GATE

DS & AI
CS & IT

Linear Algebra -I

Lecture No. 14

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Recap of previous lecture

Topic

EIGEN VECTORS

Topics to be Covered

Topic

MISCELLANEOUS

- DIAGONALISATION
- L-U Decomposition
- Reflection Mat
- Rotation Mat

HW8 Find the E Values & E Vectors of $A=\begin{bmatrix}1&2\\0&2\end{bmatrix}$

Gate + Gate + ESE

OR

If one pair of LI E.vectors of $A=\begin{bmatrix}1&2\\0&2\end{bmatrix}$ is $\begin{bmatrix}1\\a\end{bmatrix}$ & $\begin{bmatrix}1\\b\end{bmatrix}$ then $a+b=?$

0.5 Ans

OR

one pair of L.I, E. Vectors for $A=\begin{bmatrix}1&2\\0&2\end{bmatrix}$ will be ?

(a) × $[1\ 0]'$, $[-2\ 0]'$ ∵ these are LD

(b) × $[2\ 1]'$, $[-2\ -1]'$ ∵ " " LD

(c) × $[4\ 0]'$, $[2\ 3]'$ these are LI but 2^{nd} is not an E Vector for any λ

(d) ✓ $[1\ 0]'$, $[2\ 1]'$

∵ $AX=\lambda X$

$\begin{bmatrix}1&2\\0&2\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}=\lambda\begin{bmatrix}2\\3\end{bmatrix}$

$\begin{bmatrix}8\\6\end{bmatrix}=\begin{bmatrix}2\lambda\\3\lambda\end{bmatrix}\Rightarrow \left.\begin{matrix}\lambda=4\\\lambda=2\end{matrix}\right\}$ Not unique

(i) $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

$\rightarrow \lambda = 1,\ X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \end{bmatrix} \ldots\ldots \infty$

$\rightarrow \lambda = 2,\ X = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} \ldots\ldots \infty$

(ii) M-I Separately find E.Vectors and the Compare

$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

$\rightarrow \lambda = 1,\ X = \begin{bmatrix} 1 \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow a = 0$

$\rightarrow \lambda = 2,\ X = \begin{bmatrix} 1 \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} \Rightarrow b = \frac{1}{2}$

So $a + b = \frac{1}{2}$ Ans

M-II let for $\lambda = 1$, $X \begin{bmatrix} 1 \\ a \end{bmatrix}$ & for $\lambda = 2$, $X = \begin{bmatrix} 1 \\ b \end{bmatrix}$ so

$AX = \lambda X$ & $AX = \lambda X$

$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ a \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ b \end{bmatrix} = 2\begin{bmatrix} 1 \\ b \end{bmatrix}$

$\begin{bmatrix} (1+2a) \\ 2a \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1+2b \\ 2b \end{bmatrix} = \begin{bmatrix} 2 \\ 2b \end{bmatrix}$

$\Rightarrow a = 0$ & $b = \frac{1}{2}$

So Ans $= a + b = 0 + \frac{1}{2} = 0.5$

SIMILAR MATRICES → Two Matrices A & B are said to be similar if ∃ a relationship of the type $\boxed{P^{-1}AP = B}$ where P is any Invertible Mat.

Note ① Similar Matrices have same Trace, same Det, same C.Equⁿ, same E.Values as well as same Rank.

while Equivalent Matrices have same RANK & other properties may be diff.

② Similarity Relation b/n Matrices is an Equivalence Relation

i.e Reflexive, Symmetric as well as Transitive.

DIAGONALISATION → A is called Diagonalisable if it is SIMILAR to Diag Mat

i.e $\exists$ P s.t $\boxed{P^{-1}AP = D}$ & this process is called Diagonalisation.

Consider $A_{3\times3}$ & let it's E.Values are $\lambda_1, \lambda_2, \lambda_3$

& E.vectors are X_1, X_2, X_3

then Modal Mat is $P = [X_1\ X_2\ X_3]$ & let $D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

& let $|P| \neq 0$.

Let us try to Calculate,

$$P^{-1}AP = \left[\ P^{-1}\ \right]\left[\ A\ \right]\left[\ P\ \right] = \cdots\cdots = D$$

i.e $P^{-1}AP = \text{Diag. Mat} \Rightarrow$ A is Diagonalisable

this is very special Property of Mat & this property is called Diagonalisation

Note ① Use of Diagonalisation →

$\because$ $P^{-1}AP = D$

$P(P^{-1}AP)P^{-1} = PDP^{-1}$

$IAI = P.D.P^{-1}$

$\boxed{A = PDP^{-1}}$

ie $PDP^{-1} = A$

Note ② N. Cond^n for Diagonalisation:

$\boxed{\text{Number of L.I E.Vectors = order of } A}$

ie Modal Mat must be Non sing.

or $|P| \neq 0$.

③ If E.vector are LD then Modal Mat becomes singular & Diagonalisation is not possible.

Eg: Verify the process of Diagonalisation for $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

PODCAST

$P^{-1}AP = D$

we.k.that $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ → $\lambda_1 = 2,\ X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$; $\lambda_2 = 6,\ X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Consider Modal Mat $P = [X_1\ X_2] = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$ & let $D = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$ = Diag Mat

$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow M^{-1} = \frac{1}{|M|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ So $P^{-1} = \frac{1}{(-2)}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

Now, $P^{-1}AP = \frac{1}{(-2)}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \ldots\ldots = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix} = D$

Use: Calculating $PDP^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}\frac{1}{(-2)}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} = \ldots\ldots = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = A$

Q. If $A_{2\times 2}$ s.t. $A\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}-2\\2\end{bmatrix}$ & $A\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}6\\6\end{bmatrix}$ then $A=?$

(a) $\begin{bmatrix}-1 & 1\\1 & 1\end{bmatrix}$

(b) $\begin{bmatrix}4 & 2\\2 & 4\end{bmatrix}$ ✓

(c) $\begin{bmatrix}-2 & 6\\2 & 6\end{bmatrix}$

(d) $\begin{bmatrix}2 & 0\\0 & 6\end{bmatrix}$

$A\begin{bmatrix}-1\\1\end{bmatrix}=2\begin{bmatrix}-1\\1\end{bmatrix}$ & $A\begin{bmatrix}1\\1\end{bmatrix}=6\begin{bmatrix}1\\1\end{bmatrix}$

$\lambda_1=2,\ X_1=\begin{bmatrix}-1\\1\end{bmatrix}$ & $\lambda_2=6,\ X_2=\begin{bmatrix}1\\1\end{bmatrix}$

$D=\begin{bmatrix}2 & 0\\0 & 6\end{bmatrix},\ P=[X_1, X_2]=\begin{bmatrix}-1 & 1\\1 & 1\end{bmatrix},\ P^{-1}=\frac{1}{(-2)}\begin{bmatrix}1 & -1\\-1 & -1\end{bmatrix}$

Now using the concept of Diagonalisation,

$P^{-1}AP=D \Longrightarrow A=PDP^{-1}=[P][D][P^{-1}]=\dots\dots=\begin{bmatrix}4 & 2\\2 & 4\end{bmatrix}$

Q: If $A_{2\times 2}$ s.t. $A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$ & $A\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$ then find $A=?$

(a) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$

(c) $\begin{bmatrix}1 & 4\\0 & 2\end{bmatrix}$

(d) $\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}$ ✓

$A\begin{bmatrix}1\\0\end{bmatrix}=1\begin{bmatrix}1\\0\end{bmatrix}$ & $A\begin{bmatrix}2\\1\end{bmatrix}=2\begin{bmatrix}2\\1\end{bmatrix}$

$\lambda_1=1, X_1=\begin{bmatrix}1\\0\end{bmatrix}$ & $\lambda_2=2, X_2=\begin{bmatrix}2\\1\end{bmatrix}$

$D=\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$, $P=[X_1\, X_2]=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$, $P^{-1}=\frac{1}{(1)}\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}$

Now $P^{-1}AP=D \Rightarrow A=PDP^{-1}$

$=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}=\ldots\ldots=\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}$

The number of linearly independent eigen vectors

of the matrix $\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is $= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$

(a) 1 (b) 2

(c) 3 (d) 4

$Tr(A) = 10, \quad |A| = |A_1| \cdot |A_2|$

$= (-2)(12) = -24$

EValues of $A_2 = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ $\lambda = 3$, $\lambda = 4$

EValues of $A_1 = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix}$ $\lambda = \frac{3+\sqrt{17}}{2}$, $\lambda = \frac{3-\sqrt{17}}{2}$

$A_1 = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix}$

ie A has 4 different EValues

$\Rightarrow$ " " 4 LI EVectors = order (so Diag possible.

C Equ^n is $\lambda^2 - (Tr A_1)\lambda + |A_1| = 0$

$\lambda^2 - (3)\lambda + (-2) = 0 \Rightarrow \lambda = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2} = \frac{3 \pm \sqrt{17}}{2}$

M-II C.Eq of A is $|A-\lambda I| = 0$.

$$\begin{vmatrix} (2-\lambda) & 2 & 0 & 0 \\ 2 & (1-\lambda) & 0 & 0 \\ 0 & 0 & (3-\lambda) & 0 \\ 0 & 0 & 0 & (4-\lambda) \end{vmatrix} = 0$$

$$(4-\lambda)\begin{vmatrix} (2-\lambda) & 2 & 0 \\ 2 & (1-\lambda) & 0 \\ 0 & 0 & (3-\lambda) \end{vmatrix} = 0$$

$$(4-\lambda)(3-\lambda)\begin{vmatrix} (2-\lambda) & 2 \\ 2 & (1-\lambda) \end{vmatrix} = 0$$

$$(\lambda-4)(\lambda-3)\left[(2-\lambda)(1-\lambda)-4\right] = 0$$

$$(\lambda-4)(\lambda-3)\left[2-2\lambda-\lambda+\lambda^2-4\right] = 0$$

$$(\lambda-4)(\lambda-3)\left[\lambda^2-3\lambda-2\right] = 0$$

$$\lambda = 3, 4, \text{ \& } \lambda = \frac{3 \pm \sqrt{17}}{2}$$

L-U Decomposition

If we want to factorize any square Mat into the product of L.T.M & U.T.M then we have following Methods;

① Dolittle Method

A = (Unit L.T.M)(U.T.M)

$$A_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

② CROUT Method

A = (L.T.M).(unit UTM)

$$A_{3\times3} = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

③ Cholesky Method

$$A = L L^{\theta}$$

or

$$A = L L^{T}$$

where L = L.T.M

Application: (we can solve equally determined Non Homog system) $\rightarrow$

Given system $\boxed{A_{n\times n} X_{n\times 1} = B_{n\times 1}}$ — (1) $\Rightarrow X = \begin{bmatrix} - \\ - \\ \vdots \\ - \end{bmatrix} = ?$

$$(LU)X = B$$

$$L(UX) = B$$

$$LY = B \longrightarrow (2)$$

where $UX = Y \longrightarrow (3)$

Now By (2), $L_{n\times n} Y_{n\times 1} = B_{n\times 1}$

$$\begin{bmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & \cdots & & 0 \\ \vdots & & & & \\ l_{n1} & l_{n2} & \cdots & & l_{nn} \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{bmatrix} - \\ - \\ \vdots \\ - \end{bmatrix}_{n\times 1}$$

Now using Forward substitution Method.

we can calculate $Y = \begin{bmatrix} - \\ - \\ \vdots \\ - \end{bmatrix}$

P.T.O.

Now using ③, $UX = Y$

$$\begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ 0 & u_{22} & \cdots & u_{2n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & u_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Now using Backward substitution Method we can calculate

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \text{Ans}$$

Q: If $A = \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}$ is to be decomposed into the product of L.T.M & U.T.M then properly decomposed L & U Matrices are ?

(a) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 2 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}$

(d) None

M-I Using Dolittle Method $\rightarrow$

write A = (Unit LTM) (U.T.M)

$$A = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} b & c \\ 0 & d \end{bmatrix} \longrightarrow (1)$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix} = \begin{bmatrix} b & c \\ ab & ac+d \end{bmatrix}$$

$b = 1$, $c = 2$, $ab = 2 \Rightarrow a = 2$

$ac + d = 13$

$(2)(2) + d = 13 \Rightarrow d = 9$

So $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 9 \end{bmatrix}$

= (a)

M-II Using CROUT's Method →

$$A = (L.T.M)(\text{unit } U.T.M)$$

$$A = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \quad \text{--- (1)}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix} = \begin{bmatrix} a & ad \\ b & bd+c \end{bmatrix}$$

$a=1$, $b=2$, $ad=2$, $bd+c=13$

$\Downarrow$

$d=2$, $(2)(2)+c=13$

$c=9$

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = (b)$$

M-III using CHOLSKEY Method

write $A = LL^{\theta}$ or LL^{T}

$$A = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \quad \text{--- (1)}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix} = \begin{bmatrix} a^2 & ab \\ ab & b^2+c^2 \end{bmatrix}$$

$a^2=1$, $ab=2$, $b^2+c^2=13$

$a=1$, $b=2$; $4+c^2=13 \Rightarrow c^2=9$

$c=3$

By (1), $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ (c)

The matrix $[A] = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of a lower triangular matrix x[L] and an upper triangular matrix [U]. The properly decomposed [L] and [U] matrices respectively are

(a) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ ✓

In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is ______.

Using CROUT Method

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} \\ l_{21} & l_{21}u_{12}+l_{22} \end{bmatrix}$$

$$\Rightarrow l_{11} = 2,\ l_{21} = 4,\ l_{11}u_{12} = 2 \Rightarrow u_{12} = 1$$

$$l_{21}u_{12} + l_{22} = 9$$

$$(4)(1) + l_{22} = 9$$

$$l_{22} = 5 \quad \text{Ans}$$

Reflection Matrix → with angle $\theta = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(i) About X axis $(\theta = 0)$

$Rf\,M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(ii) About Y axis $(\theta = 90^\circ)$

$Rf\,M = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

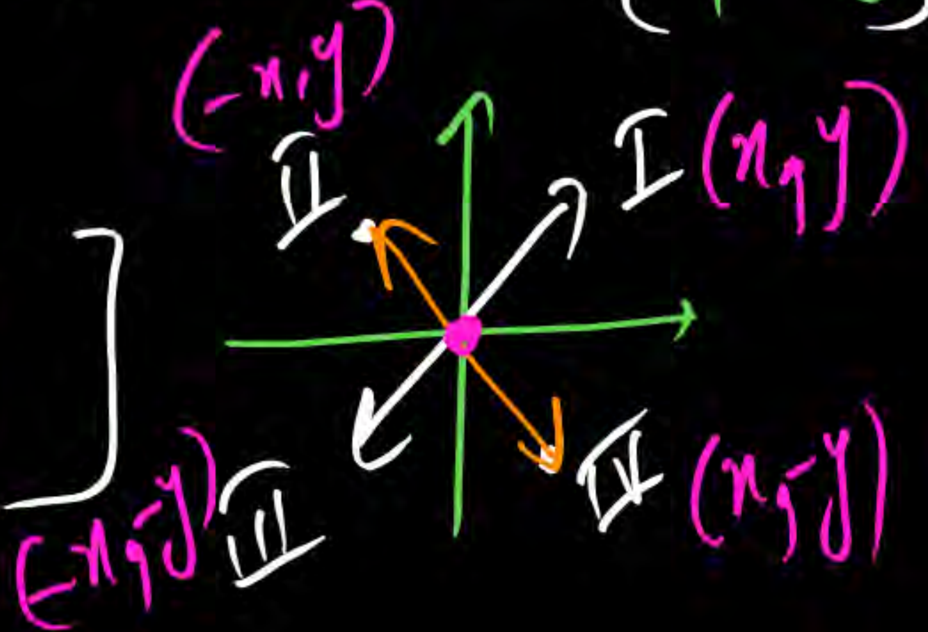
(iii) About $y = x$ $(\theta = 45^\circ)$

$Rf\,M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

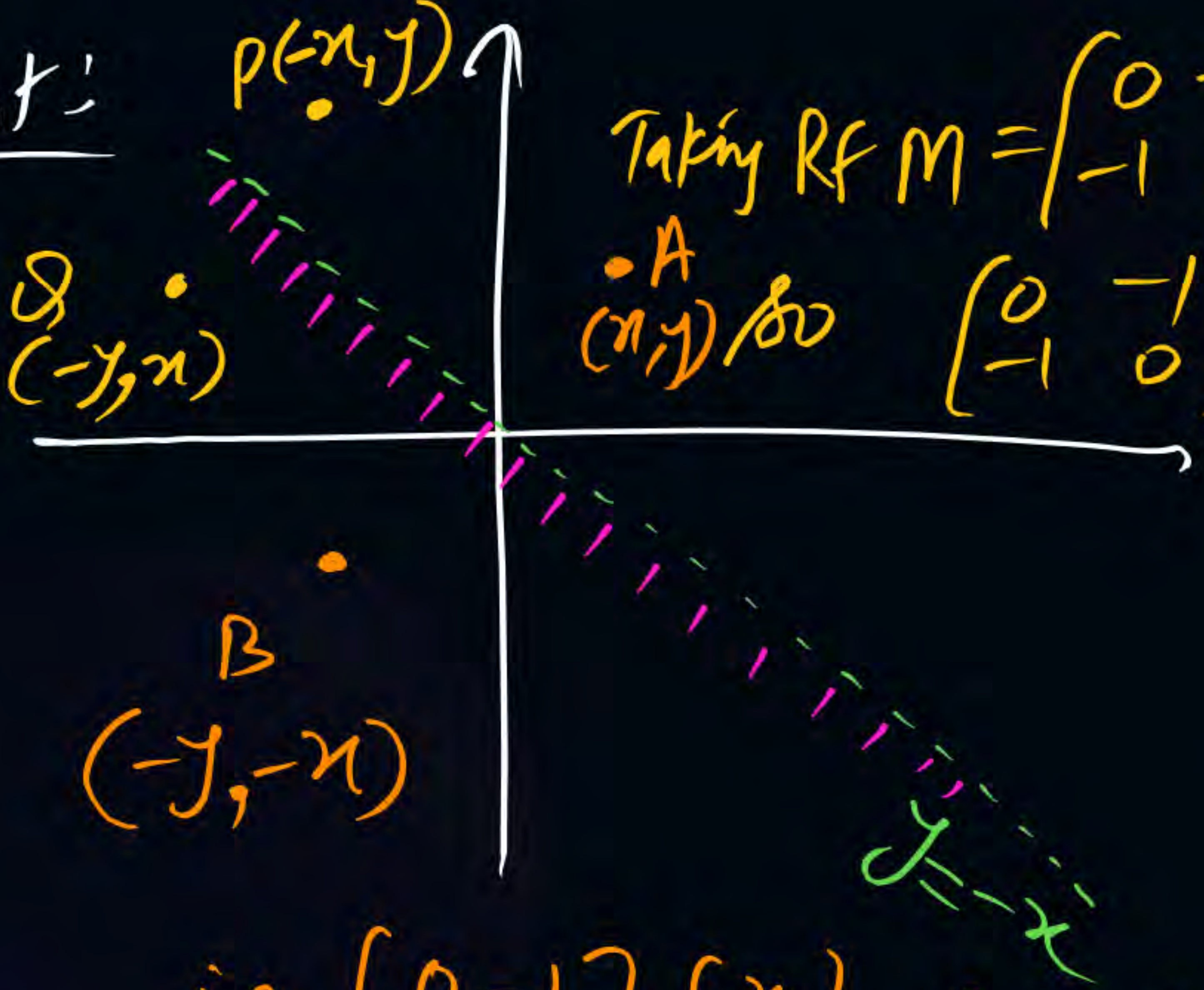
(iv) About $y = -x$ $(\theta = -45^\circ)$

$Rf\,M = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(v) Special Case : (About origin) → $Rf\,M = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$



4th Point:

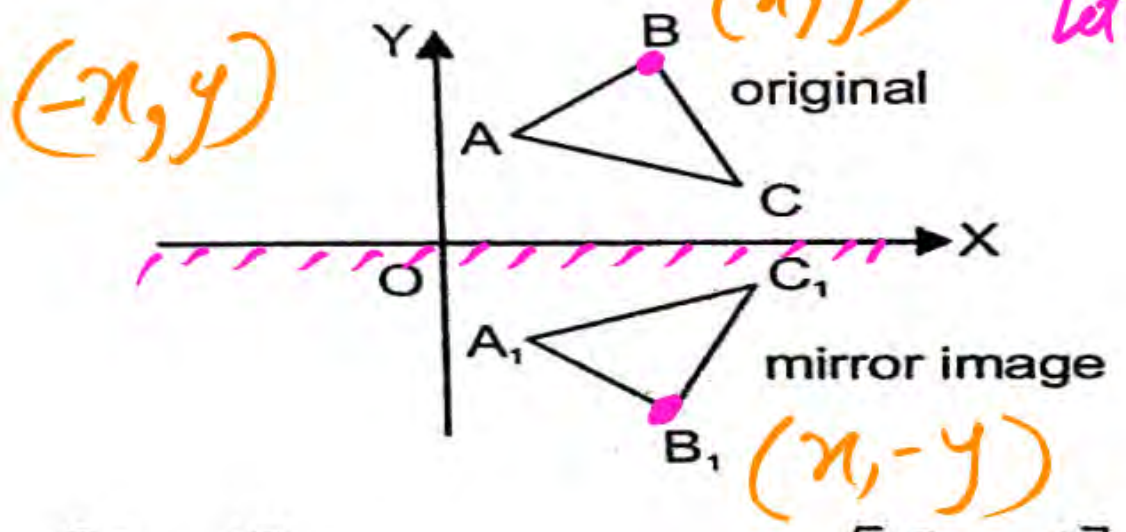


Taking Rf $M = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

so $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix} = Q$ ✓

ie $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix} = B$ ✓

The figure shows a shape ABC and its mirror image $A_1B_1C_1$ across the horizontal axis (X –axis). The coordinate transformation matrix that maps ABC to $A_1B_1C_1$ is



let $B = \begin{bmatrix} x \\ y \end{bmatrix}$

$B_1 = \begin{bmatrix} x \\ -y \end{bmatrix}$

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} \neq B_1$

(b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix} \neq B_1$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix} \neq B_1$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix} = B_1$ ✓

Note (c) is giving Mirror Image about Y axis

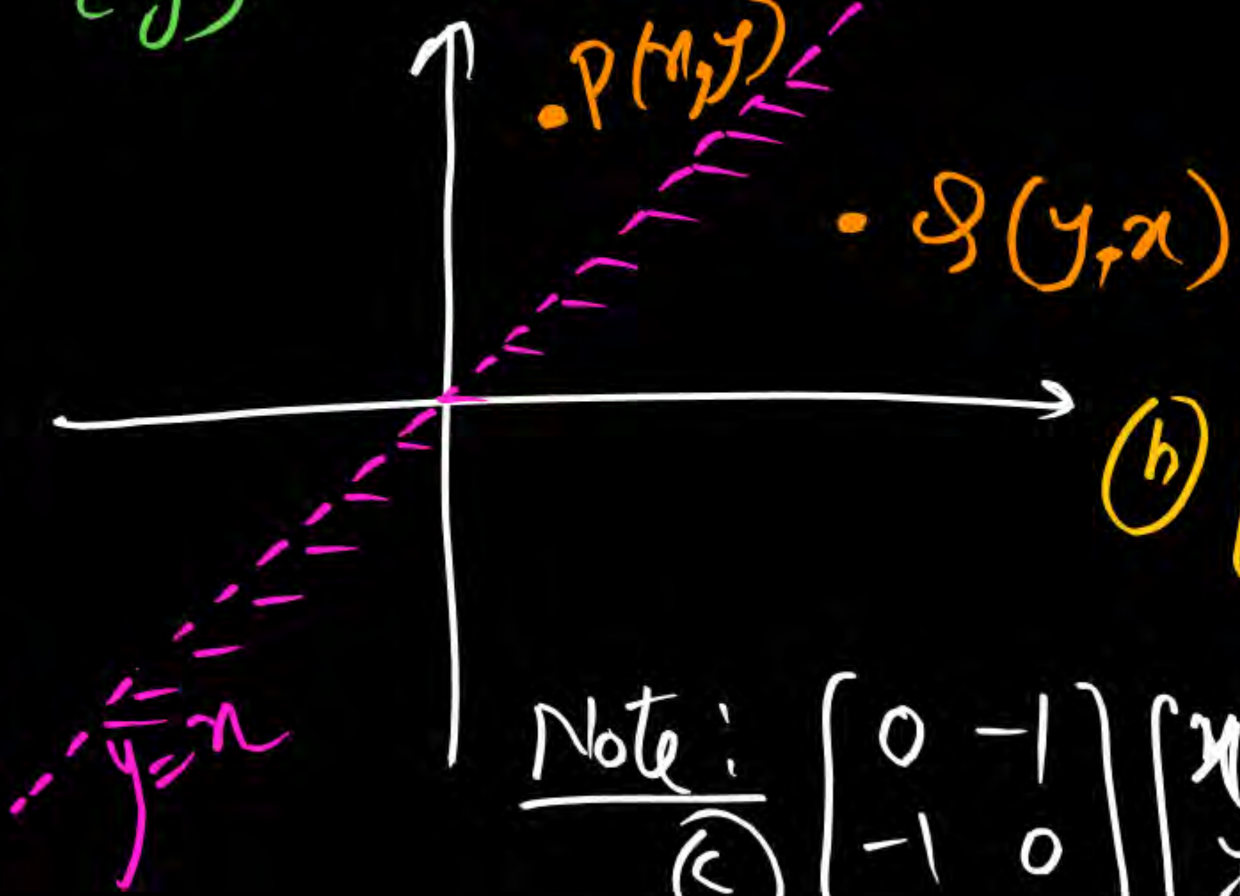
Q. The transformation Matrix to find mirror image of any point $P\begin{bmatrix}x\\y\end{bmatrix}$ in x-y plane about the line $y=x$ is ?

(a) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

(b) $\begin{bmatrix}0&1\\1&0\end{bmatrix}$ ✓

(c) $\begin{bmatrix}0&-1\\-1&0\end{bmatrix}$

(d) $\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$



(a) $\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}\neq Q$

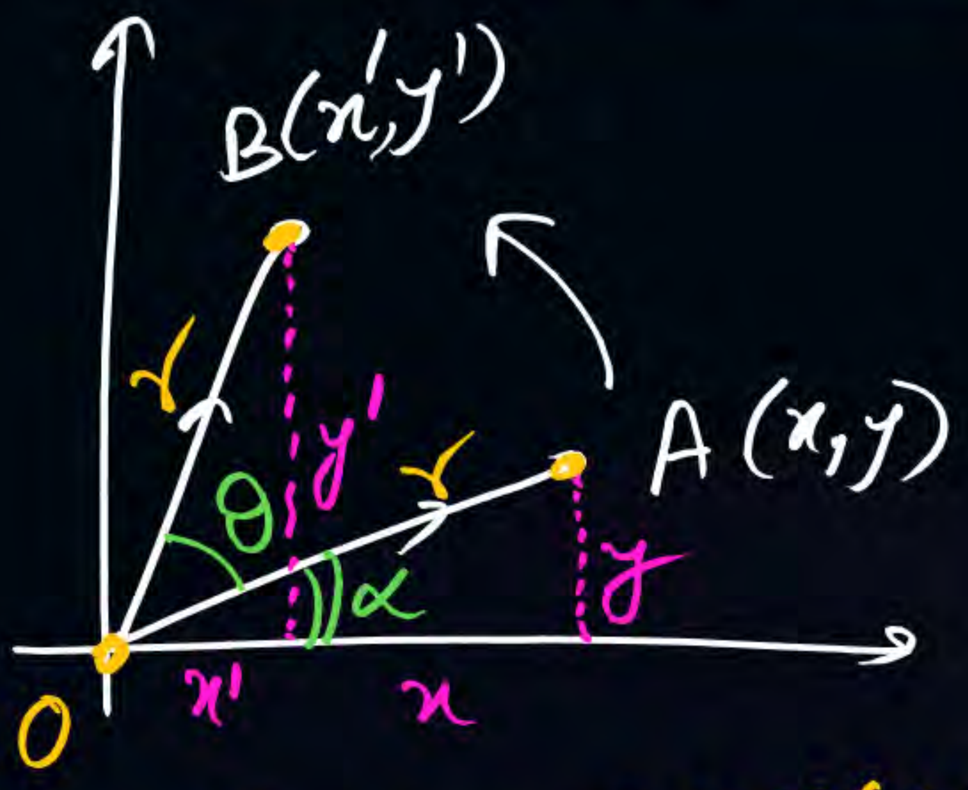
(b) $\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}y\\x\end{bmatrix}=Q$ ✓

Note: (c) $\begin{bmatrix}0&-1\\-1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-y\\-x\end{bmatrix}\neq Q$.

(d) $\begin{bmatrix}-1&0\\0&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-x\\-y\end{bmatrix}$ i.e (d) is About origin

ROTATION MATRIX

(in 2D in ACW sense) $\longrightarrow$



Similarly, $x' = r\cos(\alpha+\theta) = r[\cos\alpha\cos\theta - \sin\alpha\sin\theta]$

$y' = r\sin(\alpha+\theta) = r[\sin\alpha\cos\theta + \cos\alpha\sin\theta]$

$\frac{x}{r} = \cos\alpha \Rightarrow x = r\cos\alpha$

$\frac{y}{r} = \sin\alpha \Rightarrow y = r\sin\alpha$

where $|\vec{OB}| = |\vec{OA}| = r$

ie $x' = x\cos\theta - y\sin\theta = x\cos\theta - y\sin\theta$

$y' = y\cos\theta + x\sin\theta = x\sin\theta + y\cos\theta$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = R_{ACW}\begin{bmatrix} x \\ y \end{bmatrix}$$

ie Rot. Mat in 2D in ACW sense $= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Note: If vector is Rotated by angle θ in C.W sense then Replace $\theta \to -\theta$ in Rot Mat.

i.e. $R = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ & $\begin{pmatrix} x' \\ y' \end{pmatrix} = R_{CW} \begin{pmatrix} x \\ y \end{pmatrix}$

(*) Rotation Mat in 3D: → (in A.C.W sense by an angle θ) ⟶

Case I → (If vector is Rotating) →

(1) About X axis

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_x \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(2) About Y axis

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_y \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(3) About Z axis

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_z \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

② If Coordinate system is Rotating → (in ACW sense). Take the Transpose of Case I.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}, \quad R_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, \quad R_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Analysis: → ① Rotation Matrices are orthogonal matrix

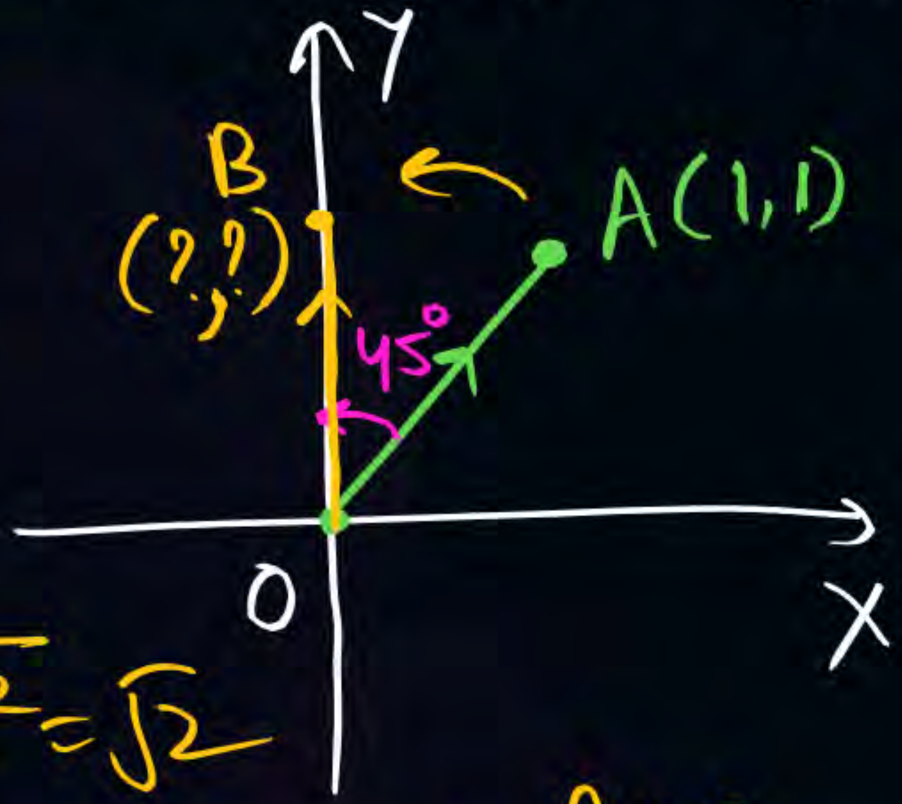
② $\mathrm{Tr}(\text{Rot Mat}) = ? = \boxed{1 + 2\cos\theta}$

Q- If the vector $\begin{bmatrix}1\\1\end{bmatrix}$ is to be rotated by 45° in an ACW sense then what are the coordinates of new vector?

(a) $\begin{bmatrix}1\\1\end{bmatrix}$ (b) $\begin{bmatrix}0\\1\end{bmatrix}$

(c) $\begin{bmatrix}0\\2\end{bmatrix}$ (d) ✓ $\begin{bmatrix}0\\\sqrt{2}\end{bmatrix}$

M-I $A = \begin{bmatrix}1\\1\end{bmatrix}$, $B = \begin{bmatrix}?\\?\end{bmatrix}$



$|\vec{OA}| = \sqrt{1^2+1^2} = \sqrt{2}$

$|\vec{OB}| = |\vec{OA}| = \sqrt{2} \Rightarrow \vec{OB} = \begin{bmatrix}0\\\sqrt{2}\end{bmatrix}$

M-II $R_{ACW} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

Here $\theta = 45°$ so $\vec{OB} = R_{ACW}\cdot\vec{OA}$

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}\cos 45° & -\sin 45°\\ \sin 45° & \cos 45°\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{bmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \begin{bmatrix}0\\2/\sqrt{2}\end{bmatrix} = \begin{bmatrix}0\\\sqrt{2}\end{bmatrix}$$ //

Q. If the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is to be rotated by 30° in ACW about x axis then find new vector?

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_x \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \left(\frac{\sqrt{3}-1}{2}\right) \\ \left(\frac{\sqrt{3}+1}{2}\right) \end{bmatrix}$$

The Cartesian coordinates of a point P in a right-handed coordinate system are (1, 1, 1). The transformed coordinates of P due to a 45° clockwise rotation of the coordinate system about the positive x-axis are

(a) $(1, 0, \sqrt{2})$ (b) $(1, 0, -\sqrt{2})$

(c) $(-1, 0, \sqrt{2})$ (d) $(-1, 0, -\sqrt{2})$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_x \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

CW

Case I (ACW about X axis)

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Case II (ACW about X axis)

$$\xrightarrow{\text{Taking Transpose}} R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \xrightarrow{\theta \to -\theta} R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$\theta = 45^\circ$

THANK - YOU

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