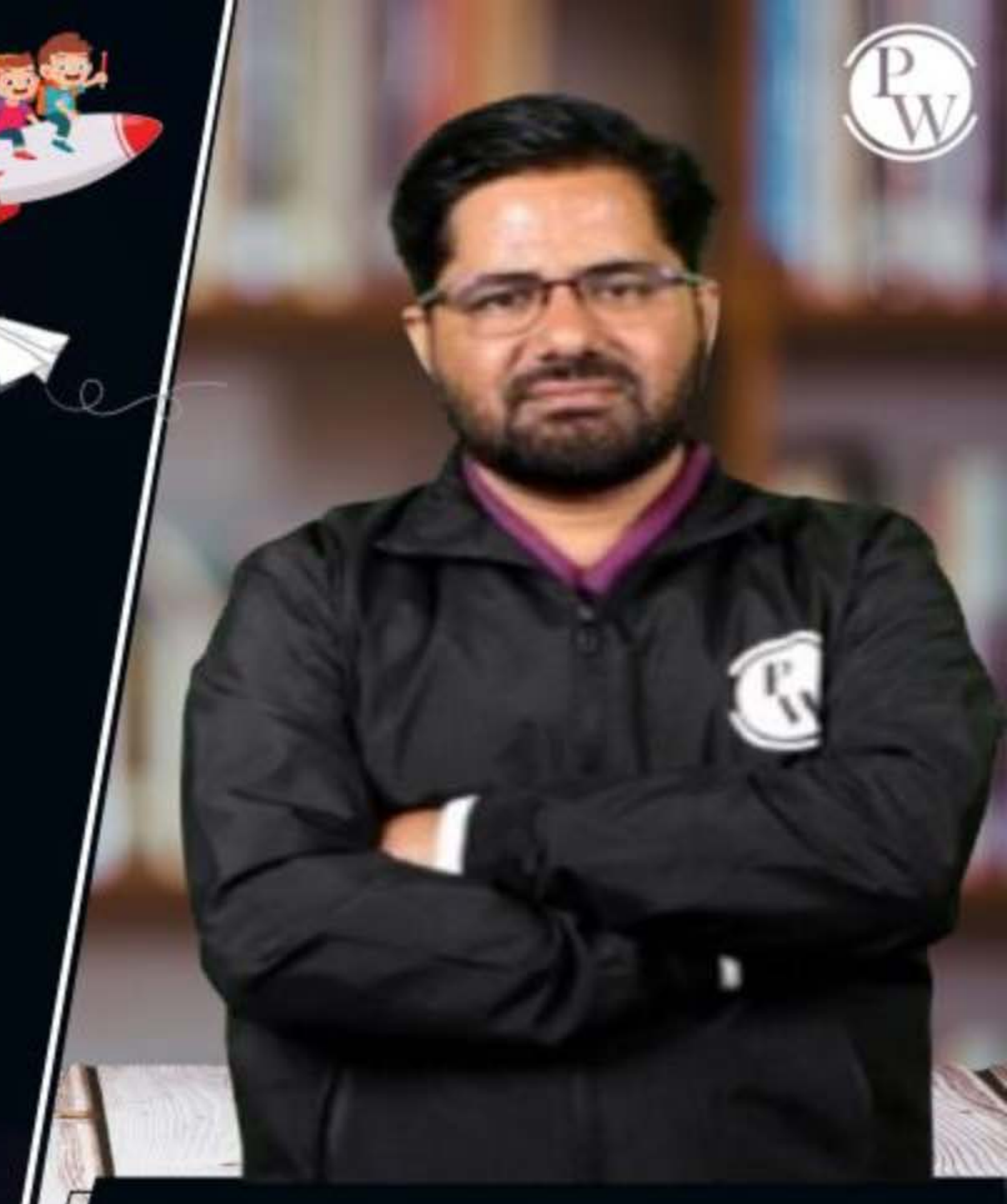
GATE

DS & AI
CS & IT

Linear Algebra

Lecture No. 07



By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

RANK of MATRIX

Topics to be Covered

Topic

→ RANK of MATRIX (continued)

→ LD & LI vectors

RANK

(*) Submatrix → By deleting some Rows or Some Columns or Both, the Matrix obtained is called Submatrix.

Def^n of Rank: "It is the order of Non singular Submatrix of Highest order that can exist in a given Mat"

Def^n In Books:

if $\boxed{\rho(A_{6\times 7}) = 4}$ then

→ $\exists$ atleast one Non Sing submatrix of order 4×4

→ Every Square submatrix of order 5×5 & 6×6 are Singular

(*) "E-operations do not alter the Rank of Matrix"

i.e. we are free to apply all three E-operations $\boxed{\text{while}}$ calculating Rank

that's why RANK is an INVARIANT property of Mat.

Equivalent Matrix → matrix obtained by applying one or more E-operations are equivalent to each other.

Q: Equivalent Matrices have Same Rank.

But they may have different Determinants

Echelon Form → (Triangular Form) →

Any Mat $A_{m \times n}$ is said to be in Echelon Form if,

① Number of Zeros before the 1^{st} Non Zero element in a Row should be in an Increasing order in the subsequent Rows.

② Every Zero Row (if exist) should occur at the bottom of a Mat.

Note: ① $\boxed{\rho(\text{Echelon form}) = \text{Number of Non Zero Rows.}}$

② Any Mat can be converted into an E-form by using E-operations.

③ It is advisable to apply only E-Row operations while converting given Mat into an E-Form. (as per our syllabus)

Flowchart of Converting given Mat into an E-Form →

① Make a_{11} unity (Not compulsory but advisable)

② Make all the elements of C_1 (that lies below a_{11}) Zero by using E-Row Operation

③ Make a_{22} unity (Not compulsory but advisable)

④ Make all the elements of C_2 (that lies below a_{22}) Zero by " " "

⑤ Make a_{33} unity & so on - - - - -

Note: Take care, In E-Form, $\boxed{a_{21} = \text{Zero.}}$

Q. (NET) $A = \begin{bmatrix} 2 & & & & & \\ & -1 & & & \text{Zero} & \\ & & 0 & & & \\ & & & 3 & & \\ & \text{Zero} & & & 4 & \\ & & & & & 0 \end{bmatrix}_{6\times 6}$ or $A = \text{diag}(2, -1, 0, 3, 4, 0)$ then $\rho(A) = ?$

(M-I) By observation, $A_1 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}_{4\times 4}$ By deleting R_3, C_3 & R_6, C_6

$\because |A_1| \neq 0$ i.e. A_1 is Non Sing of 4×4 so $\rho(A) =$ Four.

(M-II) $A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_4} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_4 \leftrightarrow R_5} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 4$

Properties of Rank –

(1) $\rho(A_{m\times n}) \le \min\{m, n\}$ eg $\rho(A_{6\times 4}) \le 4$

(2) $\rho(A_{m\times n}) \ge 1$ i.e $\boxed{1 \le \rho(A_{m\times n}) \le \min\{m, n\}}$

(3) $\rho(\text{Null Mat}) = 0$ (defined / Assumption)

(4) if $A_{n\times n}$ s.t A is Non singular then $\rho(A) = n$ & $\rho(A^{-1}) = n$

i.e $\boxed{\rho(A) = \rho(A^{-1})}$

(5) $\rho(A) = \rho(A^T) = \rho(A^\theta) = \rho(AA^T) = \rho(AA^\theta)$

~~i.e $\rho(A)$ & ρ(orthogonal Mat) are same~~

(if $AA^T = I$ then only A is called O-Mat)

(BLUNDER) / PAAP

(6) If A & B are two Matrices s.t AB is defined then

$$\boxed{\rho(AB) \le \min\{\rho(A), \rho(B)\}}$$

i.e Rank of the product can never exceeds their individual Rank.

eg if $\rho(A_{6\times 7}) = 5$ & $\rho(B_{7\times 5}) = 3$ then $\rho(AB)_{6\times 5} \le 3$

(7) $\rho(A+B) = \rho(A) + \rho(B)$ Not always True

i.e $\rho(A+B) \le \rho(A) + \rho(B)$ (True)

(8) $\rho(\text{Row Mat}) = \rho(A_{1\times n}) = 1$

$\rho(\text{Column Mat}) = \rho(B_{n\times 1}) = 1$

$\Rightarrow \rho(\text{Row} \times \text{Column}) = \rho(AB)_{1\times 1} = 1 \text{ or } 0$

$$\boxed{\rho(\text{Column} \times \text{Row}) = \rho(BA)_{n\times n} = 1 \text{ or } 0}$$

Q: if $A = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}_{1\times3}$, $B = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}_{3\times1}$ then $\rho(AB) = ?$ & $\rho(BA) = ?$

Sol:

$$AB = \begin{bmatrix} (8-2+3) \end{bmatrix}_{1\times1} = \begin{bmatrix} 9 \end{bmatrix}_{1\times1} \Rightarrow \rho(AB)_{1\times1} = 1.$$

$$BA = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}_{3\times1} \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}_{1\times3} = \begin{bmatrix} 8 & -4 & 12 \\ 4 & -2 & 6 \\ 2 & -1 & 3 \end{bmatrix}_{3\times3}$$

$\because$ it is not a Null Mat

So $\rho(BA) = 1.$

(M-II)

$$BA \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ 8 & -4 & 12 \end{bmatrix} \xrightarrow[R_3 - 4R_1]{R_2 - 2R_1} \begin{bmatrix} 2 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So $\rho(BA) = 1$

Q1: if $A = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}_{4\times4}$ then $\rho(A) = ?$ 4

$\because |A| = \dots\dots = 5$

ie A is Non Singular

Q2: if $A = \begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}_{4\times4}$ s.t $\boxed{\rho(A) = 3}$ $\Rightarrow$ A is Singular $\Rightarrow |A| = 0$

then No. of different values of μ satisfying it will be ?

(a) 0

(b) 1 = one

(c) 2 = two

(d) 3 = three ✓

(e) All (b), (c), (d)

$$\begin{vmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{vmatrix} = 0$$

$$\mu^3 - 6\mu^2 + 11\mu - 6 = 0$$

$$(\mu-1)(\mu-2)(\mu-3) = 0 \Rightarrow \mu = 1, 2, 3$$

ie for three diff values of μ, $\rho(A) = 3$

Q. if $\underbrace{\rho(A_{m\times n}) = n}_{n \le m}$, $\underbrace{\rho(B_{n\times p}) = p}_{p \le n}$ then $\rho(AB) = ?$ (a) m (b) n

(c) p (d) $n+p$

$$\boxed{p \le n \le m}$$

so $\rho(AB)_{m\times p} \le p$

Qs (2015) If $A=\begin{bmatrix} p & q \\ r & s \end{bmatrix}_{2\times 2}$, $B=\begin{bmatrix} p^2+q^2 & rp+sq \\ pr+qs & r^2+s^2 \end{bmatrix}_{2\times 2}$ s.t. $\rho(A)=N$ then $\rho(B)=$?

(a) N

(b) N-1

(c) 2N

(d) N^2

(M-I) Let us try to calculate, $AA^T=\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} p & r \\ q & s \end{bmatrix}=\begin{bmatrix} - & - \\ - & - \end{bmatrix}=B$ ☺

ie $B=AA^T \Rightarrow \rho(B)=\rho(AA^T)=\rho(A)=N$

(M-II) $|A|=ps-qr$ & $|B|=\ldots\ldots\ldots=(ps-qr)^2$

ie $|A|$ & $|B|$ will be simultaneously Zero or Non Zero.

if $|A|=0 \Rightarrow |B|=0$, $\rho(A)=N=1=\rho(B)=N$. Ans

if $|A|\neq 0 \Rightarrow |B|\neq 0$, $\rho(A)=N=2=\rho(B)=N$ Ans

M-III (GEN Z) $\rightarrow A=\begin{bmatrix} p & q \\ r & s \end{bmatrix}, B=\begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$

Case I: let $p=s=4, q=r=0$

$A=\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, B=\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

$\rho(A)=2=N \quad \rho(B)=2=N$

Case II: $p=4, q=r=s=0$,

$A=\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}, B=\begin{bmatrix} 16 & 0 \\ 0 & 0 \end{bmatrix}$

$\rho(A)=1=N \quad \rho(B)=1=N$

ie $\rho(A)=\rho(B)=N$ always.

Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}_{3\times3}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices. Then the rank of P + Q is______.

$\because |P| = 0 \Rightarrow \rho(P) = 2$, $\because |Q| = 0 \Rightarrow \rho(Q) = 1$

M-I so ~~$\rho(P+Q) = \rho(P) + \rho(Q) = 2 + 1 = 3$~~

(Blunder.)

$P+Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}_{3\times3}$

$\because |P+Q| = 0$ i.e. it is singular

so $\rho(P+Q) \neq 3$

$\Rightarrow \rho(P+Q) = 2$ Ans

Qs (CE) if $X = [x_1\ x_2\ x_3 \dots x_n]'$ is an ordered n-tuple Non Zero vector

s.t $XX' = V$ then $\rho(V) = ?$

(a) n

(b) $n-1$

(c) 1

(d) n^2

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} = \text{Column Mat}$$

$$X' = [x_1\ x_2\ x_3 \dots x_n]_{1\times n} = \text{Row Mat}$$

$$V = (XX') = (\text{Column} \times \text{Row})_{n\times n}$$

$$\rho(V) = \rho(XX')_{n\times n} = 1$$

The rank of 4 × 4 skew-symmetric matrix

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{bmatrix}_{4\times 4}$$ is ______.

HW

$|A| = 4 = (2)^2$ i.e. A is Non singular of 4×4

$\rho(A) = 4$

eg $A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{bmatrix}_{4\times 4}$ $\longrightarrow |A| = 0 = (0)^2 \Rightarrow \rho(A) \neq 4$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & -1 & -2 & -3 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{bmatrix} \xrightarrow[R_4 - 3R_1]{R_3 - 2R_1} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \end{bmatrix} \xrightarrow[R_4 + 2R_2]{R_3 + R_2} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 2$$

eg $A = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} \Rightarrow \rho(A) = 2$

eg $A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \Rightarrow \rho(A) = 2$

eg $A = \begin{bmatrix} 0 & 0 \\ -0 & 0 \end{bmatrix} \neq$ Skew symm.

$=$ Null Mat $\Rightarrow \rho(A) = 0$

eg $A = \begin{bmatrix} 0 & -3 & 0 \\ 3 & 0 & 5 \\ 0 & -5 & 0 \end{bmatrix}_{3\times 3}$

$\because |A| = 0 \Rightarrow \rho(A) \neq 3$

So $\rho(A) = 2$

$$\cancel{\begin{aligned} 2 &= 2 \\ 2^2 &= 2^2 \\ 2^2 - 2^2 &= 0 \\ (2-2)(2+2) &= 0 \\ (2+2) &= \frac{0}{2-2} \Rightarrow \frac{0}{0} = 4 \end{aligned}}$$

The rank of the matrix $\begin{bmatrix} 0 & i & -i \\ -i & 0 & i \\ i & -i & 0 \end{bmatrix}_{3\times3}$ is ______. = Skew symm Mat $\Rightarrow |A| = 0$

$\Downarrow$

$\rho(A) \neq 3$

(a) 1 (b) 2

(c) 3 (d) 4

$A_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}_{2\times2} \Rightarrow |A_1| = 0 - (-i^2) = -1$

i.e. A_1 is Non sing so $\boxed{\rho(A) = 2}$

(*) $\rho(\text{Skew symm Mat})_{n\times n} \geq 2$ i.e. $\rho(\text{Skew symm})$ can not be 1.

$\because$ Skew Symm Mat of order 1×1 ONE

Linearly Dependent & Linearly Independent Vectors →

L.D vectors → vectors are called L.D if ∃ Linear Relation b/w them.

L.I Vectors → " " " L.I if there DNE any Linear Relation b/w them.

Linear Combination of vectors → Let $X_1, X_2, X_3, \ldots, X_r$ are the given Vectors
(Linear Relationship) & $k_1, k_2, k_3, \ldots, k_r$ are the Scalars (Const)
then the Relationship of the type $\boxed{k_1X_1 + k_2X_2 + k_3X_3 + \ldots + k_rX_r = 0}$
is called Linear Combination of vectors.

Sp. Note → $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4\times1}$ then $X^2 = [\ ][\ ] =$ N.D so Relationship other than linear will never exist.

Methods of Checking the Nature of Vectors $\rightarrow$

Consider the given vectors are $x_1, x_2, x_3, \ldots, x_r$
then Construct a Matrix A as follows; $A = [x_1\, x_2\, x_3 \ldots x_r] \neq$ Row Mat

M-I General Method (always applicable) $\rightarrow$

(i) if $\rho(A) =$ No. of vectors $\Rightarrow$ Vectors are LI

(ii) if $\rho(A) <$ " " " $\Rightarrow$ " " LD

M-II Tricky Method (applicable only when A is Sq Mat) $\rightarrow$

(i) if $|A| \neq 0 \Rightarrow$ Vectors are LI

(ii) if $|A| = 0 \Rightarrow$ " " LD

Note: If there are two vectors X_1 & X_2 then No Need to use G. Method or T. Method, only use observation method

i.e Consider X_1 & X_2 are given vectors.

if $\boxed{X_1 = KX_2}$ for any K then vectors are LD

& if $X_1 \neq KX_2$ (i.e K = DNE) then " " LI

eg $X_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, $X_2 = \begin{bmatrix} -4 \\ 6 \\ -2 \end{bmatrix}$

$\because 2X_1 = -X_2$

$\boxed{2X_1 + X_2 = 0}$ LD

, eg $X_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, $X_2 = \begin{bmatrix} -4 \\ 6 \\ 0 \end{bmatrix}$

$\because X_1 \neq KX_2$ for any Non Zero K

i.e K = DNE $\Rightarrow$ LI

Q Check the Nature of vectors, $X_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \end{bmatrix}$, $X_2 = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, $X_3 = \begin{bmatrix} -6 \\ 6 \\ 6 \\ -4 \end{bmatrix}$

M-I By observation,

$$X_1 - X_2 = \begin{bmatrix} 3 \\ -3 \\ -3 \\ 2 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -6 \\ 6 \\ 6 \\ -4 \end{bmatrix} = -\frac{1}{2}X_3$$

$$\boxed{2X_1 - 2X_2 + X_3 = 0}$$

$\therefore$ Linear Relationship exist

So X_1, X_2, X_3 are L.D

M-II (W/o observation)

$$A = [X_1 X_2 X_3] = \begin{bmatrix} 2 & -1 & -6 \\ -1 & 2 & 6 \\ 0 & 3 & 6 \\ 3 & 1 & -4 \end{bmatrix}_{4\times 3}$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} -1 & 2 & 6 \\ 2 & -1 & -6 \\ 0 & 3 & 6 \\ 3 & 1 & -4 \end{bmatrix} \xrightarrow[R_4 + 3R_1]{R_2 + 2R_1} \begin{bmatrix} -1 & 2 & 6 \\ 0 & 3 & 6 \\ 0 & 3 & 6 \\ 0 & 7 & 14 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - \frac{7}{3}R_2]{R_3 \to R_3 - R_2} \begin{bmatrix} -1 & 2 & 6 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < 3$

So vectors are LD

(PYQ) the vectors $(1\ 2\ -1)'$, $(2\ 3\ 4)'$, $(0\ 1\ 2)'$, $(4\ -3\ 2)'$ are ?

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, x_4 = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

$$A = [x_1\ x_2\ x_3\ x_4] = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 2 & 3 & 1 & -3 \\ -1 & 4 & 2 & 2 \end{bmatrix}_{3\times 4}$$

$\rho(A) \leq 3$ (By prop of Rank)

i.e. $\rho(A) < 4$ definitely by common sense

$\Rightarrow \rho(A) <$ No. of vectors $\Longrightarrow$ vectors are LD

Q. Find λ for which $\exists$ a linear combination b/w the vectors; $\Rightarrow$ given vectors are LD

$\hat{i}+2\hat{j}+3\hat{k}$, $4\hat{i}+5\hat{j}+6\hat{k}$, $7\hat{i}+\lambda\hat{j}+9\hat{k}$ then $\lambda =$?

Sol: $X_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $X_2 = \begin{bmatrix}4\\5\\6\end{bmatrix}$, $X_3 = \begin{bmatrix}7\\\lambda\\9\end{bmatrix}$

$$A = [X_1 X_2 X_3] = \begin{bmatrix}1 & 4 & 7\\2 & 5 & \lambda\\3 & 6 & 9\end{bmatrix}_{3\times3}$$

By Tricky Method (for LD vectors)

$$|A| = 0 \Rightarrow \begin{vmatrix}1 & 4 & 7\\2 & 5 & \lambda\\3 & 6 & 9\end{vmatrix} = 0$$

....

$\lambda = 8$

Q. No. of different values of λ for which above vectors are LI will be ? ∞ Values.

for $\lambda = 8$, vectors are LD

So for $\lambda \neq 8$ " " LI i.e. $\lambda \in R - \{8\}$

Q. the vectors $(1\ 2\ 1)'$, $(2\ 1\ -4)'$, $(3\ -2\ 1)'$ are ortho. & L.I.

Sol: $X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}, X_3 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

$X_1 \cdot X_2 = X_2 \cdot X_3 = X_3 \cdot X_1 = 0$

$\Rightarrow X_1, X_2, X_3$ are orthogonal so these are LI also.

M-II $A = [X_1\ X_2\ X_3] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}$

$\because |A| = \text{- - - -} = -42 \neq 0$ so By Tricky Method.

Vectors are LI

VARIOUS Defⁿ of RANK :-

Defⁿ $\rho(A_{6\times 7}) = 4$

→ Mat A will have at most 4 LI Row vectors.

→ Mat A " " at most 4 LI Column vectors.

Defⁿ of Rank :- "It is the order of Non singular submatrix of Highest order that can exist in a given Mat"

Defⁿ In Books :

if $\boxed{\rho(A_{6\times 7}) = 4}$ then

→ ∃ at least one Non singular submatrix of order 4×4

→ Every square submatrix of order 5×5 & 6×6 are singular

July 27

Sir please request pw management to upload the Dpps for rank improvement batch gate DA. I saw all sections of the app but it's not uploaded anywhere

1:14 AM

July 28

I think it is in pdf section 10:00 PM

Sir actually i found it, its attached to that lecture's attachment when dpp is uploaded.
In the dpp section it's empty.
It will be helpful for others to find if you inform others in class.

10:09 PM

see in LEC3

Message

THANK - YOU

Tel:

$\boxed{\text{dr puneet sir pw}}$