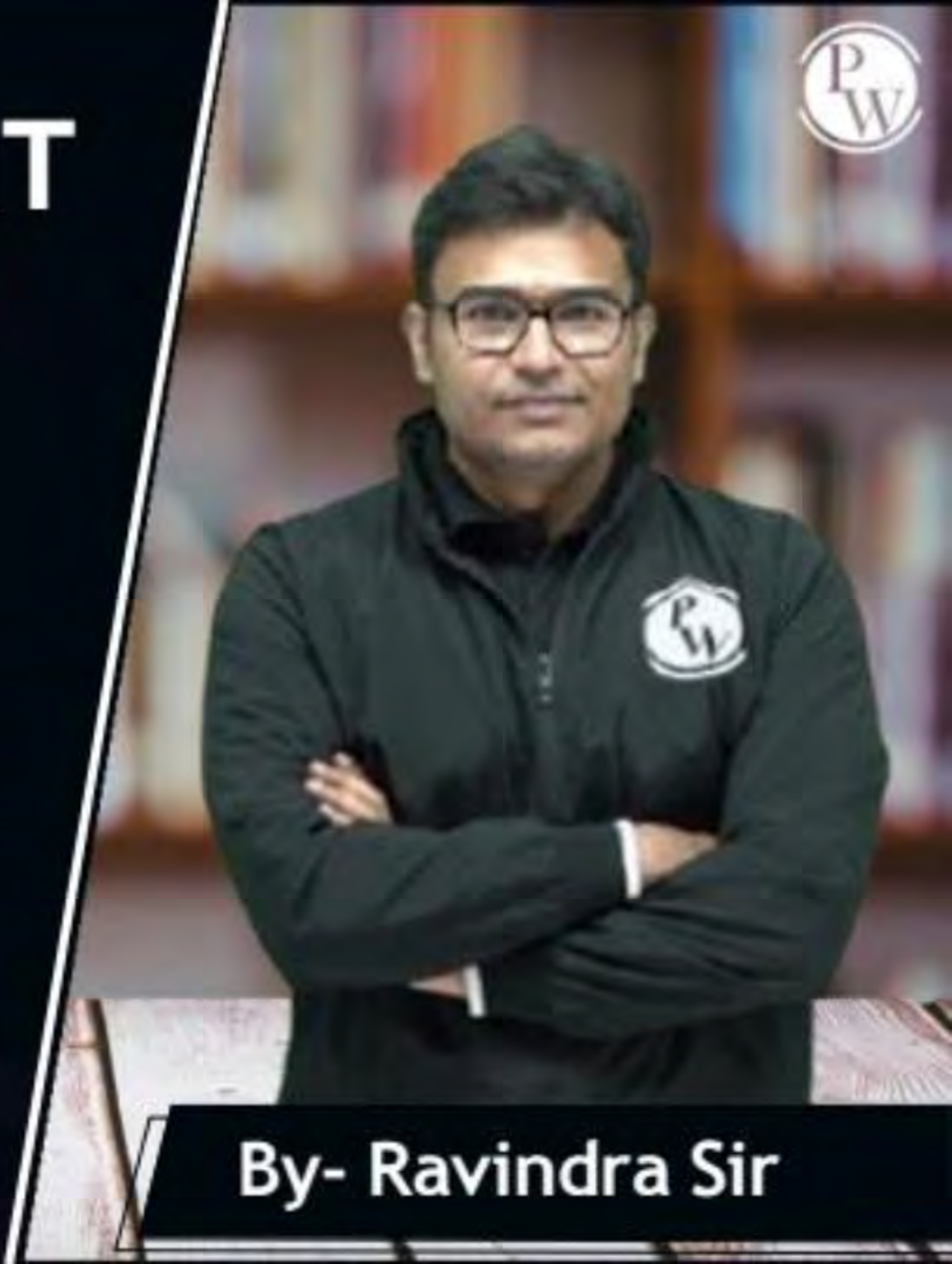
Computer Science & IT

ALGORITHMS

Algorithms

Lecture No. 06



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Recap of Previous Lecture

Topic

Topic

Topic

TC

Topics to be Covered

Topic

Topic

Topic

masters theorem

Inspiring Stories : Karla Wheelock

Background: Born in a modest family in Mexico.

Education: Studied law while climbing.

Achievements: First Latin American woman to climb the Seven Summits.

Impact: Broke barriers in mountaineering.

Inspiring Stories : Selvi

Background: From a rural village in Tamil Nadu, child bride.

Education: Studied later with help.

Achievements: Became India's first female taxi driver.

Impact: Symbol of independence for rural women.

Inspiring Stories : Maja Kazazic

Background: Lost leg in Bosnia war as a teen.

Education: Studied IT in the USA.

Achievements: Built a new life, became public speaker.

Impact: Shows healing after trauma is possible.

masters theorem :-

Let $T(n) = aT(n/b) + f(n)$

$a, b \rightarrow$ constants

$a \geq 1, b > 1,$ $f(n)$ is a function of 'n'

Case(i):- If $f(n) = O(n^{\log_b a - \epsilon})$, $\epsilon > 0$, then

$$T(n) = \Theta(n^{\log_b a})$$

X

x

Case (iii) :- If $f(n) = \Omega\left(n^{\log_b a + \epsilon}\right)$, $\epsilon > 0$, then

$$T(n) = \Theta(f(n))$$

Case (iii):- If $f(n) = \Theta\left(n^{\log_b a} \cdot (\log n)^k\right)$

where k is a constant, $k \geq 0$

$$T(n) = \Theta\left(n^{\log_b a} \cdot (\log n)^{k+1}\right)$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) \qquad n^{\log_b a}$$

1) which one is greater
2) It is greater by polynomial time

$$\Theta(\text{greater term})$$

$f(n)$ $n^{\log_b a}$

n $\underline{\underline{n^3}}$ n^2

$\Theta(n^3)$ ✓

$f(n)$ $n^{\log_b a}$

$n^{0.81}$ $n^{2.81}$ n^2

$\Theta(n^{2.81})$ ✓

$f(n) \qquad n^{\log_b a}$

$n^{50} \qquad \underline{n^{100}}$

$\Theta(n^{100})$ ✓

$f(n)$ $\qquad$ $n^{\log_b a}$

$\underline{n^2}$ $\qquad$ $\underline{n^2}$

$\Theta(n^2 \underline{\log n})$

$f(n)$ $\quad\quad$ $n^{\log_b a}$

$\underline{n\log n}$ $\quad\quad$ n

$$\Theta(n\log n \times \log n)$$

$$= \Theta(n(\log n)^2)$$

$f(n)$ $\quad\quad$ $n^{\log_b a}$

$\underline{n(\log n)^{10}}$ $\quad\quad$ n

$$\Theta\left(n(\log n)^{10} \times \log n\right)$$

$$\Theta\left(n(\log n)^{11}\right)$$

$f(n)$ $\quad\quad$ $g(n)$

$n(\log n)^5$ $\quad\quad$ n

$$\Theta\left(n(\log n)^5 \times \log n\right)$$

$$\Theta\left(n(\log n)^6\right)$$

$f(n)$ $\qquad\qquad$ $n^{\log_b a}$

$\dfrac{n^2 \log n}{n^1 \log n}$ $\qquad\qquad$ n

$\Theta(n^2 \log n)$

$f(n)$ $\qquad$ $n^{\log_5 9}$

n^4 $\qquad$ $\underline{\underline{n^5}}$

$\Theta(n^5)$

$f(n)$ $\quad$ $n^{\log_b a}$

n^3 $\quad$ n^3

$\Theta(n^3 \log n)$

Gate:- $T(n) = 8T(n/2) + n^2$

$a = 8$
$b = 2$

$n^{\log_b a} = n^{\log_2 8} = n^3$

$$f(n) \downarrow n^2 \qquad n^{\log_b a} \downarrow \underline{n^3}$$

$\Theta(n^3)$

Gate :- $T(n) = 2T(n/2) + n^2$

$a=2, b=2$

$f(n) \qquad n^{\log_b a} = n^{\log_2 2}$

$\downarrow \qquad \downarrow$

$\underline{n^2} \qquad n$

$\theta(n^2)$

Gate:-

$$T(n) = 2T(n/2) + n$$

$$f(n) \qquad n^{\log_b a} = n^{\log_2 2} = n$$

$$\downarrow \qquad \downarrow$$

$$n \qquad n$$

$$\Theta(n \log n)$$

Gate:-

$$T(n) = 7T(n/2) + n^2$$

$a = 7 \quad b = 2$

$f(n) \quad n^{\log_b a} = n^{\log_2 7} = n^{2.81}$

$\downarrow \qquad \downarrow$

$n^2 \qquad n^{2.81}$

$$\Theta(n^{2.81})$$

Gate:- $T(n) = T(n/2) + n$

$a = 1 \quad b = 2$

$f(n) \qquad n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$\downarrow \qquad \downarrow$

$n \qquad 1$

$\Theta(n)$

Gate

$$T(n) = 2T(n/2) + n\log n$$

$a=2 \quad b=2$

$$n^{\log_b a} = n^{\log_2 2} = n$$

$f(n)$

$\downarrow$

$n\log n$

$n^{\log_b a}$

$\downarrow$

n

$$\Theta(n\log n \times \log n)$$

$$= \Theta(n(\log n)^2)$$

Gate:- $T(n) = 8T(n/2) + n^3 \log n$

$f(n)$ $\downarrow$ $n^3 \log n$

$n^{\log_b a} = n^{\log_2 8} = n^3$ $\Downarrow$ n^3

$\Theta(n^3 \log n \times \log n)$

$\Theta(n^3 (\log n)^2)$

$$T(n) = 2T(n/2) + n(\log n)^9$$

$a = 2 \quad b = 2$

$$f(n) \qquad n^{\log_b a} = n^{\log_2 2}$$

$$\downarrow \qquad \downarrow$$

$$n(\log n)^9 \qquad n$$

$$\Theta(n(\log n)^9 \times \log n)$$

$$= \Theta(n(\log n)^{10})$$

$$T(n) = 2T(n/2) + n^2 \log n$$

$a = 2 \quad b = 2$

$$n^{\log_b a} = n^{\log_2 2}$$

$\downarrow$

n^1

$f(n)$

$\downarrow$

$n^2 \log n$

$$\Theta(n^2 \log n)$$

$T(n) = 2^n T(n/2) + n^n$ ✓

not Const $\leftarrow$ $a = 2^n$ $\quad b = 2$

$n^{\log_b a} = n^{\log_2 2^n} = n^n$

$f(n) \downarrow n^n$ $\quad n^{\log_b a} \downarrow n^n$

$\Theta(n^n \log n)$ ✗

MT is not applicable

$$T(n) = \underline{0.5\,T(n/2)} + n$$

$$a = 0.5 < 1$$

$$\underline{a \geq 1}$$

MT is not applicable

Gate

$$T(n) = T(\sqrt{n}) + c \quad \text{if } n > 2$$
$$= 2 \quad \text{if } n = 2$$

assume $n = 2^k \Rightarrow k = \log_2 n$

$$T(2^k) = T(2^{k/2}) + c$$

assum $T(2^k) = \underline{S(k)}$

$$S(k) = S(k/2) + c$$

$f(k)$ $\quad k^{\log_b a} = k^{\log_2 1} = k^0 = 1$

$\downarrow$ $\quad \downarrow$

c $\quad c$

$$S(k) = \theta(c \log k) = \theta(\log k)$$
$$T(2^k) = \theta(\log k)$$
$$T(n) = \theta(\log \log n)$$

$$T(n) = T(\sqrt{n}) + c \; ; \; n > 2$$
$$= 2 \; ; \; n = 2$$

$$T(n) = T(\sqrt{n}) + c$$
$$= T(n^{1/4}) + c + c$$
$$= T(n^{1/8}) + c + c + c$$
$$= T(n^{1/2^3}) + 3c$$
$$\Big\downarrow k$$
$$= T(n^{1/2^k}) + \underline{k}c$$

$$n^{1/2^k} = 2 \Rightarrow \frac{1}{2^k} \log n = 1$$
$$\Rightarrow k = \log\log n$$

$$= T(2) + \log\log n \times c = \Theta(\log\log n)$$

$$T(n) = 2T(\sqrt{n}) + \log_2 n$$

assume $n = 2^k \Rightarrow k = \log n$

$$T(2^k) = 2T(2^{k/2}) + k$$

assume $T(2^k) = S(k)$

$$S(k) = 2S(k/2) + k$$

$f(k)$ $\qquad$ $k^{\log_b a}$

$\downarrow$ $\qquad$ $\downarrow$

k $\qquad$ k

$$S(k) = \Theta(k \log k)$$

$$T(2^k) = \Theta(k \log k)$$

$$T(n) = \Theta(\log n \log \log n)$$

Design

and

analysis

of

algo

THANK - YOU