CS & IT ENGINEERING

Algorithms

Algorithms

Lecture No. 16

By- Ravindrababu Ravula Sir

Topics to be Covered

Topic Subset Sum

Topic Multi stage graph

Topic: Subset Sum

Set of set of n numbers → $\{a_1, a_2, a_3 \ldots\ldots a_n\}$

Subset of n number whose sum is ‘s’.

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Set of set of n numbers → $\{a_1, a_2, a_3 \ldots\ldots a_n\}$

Subset of n number whose sum is 's'.

Example:

{6, 2, 3, 1} set of 4 number.

is there any subset of this 4 numbers whose sum is '5'.

- {2, 3} → 2 + 3 → 5

Brute FORCE → Find all the Subsets - $O(2^n)$

Topic: Subset Sum

Set of set of n numbers → $\{a_1, a_2, a_3 \ldots\ldots a_n\}$

Subset of n number whose sum is 's'

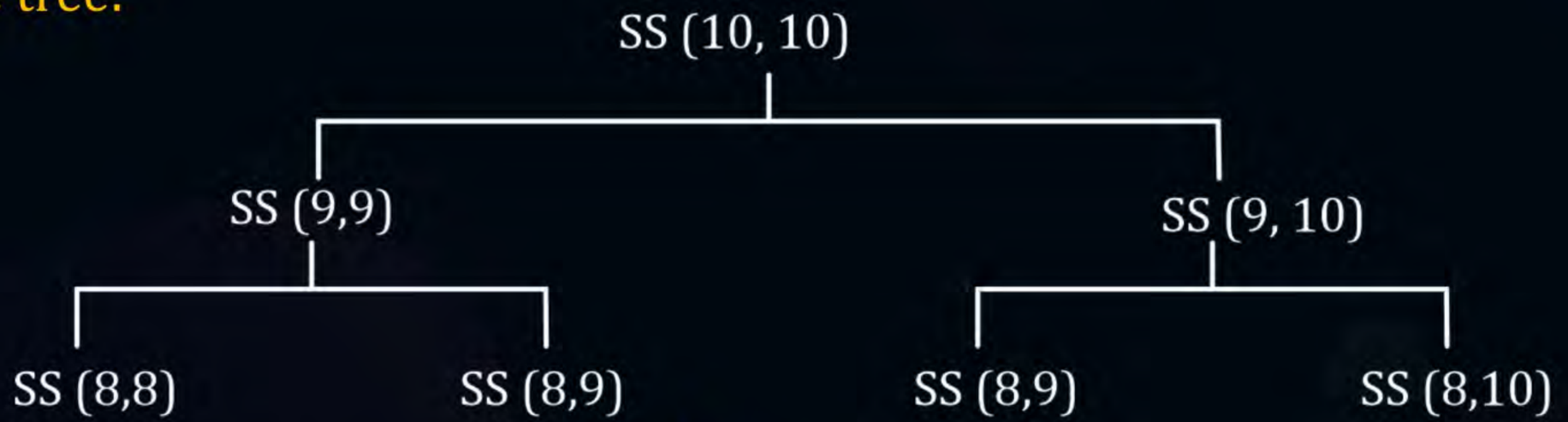
$$SS(i,s) = \begin{cases} SS(i-1, s); & s < a_i \\ SS(i-1, s-a_i) \vee & ss\ (i-1, s) \\ True\ ; & s = 0 \\ False; & i = 0,\ s \neq 0 \end{cases}$$

Topic: Subset Sum

Recursive tree:

Topic: Subset Sum

Recursive tree:



Number of node here → $O(2^n)$

SS(n,w) unique subproblem $O(nw)$

Topic: Subset Sum

Example:

{6, 3, 2, 1} → 4 element

W = 5 (sum)

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{ }

Topic: Subset Sum

Example:

{6, 3, 2, 1} → 4 element

W = 5 (sum)

	0	1	2	3	4	5
i = 0	T	F	F	F	F	F
i = 1	T					
i = 2	T					
i = 3	T					
i = 4	T					

i =	1	2	3	4
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ss(1,1) = ss(0, 1)

Topic: Subset Sum

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ss(2,1) = ss(1, 1)

Topic: Subset Sum

Example:

{6, 3, 2, 1} → 4 element

W = 5 (sum)

	0	1	2	3	4	5
i = 0	T	F	F	F	F	F
i = 1	T	F	F	F	F	F
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ss(2,2) = ss(1, 2)

Topic: Subset Sum

Example:

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	0	1	2	3	4	5
i = 0	T	F	F	F	F	F
i = 1	T	F	F	F	F	F
i = 2	T	F	F	T		
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$ss(2,3) = ss(1, 0)\ V\ ss(1, 3)$

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$$ss(2,4) = ss(1, 1) \vee ss(1, 4)$$

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$$ss(2,5) = ss(1, 2) \vee ss(1, 5)$$

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ss(3,1) = ss(2, 1)

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{6, 3, 2, 1} → 4 element

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i = 0	T	F	F	F	F	F
i = 1	T	F	F	F	F	F
i = 2	T	F	F	T	F	F
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$$SS(i,s) = \begin{cases} SS(i-1, s); & s < a_i \\ SS(i-1, s-a_i) \vee SS(i-1, s) & \\ True; & s = 0 \\ False; & i = 0, s \neq 0 \end{cases}$$

$ss(3,2) = ss(2, 0)\ V\ ss(2, 2)$

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$$ss(3,4) = ss(2, 2) \vee ss(2, 4)$$

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$ss(3,5) = ss(2, 3)\ V\ ss(2, 5)$

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$ss(4,1) = ss(3, 0)\ V\ ss(3, 1)$

Topic: Subset Sum

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	0	1	2	3	4	5
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i = 1	T	F	F	F	F	F
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i = 4	T	T	T	T	T	

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ss(4,5) = ss(3,4) V ss(3,5)

Topic: Subset Sum

Example:

{6, 3, 2, 1} → 4 element

W = 5 (sum)

O(nw) ✓

O(nw) ✓

	0	1	2	3	4	5
i = 0	T	F	F	F	F	F
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i = 2	T	F	F	T	F	F
i = 3	T	F	T	T	F	T
i = 4	T	T	T	T	T	**T**

→ Final answer
Use sum of subset = 5
True

i =	1	2	3	4
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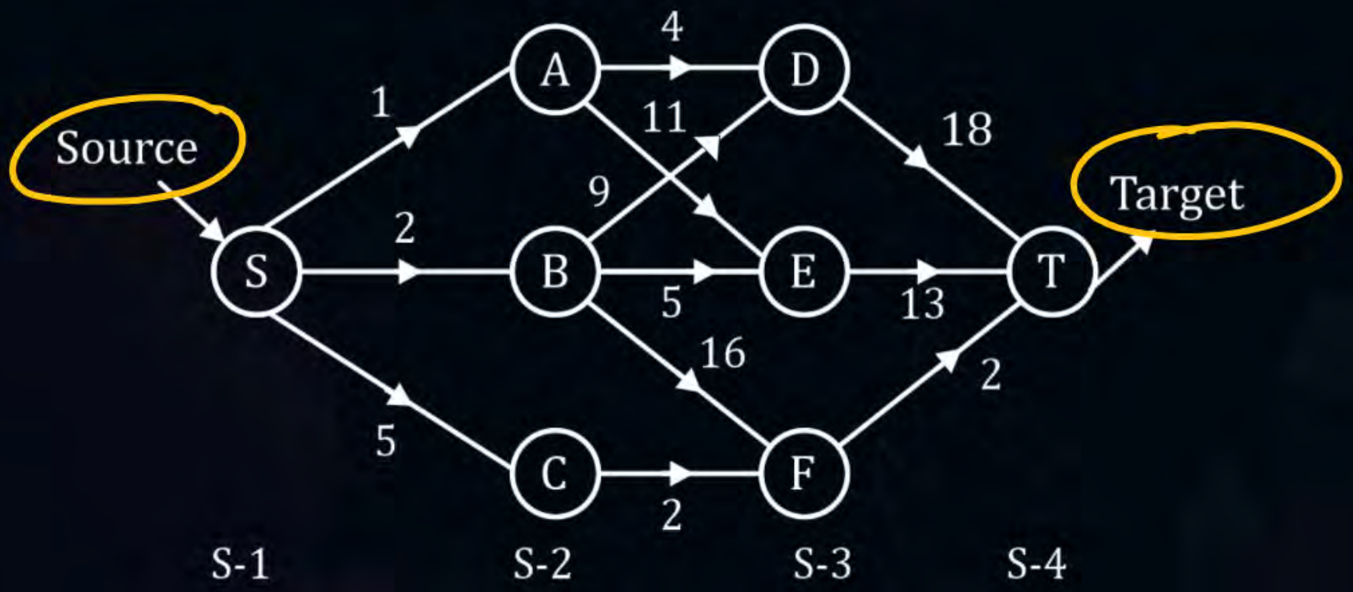
$$SS(i,s) = \begin{cases} SS(i-1, s); & s < a_i \\ SS(i-1, s-a_i) \vee SS(i-1, s) & \\ True; & s = 0 \\ False; & i = 0, s \neq 0 \end{cases}$$

Time complexity = O(nw)

Space complexity = O(nw)

Topic: Multi stage graph

Introduction to multi stage graph:



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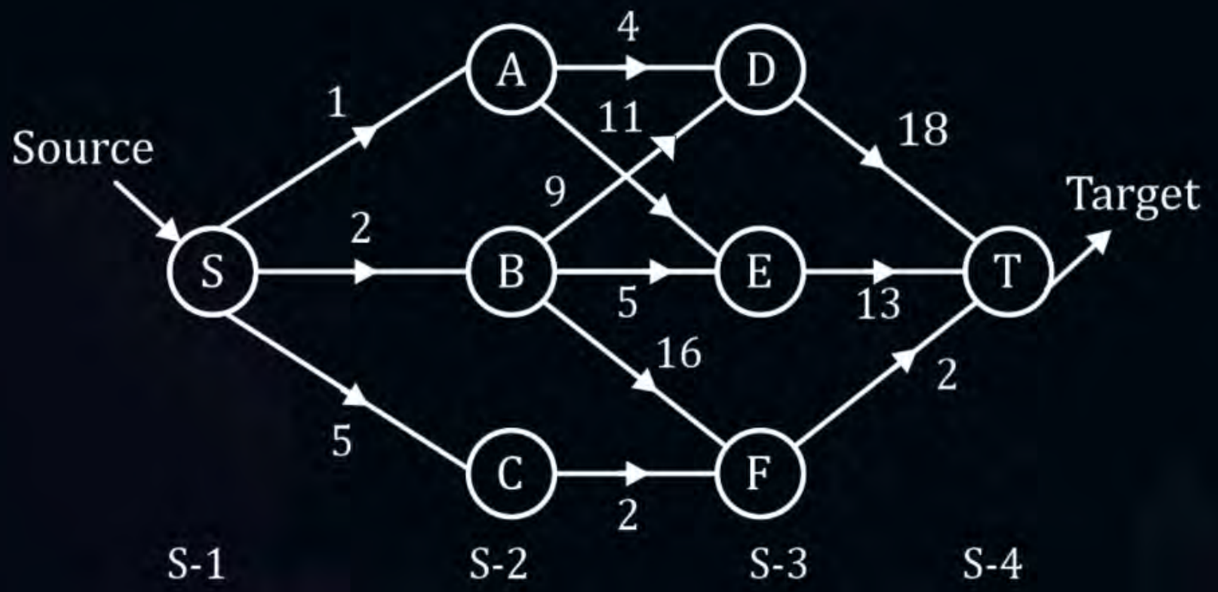


Multi stage graph:

- with in one stage there is no edge between them.
- edges only form S_i to S_{i+1} where S_i and S_{i+1} are stages

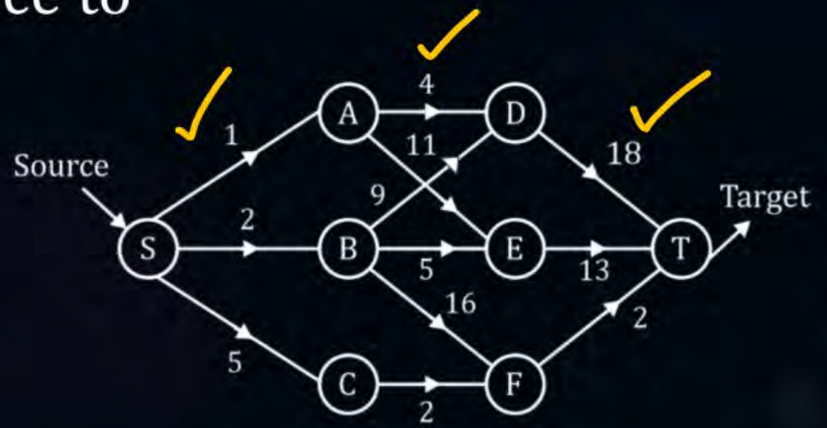
Topic: Multi stage graph

Find shortest path from S to T.

1. Apply greedy method & find shortest path from source to the target.

$S \xrightarrow{1} A \xrightarrow{4} D \xrightarrow{18} T = (1 + 4 + 18) = 23$

After apply greedy Method, we get length of 23. Bu
not shortest path between S and T.



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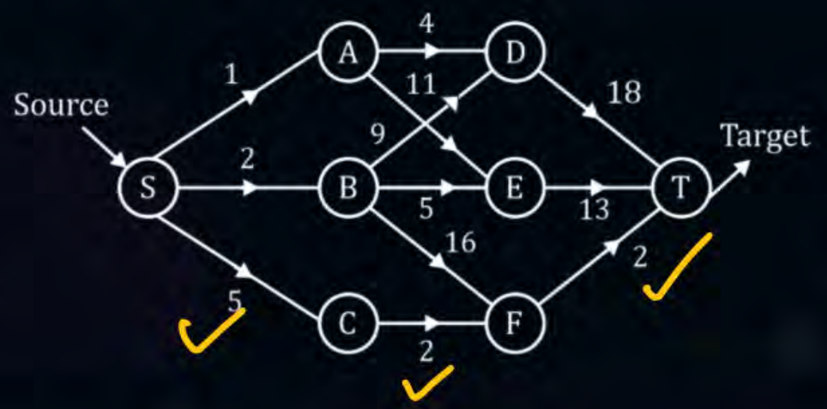


$S \xrightarrow{1} A \xrightarrow{4} D \xrightarrow{18} T = (1 + 4 + 18) = 23$

After apply greedy Method, we get length of 23. Bu not shortest path between S and T.

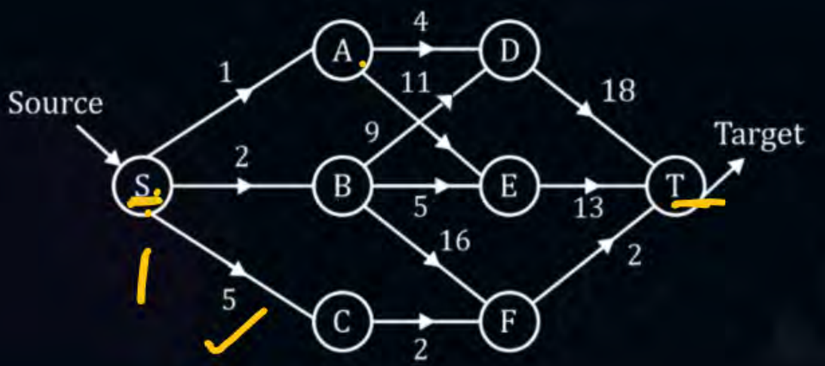
But actual shortest path is 9

$S \xrightarrow{5} C \xrightarrow{2} F \xrightarrow{2} T = (5 + 2 + 2) = 9$ ✓

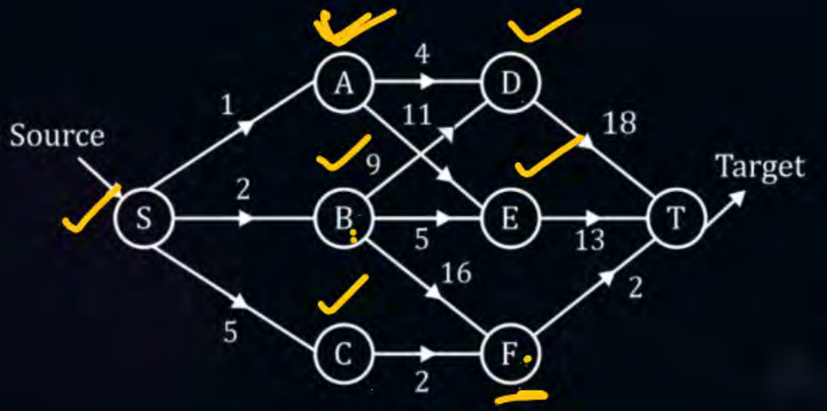
Topic: Multi stage graph

2. Apply dynamic method to find shortest path between S and T. (Structure and recursive equation)

$$M(1, S) = \min \begin{cases} S \to A + M(2, A) \\ S \to B + M(2, B) \\ S \to C + M(2, C) \end{cases}$$

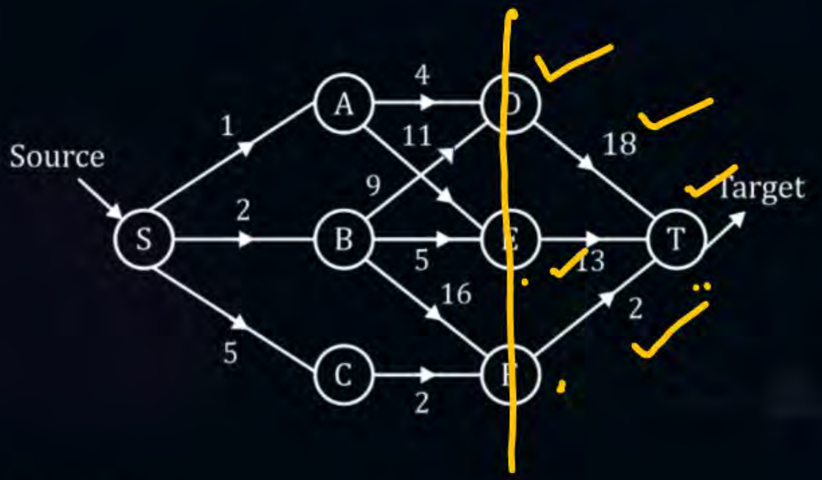


Topic: Multi stage graph



Recursive tree

Topic: Multi stage graph



Recursive tree

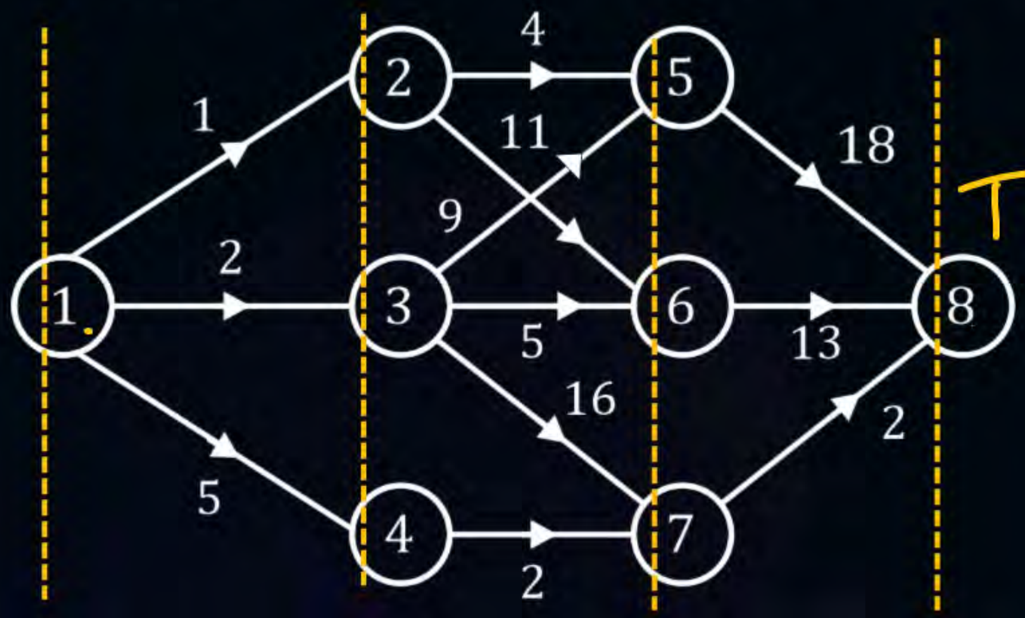
M (N–1, i) = i → T Stop

Number level= K Space complexity = O(K)

Time complexity = $O(k^n)$ → (exponential time)

Topic: Multi stage graph

- Apply Bottom up DP algorithm to reduce time complexity by reduce overlapping.

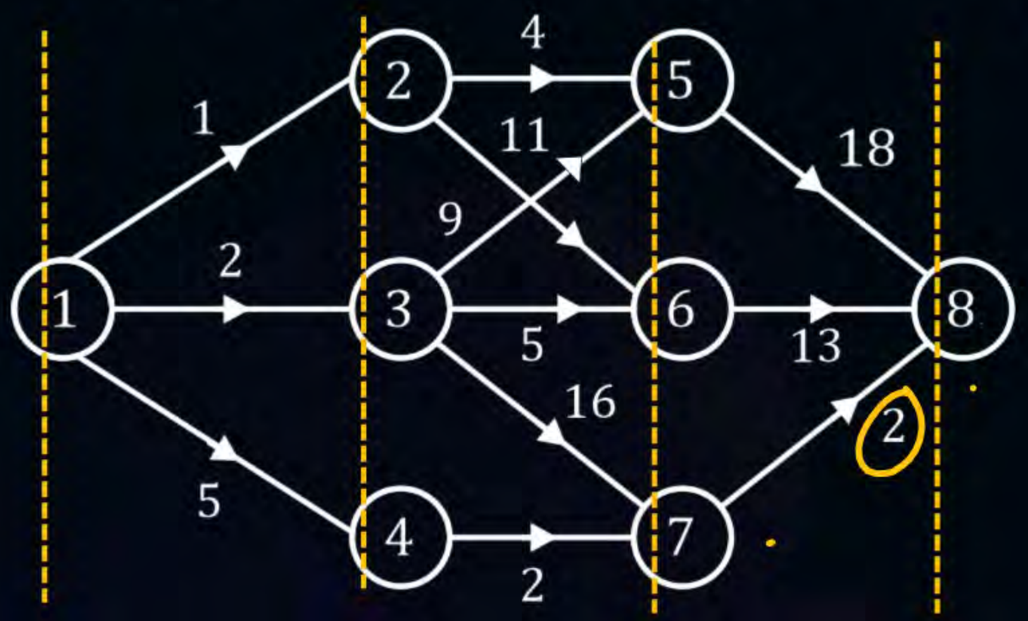


$$T[i] = \min \text{cost}_{\text{for all } (i+1) \text{ to } n}(i, j) + T[j]$$

Number of sub problem = number of vertices

Topic: Multi stage graph

- Apply Bottom up DP algorithm to reduce time complexity by reduce overlapping.



(array) T

1	2	3	4	5	6	7	8
						2	0

$T[i] = \min \text{cost}_{\text{for all (i+1) to n}}(i, j) + T[j]$

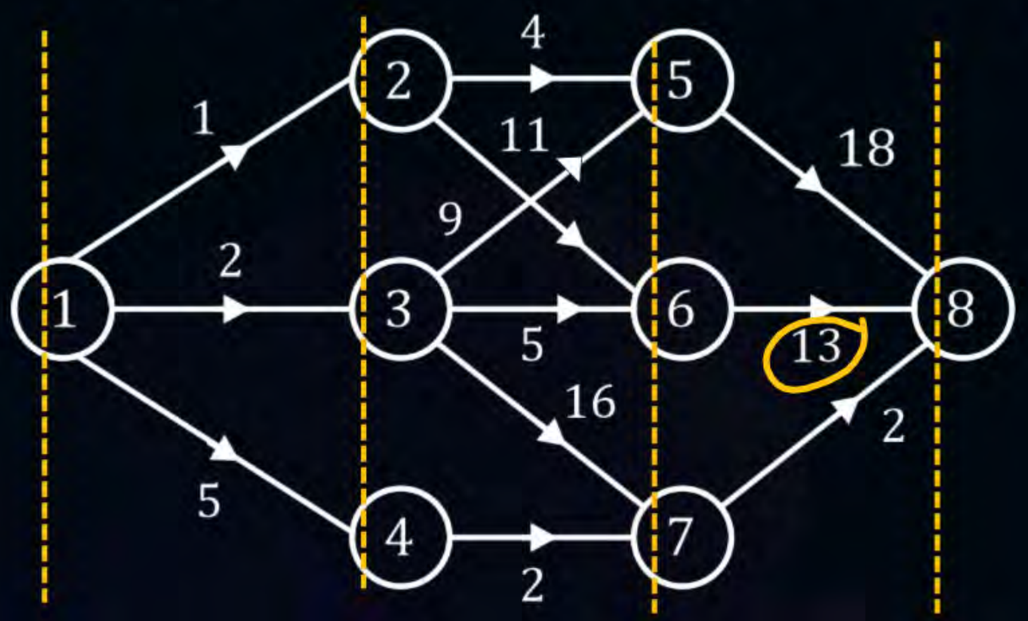
$T[8] = 0$

$T[7] = (7,8) + T[8] = 2$

Number of sub problem = number of vertices

Topic: Multi stage graph

- Apply Bottom up DP algorithm to reduce time complexity by reduce overlapping.



(array) T

1	2	3	4	5	6	7	8
					13	2	0

$T[i] = \min \text{cost}_{\text{for all } (i+1) \text{ to } n}(i, j) + T[j]$

$T[6] = (6, 8) + T[8] = 13 + 0 = 13$

Number of sub problem = number of vertices

Topic: Multi stage graph

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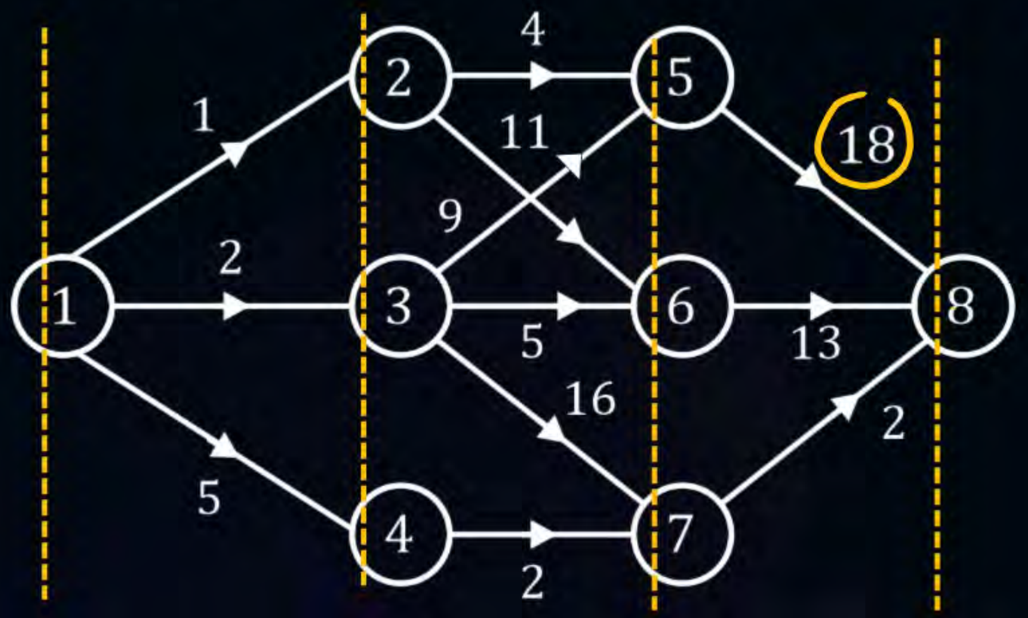


(array) T

1	2	3	4	5	6	7	8
				18	13	2	0

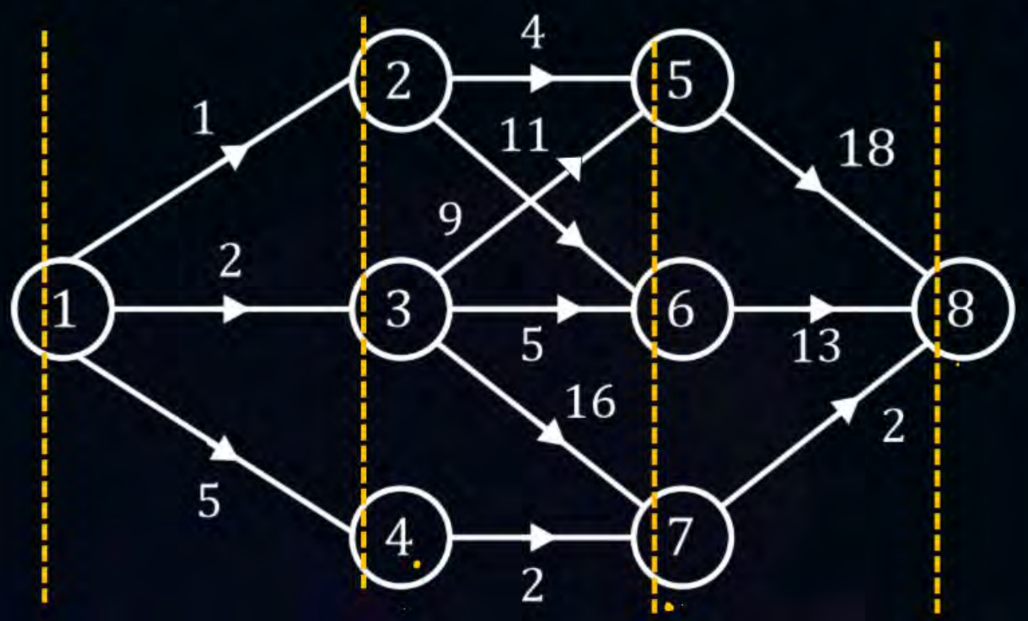
$T[i] = \min cost_{\text{for all } (i+1) \text{ to } n}(i, j) + T[j]$

$T[5] = (5, 8) + T[8] = 18 + 0 = 18$

Number of sub problem = number of vertices

Topic: Multi stage graph

- Apply Bottom up DP algorithm to reduce time complexity by reduce overlapping.



(array) T

1	2	3	4	5	6	7	8
			4	18	13	2	0

$T[i] = \min \text{cost}_{\text{for all (i+1) to n}}(i, j) + T[j]$

$T[4] = (4, 7) + T[7] = 2 + 2$

Number of sub problem = number of vertices

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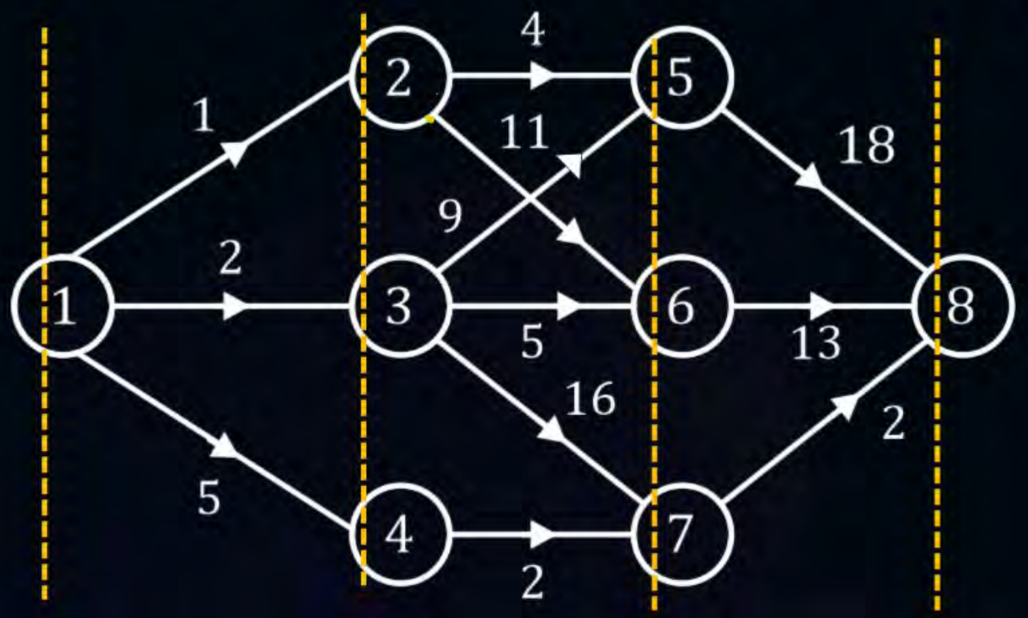

(array) T

1	2	3	4	5	6	7	8
		18	4	18	13	2	0

$T[i] = \min \text{cost}_{\text{for all } (i+1) \text{ to } n}(i, j) + T[j]$

$$T[3] = \min \begin{cases} (3,5) + T[5] = 11 + 18 \\ (3,6) + T[6] = 5 + 13 \\ (3,7) + T[7] = 16 + 2 \end{cases}$$

Number of sub problem = number of vertices

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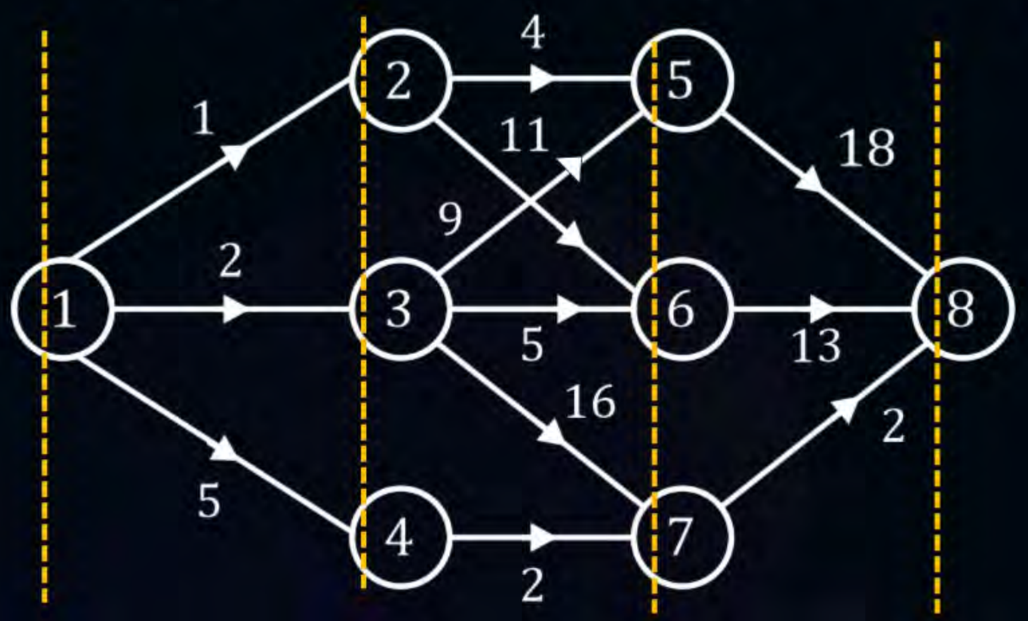

(array) T

1	2	3	4	5	6	7	8
	22	18	4	18	13	2	0

$T[i] = \min \text{cost}_{\text{for all } (i+1) \text{ to } n}(i, j) + T[j]$

$T[2] = (2, 5) + T[5] = 4 + 18$

$(2, 6) + T[6] = 11 + 13$

Number of sub problem = number of vertices

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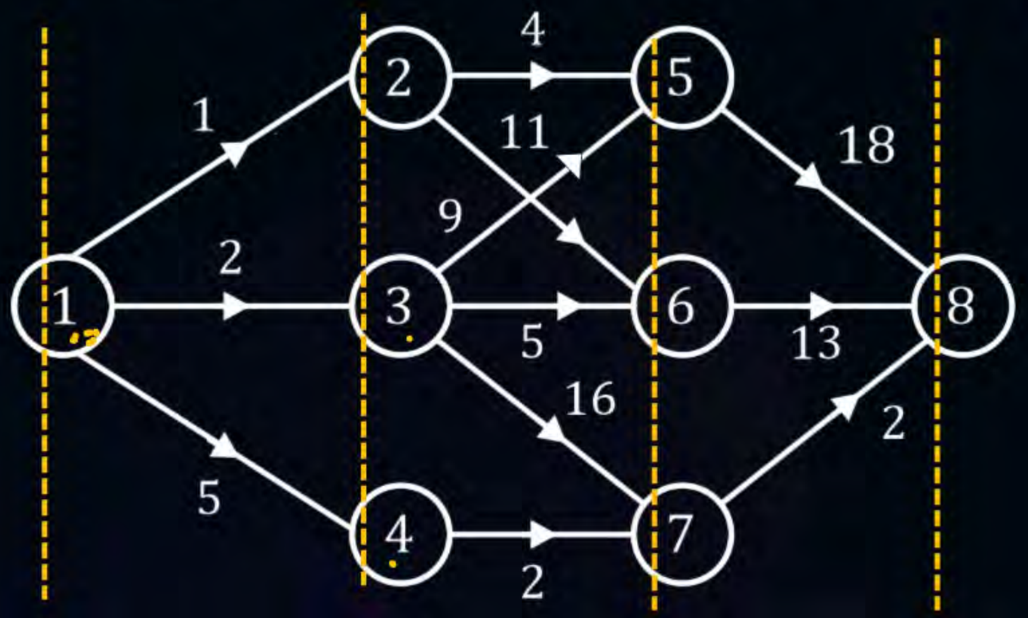

(array) T

1	2	3	4	5	6	7	8
9	22	18	4	18	13	2	0

$T[i] = \min \text{cost}_{\text{for all (i+1) to n}}(i, j) + T[j]$

$T[1] = (1, 2) + T[2] = 1 + 22$

$(1, 3) + T[3] = 2 + 18$

$(1, 4) + T[4] = 5 + 4$

Number of sub problem = number of vertices

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$T[i] = \min \text{cost}_{\text{for all } (i+1) \text{ to } n}(i, j) + T[j]$

(array) T

1	2	3	4	5	6	7	8
9	22	18	4	18	13	2	0
7	6	5	4	3	2	1	0

Time complexity = number of subproblem × time take to each problem

$= O(n^2)$

$= O(v^2)$

Time complexity $= O(e)$

THANK - YOU