# DATA SCIENCE &

ARTIFICIAL INTELLIGENCE

Not for (CS/1T)

Linear Algebra - I

Lecture No. 03



### Recap of previous lecture











Topic

#### **Topics to be Covered**







Topic

VECTOR SPACE

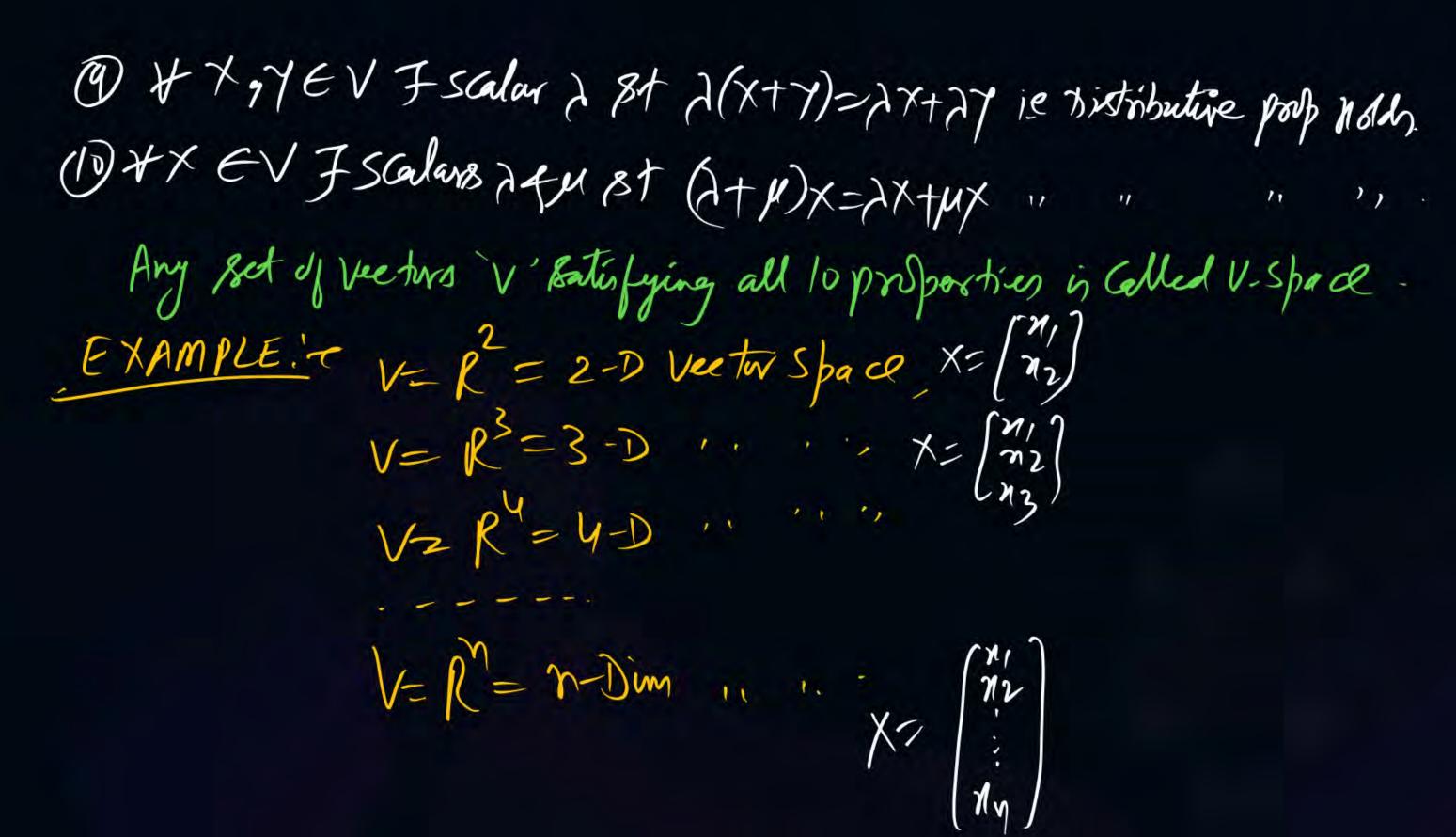
- SUBSPACE
- BASIS
- -> DIMENSION OF VECTOR SPACE
- Concept of SPANNIG in Vector Space.

#### VECTOR SPACE



## Defin - Any sect of voctors Vin Called V. Space of it boutisty following properties;

- 1) Let 7.97 EV => X+7 EV is Closure property of vector addition Holds.
- (2) XX,7EV=>XXY=7XX is Commutative prop of Vaddition Holds.
- 3 + xxx, 2 EV => (x+x)+z=x+(y+z) is Associative propoly vaddition nolds.
- 9 XXEV JOEV 8+X+0=X is additive Identify emint
- 3) XXEV J-XEV St X+(-X)=0 is Additive inverse enist.
- 6 HXEV F Scalar C 8+ CXEVie Clasure prop for scalar Multiplication Nolds.
- O + X = V = Scalars > fa st > (UX)=(NU)X is Asso. Prop for Scalar Multi Holds.
- (8) +XEV 7 scalar 1 1.1 1.XEV is Multiplicative identity engit.



Pw

Verification: - Consider V= R= S[y]; x, y eR}

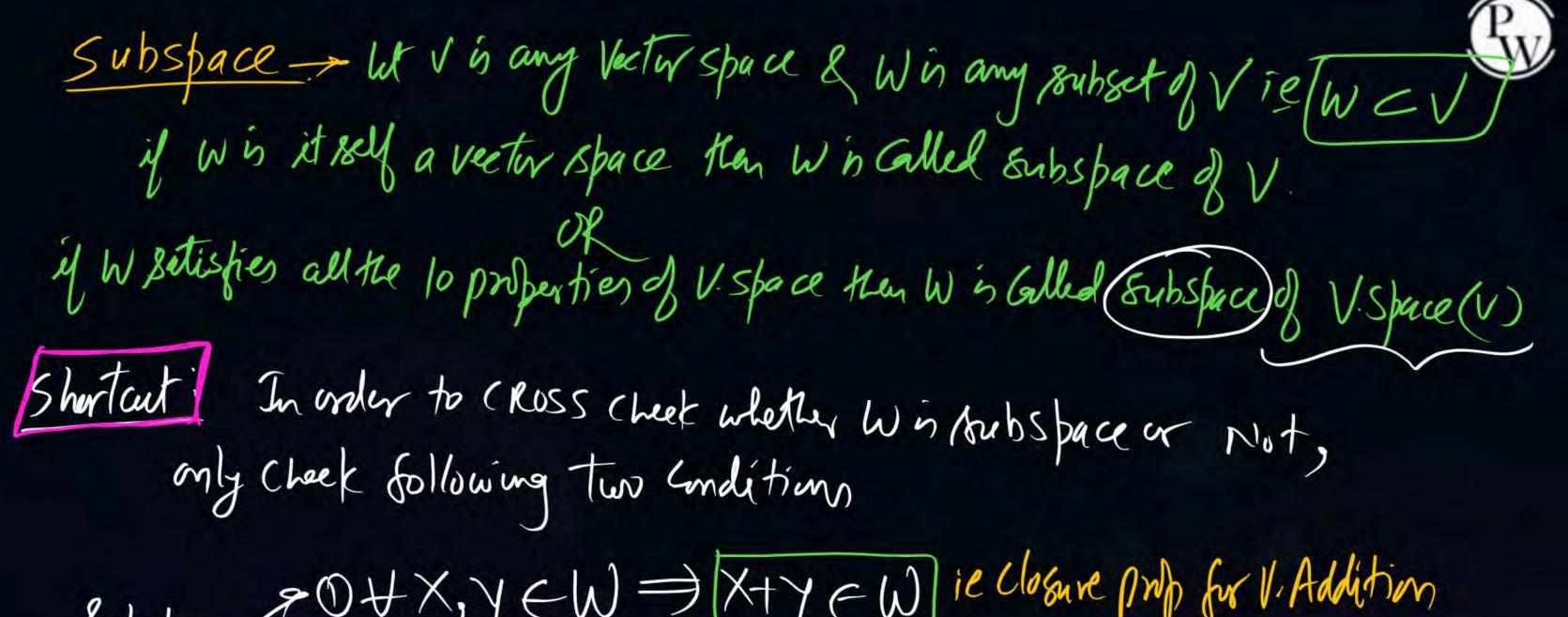


1) Let 
$$x=\begin{bmatrix}2\\3\end{bmatrix}$$
,  $7=\begin{bmatrix}-1\\2\end{bmatrix}$  Kun  $x+y=\begin{bmatrix}1\\5\end{bmatrix}\in\mathbb{R}^2$  is Clubure prop for  $V$ . Addition holds.

(3) Let 
$$Z = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$
 then  $(x+7)+Z = \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$  i.e. Associative prof for  $(x+(y+z)) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$  V. addition holds

(3) 
$$(-1)^{-1} = (-1)^{-1} =$$

6) let c=5, x={3} then cx=5x={10 | FR is Closure prop for Scalar Midds Holds. ωt λ=3,μ=-5 then λ(μx)= 3(-5x)=3(-10)=(-30) = (-30) = (-30) = (-45)  $4(AM)X = (3(-5))X = -15\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}-30\\-45\end{bmatrix} \in \mathbb{R}^2$ ie A(MX) = (AM)X so Associative pm) for scalar Multi Holds. (8) Let  $C=1 \Rightarrow CX=1.X=1.\begin{bmatrix} 2 \\ 3 \end{bmatrix}=\begin{bmatrix} 2 \\ 3 \end{bmatrix}=X\in \mathbb{R}^2$  is Multi Identity exist. (9) Let  $\lambda = 3, x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, 7 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$  then  $\lambda (x+y) = 3(\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}) = 3[\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ (10)  $\lambda = 3, M = -5, x = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = A(1+M)x = (3-5)x = -2x$  is both types of Pointerbutine Proposition Holds.



Poubspace POHX, YEW = X+YEW is closure prof for V. Addition (2) HYEW, Fscalar C St CXEW is Closure prof for scalar Multi. (M) Let Mn denote the vector space of all nxn MS real matrices consider the following subsets

- 1. W, = {A E Mm 1 A2 = I3, where I is the identity
- 2. W= = & A & Mn 1 rank (A) = 13
- 3. W3 = {AE Mn 1 trace (A) = 09
- 4. W4 = & A+BIAEM, B is a fixed materix in Mn3

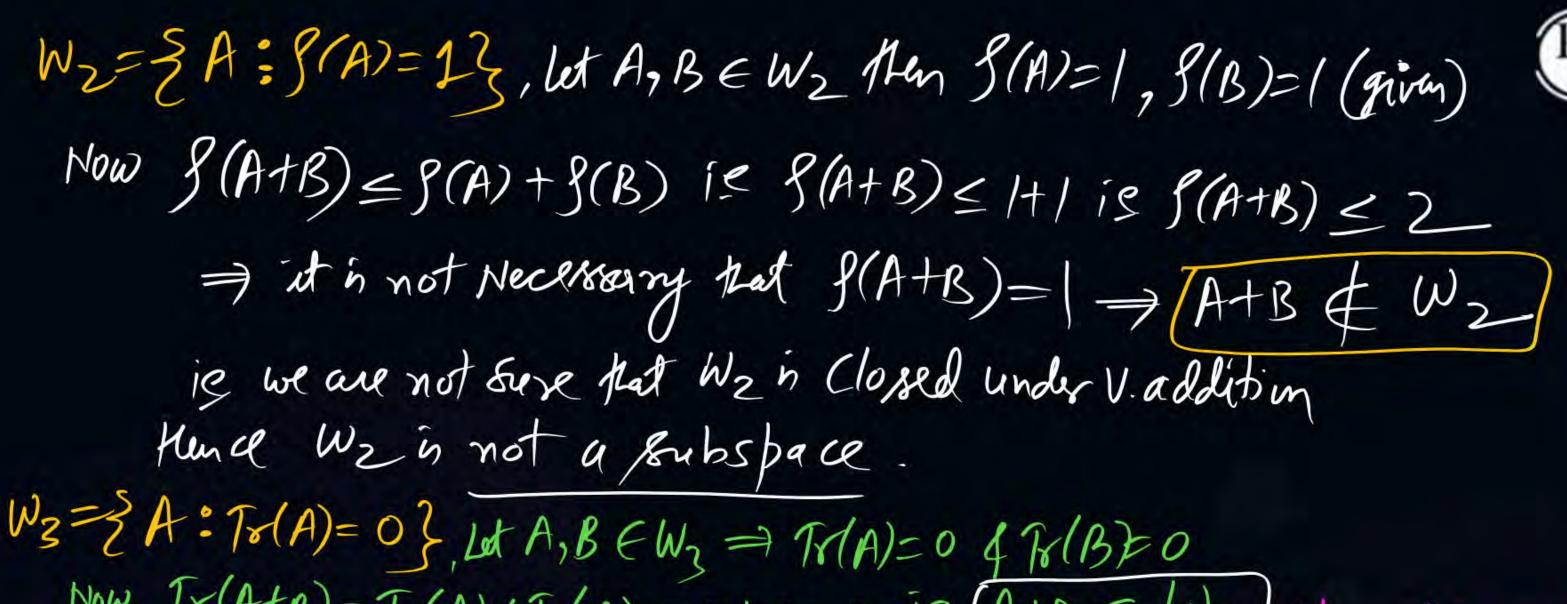
which of the following estatements is correct?

- (a) W, is a linear subspace of Mm:
- 6) We is a linear subspace of Mm.
- a) Wy is a linear subspace of Mn(T)

  Terro materix:

ie Win Hot Closed under vector addition Hence W, is not subspace

(1) W1= 2 A EMmm. A=Iflet A,B & W, then A=IrB=I (given) Now,  $(A+B)^2 = A^2 + B^2 + AB + BA = I + I + AB + BA = I$ is (A+B) is Not Necessarily Involution Mult is  $(A+B)^2 + I = A+B \in W$ 



Now Tr(A+B)=0, Let  $A,B \in W_3 \Rightarrow Tr(A)=0$  of Tr(B)=0Now Tr(A+B)=Tr(A)+Tr(B)=0+0=0 is  $A+B \in W_3$  (lower holy Holds. Again Tr(CA)=CTr(A)=((0)=0 is  $CA \in W_3$ ) (lower holy Holds. Hurce  $W_3$  is a publishace of  $M_{nxy}$  Wy= { AtB : Bis a fined Mid}.
We have two possibilities for B geithr B=0=100) bor B = 0=(01/162).[5] if we take B = 0 thin je additive identity DNE in Wy. & Hence Wy is not a Veeder Space. A+B+A 9 whice B=0 then A+B=A+0=A Redditive identity exist (i)
& thence by law be a vector space only when we take B= 0= fined Matrin 2) Let S,, S2, S3 be sets of real - valued functions 051:3f(a):f(3)=07 defined as: S1 = & f1f(3) = 03 to 51 (n), f2(n) (=5, 1hm Sz = {g/g(x) = x+1 for all x < R} S3 = & h 1h(x) = C for some instant ce K} fi(3)=04 f2(3)=0 which of the following statements is coverect? Now (fi+f2)(n)=fi(n)+f2(n) a) Si is a vector space & Sz is not a vector space. (1) Sz is a vector space & Sz is not a vector space. 2040=0 9/5, & S3 are vectorspaces, but S2 is not a vector 18fitf2ES1 d) S2 & S3 are wester spaces but S, is not a wester of fife (S) =) (1+fz) (S) Now (cf)(3)=(f(3)=((0)=0 Am = ( But in ase of MSD ie cf ESI Afts, Floder ( St Cf E S) M-940 both. Huyus, jav. space.





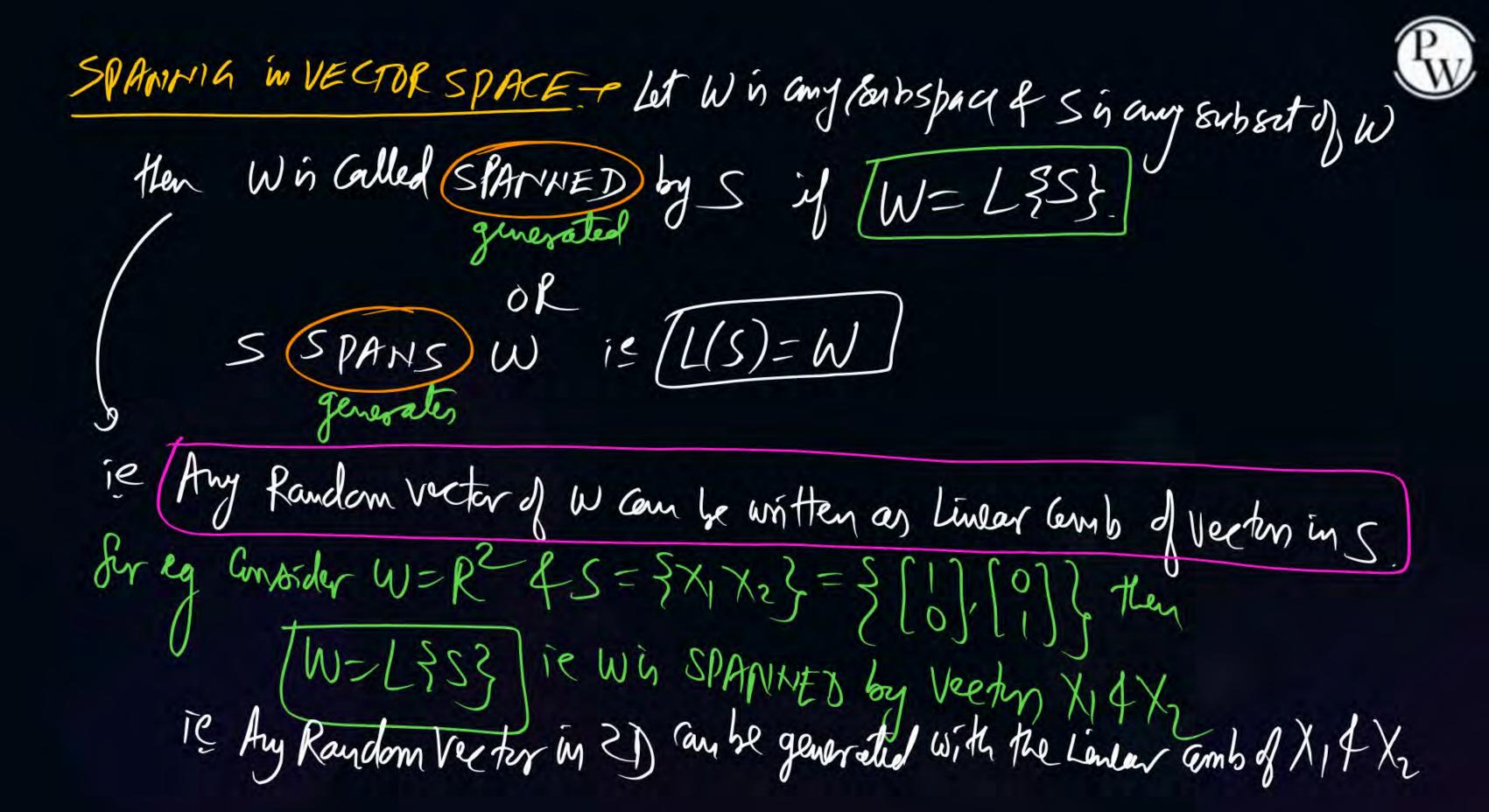
let h1(n), h2(n) E53 Ken h(n)=C, h2(n)=C Now (h,+hz)(n)=h1(n)+hz(n) = c+c=zc=G=)(h+hz) E53 is (thi, hi = 53 ) hithi = 53 / is closure Holds. Let h(n) ES3 then h(n) = C Let  $\lambda$  is any scalar then  $(\lambda h)(n) = \lambda (h(n)) = \lambda (c) = (3 = )(\lambda h) \in S_2$ 19 Ft h Esz f scalar & St Ah Esz Je clobuse Holds. Hunce Szis also a V. Space.

52= 39(n): 7(n)= n+1}

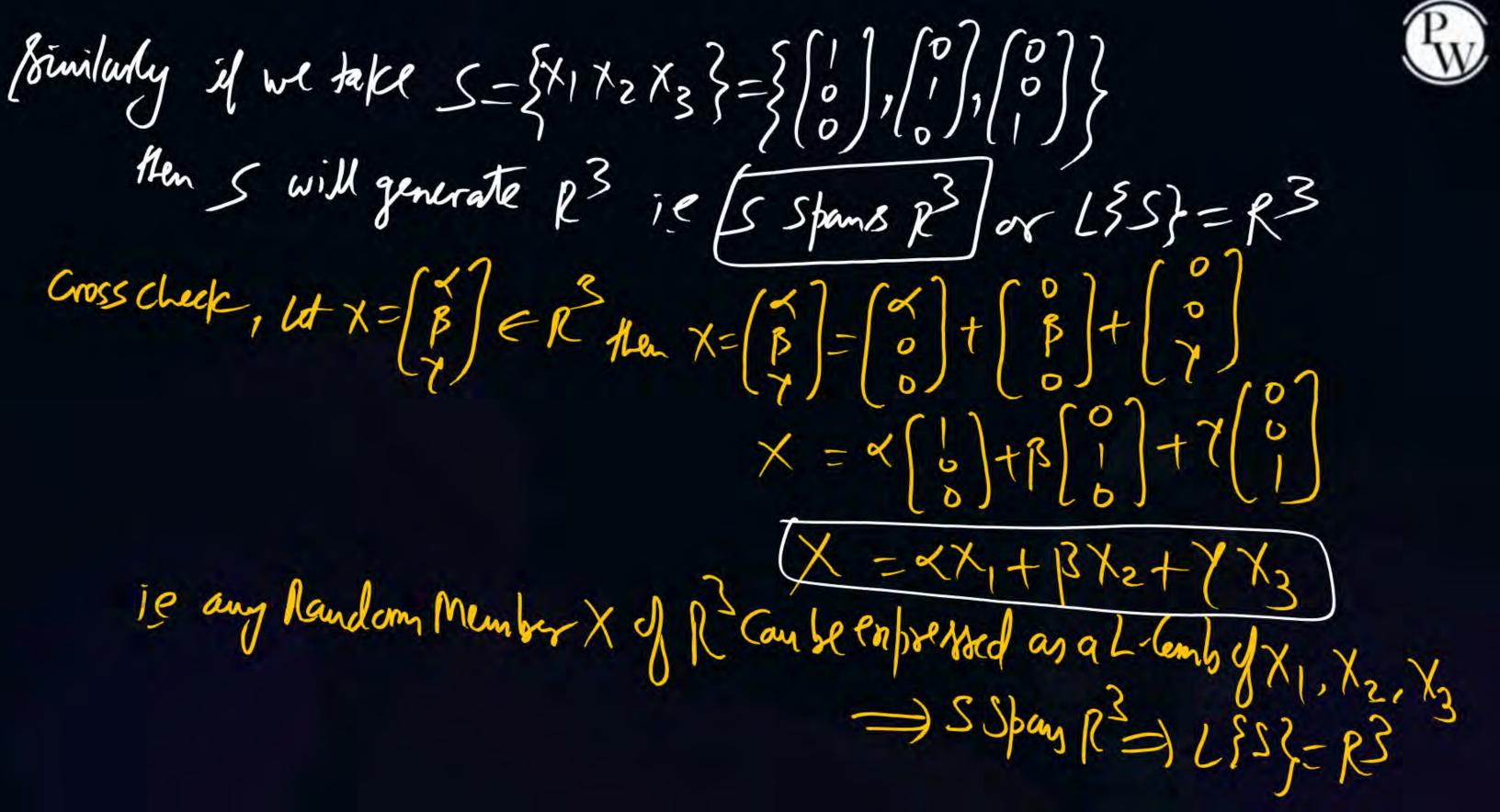


If  $g_1(n) \notin g_2(n) \in S_2$  thun  $g_1(n) = n+1$   $(g_2(n) = n+1)$ How  $(g_1 + g_2)(n) = g_1(n) + g_2(n) = (n+1) + (n+1) = 2(n+1) \neq n+1$ is  $g_1 + g_2 \neq S_2 \Rightarrow$  Closux prop for V. Addition Not holds.  $80 \cdot S_2$  is not a V. Space.

linear span - tet sin any set of verbus then set of all linear Combinations of pectus in S in Called Linear span of 5 ?? and it is denoted by 135} Let  $S = \{X_1, X_2, X_3, X_4\}$  then all the linear tembination of these vectors can be written as  $(K_1X_1 + K_2X_2 + K_3X_3 + K_4X_4)$  where  $K_i \in \mathbb{R}$ . then LFS}={(Kititkettettestettestytty); KiER}=linearspans
of S.



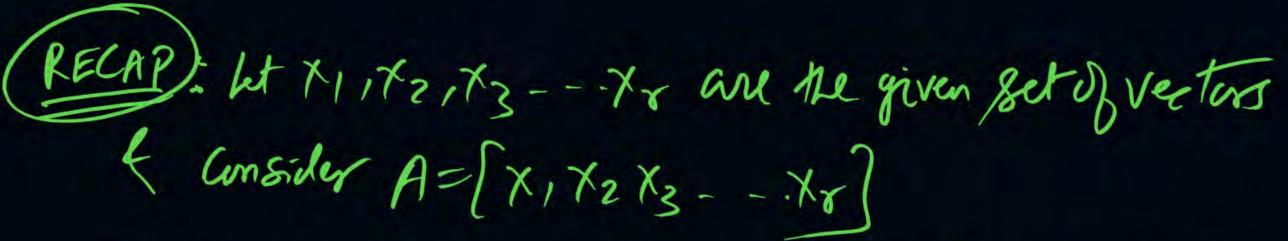
VERIFICATION: 5= { \*1,1 \*2}= { [] } [] } Hen S spans  $R^2$  ie  $L(5)=R^2$ Let  $X = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \in \mathbb{R}^2$  Ken  $\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 2 X_1 - 3 X_2$ is ne have justified that, Any Random Memberd R2 can be enpressed as a linear Combination of X14X2 T. (35)-R2 S generates R2 or S Spans R2

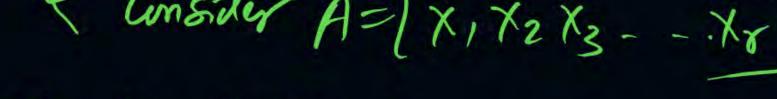




13ASIS of Vector space - ld Winamy Bubspace of Sin amy given set of vectors. Hen Sis Called BASIS for Wig Spans W E) Scontain (I-I) set of vectors for eg Basis for  $W=R^2$  is  $S_1=\{X_1X_2\}=\{[0],[1]\}$ :: L $\{S_1\}=R^2$  of  $S_1$  contains LI voctors (By Shservation) again, Basis for  $w=R^3$  6 Sz= $\{X_1X_2X_3\}=\{\{0\},\{0\},\{0\}\}\}$ ":  $\{\{S_2\}=R^3\}$  & Sz Contains LI Vectors (By Tricky Method)

· S2 = 0





1) General Method

(i) A S(A) = No. of vector =) (II)

(ii) if f(A) < " " => LD

(2) Tricky Method (4Ais Sq. Met)

(i) if  $|A| \neq 0 \implies \text{vectors are}(II)$ (ii) if  $|A| = 0 \implies 1 \implies 2D$ 

Note: In Cased Proveeten X14X2, there is no Need to follow G. Method or Tricky Method, only follow observation Method.



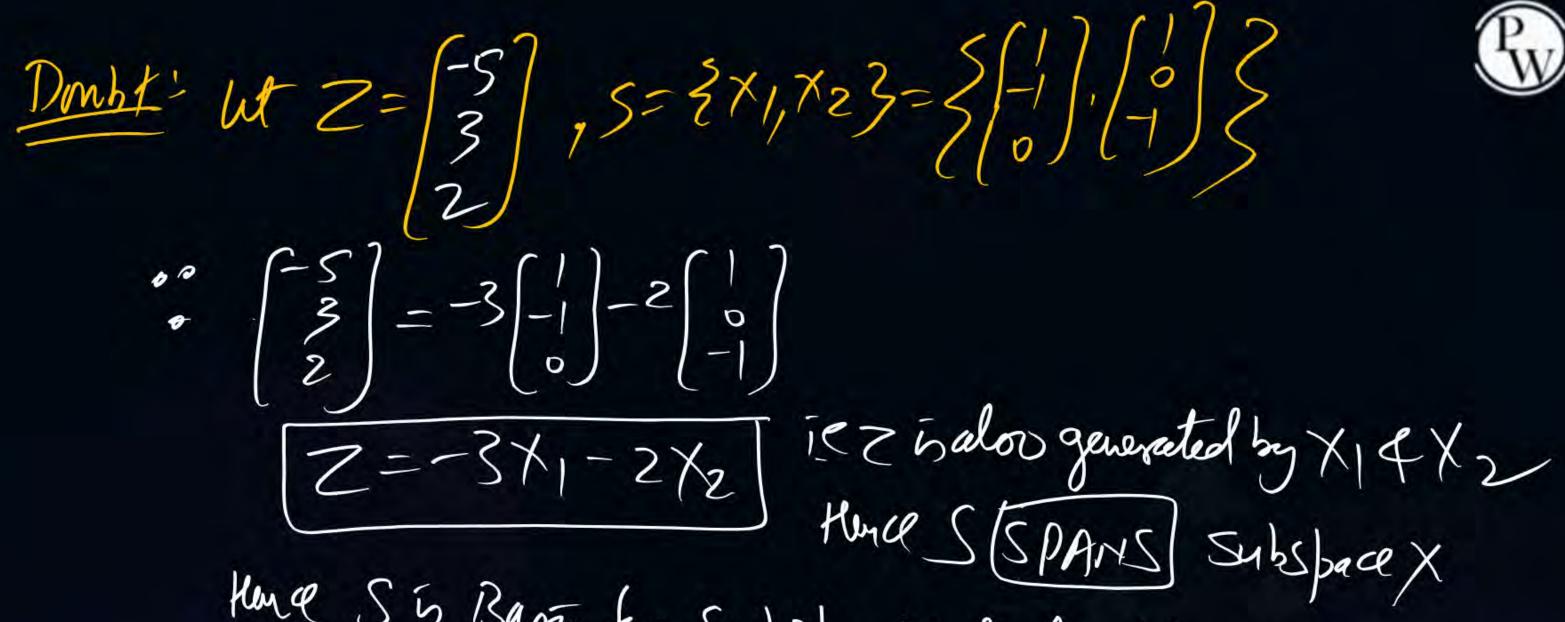
Dimenssion of vector (Space) = Number of vectors in it's BASIS for eg din(R2)=2 0: Bans for R2 = 3 6 / [1] 3= 3 × 1, 1 × 23 "  $\dim(R^3) = 3$  ": Basis for  $R = S[0], [0], [0], [0] = \{x_1, x_2, x_3\}$ Let win kubspaced R Hundim W=? = Not Necessarily 3 it will defend on the Basis of W (will be discussed later) If Vinany Veeters pace 1.+ ety Bain = {E1, E2, G, Fy, Es} then dim(V) = No. of vectors in Basis = 5

De Consider the set of Column rectors defined by X=3 (1/12/13); [1/11/12/13] then S= { (1-10), (10-1) } in Bassis for Subspace X? (4ES) MI : X C R3 80 X is subspace of R5 defined as. Clearly X14 X2 we (LI) by
Shservation) Hence 2 and holds.
Now Let us take any Random vector of  $X = \begin{cases} n_2 \\ n_3 \end{cases}; n_1 + n_2 + n_3 = 0 \end{cases}$ Now  $S = \{ \{ \{ \{ \} \}, \{ \{ \} \} \} \} = \{ \{ \{ \} \}, \{ \} \}$ 8abspace X bought is y= -3 then will be Basis for X 4 & Swill SPANX

Y= 3X1-X2 bo y (our be)

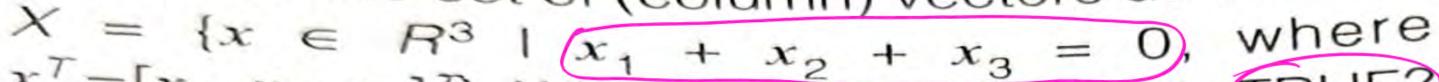
S & Basis for Subspace X & din (X)= 2

Weather the Span X three SSPANX



tune S is Barin for Subspace X of clin (X)=No-yvertining





- $x^T = [x_1, x_2, x_3]^T$ . Which of the following is TRUE?
- (a)  $\{[1, -1, 0]^T, [1, 0, -1]^T\}$  is a basis for the subspace X. T
- (b)  $\{[1, -1, 0]^T, [1, 0, -1]^T\}$  is a linearly independent set, but it does not span X and therefore is not a basis of X
- (c) X is not a subspace for R3
- (d) None of the above



If the vectors,  $e_1=(1,0,2), e_2=(0,1,0)$  and  $e_3=(-2,\bar{0},1)$  from an orthogonal basis of the three sequences of the three sequences  $u=(4,3-3)\in\mathbb{R}^3$  can be expressed as

(a) 
$$u = -\frac{2}{5}e_1 - 3e_2 - \frac{11}{5}e_3$$

(b) 
$$u = -\frac{2}{5}e_1(-3e_2) + \frac{11}{5}e_3$$

(c) 
$$u = -\frac{2}{5}e_1 + 3e_2' + \frac{11}{5}e_3$$

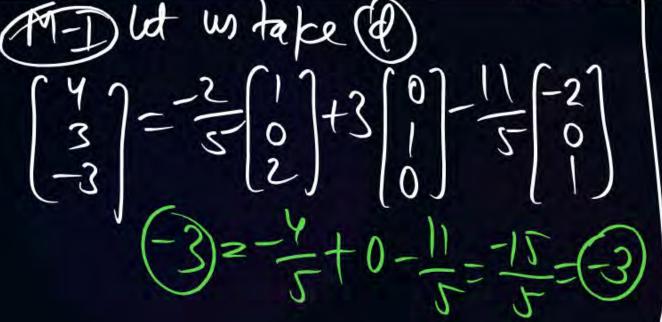
(d) 
$$u = -\frac{2}{5}e_1 + 3e_2 - \frac{11}{5}e_3$$

= Bans of R<sup>3</sup> (given)

: 'Sin Bury for R<sup>3</sup> 80 [Swill SPAN R<sup>3</sup>]

Any Random Vector of R<sup>3</sup> (see U=[3])

Can be empressed on a linear Combination of vector in



Le M-II) HW



## THANK - YOU