DS & AI
CS & IT

Probability
Lecture - 02

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Recap of previous lecture

Topic

PERMUTATION - COMBINATION

(Complete)

Topics to be Covered

Topic PROBABILITY (Part-1)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

① The No. of factors of $5^2 6^3 7^4$ are ?

ⓐ 60 ⓒ 24

ⓑ 240 ⓓ 238

$5^2 6^3 7^4 = \cancel{(5^{0,1,2})} \times (6^{0,1,2,3}) \times (7^{0,1,2,3,4})$

So ~~Total Factors = 3 × 4 × 5 = 60~~ $\because 5^2 6^3 7^4$ is not in Prime form

(M-II) $5^2 6^3 7^4 = 5^2 \times (2\times3)^3 \times 7^4 = 5^2 \times 2^3 \times 3^3 \times 7^4$

So Total factors = 3 ways × 4 ways × 4 ways × 5 ways = 240 ways.

② In How many ways one or more selections can be made from ?

5, 5, 6, 6, 6, 7, 7, 7, 7

Sol: Total Selections or Rejections = (3 ways) × (4 ways) × (5 ways) = 60 ways.

So $\underbrace{\text{one or more}}$ selection can be done by = 60 − 1 = 59 ways.

(3) How many 9 digit Numbers = Arrangement. can be made using the digits

5, 5, 6, 6, 6, 7, 7, 7, 7 (Toys = RNA)

Total 9 digit Nos $= \frac{9!}{2!\,3!\,4!} = 1260$ Nos.

(4) How many 4 digit Numbers (>6000) can be made using 5, 5, 6, 6, 6, 7, 7, 7, 7

Sol: ∵ Given digits = Nine & they are asking 4 digit No

So No Direct formula exist.

we will calculate it By Making Cases.

HW

Ans = 51

5, 5, 6, 6, 6, 7, 7, 7, 7 (Toys)

Total four digit Nos (>6000) = $\underset{P_1}{\underline{2\text{ways}}} \times \underset{P_2}{\underline{3\text{ways}}} \times \underset{P_3}{\underline{3\text{ways}}} \times \underset{P_4}{\underline{3\text{ways}}}$ = $\underbrace{\text{54 Numbers.}}$ it may be WRONG.

(6 or 7) (5 or 6 or 7) (5 or 6 or 7) (5 or 6 or 7)

So we will try to CROSS check it.

we have some Numbers which are wrongly Counted in these 54 Nos.

for eg

6 555 → X
6 666 → X
7 555 → X

ie these three Nos are not formed

So Req Ans = 54 − 3 = 51 Nos.

Qs How many 4 digit Nos (> 3000) can be made using the digits
2, 2, 3, 3, 3, 4, 4, 4, 4

(a) $\frac{9!}{2!\,3!\,4!}$

(b) 54

(c) 51

(d) 60

wrongly counted Nos $= \underset{P_1}{\underline{2 \text{ way}}} \times \underset{P_2}{\underline{3 \text{ way}}} \times \underset{P_3}{\underline{3 \text{ way}}} \times \underset{P_4}{\underline{3 \text{ way}}} = 54$ Nos

& wrong Nos are

3 222 → ✗

3 333 → ✗

4 222 → ✗

So Ans $= 54 - 3 = 51$

PROBABILITY (possibility / chance).

(1) % = Base is of 100 units
Prob = Base is of 1 unit.
Proportion = Base is of again 1 unit

eg if in a Milk & water sol, $M:W = 3:4$

then
Prop of $M = \frac{3}{3+4} = \frac{3/7}{1}$
Prop of $W = \frac{4}{3+4} = \frac{4/7}{1}$

eg If we have 70 ltrs of M & W solⁿ in which $\boxed{M:W = 3:4}$ then find exact Quantity of M & W in that mixture ?

$\Rightarrow$ Q. of $M = \frac{3}{7} \times 70$ ltrs $= 30$ ltrs
Q of $W = \frac{4}{7} \times 70$ ltrs $= 40$ ltrs

(2) Random Experiment → whenever we are not sure about the outcome of an Experiment then such types of Experiments are called R. Exp. for eg; Tossing a coin, throwing a die, selection of card from pack of cards etc.

(3) Sample space → If we write total possible outcomes of any Random Exp in set form then this set is called sample space.

(4) Event → Any subset of sample space is called an event.

if No. of elements in S. Space = N then

Total No of Events associated with S = Total No. of subsets = 2^N

eg $S_{Die} = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

let $E_1 = \{1, 3, 5\} = \{$odd No occurs$\}$,

$E_2 = \{2, 4, 6\} = \{$Even No "$\}$

$E_3 = \{1, 2, 3, 4\} = \{$No ≤ 4 occurs$\}$

$E_4 = \{3, 6\} = \{$No divisible by 3$\}$

⋮

& so on - - - - .

these are called Events associated with S.

& Total No. of events = ? $= 2^6 = 64$

Note: $A = \{a, b, c\} \Rightarrow n(A) = 3$

Various subsets are ;

$\{a\}, \{b\}, \{c\},$

$\{a, b\}, \{b, c\}, \{c, a\}$

ϕ , $\{a, b, c\}$

Total subsets of $A = ? = 2^3 = 8$

Hence Total Events = ? = 8

Impossible Event → $\because \phi \subset S$ so ϕ is also an event & it is called Impossible Event

$\Leftarrow$ $\boxed{P(\phi) = 0}$

Sure Event / Certain Event → $\because S \subseteq S$ so S is also an Event

& it is called Sure Event i.e. $\boxed{P(S) = 1}$

Note: ① $0 \leq P(E) \leq 1$, ② P(Something occurs) = 1

③ P(Nothing occurs) = 0, ④ P(given statement) = 1

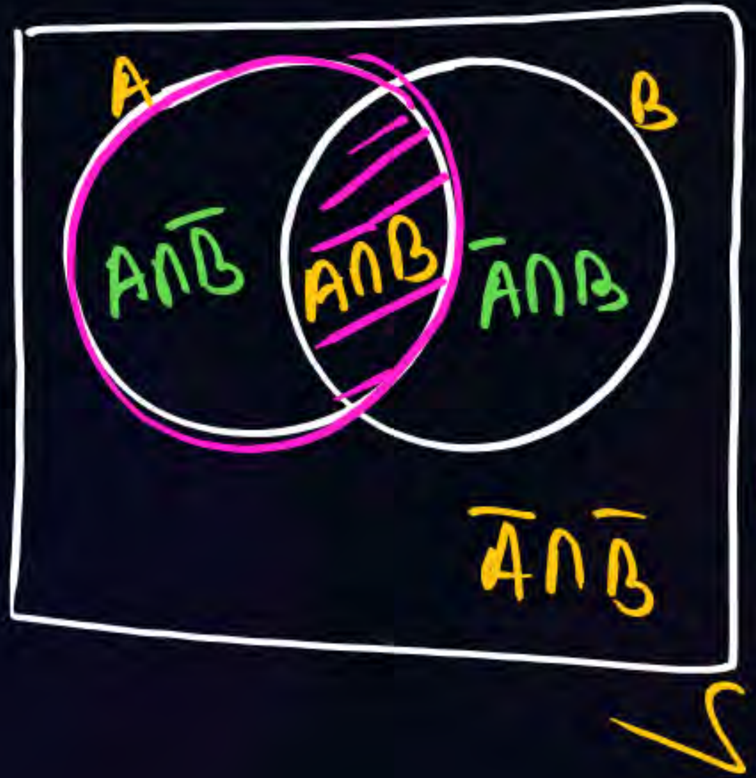
⑤ P(Death) = 1, ⑥ P(GOD) = 1

Some special Discussion →

① $P(\text{either } A \text{ or } B \text{ or Both}) = P(\text{at least one of } A \text{ or } B) = P(A\cup B)$

② $P(\text{Both } A \,\&\, B \text{ occurs}) = P(\text{simultaneous occurrence of } A \,\&\, B) = P(A\cap B)$

③ $P(\text{Neither } A \text{ Nor } B) = P(\text{None of } A \,\&\, B) = P(\bar{A}\cap\bar{B})$



(i) $A\cup B = (A\cap\bar{B})\cup(A\cap B)\cup(\bar{A}\cap B)$

(ii) only $A = A\cap\bar{B}$

(iii) only $B = \bar{A}\cap B$

(iv) $\bar{A}\cap\bar{B} = \text{Neither } A \text{ Nor } B$

① Addition Theorem of Prob. :→ $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

② Multiplication Theorem of Prob → $\boxed{P(A\cap B)=P(A/B)\cdot P(B)}$

③ P(Neither A Nor B) = 1 − P(either A or B or Both)

$$\boxed{P(\bar{A}\cap\bar{B})=1-P(A\cup B)}$$

④ P(either A or B or Both) = 1 − P(Neither A Nor B)

$\boxed{\text{P(atleast one of A or B)} = 1-\text{P(None)}}$

Mutually Exclusive Events →

If, two events can't occur simultaneously, then these are called M.E. Events

OR

If occurrence of one event prevents the occurrence of other event & vice versa than events are called ME Events. ie

If A & B are ME then only one can occur at a time

Mathematically: if E_1 & E_2 are ME events then $E_1 \cap E_2 = \phi$

Conclusion: if E_1 & E_2 are ME then

$\rightarrow P(E_1 \cap E_2) = 0$

$\rightarrow P(E_1 \cup E_2) = P(E_1) + P(E_2) - 0$

eg $S_D = \{1, 2, 3, 4, 5, 6\}$ & let us consider following events

$E_1 = \{1, 3, 5\}$ $\because E_1 \cap E_2 = \phi \Rightarrow E_1$ & E_2 are M.E & $P(E_1 \cap E_2) = 0$

$E_2 = \{2, 4, 6\}$ $\because E_2 \cap E_3 \neq \phi \Rightarrow E_1$ & E_2 are Not M.E

$E_3 = \{1, 2, 3, 4\}$ $\because E_1 \cap E_3 \neq \phi \Rightarrow E_1$ & E_3 are Not ME

$E_4 = \{2, 4\}$, $\because E_1 \cap E_4 = \phi \Rightarrow E_1$ & E_4 are also ME But $E_1 \cup E_4 \neq S$

i.e it is not Necessary that, in Case of ME Events, you will get their union as S. Space

$E_4 = \{x : 1 < x < 5$ & x is divisible by $2\}$

Independent Events. → If occurrence or Non occurrence of one event does not alter the occurrence or Non occurrence of other Event then Events are Called Independent events

$\Longleftrightarrow$

Mathematically: If A & B are Ind Events then $\Longleftrightarrow$ $\boxed{P(A \cap B) = P(A) \cdot P(B)}$

eg; $S_{Coin} = \{H, T\}$, $S_{Die} = \{1, 2, 3, 4, 5, 6\}$

$A = \{H\} \Rightarrow P(A) = \frac{1}{2}$

$B = \{1, 2, 3, 4\} \Rightarrow P(B) = \frac{4}{6}$

then $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\because A \,\&\, B \text{ are Ind. Events.}} = \frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$

PODCAST :- (Deep SEEK)

① ME events are associated with SAME sample space
while Ind " " " " DIFFERENT sample spaces

② In case of Ind events, we can observe that

$\boxed{A \cap B = \phi}$ then why not these are ME ??

∵ we are forcefully drawing a wrong conclusion.
as the concept of Intersection is applicable with in the same S.Sp.
→ that's why it is WRONG conclusion.

③ Same sample space < A & B may be ME
" " may not be ME

(4) Events formed by individual elements of S. space are ME (T)

eg $S_D = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1\}, E_2 = \{2\}, E_3 = \{3\}, E_4 = \{4\}, E_5 = \{5\}, E_6 = \{6\}$

$\because E_i \cap E_j = \phi \ \forall \ i \ \& \ j \Rightarrow E_i \ \& \ E_j$ are ME.

eg $S_{coin} = \{H, T\}, E_1 = \{H\}, E_2 = \{T\}$

$\because E_1 \cap E_2 = \phi \Rightarrow E_1 \ \& \ E_2$ are ME

(5) If two Events E_1 & E_2 are associated with different S space then question of their ME Nature doesn't arise.

(M. Imp) **Nature of Elements in S. Space** → If our R. Exp is Repeated n times then elements of S. Space are in the form of ordered n-tuple.

eg If Die is thrown once then $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$ outcomes

eg if " " twice then

$$S = \begin{Bmatrix} (11) & (12) & (13) & \cdots & (16) \\ (21) & (22) & \cdots & \cdots & (26) \\ (31) & (32) & \cdots & \cdots & (36) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & (66) \end{Bmatrix} \Rightarrow n(S) = \underset{D_1}{6} \times \underset{D_2}{6} = 36 \text{ ordered pair}$$

eg if a coin is tossed 5 times then $S = ? = \{(HHHHH), (HHHHT), \ldots, (TTTTT)\} \Rightarrow n(S) = 2\times2\times2\times2\times2 = 2^5 = 32$ tuples

eg A couple has 3 kids, then

$$S = \begin{Bmatrix} (BBB), & (BBG), & (BGB), & (BGG) \\ (GBB), & (GBG), & (GGB), & (GGG) \end{Bmatrix} \Rightarrow n(S) = \underset{C_1}{2} \times \underset{C_2}{2} \times \underset{C_3}{2} = 8 \text{ Triplets}$$

Note

(1) A Coin is tossed thrice

&

3 coins are tossed simultaneously

In both cases S.Sp would be SAME

$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$

(2) Favourable Event :- = {which is Required should be assumed as fav}

③ Methods of Solving Questions →

App I: By writing all the elements of S. Space & Favourable event (E) in set form, we can find $\boxed{P(E) = \dfrac{n(E)}{n(S)}}$

App II: If it is not easy to write S. Space then directly find fav. number of cases and Total Number of cases by using the concept of P & C & then $\boxed{\text{Req Prob} = \dfrac{\text{fav Cases}}{\text{Total Cases}}}$

App III: By using some standard Results & Standard Definitions.

Whenever in a Question, given information is in the form of Probability then use App III.

Short RECAP -

Operation	P&C	Prob	Formula	ME	Ind.
Either or	Plus	union	Addition Th	$P(A\cup B)=P(A)+P(B)$	*
AND	Multiply	Intersection	Multi Th	$P(A\cap B)=0$	$P(A\cap B)=P(A)\cdot P(B)$

Addition Th: $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

* for Independency: $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

for ME: $P(A\cup B)=P(A)+P(B)-0$

F.Q-1 A Die is thrown twice then write it's S.Space.

$$S=\begin{Bmatrix}(11)(12)(13)\ldots(16),(21)(22)\ldots(26)\\(31)(32)(33)\ldots(36),(41),(42)\ldots(46)\\(51)(52)(53)\ldots(56),(61),(62)\ldots(66)\end{Bmatrix} \Rightarrow n(S)=6\times 6=36 \text{ pair}$$

① Find the prob that, Sum of outcomes is 8 ?

App I

$$A=\{\text{Sum is } 8\}=\begin{Bmatrix}(\cancel{17})(26),(35),(44)\\(\cancel{71})\ (62),(53),(4\cancel{4})\end{Bmatrix}=5 \Rightarrow P(A)=\frac{n(A)}{n(S)}=\frac{5}{36}$$

② Find the prob that sum of outcomes is 9 ?

App I

$$B=\{\text{Sum is } 9\}=\begin{Bmatrix}(36)(45)\\(63)(54)\end{Bmatrix}=4 \Rightarrow P(B)=\frac{4}{36}$$

③ Find the prob that sum is Both 8 & 9 ?

App III

$\because A\cap B=\phi$ i.e A & B are ME So $P(A\cap B)=P(\phi)=0$

(4) Find the Prob that Sum is either 8 or 9 ?

App III $P(A \cup B) = ? = P(A) + P(B) - P(A \cap B)$

$$= \frac{5}{36} + \frac{4}{36} - 0 = \frac{9}{36} = \frac{1}{4}$$

(5) Find the Prob that Sum is Neither 8 Nor 9 ? $= 1 - P(\text{either 8 or 9})$

Gate App III

ie $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - \frac{1}{4} = \frac{3}{4}$

(6) Find the Prob that both the outcomes are identical ?

Gate App I

$C = \{\text{Both outcomes are identical}\}$

$= \{(11), (22), (33), (44), (55), (66)\} = 6$ So $P(C) = \frac{6}{36} = \frac{1}{6}$

(7) Find the Prob that Product of the outcomes will be a perfect square ?

Gate App I

(a) 0 (b) $\frac{1}{6}$ (c) $\frac{2}{9}$ ✓ (d) 1

$D = \{(11), (22), (33), (44), (55), (66), (14), (41)\} = 8$ So $P(D) = \frac{8}{36} = \frac{2}{9}$

⑧ $P(\text{sum of outcomes is divisible by } 4) = ? = P(\text{sum} = 4 \text{ or } 8 \text{ or } 12)$

$$= \frac{3+(13-8)+(13-12)}{36} = \frac{9}{36}$$

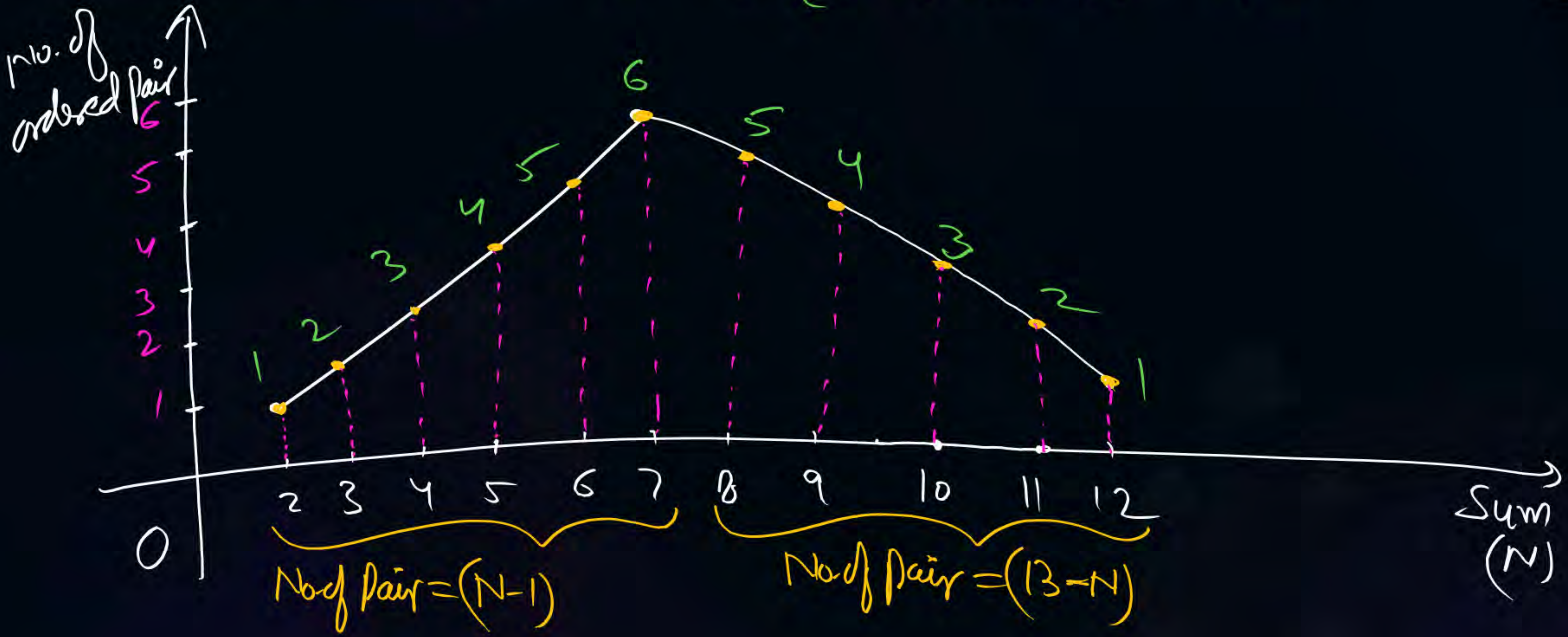
⑨ $P(\text{sum of outcomes is prime number}) = ?$

$$= P(\text{sum} = 2 \text{ or } 3 \text{ or } 5 \text{ or } 7 \text{ or } 11) = \frac{1+2+4+6+(13-11)}{36} = \frac{15}{36}$$

⑩ $P(\text{sum exceeds } 9) = ? = P(\text{sum} = 10 \text{ or } 11 \text{ or } 12)$

$$= \frac{(13-10)+(13-11)+(13-12)}{36} = \frac{3+2+1}{36} = \frac{6}{36} = \frac{1}{6}$$

Shortcut to Count Fav Pair → {Sum = 7} = {(1 6) (2 5) (3 4) (6 1) (5 2) (4 3)} = 6 pair



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Thank you

Keep Hustling!