DS & AI
CS & IT

Probability & Statistics - I

Probability
Lecture : 05

Recap of previous lecture

Topic

BASICS of PROBABILITY (Part-4)

(Conditional Probability)

Topics to be Covered

Topic

BASICS of PROBABILITY (Part 5)

1. Law of Total Prob
2. BAYE'S Theorem

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

" If, what if, AGAR, YADI, TOH, ... "

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

Short RECAP

Operation	P&C	Prob	Formula	ME	Ind.
Either or	Plus	union	Addition Th	$P(A\cup B)=P(A)+P(B)$	*
AND	Multiply	Intersection	Multi Th	$P(A\cap B)=0$	$P(A\cap B)=P(A)\cdot P(B)$

Addition Th: $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

* for Independency: $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

for ME: $P(A\cup B)=P(A)+P(B)-0$

Q. Parcels are sending from Sender S to Receiver R sequentially through two post offices. The Probability of loosing an incoming parcel by each P.O is $\frac{1}{5}$ independently of all other parcels.

Given that, $\underbrace{\text{Parcel is lost}}_{\text{Condition}}$ then Find the prob that it was lost by 2^{nd} P.O?

$\Rightarrow$ R.E+P.

RECAP

(a) 1/5

(b) 4/5

(c) 4/25

(d) 4/9 ✓

Analysis: let No. of parcels sended by sender = 25 P.

ie $P(L) = \frac{1}{5}$

$P(NL) = 1 - \frac{1}{5} = \frac{4}{5}$

$$\underset{(25)}{\text{Sender}} \longrightarrow PO\text{-}1\,(20) \xrightarrow[NL=20]{\text{lost}=5} P.O.2\,(20) \xrightarrow[NL=16]{\text{lost}=4} \text{Reciever}\,(16P)$$

$$\underset{(50)}{\text{Sender}} \longrightarrow \underset{(50)}{PO\text{-}1} \xrightarrow[NL=40]{L=10} \underset{(40)}{PO.2} \xrightarrow[NL=32]{L=8} \underset{(32)}{\text{Reciver}}$$

Appr III (using Conditional Prob) $\rightarrow$ original Prob $= P(S) = 1$.

Reduced Prob $= P(\text{Condition}) = P(\text{parcel is lost})$ $\rightarrow$ ME

RECAP

$= P\{\text{either (Lost by } 1^{st}) \text{ or (Lost by } 2^{nd} \text{ P.O)}\}$

$= P(\text{Lost by } 1^{st} \text{ PO}) + P(\text{Lost by } 2^{nd} \text{ P.O})$

$= P(\text{Lost by } 1^{st}) + P\{(\text{NL by } 1^{st}) \;\&\; (\text{Lost by } 2^{nd})\}$

$= \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{9}{25}.$

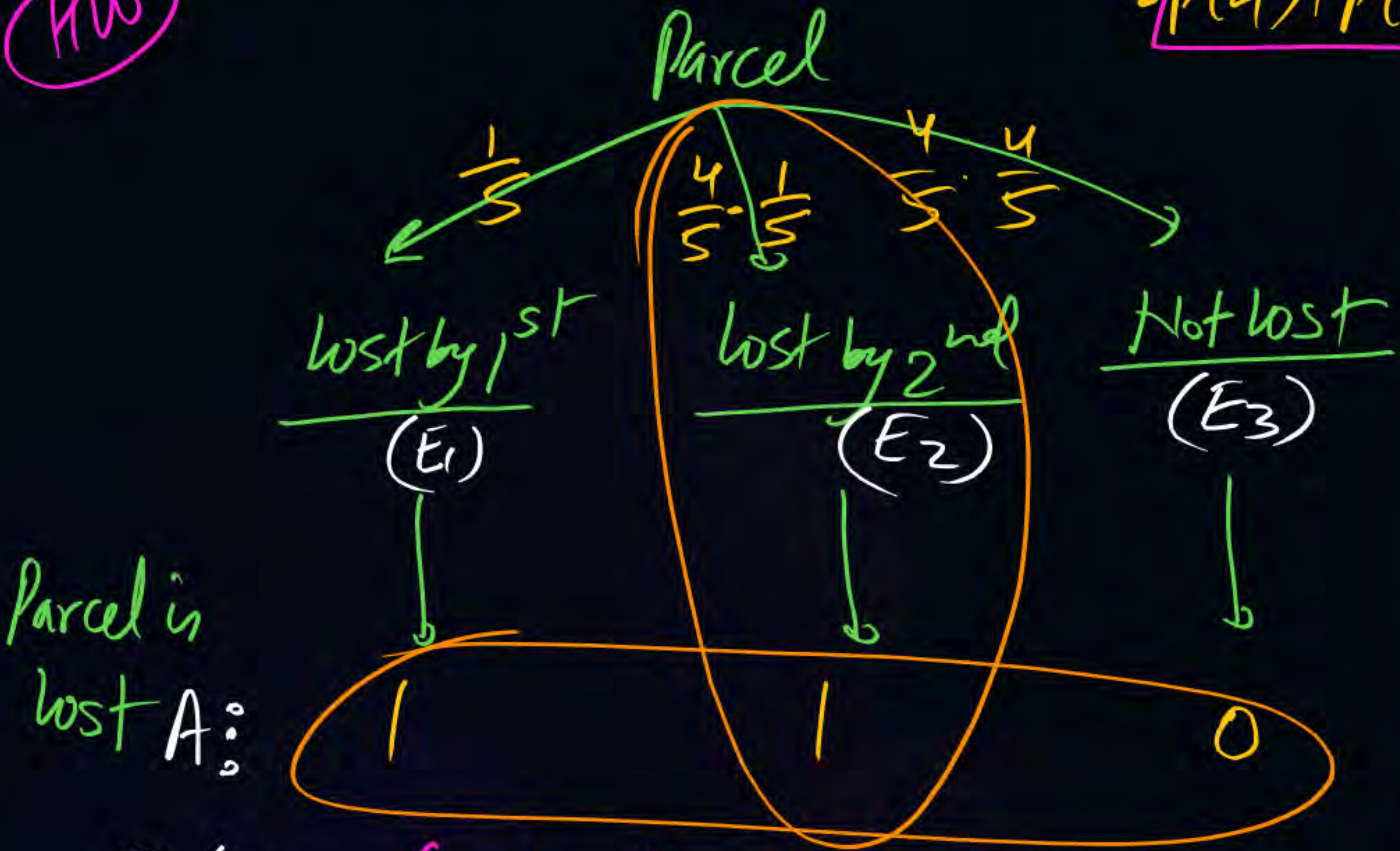
Fav Prob $= P(\text{Lost by } 2^{nd}) = P(\text{NL by } 1^{st} \;\&\; \text{L by } 2^{nd}) = \frac{4}{5} \times \frac{1}{5} = 4/25 = \frac{4/25}{1}$

Hence Cond^n Prob $= \frac{\text{Fav Prob}}{\text{R-Prob}} = \frac{4/25}{9/25} = \frac{4}{9}$

M-II (using Baye's Th) $\rightarrow$ $A = \{\text{Parcel is Lost}\}$

HW

$\therefore \boxed{P(E_1)+P(E_2)+P(E_3)} = \frac{1}{5}+\frac{4}{25}+\frac{16}{25} = 1$



$$P(A) = \left(\frac{1}{5}\times 1\right) + \left(\frac{4}{25}\times 1\right) + \left(\frac{16}{25}\times 0\right) = \frac{9}{25}$$

$$P\left(\text{Lost by 2nd} / \text{Parcel is lost}\right)$$

$$= P(E_2/A) = \frac{\frac{4}{25}\times 1}{\frac{9}{25}} = \frac{4}{9}$$

Note: $P(A/E_1) = P\left(\frac{\text{Parcel is lost}}{\text{lost by 1st}}\right) = 1$

$P(A/E_2) = P\left(\frac{\text{Parcel is lost}}{\text{lost by 2nd}}\right) = 1$

Formulatic Approach of Conditional Prob → RECAP

① $\boxed{P(A/B) = \dfrac{P(A\cap B)}{P(B)}}$ it is the prob of A when B has already occurred.

② $P(B/A) = \dfrac{P(B\cap A)}{P(A)}$ it is the prob of B when A has already occurred

③ $P(A\cap B/C) = \dfrac{P(A\cap B\cap C)}{P(C)}$ it is the prob of simultaneous occurrence of A & B when C has already occurred.

Sp. Note:- If A & B are Ind then

$P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{P(A)\cdot P(B)}{P(B)} = P(A)$

$P(B/A) = P(B)$

i.e In Case of Independency, Condition has NO Significance.

V.V. Special Note :- To check the Independency of Events, we have following three methods;

RECAP

M-I By defⁿ. (Best Method)

M-II if $[P(A \cap B) = P(A) \cdot P(B)] \Longleftrightarrow$ then [A & B are Called Ind]

M-III if $[P(A/B) = P(A)]$ then $\Longleftrightarrow$ [A & B are Ind.]

(*) The Relation of Dependency or Independency is a Vice-Versa Relation.

A box contains 4 while balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is

(a) 1/3 (b) 3/7

(c) 1/2 (d) 4/7

$\begin{bmatrix} 4W \\ 3R \end{bmatrix} \xrightarrow{W} \begin{bmatrix} 3W \\ 3R \end{bmatrix}$

* "one by one w/o Replacement"

Apb III Req Prob $= P[W \cap R]$

$= 1 \times \frac{3}{6} = \frac{1}{2}$

IPW

Apb I

original SSp = { }

R-SSp = { }

fav cases = { }

then Ans = ??

M-II / App I → $\begin{bmatrix} 4W \\ 3R \end{bmatrix}$ Balls are drawn one by one w/o Replacement.

$S = \{(\underline{W}W), (\underline{W}R), (RW), (RR)\} = 4$ pair

R.S.Sp $= \{(\underline{W}W), (\underline{W}R)\} = 2$ pair

fav pair $= \{(\underline{W}R)\} = 1$

So Conditional Prob $= \dfrac{\text{fav pair}}{\text{R.S.Sp}} = \dfrac{1}{2}$

App III Req Prob $= P[W \cap R]$

$= 1 \times \dfrac{{}^3C_1}{{}^6C_1} = 1 \times \dfrac{3}{6} = \dfrac{1}{2}$

$\begin{bmatrix} 4W \\ 3R \end{bmatrix} \xrightarrow{W} \begin{bmatrix} 3W \\ 3R \end{bmatrix}$

= unfair = BIASED = Loaded Die

Qs (2009/EE) Consider an $\boxed{\text{unbalanced die}}$ numbered 1 to 6.

The prob of an odd face is 90% the prob of an Even face. &

The prob of any even numbered face is same.

The prob of an Even face, given that face value exceeds 3 is 0.75

then find the prob that face value exceeds 3?

(a) 10/19

(b) 10/57

(c) 8/3

(d) 80/171

$P(\text{odd face}) = \frac{90}{100} P(\text{even face})$

$P(2) = P(4) = P(6) = x$ (let)

$P\left[\frac{\text{Even } f}{f>3}\right] = 0.75$

$P[f>3] = ?$

$S = \{1, 2, 3, 4, 5, 6\}$

ie Individual outcomes are ME

w.k. that $P(\text{odd } f) + P(\text{even } f) = P(S)$

$\frac{9}{10} P(\text{even } f) + P(\text{even } f) = 1$

$P(\text{even } f)\left[\frac{9}{10} + 1\right] = 1 \Rightarrow P(\text{even } f) = \frac{10}{19}$

i.e. $P(\text{even } f) = \frac{10}{19}$

$P(2 \text{ or } 4 \text{ or } 6) = \frac{10}{19}$

$P(2) + P(4) + P(6) = \frac{10}{19}$

$x + x + x = \frac{10}{19}$

$x = \frac{10}{57}$

w.k.that, $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Similarly $P\left[\text{Even } f / f > 3\right] = 0.75$

$\frac{P[(\text{Even } f) \cap (f > 3)]}{P(f > 3)} = \frac{3}{4}$

$\frac{P(4 \text{ or } 6)}{P(f > 3)} = \frac{3}{4}$

$\frac{P(4) + P(6)}{P(f > 3)} = \frac{3}{4}$

$\frac{x + x}{P(f > 3)} = \frac{3}{4}$

$P(f > 3) = \frac{8x}{3} = \frac{8}{3} \times \frac{10}{57}$

$= \frac{80}{171}$

(d)

General Discussion

① E_1 & E_2 are ME $\Longrightarrow$ $E_1 \cap E_2 = \phi$ → $P(E_1 \cap E_2) = 0$

→ $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

② E_1 & E_2 are Exhaustive $\Rightarrow$ $E_1 \cup E_2 = S$

③ Let E_1 & E_2 are ME as well as Exhaustive then $\boxed{P(E_1) + P(E_2) = 1}$

$E_1 \cap E_2 = \phi$

$E_1 \cup E_2 = S$

$\Downarrow$

$P(E_1 \cup E_2) = P(S)$

$P(E_1) + P(E_2) = 1$

④ if E_1, E_2, E_3 are ME & Exhaustive then $\boxed{P(E_1) + P(E_2) + P(E_3) = 1}$

Exhaustive Events →

if $\boxed{E_1 \cup E_2 \cup E_3 = S}$ $\Longleftrightarrow$ then [E_1, E_2, E_3 are called Exhaustive events]

M.E Events : if $\boxed{E_i \cap E_j = \phi}$ $\forall\, i \neq j$ then $\Longleftrightarrow$ [E_1, E_2, E_3 are called ME Events.]

ME & Exhaustive Events →

if $\boxed{\underset{\text{ME Nature}}{E_i \cap E_j = \phi} \;\&\; \underset{\text{Exhaustive Nature}}{E_1 \cup E_2 \cup E_3 = S}}$ $\Longleftrightarrow$ then [Events are called ME as well as Exhaustive]

Conclusion :→ $P(E_1 \cup E_2 \cup E_3) = P(S) \Longrightarrow \boxed{P(E_1) + P(E_2) + P(E_3) = 1}$

it is the Necessary & sufficient cond^n for ME & Exhaustive events.

eg: $S_{Die} = \{1, 2, 3, 4, 5, 6\}$ → $E_1 = \{1, 3, 5\}$, $P(E_1) = \frac{1}{2}$

→ $E_2 = \{2, 4, 6\}$, $P(E_2) = \frac{1}{2}$

$\because E_1 \cap E_2 = \phi \Rightarrow$ ME & $E_1 \cup E_2 = S$ is Exhaustive

$\therefore \boxed{P(E_1) + P(E_2) = 1}$ Hence verified.

eg $S_{Die} = \{1, 2, 3, 4, 5, 6\}$ → $E_1 = \{1, 2\}$, $P(E_1) = \frac{2}{6} = \frac{1}{3}$

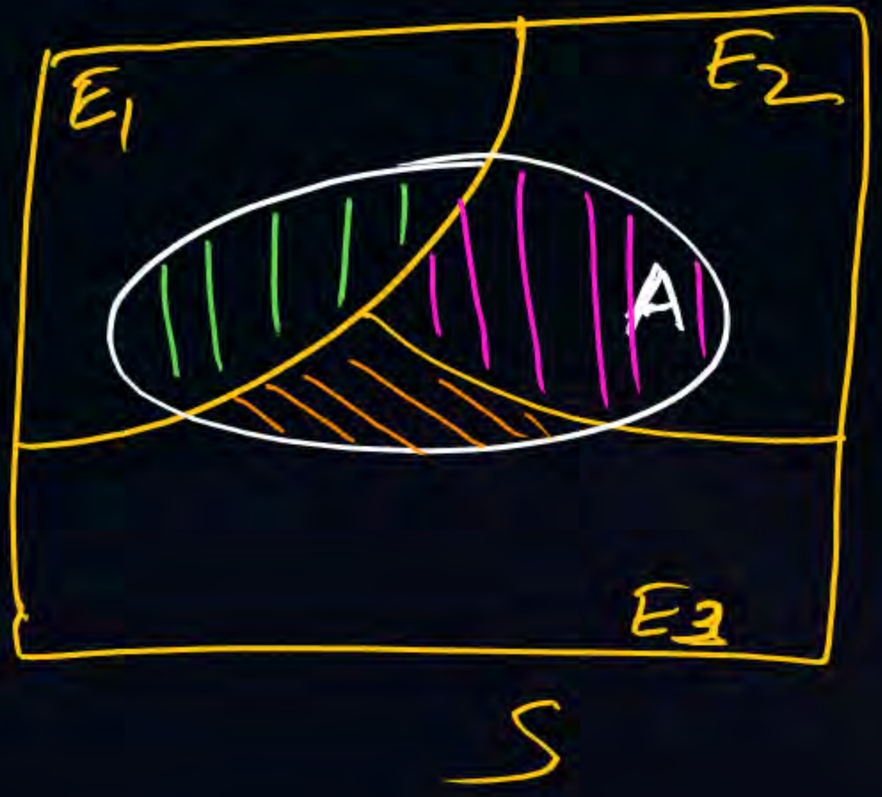
→ $E_2 = \{3\}$, $P(E_2) = \frac{1}{6}$

→ $E_3 = \{4, 5, 6\}$, $P(E_3) = \frac{3}{6} = \frac{1}{2}$

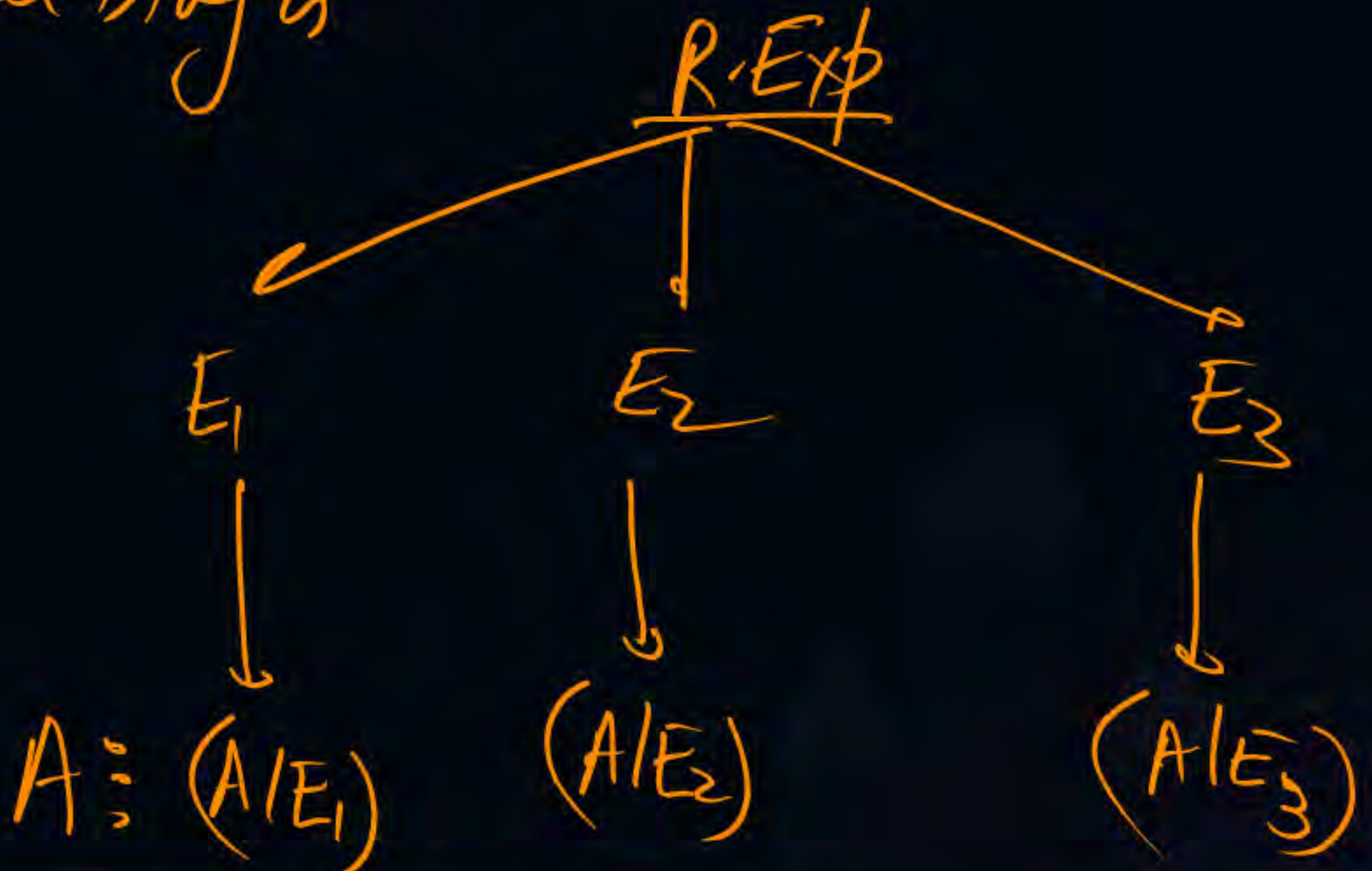
$\because E_i \cap E_j = \phi$ is ME, & $E_1 \cup E_2 \cup E_3 = S$ is Exhaustive

$\therefore P(E_1) + P(E_2) + P(E_3) = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = 1$ Hence Verified

Law of Total Prob → Let E_1, E_2, E_3 are ME & Exhaustive events associated with S.Space S & A is an Event which can occur with all E_1, E_2, E_3 i.e



& it's Tree Diag is

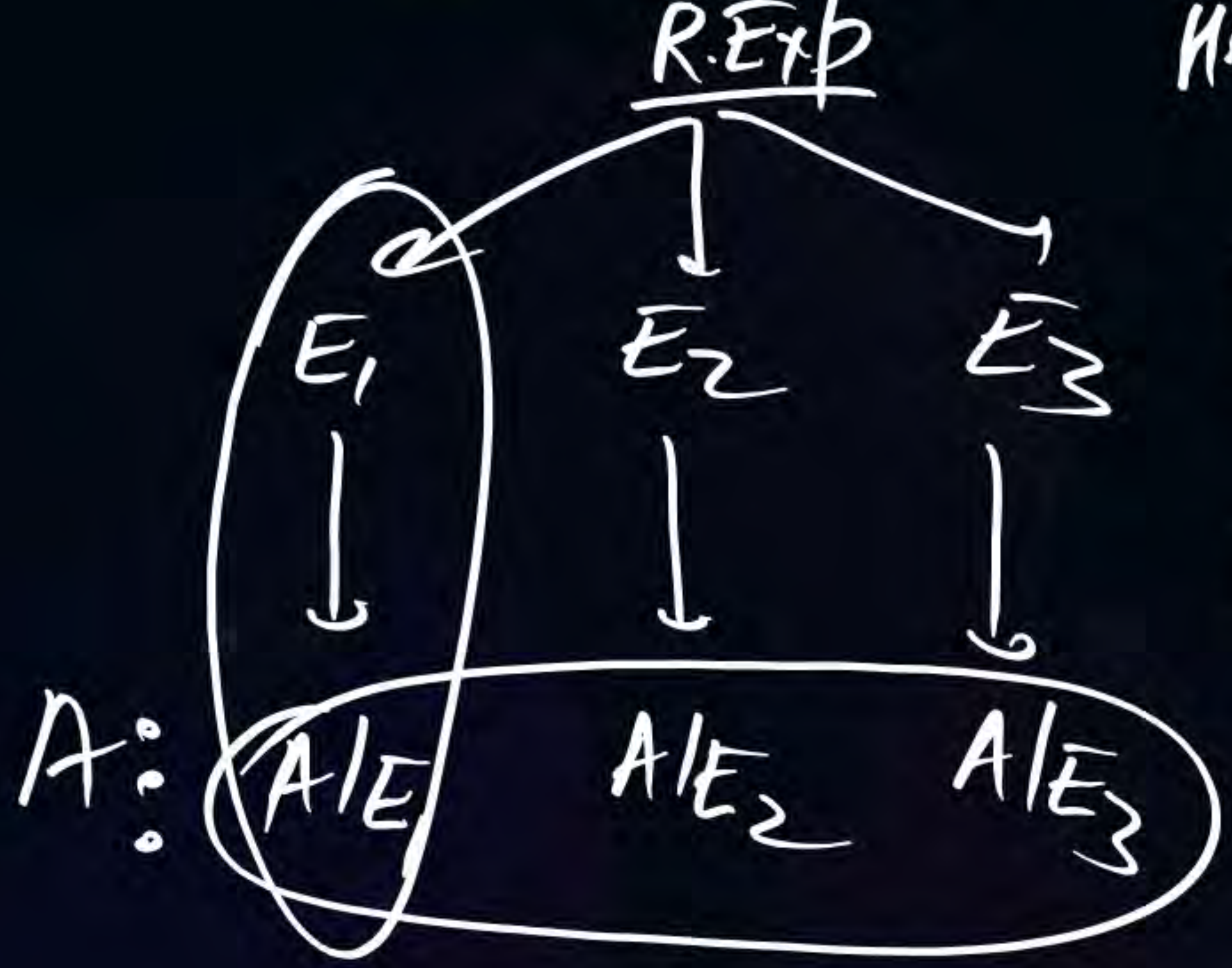


then $$\boxed{P(A) = P(E_1) \cdot P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

Baye's Theorem (Inverse prob theorem) →

(this theorem is useful to solve complex questions of conditional probability.)
(= tough)

Theory same as above



Here $$\boxed{P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} \quad (1)$$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(A)}$$

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(A)}$$

$$P(E_3/A) = \frac{P(E_3)P(A/E_3)}{P(A)}$$

Baye's Th.

Important Points :-

① Necessary condition for the existence of Law of Total Prob & Baye's Th is Associated events must be ME & Exhaustive.

② In Law of Total Prob : A = { Assume that event as A which is Required }

③ In Baye's Th : A = { Assume that event as A, which is given as condition }

④ If in a Question, there is a feeling of $\boxed{\text{CROSS Check}}$ the given Condition we can use Baye's Th.
& if we have No Condition in a Question (or No Feeling of CROSS Check) then use Law of Total Prob.

Q. Computers are supplied to an organisation according to Chart; → R.Exp.

Company	% of Computers supplied	Prob of being Defective
X	60%	$0.01 = 1/100$
Y	30%	$0.02 = 2/100$
Z	10%	$0.03 = 3/100$

① Find the Prob that supplied Computer is defective.

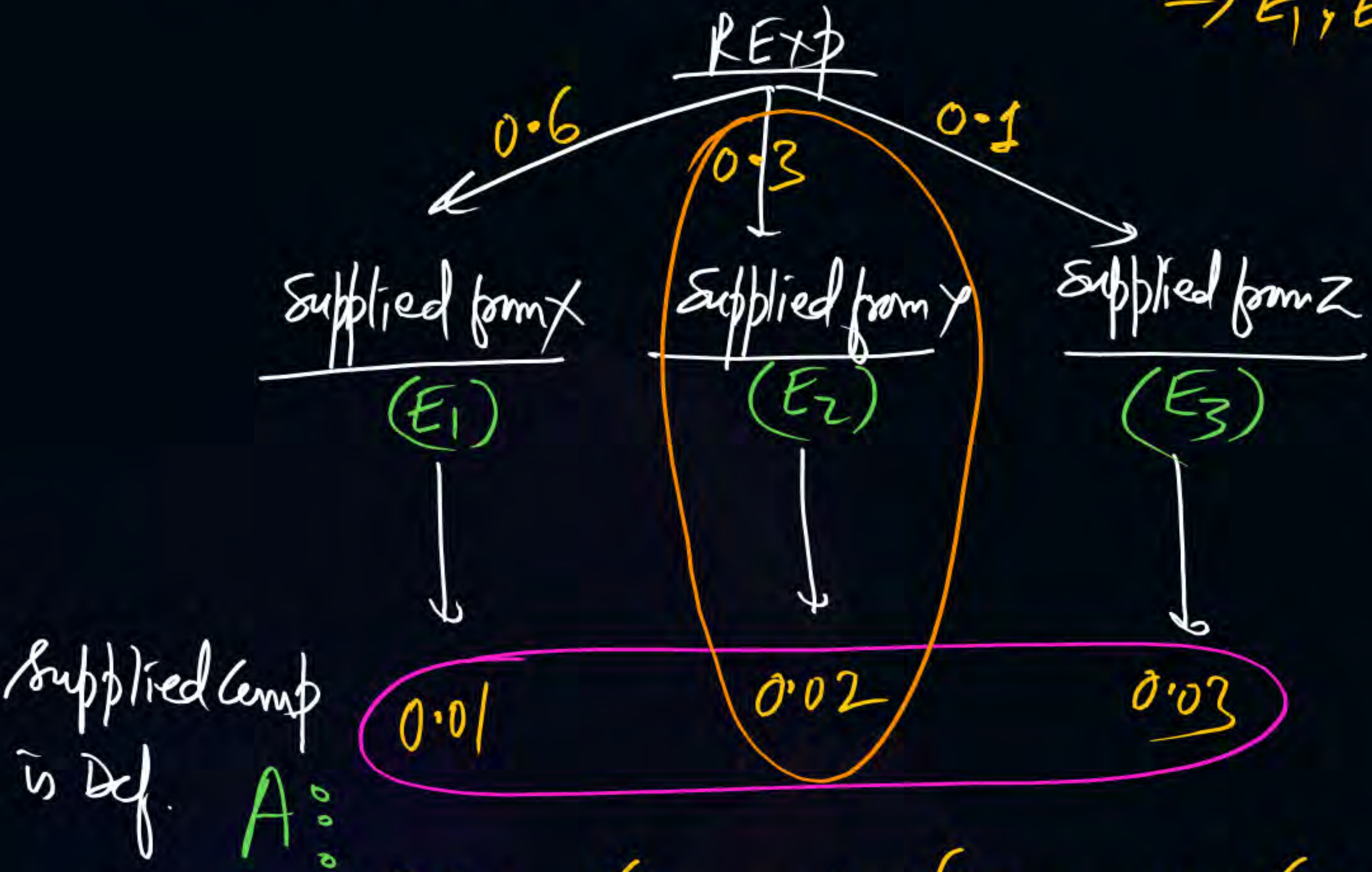
② If supplied Comp is Defective then Find the Prob that it was supplied from Company Y.

Condition

$A = \{\text{Supplied Comp is Defective}\}$

$\because P(E_1) + P(E_2) + P(E_3) = 0.6 + 0.3 + 0.1 = 1$

$\Rightarrow E_1, E_2, E_3$ are ME & Exhaustive



$$P(E_2/A) = \frac{P(\text{V.Path})}{P(\text{H.Path})} = \frac{P(\text{Fav Path})}{P(\text{Conditional Path})} = \frac{0.3 \times 0.02}{0.015} = \frac{6}{15} \quad \underline{\text{Ans}}$$

① So $P(A) = (0.6 \times 0.01) + (0.3 \times 0.02) + (0.1 \times 0.03) = 0.015 = \frac{15}{1000}$ $\underline{\text{Ans}}$

Note :- (1) Also Evaluate $P(E_1/A) = ? = \frac{\text{fav path}}{P(A)} = \frac{0.6 \times 0.01}{0.015} = \frac{6}{15}$

& $P(E_3/A) = ? = \frac{\text{fav path}}{P(A)} = \frac{0.1 \times 0.03}{0.015} = \frac{3}{15}$

(2) out of 1000 supplied computers only 15 are defective & out of 15 defective computers, 6 are coming from X, 6 from Y & 3 from Z.

General Note useful for Next Q :-

If we are using Guess Method to ans an objective type question having 4 choices in which only one is correct then $P(\text{Choosing Correct Ans}) = \frac{{}^1C_1}{{}^4C_1} = \frac{1}{4}$

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver and a truck driver is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

= R.Exp.

Condition

(a) 0.019

(b) 0.086

(c) 0.19

(d) 0.86

$A = \{$ Insured person meets with an Accident $\}$

$P(E_1) = \frac{2000}{12000}$, $P(E_2) = \frac{4000}{12000}$, $P(E_3) = \frac{6000}{12000}$

$$\begin{array}{ccccc} & & \text{R.Exp} & & \\ & \swarrow \frac{1}{6} & \downarrow \frac{1}{3} & \searrow \frac{1}{2} & \\ \text{S.D } (E_1) & & \text{C.D } (E_2) & & \text{T.D } (E_3) \\ \downarrow & & \downarrow & & \downarrow \\ 0.01 & & 0.03 & & 0.15 \end{array}$$

Person meets with an Accident A:

$\because P(E_1) + P(E_2) + P(E_3) = \frac{1}{6} + \frac{1}{3} + \frac{1}{2} = 1$

Hence Necessary & Suff. Condn Holds

$$P(E_1/A) = \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15}$$

$$= 0.019$$

(ii) Also find the prob that person meets with an Accident = ?

$$P(A) = ? = \left(\frac{1}{6} \times 0.01\right) + \left(\frac{1}{3} \times 0.03\right) + \left(\frac{1}{2} \times 0.15\right)$$

$$= 0.086 = \frac{86}{1000} = \frac{1032}{12000}$$

out of 1000 Insured persons, 86 Persons will meet with an Accident.

i.e " " 12000 Insured " , 1032 Persons " " " "

Q. In a town 10% of the population is Covid +ve. A new diagnostic kit arrives in the market. This kit correctly identifies Covid +ve individual 95% of time & Covid −ve individual 89% of time.
A person is tested by this kit & is found to be +ve then find the prob that person is actually +ve

R.E+P

condition

Sol: $A = \{$Person is declared +ve by kit$\}$

$A/E_1 = \{$Person declared +ve by kit when (Person is +ve)$\}$

$= \{$kit is giving true Result for +ve Person$\} \Rightarrow P(A/E_1) = 0.95$

$A/E_2 = \{$Person declared +ve by kit when Person is −ve$\}$

$= \{$kit is giving wrong Result for −ve Person$\} \Rightarrow P(A/E_2) = 0.11$

Person

0.1 Covid +ve (E_1)

0.9 Covid −ve (E_2)

Person declared +ve by kit A:

$A/E_1 = 0.95$ $\quad$ $A/E_2 = 0.11$

$$P(A) = (0.1 \times 0.95) + (0.9 \times 0.11) = 0.194 = \frac{194}{1000}$$

$$P(\text{Actually +ve}) = P\left(E_1/A\right)$$

$$= \frac{0.1 \times 0.95}{0.194} = \frac{95}{194}$$

Ans

General Observation →

① If we have 100 Covid +ve persons Tested by kit

→ P(+vely declared) = 95 (True Result)

→ P(−vely declared) = 5 (False Result)

② If we have 100 Covid −ve persons tested by kit

→ P(−vely declared) = 89 (True Result)

→ P(+vely declared) = 11 (False Result)

out of 1000 persons tested by kit, 194 are declared +ve by kit & out of 194 +vely declared persons by kit, 95 are Actually +ve.

PODCAST (Deep Analysis) →

① 1000 persons → 100 Actually +ve $\xrightarrow{95\%}$ 95 are declared +ve by kit (True)

100 Actually +ve $\xrightarrow{5\%}$ 5 are declared −ve by kit (False)

1000 persons → 900 Actually −ve $\xrightarrow{89\%}$ 801 are declared −ve by kit (True)

900 Actually −ve $\xrightarrow{11\%}$ 99 are declared +ve by kit (False)

② 1000 persons → 194 are declared +ve by kit → 95 are actually +ve

194 are declared +ve by kit → 99 are actually −ve

1000 persons → 806 are declared −ve by kit → 801 are actually −ve

806 are declared −ve by kit → 5 are actually +ve

Conclusion: Hence Sir has justified the THUMB RULE of Class.

Q. In a town there are equal number of Man & Woman in which 80% M & 50% W are employed. A person is selected at Random then find the prob that person is an unemployed person

R.Exp $\Rightarrow$ this Quest is Based on L.T.P (given by S.Y).

(a) 65%

(b) 50%

(c) 35%

(d) None

Person

$\xrightarrow{0.5}$ Man (E_1) $\longrightarrow$ $A/E_1 = 0.2$

$\xrightarrow{0.5}$ Woman (E_2) $\longrightarrow$ $A/E_2 = 0.5$

A:

A = { Person is an unemployed person }

So $P(A) = (0.5 \times 0.2) + 0.5 \times 0.5$

$= 0.35 = 35\%$ chance.

Q. A Person is known to speak Truth 3 out of 4 times.

(HW) He throw a die & Reports that it is six then Find the prob that it is Actually six?

Ans = $\frac{3}{8}$

Solve it by two diff Methods

Q:- From a pack of 52 Cards, while shuffeling one Card is lost & then two Cards are drawn at Random then find the prob that both the selected Cards are of Spade?

HW

Ans = $\frac{1}{17}$

NIRMAN
2025

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Thank

you

Keep Hustling!