \times 8! = 8!$

Eg: In how many ways 8 persons can be seated on 5 chairs = ? = senseless Questions

Permutation :→ (Selection & Arrangement both) ⟶

If in a Question, Counting is Based on, Selection as well as on Arrangement also then use this Rule.

$${}^nP_r = \frac{n!}{(n-r)!} = \boxed{{}^nC_r \times r!}$$

eg $${}^{11}P_3 = {}^{11}C_3 \times 3! = 11\times10\times9$$

eg $${}^{12}P_4 = {}^{12}C_4 \times 4! = 12\times11\times10\times9$$

while $${}^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}, \quad {}^{12}C_4 = \frac{12\times11\times10\times9}{4\times3\times2\times1}$$

$$\boxed{{}^nP_0 = 1, \quad {}^nP_n = n!, \quad {}^nP_1 = n}$$

eg $${}^6P_0 = 1, \quad {}^6P_6 = 6!, \quad {}^6P_1 = 6$$

eg $${}^8P_0 = 1, \quad {}^8P_8 = 8!, \quad {}^8P_1 = 8$$

Eg: Consider three letters a, b, c then

Various combinations = ? $= {}^3C_3 = 1$ ie only (a b c)

ie (abc) = (acb) = (bac) = (bca) = (cab) = (cba)

Various Arrangements = ?

M-I : $= \underset{P_1}{\underline{3}} \times \underset{P_2}{\underline{2}} \times \underset{P_3}{\underline{1}}$ = 6 arrangements

M-II : $= 3! = 6$, M-III $= {}^3C_3 \times 3! = 6$

eg (a b c), (a c b), (b c a), (b a c), (c a b), (c b a)

Ex of Combination :

→ formation of team,

→ " of Committee

→ No. of Handshakes.

→ No. of St. lines & Δ's

→ No. of 11^{gms}

. - - - - -

Ex of Perm :- If in a Question there is a feeling of interchanging things then use ${}^{n}P_{r}$.

→ formation of words.

→ " of Numbers.

→ Seating arrangement.

→ Formations of photographs

→ " of signals.

GAZAB KA Conclusion →

① if $n > r$ & RNA, then Multi Rule = Perm. Rule

② if $n = r$ & RNA, Then Multi Rule = Perm Rule = Factorial Rule

③ if RA, then only use Multi Rule.

i.e the Concept of ${}^nC_r$, ${}^nP_r$ & $r!$ is applicable only when RNA

Q How many 4 letter words (w or w/o meaning) can be formed using the letters of the word 'EQUATION' $= \underset{P_1}{8} \times \underset{P_2}{7} \times \underset{P_3}{6} \times \underset{P_4}{5} = {}^8C_4 \times 4! = {}^8P_4$

= Toys (RNA)

M-I M-II M-III

Q How many 5 letter words can be formed using the letters of the word 'LOGARITHMS'

= Toys = (RNA)

$n = 10$

$= \underset{P_1}{10} \times \underset{P_2}{9} \times \underset{P_3}{8} \times \underset{P_4}{7} \times \underset{P_5}{6} = {}^{10}C_5 \times 5! = {}^{10}P_5$

M-I M-II M-III

Qs (5) persons entered in a lift at Ground Floor in an 8 floor House then in how many ways they can leave the lift ?

① At any floor = ? $= \underset{P_1}{\underline{7}} \times \underset{P_2}{\underline{7}} \times \underset{P_3}{\underline{7}} \times \underset{P_4}{\underline{7}} \times \underset{P_5}{\underline{7}} = 7^5$ ways

(RA) = No Restriction

Persons = 5

floor = 7

② At different floors = ? $= \underset{P_1}{\underline{7}} \times \underset{P_2}{\underline{6}} \times \underset{P_3}{\underline{5}} \times \underset{P_4}{\underline{4}} \times \underset{P_5}{\underline{3}} = {}^7C_5 \times 5! = {}^7P_5$

(RNA)

③ P (they all will leave the lift at different floors) = ? $= \dfrac{\text{fav}}{\text{Total}} = \dfrac{{}^7P_5}{7^5}$

Q. there are 5 flags of different colours then how many different signals can be formed using f_1, f_2, f_3, f_4, f_5

(1) 3 flags at a time = ?

(RNA)

$$= \frac{5 \text{ ways}}{P_1} \times \frac{4 \text{ ways}}{P_2} \times \frac{3 \text{ ways}}{P_3} = {}^5C_3 \times 3! = {}^5P_3$$

Q. there are 5 flags of different colours then how many different signals can be formed using

(2) Any Number of flags at a time?

we can make signal either by using

= (1 f) or (2 f) or (3 f) or (4 f) or (5 f) at a time

$= (5) + (5\times4) + (5\times4\times3) + (5\times4\times3\times2) + (5\times4\times3\times2\times1) = 325$

M-II $= ({}^5C_1\times1!) + ({}^5C_2\times2!) + ({}^5C_3\times3!) + ({}^5C_4\times4!) + ({}^5C_5\times5!) = 325$

M-III $= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5 = 325$

(3) using at least 3 f at a time = ? = ${}^5P_3 + {}^5P_4 + {}^5P_5 = 300$ ✓

$= 60 + 120 + 120$

(4) " at most 2 flags at a time = ? = ${}^5P_1 + {}^5P_2 = 25$

The number of words of four letters containing equal number of vowels and consonants (Repetition Not allowed)

(a) 60×210 (b) 210×243

(c) 210×315 (d) 630 ~~(e)~~ 50400

1. ([redacted])

WRONG APP.:-

$Ans = \cancel{{}^5P_2 \times {}^{21}P_2}$

(Vowels are selected and Arranged also) (Consonant are selected & Arranged also)

But separately

Vowels $= 5 \Rightarrow$ No. of ways of selecting $2V = {}^5C_2$

Consonants $= 21 \Rightarrow$ " " " $2C = {}^{21}C_2$

Req Ans (RNA) $= \{$equal No. of Vowels & Consonants$\} = \underbrace{{}^5C_2 \times {}^{21}C_2}_{\text{(selection of 4 letters)}} \times \overbrace{4!}^{\text{Arrangement.}} = 50400$

Concept of At least and at Most →

if we want to distribute 5 Apple among kids (where distribution is Totally Based on our choice) then various possibilities are as follows;

=None

(0 App) or (1 App) or (2 App) or (3 App) or (4 App) or (5 App) = Total possibilities

At Most 2 App.

At least 3 App.

At Most 1 App.

At least 2 App.

At least 1 App.

Let $X = \{\text{No. of Apples distributed}\}$ then

(At Most) (At least)

① $(X \leq 1) + (X \geq 2) =$ Total Possibilities

② $(X \leq 2) + (X \geq 3) =$ "

③ $(X \leq 3) + (X \geq 4) =$ "

④ $(X = 0) + (X \geq 1) =$ Total $\Longrightarrow$ $\boxed{\text{At least one} = \text{Total} - \text{None}}$

(*) Whether we should include or exclude 'None' in At Most, it totally depends on the Nature of Question.

Q. Three dices are thrown simultaneously then find the possible number of outcomes in which at least one dice show digit '4'?

Total outcomes (RA) $= \underset{D_1}{\underline{6 \text{ outcomes}}} \times \underset{D_2}{\underline{6 \text{ outcomes}}} \times \underset{D_3}{\underline{6 \text{ outcomes}}} = 216$ outcomes

No. of outcomes in which No Die show digit '4' $= \underset{D_1}{\underline{5}} \times \underset{D_2}{\underline{5}} \times \underset{D_3}{\underline{5}} = 125$ outcomes

So No. of outcomes in which At least one die show digit '4'

$= \boxed{\text{Total outcomes} - \text{No die show digit 4}}$

$= 216 - 125 = 91$

M-II No. of outcomes in which at least one die show digit '4'

PODCAST

= (exactly one die is showing '4') $\longrightarrow$ ${}^3C_1 \times \underline{1} \times \underline{5} \times \underline{5} = 3 \times 25 = 75$ cases

OR

(exactly 2 dice are showing '4') $\longrightarrow$ ${}^3C_2 \times \underline{1} \times \underline{1} \times \underline{5} = 3 \times 5 = 15$ cases

OR

(All 3 dice are showing '4') $\longrightarrow$ ${}^3C_3 \times \underline{1} \times \underline{1} \times \underline{1} = 1$ case

---

Total $= 75 + 15 + 1 = 91$ cases.

Q: How many three digit Nos are there such that at least one of the digit is 7 ?

(digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

(M-I) Total 3 digit No's (RA) $= \underset{P_1}{9} \times \underset{P_2}{10} \times \underset{P_3}{10} = 900$ Nos

Total 3 digit Nos in which (RA) $\underbrace{\text{7 is not coming even once}}_{\text{(None)}} = \underset{P_1}{8} \times \underset{P_2}{9} \times \underset{P_3}{9} = 648$ Nos

At least one of the digit is 7 $= \boxed{\text{Total} - \text{None}}$

$= 900 - 648 = 252$ Nos.

(M-II) By Making Cases → HW

Thank you

Keep Hustling!