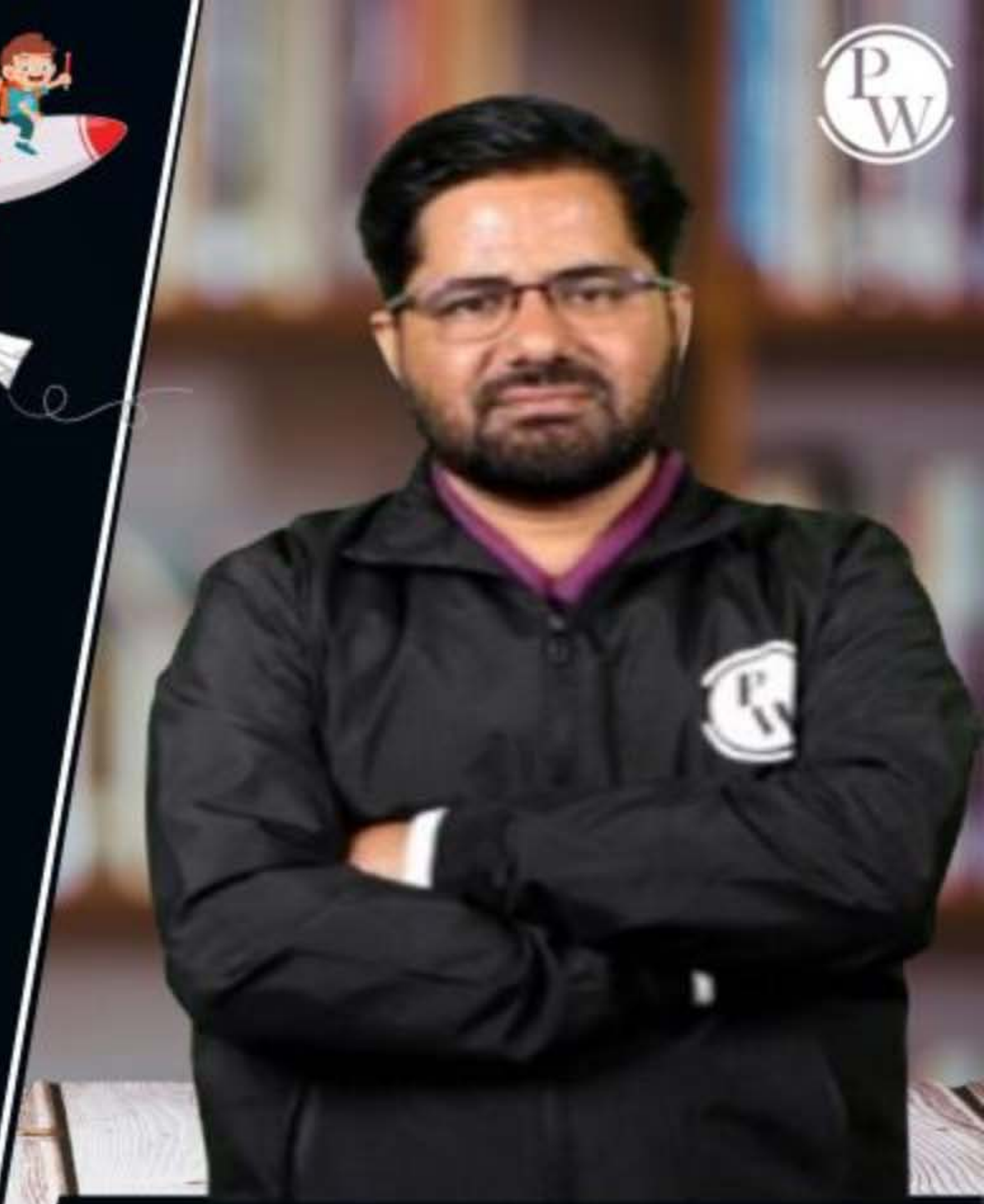
DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE

Not for CS/IT

Linear Algebra- II

Lecture No. 04



By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

BASICS OF VECTOR SPACE

(Part 1)

Topics to be Covered

Topic

(1) Remaining Part of Vector space

(2) Partition Matrix

If the vectors, $e_1 = (1,0,2), e_2 = (0,1,0)$ and $e_3 = (-2,0,1)$ from an orthogonal basis of the three dimensional real space $\mathbb{R}^3$, then the vector $u = (4,3-3) \in \mathbb{R}^3$ can be expressed as

(a) $u = -\frac{2}{5}e_1 - 3e_2 - \frac{11}{5}e_3$

(b) $u = -\frac{2}{5}e_1 - 3e_2 + \frac{11}{5}e_3$

(c) $u = -\frac{2}{5}e_1 + 3e_2 + \frac{11}{5}e_3$

(d) $u = -\frac{2}{5}e_1 + 3e_2 - \frac{11}{5}e_3$

$S = \{e_1\ e_2\ e_3\}$

$= \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

= Basis of R^3 (given)

∵ S is Basis for R^3 so $\boxed{\text{S will SPAN } R^3}$

⇒ Any Random Vector of R^3 (say $u = \begin{bmatrix} 4 \\ 3 \\ -3 \end{bmatrix}$)

Can be expressed as a Linear Combination of Vector in S

ie $\boxed{u = k_1 e_1 + k_2 e_2 + k_3 e_3}$ — (1)

$$\begin{bmatrix} 4 \\ 3 \\ -3 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + k_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

(M-I) Let us take (d)

$$\begin{bmatrix} 4 \\ 3 \\ -3 \end{bmatrix} = \frac{-2}{5} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \frac{11}{5} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$-3 = -\frac{4}{5} + 0 - \frac{11}{5} = \frac{-15}{5} = -3$$

$$k_1\begin{bmatrix}1\\0\\2\end{bmatrix}+k_2\begin{bmatrix}0\\1\\0\end{bmatrix}+k_3\begin{bmatrix}-2\\0\\1\end{bmatrix}=\begin{bmatrix}4\\3\\-3\end{bmatrix}$$

$$1.k_1+0k_2-2k_3=4$$
$$0k_1+1.k_2+0k_3=3$$
$$2k_1+0k_2+1.k_3=-3$$

$$\begin{bmatrix}1&0&-2\\0&1&0\\2&0&1\end{bmatrix}\begin{bmatrix}k_1\\k_2\\k_3\end{bmatrix}=\begin{bmatrix}4\\3\\-3\end{bmatrix}$$

$$\boxed{SX=U}$$

it is equally determined Non Homog System,

$\because |S|=\begin{vmatrix}1&0&-2\\0&1&0\\2&0&1\end{vmatrix}\neq 0 \Rightarrow$ unique sol. exist.

i.e $X=\begin{pmatrix}k_1\\k_2\\k_3\end{pmatrix}$ = unique values of k_1,k_2,k_3 exist.

$$[S:U]=\begin{bmatrix}1&0&-2&:&4\\0&1&0&:&3\\2&0&1&:&-3\end{bmatrix}\xrightarrow{R_3-2R_1}\begin{bmatrix}1&0&-2&:&4\\0&1&0&:&3\\0&0&5&:&-11\end{bmatrix}$$

i.e $SX=U \Rightarrow \begin{bmatrix}1&0&-2\\0&1&0\\0&0&5\end{bmatrix}\begin{bmatrix}k_1\\k_2\\k_3\end{bmatrix}=\begin{bmatrix}4\\3\\-11\end{bmatrix}$

$$\begin{bmatrix}(k_1-2k_3)\\k_2\\5k_3\end{bmatrix}=\begin{bmatrix}4\\3\\-11\end{bmatrix}\Rightarrow k_3=\frac{-11}{5},\quad k_2=3$$

$$k_1-2k_3=4\Rightarrow k_1=4+2\left(\frac{-11}{5}\right)=\frac{-2}{5}$$

So By (1) $U=-\frac{2}{5}e_1+3e_2-\frac{11}{5}e_3$. (d)

PROPERTIES of BASIS & DIMENSION →

① Basis of any vector space is not unique (T)

② If $\boxed{\dim(\text{V.Space}) = n}$ then $\boxed{\text{Any set of } n \text{ L.I vectors forms Basis}}$

for eg w.k.that $\dim(\mathbb{R}^3) = 3 \Rightarrow$ Any set of 3 L.I vectors forms Basis for $\mathbb{R}^3$

Explaination → for the vector space $V = \mathbb{R}^3$, standard Basis is $S_1 = \{e_1, e_2, e_3\}$

ie $S_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

if we take $S_2 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \right\}$ then S_2 is also Basis for $\mathbb{R}^3$

$= \{X_1 X_2 X_3\}$

CROSS CHECK: $\because |S_2| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{vmatrix} = \cdots\cdots = -42$ i.e $|S_2| \neq 0 \Rightarrow$ By Tricky Method X_1, X_2, X_3 are LI.

Let us take any Random vector $B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \in R^3$ where α, β, γ are Real Nos (i.e SCALARS)

Now taking the assumption that B is generated by X_1, X_2, X_3

then $X = K_1X_1 + K_2X_2 + K_3X_3$

OR $\boxed{K_1X_1 + K_2X_2 + K_3X_3 = B}$ — (1)

$K_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + K_2 \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} + K_3 \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$

$S_2 X = B$

$\because |S_2| \neq 0$ so By matrix Method, unique soln exist i.e

it is possible to find unique values of K_1, K_2, K_3 ie (1) is possible or our Assumption is Correct $\Rightarrow \boxed{S_2 \text{ SPANS } R^3} \Rightarrow S_2$ is Basis eg: Consider the vector space $V = R^4$ then it's Standard Basis is

$$S_1 = \{ e_1, e_2, e_3, e_4 \} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\} \Rightarrow \dim(R^4) = 4.$$

it's another Basis are as follows;

① $$S_2 = \left\{ \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \\ 2 \end{pmatrix} \right\} = \{ X_1, X_2, X_3, X_4 \}$$

$\because |S_2| = \cdots\cdots = 88$

ie $|S_2| \neq 0 \Rightarrow X_1, X_2, X_3, X_4$ are LI & hence S_2 forms Basis for R^4.

② $$S_3 = \left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \right\} = \{ Y_1, Y_2, Y_3, Y_4 \}$$

$\because |S_3| = \cdots\cdots = 5$ ie $|S_3| \neq 0$

$\Rightarrow Y_1, Y_2, Y_3, Y_4$ are LI

Hence S_3 also forms Basis for R^4.

eg: let us take $S_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\} = \{z_1\ z_2\ z_3\ z_4\}$

then S_4 also FORMS Basis for R^4. (False)

M-I $\because$ $\boxed{z_4 = z_1 + z_2 + z_3}$ i.e z_4 depends on z_1, z_2, z_3

or z_1, z_2, z_3, z_4 are LD so can not form Basis.

M-II $|S_4| = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0 \Rightarrow z_1, z_2, z_3, z_4$ are LD

(By Tricky Method)

It is given that $X_1, X_2, \ldots X_M$ are M non-zero, orthogonal vectors. The dimension of the vector space spanned by the 2M vectors $\{X_1, X_2, \ldots X_M, -X_1, -X_2, \ldots -X_M\}$ is

$S = \{x_1, x_2, x_3, \ldots, x_M, -x_1, -x_2, \ldots, -x_M\}$

ATQ, $\boxed{L(S) = V}$

(a) 2M

(b) M + 1

(c) M

(d) dependent on the choice of $X_1, X_2, \ldots X_M$

w.k.that orthogonal vectors are LI also.

i.e out of 2M vectors only $X_1, X_2, X_3, \ldots, X_M$ are LI.

So Basis $= \{X_1, X_2, X_3, \ldots, X_M\}$, $\Rightarrow \dim(V) = M$

If $v_1, v_2, \ldots, v_6$ are six vectors in R^4, which one of the following statements is False?

$\Rightarrow \dim(R^4) = 4.$

(a) If $\{v_1, v_3, v_5, v_6\}$ spans R^4, then it forms a basis for R^4. (T)

$= LD.$

(b) These vectors are not linearly independent. (T) $\because \dim = 4$

(c) It is not necessary that these vectors span R^4. (T)

(d) Any four of these vectors form a basis for R^4. (F)

$\because$ it is not necessary that these 4 selected vectors are also LI

$P = \begin{bmatrix} -10 \\ -1 \\ 3 \end{bmatrix}^T$, $Q = \begin{bmatrix} -2 \\ -5 \\ 9 \end{bmatrix}^T$ and $R = \begin{bmatrix} 2 \\ -7 \\ 12 \end{bmatrix}^T$ are three vectors. An $\boxed{\text{orthogonal}}$ set of vectors having a span that contains P, Q, R is

(a) $\begin{bmatrix} -6 \\ -3 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$ ✓ $\Rightarrow X_1 \cdot X_2 = 0$ i.e orthogonal

(b) $\begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 7 \\ -11 \end{bmatrix}$ $\begin{bmatrix} 8 \\ 2 \\ -3 \end{bmatrix}$ ✗ $\because X_1 \cdot X_2 \neq 0$

(c) $\begin{bmatrix} 6 \\ 7 \\ -1 \end{bmatrix}$ $\begin{bmatrix} -3 \\ 2 \\ -2 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 9 \\ -4 \end{bmatrix}$ ✗ $\because X_1 \cdot X_2 \neq 0$

(d) $\begin{bmatrix} 4 \\ 3 \\ 11 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 31 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$ ✗ $\because X_1 \cdot X_2 \neq 0$

M-II $X_1 = \begin{bmatrix} -6 \\ -3 \\ 6 \end{bmatrix}$, $X_2 = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$

Let $S = \{X_1, X_2\}$ then $P, Q, R \in L\{S\}$

$\because P = X_1 - X_2$ so $P \in L\{S\}$

$\because Q = X_1 + X_2$ so $Q \in L\{S\}$

& $R = X_1 + 2X_2$ so $R \in L\{S\}$

Hence S $\boxed{\text{SPANS}}$ P, Q, R

& S contains orthogonal vectors also

So Correct Ans is (a)

The following vector is linearly dependent upon the solution to the previous problem

(a) $\begin{bmatrix} 8 \\ 9 \\ 3 \end{bmatrix} = x_3$

(b) $\begin{bmatrix} -2 \\ -17 \\ 30 \end{bmatrix} = x_4 = 3x_1 + 4x_2$

MAG

(c) $\begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix} = x_5$

(d) $\begin{bmatrix} 13 \\ 2 \\ -3 \end{bmatrix} = x_6$

Solution of Last Question is

$S = \{x_1 \, x_2\} = \left\{ \begin{bmatrix} -6 \\ -3 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} \right\}$

(a) $A = [x_1 \, x_2 \, x_3] = \begin{bmatrix} -6 & 4 & 8 \\ -3 & -2 & 9 \\ 6 & 3 & 3 \end{bmatrix}$

$\because |A| = \text{-------} \neq 0$ ie LI

(b) $B = [x_1 \, x_2 \, x_4] = \begin{bmatrix} -6 & 4 & -2 \\ -3 & -2 & -17 \\ 6 & 3 & 30 \end{bmatrix}$

$\because |B| = \text{------} = 0$ ie x_1, x_2, x_4 are LD

(c) $C = [x_1 \, x_2 \, x_5] \because |C| \neq 0$ LI

(d) $D = [x_1 \, x_2 \, x_6] \because |D| \neq 0$ LI

PARTITION MATRIX

Application :→ Find det & E-values of

$$A=\begin{bmatrix}2&4&0&0\\0&-3&0&0\\0&0&1&-2\\0&0&3&2\end{bmatrix}_{4\times4}$$

(M-I) using Conventional App

$|A| = \text{-----} = \text{Do yourself.}$

& $|A-\lambda I| = 0 \text{ -----} \to \lambda = \text{Do yourself}$

(M-II)
$$A=\left[\begin{array}{cc|cc}2&4&0&0\\0&-3&0&0\\\hline 0&0&1&-2\\0&0&3&2\end{array}\right]=\begin{bmatrix}A_1&0\\0&A_2\end{bmatrix}$$

$|A| = |A_1|\cdot|A_2| = (-6)(2+6) = -48.$

E Values of $A_1=\begin{bmatrix}2&4\\0&-3\end{bmatrix}$ are $\lambda = 2 \,\&\, -3$

E Values of $A_2=\begin{bmatrix}1&-2\\3&2\end{bmatrix}$ are $\lambda = ?, ?$

$\lambda^2-(Tr)\lambda+(|A|)=0$

$\lambda^2-(3)\lambda+(8)=0$

$$\lambda=\frac{3\pm\sqrt{9-32}}{2}=\frac{3\pm\sqrt{-23}}{2}=1.5\pm i\frac{\sqrt{23}}{2}$$

PARTITION MATRIX (BLOCK MATRIX)

eg
$$\left[\begin{array}{cc|cc|c} 1 & 2 & -3 & 0 & 1 \\ \hline 2 & 0 & 1 & 2 & -1 \\ 3 & 1 & 2 & -1 & 0 \\ \hline 1 & 2 & 1 & 3 & 4 \end{array}\right]_{4\times 5}, \quad \left[\begin{array}{cc|cc|c} 1 & 2 & -3 & 0 & 1 \\ 2 & 0 & 1 & 2 & -1 \\ \hline 3 & 1 & 2 & -1 & 0 \\ 1 & 2 & 1 & 3 & 4 \end{array}\right]_{4\times 5}, \quad \left[\begin{array}{cc|cc|c} 2 & -1 & 3 & 4 & 0 \\ 1 & 2 & 0 & -1 & 2 \\ \hline 3 & -1 & 2 & 1 & 4 \\ 0 & 1 & 3 & 1 & -3 \\ \hline 1 & 2 & -1 & 2 & 1 \end{array}\right]_{5\times 5}$$

these Matrices are called partition Matrices.

Def^{n} – with the help of Horizontal & Vertical lines, we can convert given Matrix into a Partition Matrix.

Square Block Matrix →

eg $\left[\begin{array}{cc|cc} 1 & -2 & 4 & 3 \\ 2 & 0 & 1 & 4 \\ \hline 0 & 1 & -1 & 7 \\ 3 & 1 & 0 & 5 \end{array}\right]_{4\times4}$, eg $\left[\begin{array}{cc|cc|c} 1 & 3 & 1 & 6 & -2 \\ 0 & -1 & 5 & 7 & 0 \\ \hline 0 & 0 & 2 & 4 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ \hline 0 & 0 & 0 & 0 & -3 \end{array}\right]_{5\times5}$, eg $\left[\begin{array}{ccc|ccc} 1 & -2 & 4 & 1 & 3 & 2 \\ 2 & 4 & -1 & 0 & 1 & 4 \\ 1 & 1 & 2 & -1 & 0 & 3 \\ \hline -2 & 3 & 4 & 1 & 2 & 7 \\ 7 & 3 & 4 & -1 & 2 & 3 \\ 0 & 7 & 5 & 2 & -1 & 4 \end{array}\right]_{6\times6}$

eg $\left[\begin{array}{cc|cc|cc} 1 & 2 & 4 & -5 & 0 & 7 \\ 2 & 4 & 1 & -1 & 2 & 7 \\ \hline 0 & 7 & 5 & 1 & 2 & 3 \\ 4 & 1 & 0 & 2 & -1 & 4 \\ \hline 1 & 2 & -1 & 4 & 2 & 1 \\ 7 & 2 & 4 & -1 & 2 & 0 \end{array}\right]_{6\times6}$

Def^n: there are two necessary conditions

(1) No. of Horizontal lines = No of vertical lines

(2) Diagonal blocks must be square Matrices
(Not Necessarily of same order)

Block DIAGONAL MATRIX →

$$A = \begin{bmatrix} A_1 & 0 & 0 & \cdots & 0 \\ 0 & A_2 & 0 & \cdots & 0 \\ 0 & 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & A_n \end{bmatrix}_{n\times n}$$

① if Blocks exist only at Diag positions

② All the off Diag Blocks must be Null Matrices.

App: $\boxed{|A| = |A_1|\cdot|A_2|\cdot|A_3|\ldots\ldots|A_n|}$

= Product of Determinants of Diag Blocks.

eg $A = \left[\begin{array}{cc:cc} 4 & -2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 2 \end{array}\right]_{4\times4} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$

$|A| = (8)(2-0) = 16$

eg $A = \left[\begin{array}{cc:cc:cc} 4 & 2 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 7 & 1 \\ 0 & 0 & 0 & 0 & -5 & 2 \end{array}\right]_{6\times6} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix}$

$|A| = (-6)(-10)(19)$
$= 1140$

eg $A = \left[\begin{array}{cc:ccc:c} 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & -1 & 0 & 8 & 0 \\ 0 & 0 & 3 & 2 & 7 & 0 \\ 0 & 0 & 0 & 4 & -6 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 8 \end{array}\right]_{6\times6} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix}$

$|A| = (-2)(136)(8)$
$= -2176$

BLOCK U.T.M

If all the Blocks that lies below the Diagonal Blocks are Null Matrices then these Matrices are Block U.T.M

$$\boxed{|A| = \text{Product of Det of Diag Blocks}}$$

eg
$$A = \left[\begin{array}{cc|cc|c} 1 & 3 & 1 & 6 & -2 \\ 0 & -1 & 5 & 7 & 0 \\ \hline 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & -3 \end{array}\right] = \begin{bmatrix} A_1 & A_2 & A_3 \\ 0 & A_4 & A_5 \\ 0 & 0 & A_6 \end{bmatrix}$$

$$|A| = (-1)(-6)(-3) = -18$$

BLOCK L.T.M

if all the Matrices that lies above the Diag Blocks are Null Matrices then Matrices are Called Block L.T.M.

$$\boxed{|A| = \text{Product of Det of Diag Blocks}}$$

eg
$$A = \left[\begin{array}{cc|cc|c} 1 & 3 & 0 & 0 & 0 \\ 5 & -1 & 0 & 0 & 0 \\ \hline 0 & 7 & 2 & 4 & 0 \\ 3 & 6 & 1 & -1 & 0 \\ \hline -2 & 4 & 1 & 3 & -3 \end{array}\right] = \begin{bmatrix} A_1 & 0 & 0 \\ A_2 & A_3 & 0 \\ A_4 & A_5 & A_6 \end{bmatrix}$$

$$|A| = (-16)(-6)(-3) = -288$$

Q. if $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -1 & 5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 3 \end{bmatrix}_{4\times 4}$ then $|A| = ?$

Gate

$$= \begin{vmatrix} 1 & 2 & 0 & 0 \\ -1 & 5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 3 \end{vmatrix}$$

$$= (5+2)(6+0) = 42$$

Qus) Consider the block diagonal matrix A given.

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \Rightarrow A^{-1} = \begin{pmatrix} A_1^{-1} & 0 & 0 \\ 0 & A_2^{-1} & 0 \\ 0 & 0 & A_3^{-1} \end{pmatrix}$$

Tell what is A^{-1} ?

a) $\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1/5 \end{bmatrix}$ ✗

(b) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{5} & 0 & 0 & 0 \\ -\frac{1}{5} & \frac{3}{5} & 0 & 0 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$ ✗

c) $\begin{bmatrix} \frac{2}{5} & -\frac{1}{5} & 0 & 0 & 0 \\ -\frac{1}{5} & \frac{3}{5} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} \end{bmatrix}$ ✓

(d) A is invertible

$$A_1 = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow A_1^{-1} = \frac{1}{5}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 4 & 2 \\ 0 & 3 \end{bmatrix} \Rightarrow A_2^{-1} = \frac{1}{12}\begin{bmatrix} 3 & -2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{6} \\ 0 & \frac{1}{3} \end{bmatrix}$$

Q. If $A = \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 2 & 0 \\ \hline 0 & 0 & 1 \end{array}\right]$, $B = \left[\begin{array}{ccc|c} -1 & 1 & 2 & 5 \\ 3 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & -1 \end{array}\right]$ then evaluate AB using Indicated partitioning.

3×3 $\qquad\qquad$ 3×4

$$= \begin{bmatrix} P_{22} & Q_{21} \\ O_{12} & R_{11} \end{bmatrix} \qquad = \begin{bmatrix} S_{23} & T_{21} \\ O_{13} & U_{11} \end{bmatrix}$$

$$AB = \begin{bmatrix} P_{22} & Q_{21} \\ O_{12} & R_{11} \end{bmatrix} \cdot \begin{bmatrix} S_{23} & T_{21} \\ O_{13} & U_{11} \end{bmatrix} = \begin{bmatrix} (P_{22}S_{23} + Q_{21}O_{13}) & (P_{22}T_{21} + Q_{21}U_{11}) \\ (O_{12}S_{23} + R_{11}O_{13}) & (O_{12}T_{21} + R_{11}U_{11}) \end{bmatrix}_{3\times 4}$$

$$= \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}_{3\times 4}$$

15) Given a 3×3 block matrix

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ 0 & A_{22} & A_{23} \\ 0 & 0 & A_{33} \end{bmatrix} \Rightarrow \boxed{|A| = |A_{11}| \cdot |A_{22}| \cdot |A_{33}|}$$

where A_{11}, A_{22}, A_{33} are square matrices. If A is upper triangular & invertible, which of the following must be correct

(a) A_{11}, A_{22} & A_{33} must be invertible (T)

(b) A_{11}, A_{22}, A_{33} & A_{23} must be invertible

(c) $\det(A_{12}) = \det(A_{23}) = 0$

(d) $A_{12}, A_{23}, A_{11}, A_{22}, A_{33}$ must be invertible

Q. The orthogonal Quadratic form of $Q(x) = \underbrace{4x_1^2 + 4x_2^2 + x_3^2 - 2x_1x_2}$ will be?

2021

(a) $+3y_1^2 + 2y_2^2 + 4y_3^2$ $\because |A| = 15$ ×

(b) $6y_1^2 + 5y_2^2 + 2y_3^2$ $\because |A| = 15$ ×

(c) $3y_1^2 + 5y_2^2 + 1 \cdot y_3^2$ ✓

(d) $y_1^2 - y_2^2 + 6y_4^2$ $\because Tr = 9$ & $|A| = 15$ ×

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \to Tr = 9 \\ \to |A| = 15 \end{matrix}$$

How to find E Values of A $\boxed{\text{Quickly}}$??

Simply By using the Concept of partition Mat.

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \Rightarrow \lambda = 3, 5, 1.$$

$$A_1 = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \Rightarrow \lambda^2 - (8)\lambda + (15) = 0$$

$$(\lambda - 3)(\lambda - 5) = 0 \Rightarrow \lambda = 3, 5$$

The number of linearly independent eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is

4×4

(a) 1 (b) 2

(c) 3 (d) 4

M-I C-Equⁿ is $|A - \lambda I| = 0$

(GANDA METHOD)

$\lambda = 3, 4, \frac{3 \pm \sqrt{17}}{2}$

BMAG wala Method →

$$A = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$$

E-Values of $A_2 = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ are $\lambda = 3 \,\&\, 4$

E Values of $A_1 = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix}$ are $\lambda = \frac{3 \pm \sqrt{17}}{2}$

$\lambda^2 - (3)\lambda + (-2) = 0$

$\lambda = \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}$

i.e. all the E Values are diff ⇒ E Vectors are also LI

$$\boxed{n! = n(n-1)(n-2)\cdots\times 3\times 2\times 1}, \quad n \in N \ \& \ \underset{\text{Defined}}{\boxed{0! = 1}}$$

$\boxed{0! = 1}$ → sp. Case

$1! = 1\times 1 = 1$

$2! = 2\times 1 = 2$

$3! = 3\times 2\times 1 = 6$

$4! = 4\times 3\times 2\times 1 = 24$

$5! = 5\times 4\times 3\times 2\times 1 = 120$

$\div 5$, $\div 4$, $\div 3$, $\div 2$, $\div 1$

(*) Consider $X_1, X_2, X_3 \cdots X_r$ are given vectors

& $K_1, K_2, K_3 \cdots K_r$ are SCALARS.

then $\boxed{K_1X_1 + K_2X_2 + K_3X_3 + \cdots + K_rX_r = 0}$ is called (1)

Linear comb of vectors.

(i) for LD vectors, $K_1, K_2, K_3 \cdots K_r$ are $\boxed{\text{not all zero}}$ simultaneously

(ii) for LI vectors, Relationship (1) DNE exist

OR

Relationship (1) exist only when $\underbrace{K_1 = K_2 = K_3 = \cdots = K_r = 0}_{\text{All will zero simultaneously}}$.

ie we have a relationship of the type,

$\boxed{0X_1 + 0X_2 + 0X_3 + \cdots + 0X_r = 0}$ Senseless Relationship.

THANK - YOU

Tel: $\boxed{\text{dr puneet sir pw}}$