DS & AI
CS & IT

Probability & Statistics - I

Probability

Lecture No. 04

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

BASICS OF PROBABILITY

(Part-2)

Topics to be Covered

Topic

BASICS of PROBABILITY

(Part-3)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

Short RECAP

Operation	P&C	Prob	Formula	ME	Ind.
Either or	Plus	union	Addition Th	$P(A \cup B) = P(A) + P(B)$	*
AND	Multiply	Intersection	Multi Th	$P(A \cap B) = 0$	$P(A \cap B) = P(A) \cdot P(B)$

Addition Th: $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

* for Independency: $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

for ME: $P(A \cup B) = P(A) + P(B) - 0$

Q8 A coin is tossed 10 times then find the prob that

(1) exactly 3 H will occur ?

(2) 4^{th} Head will occur in 9^{th} toss ? $= \frac{7}{64}$

HW

Total cases in S-Space $= 2\times2\times2\times\cdots\times2 = 2^{10} = 1024$ tuples.

(1) fav. tuples = { exactly 3H will occur } = { eg (HTTHTTTTTH) ... } $= \frac{10!}{7!\,3!} = {}^{10}C_3$

So Req Prob $= \frac{\text{fav}}{\text{Total}} = \frac{{}^{10}C_3}{2^{10}} = \frac{120}{1024}$

App II: fav cases $= \{4^{th}$ H will occur in 9^{th} toss$\}$

$$= \{ \text{eg} \big(\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{8 \text{ tosses}}^{\text{exactly } 3H} \ \underset{9^{th}}{\underline{H}} \ \underset{10^{th}}{\underline{H/T}} \big) \ldots \ldots \} = {}^8C_3 \times 1 \times 2 \text{ ways}$$

$$\text{So Req prob} = \frac{\text{fav}}{\text{Total}} = \frac{{}^8C_3 \times 1 \times 2}{2^{10}} = \left(\frac{{}^8C_3}{2^8}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{2}{2}\right)$$

App III: P(4^{th} H in 9^{th} toss) = P(getting exactly 3H in 8 tosses) $\times$ P(H in 9^{th} toss) $\times$ P(something occurs in 10^{th} toss)

$$= \left(\frac{{}^8C_3}{2^8}\right) \times \left(\frac{1}{2}\right) \times (1) = \frac{112}{1024} = \frac{7}{64}$$

$$= \frac{\left(\frac{8!}{3!5!}\right)}{2^8} \times \frac{1}{2} \times 1$$

A bag contains 4 black balls and 3 white balls. Balls are picked up one by one at random and placed in row. The chance that the balls are alternatively of different colours is ____.

w/o Replacement

[Ans:]

(a) $\frac{1}{7}$ (b) $\dfrac{\frac{7!}{4!3!}}{7^7}$ (c) $\frac{1}{35}$ (d) None

Arrangement. (RNA)

Case I : B W B W B W B (✓)

Case II : W B W B W B B → Not possible

$\underline{B\,B}$?

4B
3W

Req Prob = P[BWBWBWB]

$= \frac{\cancel{4}}{7} \cdot \frac{\cancel{3}}{\cancel{6}} \cdot \frac{\cancel{3}}{5} \cdot \frac{\cancel{2}}{\cancel{4}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{1} = \frac{1}{35}$

(M-I) Total arrangements $= \frac{7!}{4!3!}$

fav. arrangement = 1

So Prob $= \frac{\text{fav}}{\text{Total}} = \dfrac{1}{\frac{7!}{4!3!}} = \frac{4!3!}{7!}$

Independent Events → If occurrence or Non occurrence of one event does not alter the occurrence or Non occurrence of other Event then Events are Called Independent events

$\Longleftrightarrow$

Mathematically: If A & B are Ind Events then $\Longleftrightarrow$ $\boxed{P(A\cap B) = P(A)\cdot P(B)}$

eg; $S_{Coin} = \{H, T\}$, $S_{Die} = \{1, 2, 3, 4, 5, 6\}$

$A = \{H\} \Rightarrow P(A) = \frac{1}{2}$, $B = \{1, 2, 3, 4\} \Rightarrow P(B) = \frac{4}{6}$

then $\underbrace{P(A\cap B) = P(A)\cdot P(B)}_{\because \text{ A \& B are Ind. Events}} = \frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$

RECAP

A bag contains 5 red and 7 black balls and a second contains 4 blue and 3 green balls. A ball is taken out from each bag. Find the probability that

(i) one ball is red and other blue (a) $\frac{5}{21}$ (b) $\frac{10}{21}$ (c) $\frac{1}{84}$ (d) $\frac{1}{20}$

(ii) one ball is black and other green (a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{1}{21}$ (d) $\frac{1}{4}$

I: 5R, 7 Black

II: 4 Blue, 3G

Both the Bags are Ind

(1) $P(R \cap \text{Blue}) = \frac{5}{12} \times \frac{4}{7} = \frac{5}{21}$

(2) $P(\text{Black} \cap \text{Green}) = \frac{7}{12} \times \frac{3}{7} = \frac{1}{4}$

unfair

A loaded dice has following probability distribution of occurrences

Dice Value	1	2	3	4	5	6
Probability	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$

If three identical dice as the above are thrown, the probability of occurrence of values 1, 5 and 6 on the three dice is

(a) Same as that of occurrence of 3, 4, 5

(b) Same as that of occurrence of 1, 2, 5

(c) 1/128

(d) 5/8

[redacted] [redacted]

All three dice are Independent.

Now $P(1 \cap 5 \cap 6) = \frac{1}{4} \times \frac{1}{8} \times \frac{1}{4} = \frac{1}{128}$ — (1)

(a) $P(3 \cap 4 \cap 5) = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} \neq \frac{1}{128}$ ✗

(b) $P(1 \cap 2 \cap 5) = \frac{1}{4} \times \frac{1}{8} \times \frac{1}{8} \neq \frac{1}{128}$ ✗

A fair dice is rolled twice. The probability that an odd number will follow an even number is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

App II

$P(\text{Even} \cap \text{odd}) = ?$

Both the R-Exp are Ind.

$= P(\text{E on } 1^{st}) \times P(\text{O on } 2^{nd})$

$= \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

App I $S = \{(11)(12) \ldots (16)(21) \ldots (26) \ldots (66)\} = 36$ pair

fav $= \{(21)(23)(25)(61)(63)(65)(41)(43)(45)\} = 9$ pair So Req Prob $= \frac{9}{36} = \frac{1}{4}$

X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^c) = 0.7$. Which one of the following is the value of $P(X \cup Y)$?

(a) 0.7
(b) 0.5
(c) 0.4
(d) 0.3

$$P(X\cup Y) = P(X) + P(Y) - P(X\cap Y)$$
$$= 0.4 + 0.5 - (0.4)(0.5)$$
$$= 0.9 - 0.20 = 0.7$$

$$P(X\cup \bar{Y}) = 0.7$$
$$P(X) + P(\bar{Y}) - P(X\cap \bar{Y}) = 0.7$$
$$0.4 + (1 - P(Y)) - P(X).P(\bar{Y}) = 0.7$$
$$-P(Y) - P(X)P(\bar{Y}) = -0.7$$
$$-P(Y) - P(X)[1 - P(Y)] = -0.7$$
$$-P(Y) - 0.4 + 0.4P(Y) = -0.7$$
$$(-1 + 0.4)P(Y) = -0.7 + 0.4$$
$$-0.6P(Y) = -0.3 \Rightarrow P(Y) = \frac{1}{2}$$

M-II

$X = 0.4$

$\bar{X} = 0.6$

$Y = ?$

$\bar{Y} = ?$

$X \cup \bar{Y} = 0.7$

$X \cap \bar{Y} + \bar{X} \cap \bar{Y} = 0.7$

will be discussed Later

Mutually Exclusive Events → RECAP

If, two events $\boxed{\text{can't occur simultaneously}}$, then these are called M.E. Events

OR

If occurrence of one event prevents the occurrence of other event & vice versa than events are called ME Events. ie

If A & B are ME then $\boxed{\text{only one can occur at a time}}$

Mathematically: if E_1 & E_2 are ME events then $\boxed{E_1 \cap E_2 = \phi}$

Conclusion: if E_1 & E_2 are ME then

$\to P(E_1 \cap E_2) = 0$

$\to P(E_1 \cup E_2) = P(E_1) + P(E_2) - 0$

eg $S_D = \{1, 2, 3, 4, 5, 6\}$ & let us consider following events

$E_1 = \{1, 3, 5\}$; $\because E_1 \cap E_2 = \phi \Rightarrow E_1$ & E_2 are M.E & $P(E_1 \cap E_2) = 0$

$E_2 = \{2, 4, 6\}$; $\because E_2 \cap E_3 \neq \phi \Rightarrow E_1$ & E_2 are Not M.E

$E_3 = \{1, 2, 3, 4\}$; $\because E_1 \cap E_3 \neq \phi \Rightarrow E_1$ & E_3 are Not ME

$E_4 = \{2, 4\}$, $\because E_1 \cap E_4 = \phi \Rightarrow E_1$ & E_4 are also ME But $E_1 \cup E_4 \neq S$

i.e it is not Necessary that, in Case of ME Events, you will get their union as S. Space

$E_4 = \{x : 1 < x < 5$ & x is divisible by $2\}$

RECAP

Assertion (A) : The probability of drawing either an ace or a king from a deck of card in a single draw is $\frac{2}{13}$ True $\because A \cap K = \phi \Rightarrow A$ & K are M.E.

Reason (R) : For two events E_1 and E_2 which are not mutually exclusive, the probability is given by

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$ True (By addition Th)

(a) A and R are true, R is the correct explanation of A

(b) A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

$$P(A \cup K) = P(A) + P(K) - P(A \cap K)$$
$$= \frac{4}{52} + \frac{4}{52} - \frac{0}{52}$$
$$= \frac{2}{13}$$

where $P(Ace) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52}$

$P(King) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52}$

Assertion (A) : The probability of drawing either an ace or a king from a deck of card in a single draw is $\frac{2}{13}$. True

A K 52 Cards

Reason (R) : For two events E_1 and E_2 which are not mutually exclusive, the probability is given by

$$P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$ (True)

(a) A and R are true, R is the correct explanation of A

(b) A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

$A = \{Ace\} \Rightarrow P(A) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$K = \{king\} \Rightarrow P(K) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

$\because A \cap K = \phi \Rightarrow P(A \cap K) = 0$

$P(A \cup K) = P(A) + P(K) - 0$

$= \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$

Two dice are tossed. One dice is regular and the other is biased $_{= \text{unfair}}$ with probabilities $P(1) = P(6) = 1/6$, $P(2) = P(4) = 0$ and $P(3) = P(5) = 1/3$. The probability of obtaining a sum of 4 is

(a) 1/9 (b) 1/12 (c) 1/18 (d) 1/24

Don't Try to apply Shortcut in this Question bcoz 2nd Die is Not Regular

(umbrella wala)

for Regular die, $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

$$P(\text{sum is } 4) = P\left[\overset{R\ B}{(1,3)} \text{ or } \overset{R\ B}{(3,1)} \text{ or } \overset{R\ B}{(2,2)}\right] \rightarrow \text{ME.}$$

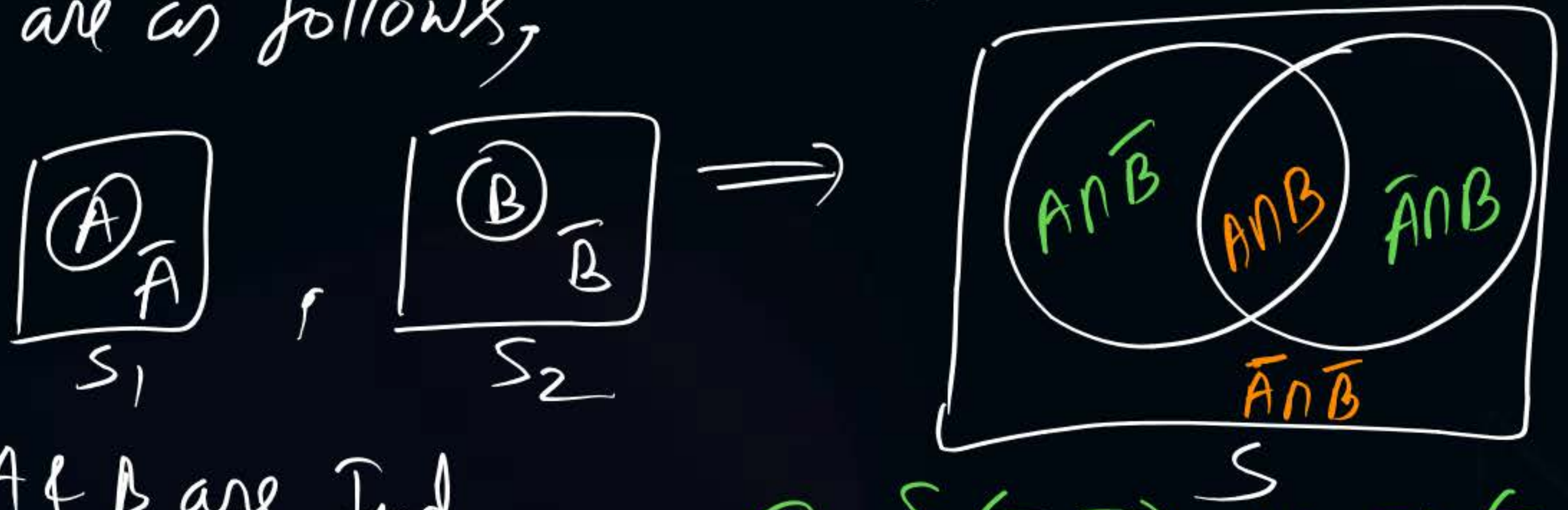
$$= \frac{1}{6} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times 0$$

$$= \frac{1}{18} + \frac{1}{36} + 0 = \frac{2+1}{36} = \frac{3}{36} = \frac{1}{12}$$

$\because$ Both die are Ind so we can Multi also.

Concept of ME and Independency in a Single Question →

Consider A & B are firing at the Target once then various possibilities are as follows,



A & B are Ind

A & $\bar{B}$ " "

$\bar{A}$ & B " "

$\bar{A}$ & $\bar{B}$ " "

$$S = \{ \underbrace{(\bar{A} \cap \bar{B})}_{=E_1}, \underbrace{(A \cap \bar{B})}_{=E_2}, \underbrace{(\bar{A} \cap B)}_{=E_3}, \underbrace{(A \cap B)}_{=E_4} \}$$

$\because$ E_1, E_2, E_3, E_4 are Individual elements of this New S. Space

So these 4 events are ME.

Analysis: Various possibilities are ; $S = \{\bar{A}\cap\bar{B}, A\cap\bar{B}, \bar{A}\cap B, A\cap B\}$

(None will hit) or (A hit & B missed) or (A missed & B hit) or (Both hit) = Total possibilities

$$(\bar{A}\cap\bar{B}) \cup \underbrace{(A\cap\bar{B}) \cup (\bar{A}\cap B)}_{\text{only one will hit / Exactly one will hit.}} \cup (A\cap B) = S$$

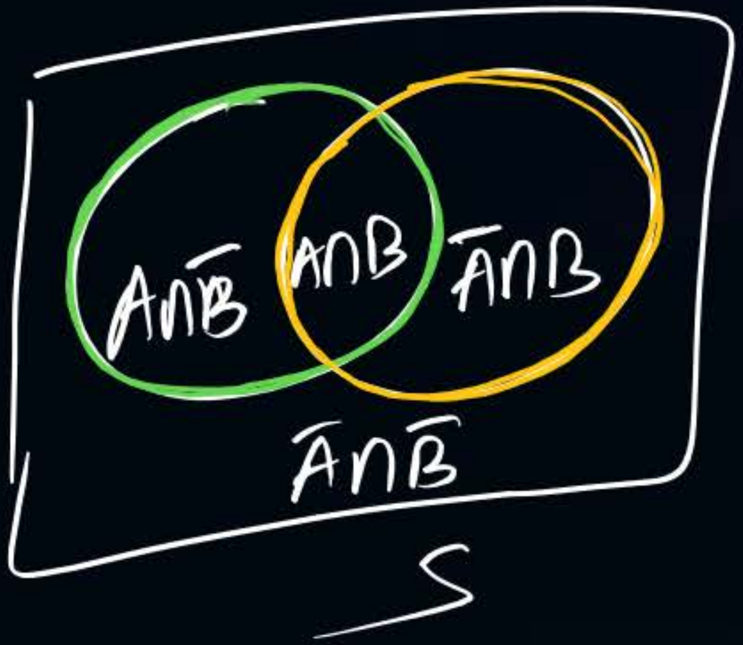
$$\underbrace{(A\cap\bar{B}) \cup (\bar{A}\cap B) \cup (A\cap B)}_{\text{At least one will hit.}}$$

$E_1 \cup E_2 \cup E_3 \cup E_4 = S \Rightarrow P(E_1 \cup E_2 \cup E_3 \cup E_4) = P(S)$

$\because E_1, E_2, E_3, E_4$ are (ME) so $P(E_1)+P(E_2)+P(E_3)+P(E_4) = 1$

i.e $P(\text{something occurs}) = 1$

*



$A\cap\bar{B} = A - A\cap B$ = only A

$\bar{A}\cap B = B - A\cap B$ = only B

$\bar{A}\cap\bar{B}$ = Neither A Nor B

= None of A & B

$$\underbrace{A\cup B}_{\text{Either A or B or Both}}$$

(at least one of A or B)

- $(A\cap\bar{B}) + (\bar{A}\cap B) + (A\cap B)$
- $S - (\bar{A}\cap\bar{B})$
- $A + B - A\cap B$ ✓

F.Q3 →

There are two Gangsters Munna Mobile (A) & Pappu Pazer (B), They both fire at the target once with probability of their hitting is $\frac{4}{5}$ & $\frac{3}{4}$ resp.

Sol: $P(A)=\frac{4}{5}$, $P(B)=\frac{3}{4}$, A & B are Ind & $S=\{\underbrace{(\bar{A}\cap\bar{B})}_{=E_1}, \underbrace{(A\cap\bar{B})}_{=E_2}, \underbrace{(\bar{A}\cap B)}_{=E_3}, \underbrace{(A\cap B)}_{=E_4}\}$

$P(\bar{A})=\frac{1}{5}$, $P(\bar{B})=\frac{1}{4}$,

(1) Find the prob that Both will hit = ? $= P(E_4) = P(A\cap B) = \frac{4}{5}\times\frac{3}{4} = \frac{3}{5}$

(2) " " " " None will hit = ? $= P(E_1) = P(\bar{A}\cap\bar{B}) = \frac{1}{5}\times\frac{1}{4} = \frac{1}{20}$

(3) " " " " Atleast one will hit = ? $= 1 - P(\text{None will hit}) = 1 - \frac{1}{20} = \frac{19}{20}$

(4) " " " " target will be hit = ? $= P(\text{at least one will hit}) = \frac{19}{20}$.

(5) Find the prob that $\underline{\text{either of them will hit}} = ? = P(A \cup B)$

(either A or B or Both)

$= P(A) + P(B) - P(A \cap B)$

$= \frac{4}{5} + \frac{3}{4} - \frac{3}{5} = \frac{19}{20}$ (same as (4))

(6) find the prob that $\underbrace{\text{A hit \& B missed}}_{\text{only A will hit}} = ? = P(E_2) = P(A \cap \bar{B}) = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$

(7) Find the prob that $\underbrace{\text{only one will hit}}_{\text{(only A or only B)}} = ? = P(E_2 \cup E_3)$ $\{\because E_2 \,\&\, E_3 \text{ are ME}\}$

$E_2 \cup E_3$

$= P(E_2) + P(E_3) - 0$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$ $\{\because A \,\&\, B \text{ are Ind}\}$

(8) If $\underbrace{\text{exactly one person hit}}_{\text{Condition}}$ then Find the prob that A hit & B missed = ?

Have patience.

SET THEORY

Q out of 1000 integers from 1 to 1000 (Both Inclusive). How many integers are

(1) divisible by 3 or 5 or 7 = 543

(2) " " 3 4 5 & 7 Ans = 9

(3) " " Neither 3, nor 5 nor 7 (= 457)

(4) " " only one of them (229 + 76 + 115 = 420)

(5) " " 3 & 5 but Not by 7 = 57

only(7) = 142 − (38 + 9 + 19) = 76

$n(3 \cup 5 \cup 7) = 229 + 76 + 115 + (57 + 38 + 19) + 9$
$= 543$

$n(\bar{3} \cap \bar{5} \cap \bar{7}) = 1000 - 543 = 457$

$n(3) = \frac{1000}{3} = 333$

$n(5) = \frac{1000}{5} = 200$

$n(7) = \frac{1000}{7} = 142$

$n(3 \cap 5) = \frac{1000}{15} = 66$

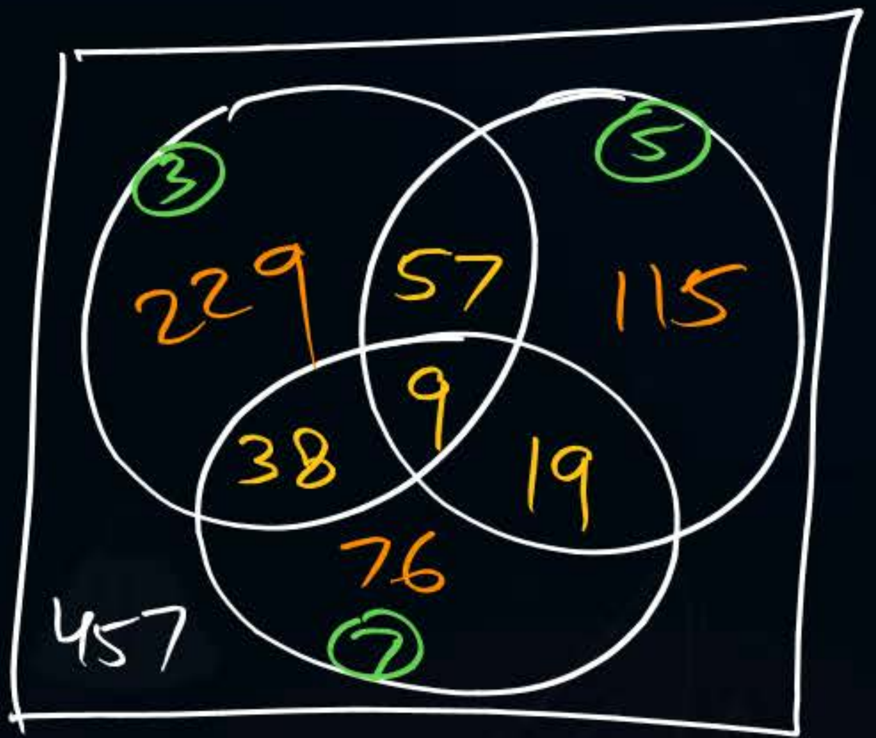
$n(5 \cap 7) = \frac{1000}{35} = 28$

$n(7 \cap 3) = \frac{1000}{21} = 47$

$n(3 \cap 5 \cap 7) = \frac{1000}{105} = 9$

only(3) = 333 − (57 + 9 + 38) = 229

only(5) = 200 − (57 + 9 + 19) = 115



S = 1000

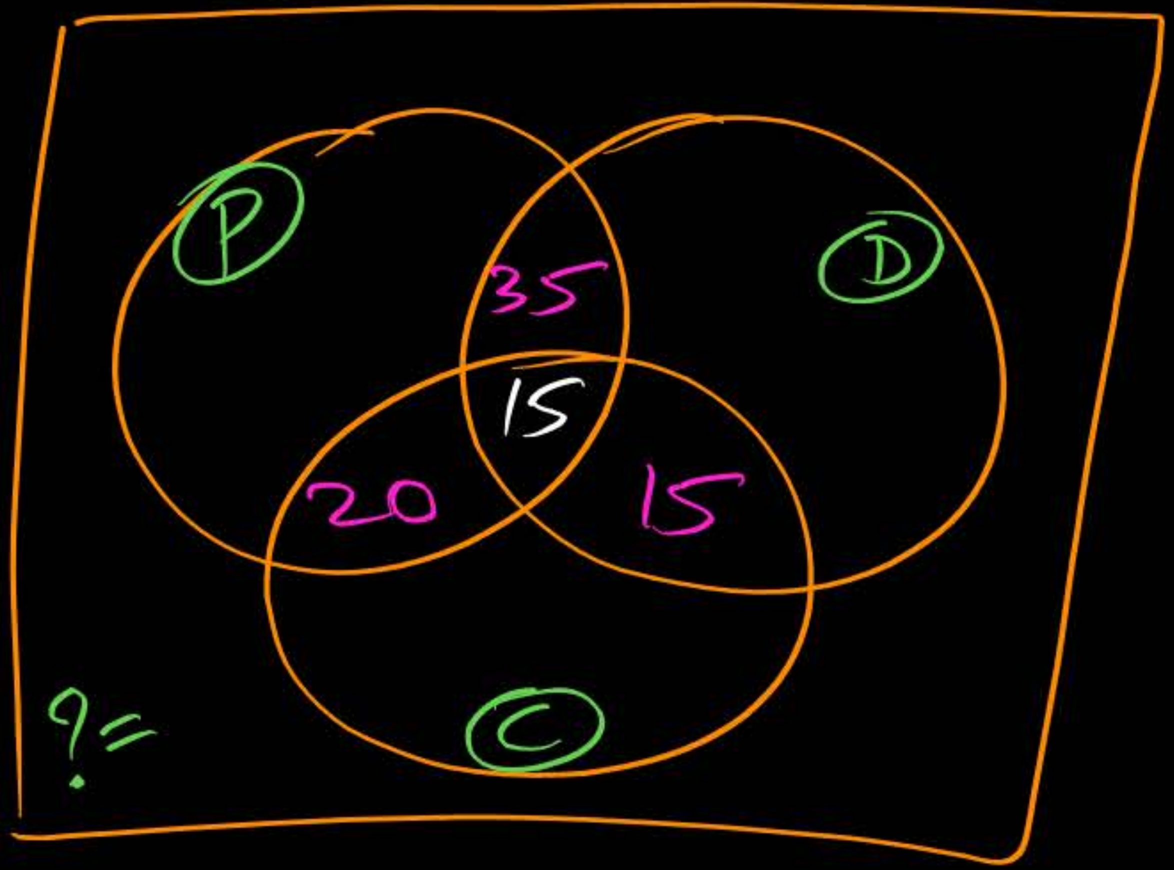
In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students have taken both programming languages and computer organization, 30 students have taken both data structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses?

(a) 15 (b) 20

(c) 25 (d) 35

$\bar{P} \cap \bar{D} \cap \bar{C} = ?$

Sol:



NIRMAN

2025

Dr Puneet Sir pw

@DRPUNEETSIRPW

Thank

you

Keep Hustling!