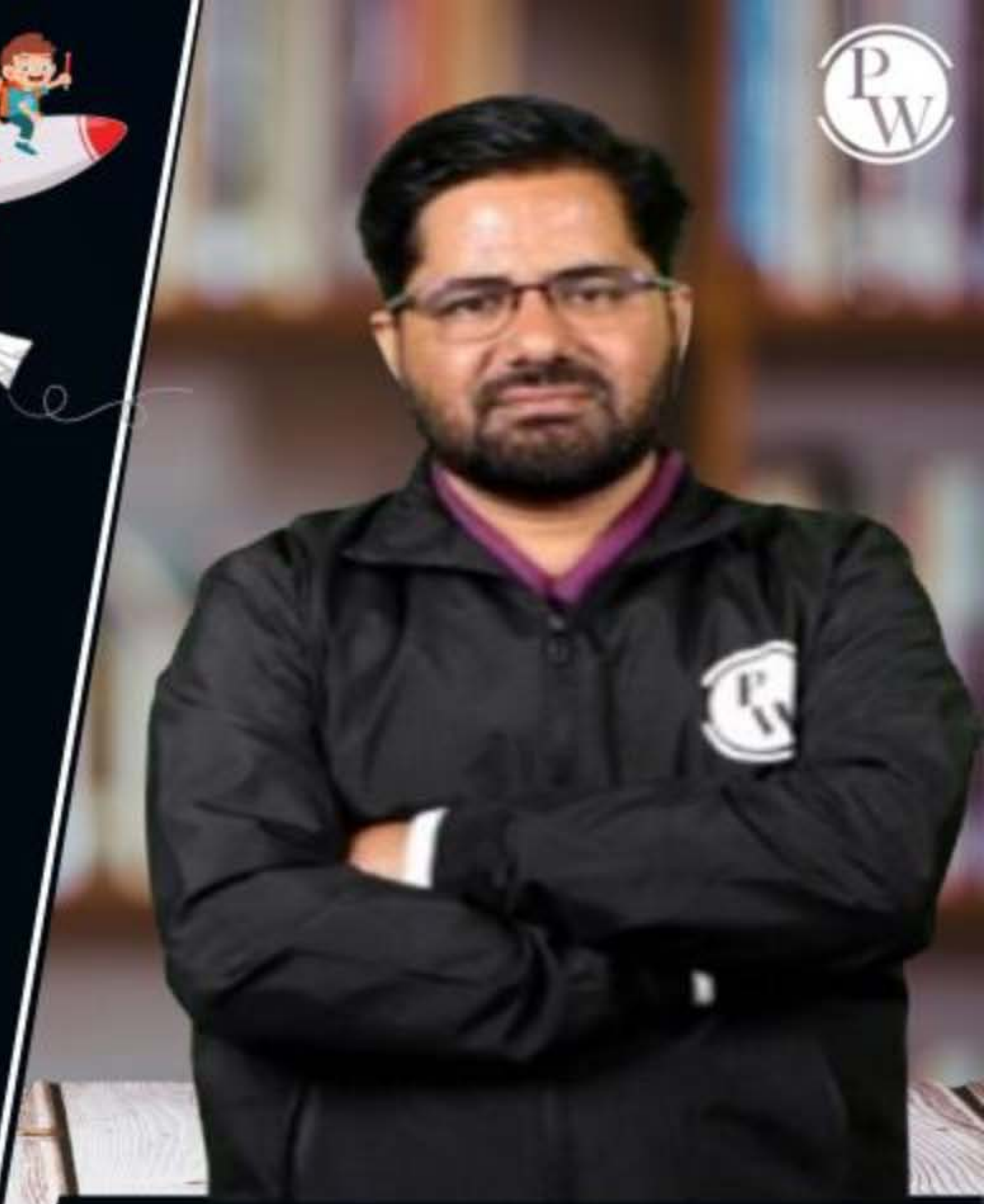
GATE

DS & AI
CS & IT

Linear Algebra

Lecture No. 13



By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

① EIGEN VALUES

② Cayley Hamilton Theorem

Topics to be Covered

Topic EIGEN VALUES - EIGEN VECTORS

1. EIGEN VECTORS
2. DIAGONALISATION.

EIGENVECTORS: If $A_{n\times n}$ s.t. $\boxed{AX = \lambda X}$ or $\boxed{(A-\lambda I)X = 0}$

$\lambda \to$ E-Value, $X \to$ Eigen Vector.

eg $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

$\lambda_1 = 2$, $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\because AX_1 = 2X_1$

$\lambda_2 = 6$, $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\because AX_2 = 6X_2$

Now our aim is to know the procedure of finding E vectors?

(Have patience)

Note: 1st I will write all the properties at one place & then explain procedure.

Properties of E-Vectors →

(1) if X is an E Vectors for λ then (KX) is also an E Vector for λ. $\Longleftrightarrow$

Proof: $AX = \lambda X \Rightarrow (A - \lambda I)X = 0$ —— (1) $(K \neq 0)$

$$K(A - \lambda I)X = K \cdot 0$$

$$(A - \lambda I)(KX) = 0 \text{ —— (2)}$$

ie we are free to Multiply or divide with any constant in case of E. vectors.

(2) Shortcut Method of finding E.Vectors: (if options are given)

" λ should be unique in all Comparisions & there should not be any INVALID Result "

(3) E-Vectors corresponding to different E-Values of Symm Mat are orthogonal

& Hence they are LI also

$\Rightarrow$ A is Symm.

Q if $A_{3\times3}$ s.t $A^T = A$ and a, b are the E-Values & Corresponding

(PYQ) E-Vectors are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ & $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ then for $a \neq b$ the value of

E Values are different

$x_1y_1 + x_2y_2 + x_3y_3 = \underline{\ ?\ }$

Sol: By above Prop, $X \cdot Y = 0 \Rightarrow x_1y_1 + x_2y_2 + x_3y_3 = 0$ Ans

(4) For Different E-Values, Corresponding E-Vectors are also L.I (T)

(5) If E-Value Repeats (then Headache will start) then Corresponding E Vectors may be LI or may be LD

(6) **MODAL MATRIX** :→ Matrix formed by E-Vectors is Called Modal Mat

Let $A_{3\times3}$ is the given Mat & X_1, X_2, X_3 are the E-Vectors then

Modal Mat is $P = [X_1 \; X_2 \; X_3]$

(7) Algebraic Multiplicity (A.M) Number of times particular eigen value repeats is called it's A.M

(8) Geometric Multiplicity (G.M) Number of L.I E vectors for particular E Value λ is called it's G.M

(9) Shortcut of Finding GM (for λ) $\rightarrow$ Let λ is the Repeated E Value then

$$\text{GM of } \lambda = \boxed{\text{order} - \rho(A - \lambda I)}$$

(Counting of L.I E.Vectors for λ)

Note: For Non Repeated E. Value $\begin{cases} AM = 1 \\ GM = 1 \end{cases}$

Qs Find the E. Values & E. Vectors of $A=\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}$

Sol: C. Equⁿ is $\lambda^2-(TrA)\lambda+(|A|)=0 \Rightarrow \lambda^2-(8)\lambda+(12)=0 \Rightarrow \lambda=2\ \&\ 6.$

E. Vector for $\lambda=2$:→

Consider $AX=\lambda X$

$$(A-\lambda I)X=0$$

$$(A-2I)X=0$$

$$\begin{bmatrix}(4-2) & 2\\ 2 & (4-2)\end{bmatrix}X=0$$

$$\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}X=0$$

R_2-R_1

$$\begin{bmatrix}2 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}(2x_1+2x_2)\\ 0\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \Rightarrow \left.\begin{matrix}2x_1+2x_2=0\\ \&\ 0=0\end{matrix}\right\}$$

ie $x_1=-x_2$ Let $x_2=K$, $x_1=-K$

So $X=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}-K\\ K\end{bmatrix}=K\begin{bmatrix}-1\\ 1\end{bmatrix}$

ie for $\lambda=2$, $X=\begin{bmatrix}-1\\ 1\end{bmatrix}$

E-Vector for $\lambda = 6$ $\rightarrow$

Consider $AX = \lambda X$

$$(A - \lambda I)X = 0$$

$$(A - 6I)X = 0$$

$$\begin{bmatrix} (4-6) & 2 \\ 2 & (4-6) \end{bmatrix} X = 0$$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} X = 0$$

$R_2 + R_1$ $$\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (-2x_1 + 2x_2) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{aligned} -2x_1 + 2x_2 &= 0 \\ 0 &= 0 \end{aligned} \right\} \Rightarrow x_1 = x_2 = K \text{ (let)}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} K \\ K \end{bmatrix} = K \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

i.e. for $\lambda = 6$, $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

i.e. for $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ $\rightarrow$ $\lambda_1 = 2$, $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$; $\lambda_2 = 6$, $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

ANALYSIS (PODCAST) →

(1) $A=\begin{bmatrix}4&2\\2&4\end{bmatrix}$

$\lambda_1=2,\ X_1=\begin{bmatrix}-1\\1\end{bmatrix} \longrightarrow \begin{bmatrix}1\\-1\end{bmatrix},\begin{bmatrix}-2\\2\end{bmatrix},\begin{bmatrix}-3\\3\end{bmatrix},\begin{bmatrix}-1/5\\1/5\end{bmatrix},\begin{bmatrix}0.5\\-0.5\end{bmatrix}\ldots\ldots \infty$ E.V exist

But all are LD on X_1

$\lambda_2=6,\ X_2=\begin{bmatrix}1\\1\end{bmatrix} \longrightarrow \begin{bmatrix}-1\\-1\end{bmatrix},\begin{bmatrix}2\\2\end{bmatrix},\begin{bmatrix}1/4\\1/4\end{bmatrix},\begin{bmatrix}-5\\-5\end{bmatrix},\ldots\ldots \infty$ EV. exist.

But all are L.D on X_2

(2) Here X_1 & X_2 are LI (By observation)

(3) In fact, we are getting two L.I Families of E Vectors. i.e if we take one vector from 1st family & another vector from 2nd family then both will be LI

(4) Mat A has at Most two Ind E Vectors $\boxed{\text{at a time}}$

(5) $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ = Symm Mat

$\lambda = 2,\ X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\lambda = 6,\ X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Now $X_1 \cdot X_2 = (-1)(1) + (1)(1) = 0$

ie X_1 & X_2 are orthogonal.

(6) one pair of L.I, E. Vectors of $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ will be?

(a) × $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ ∵ these are L.D

(b) × $\begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ ∵ these are LD

(c) × $\begin{bmatrix} 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ these are LI But 2nd is not an E. Vector

(d) ✓ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ these are LI as well as E Vectors

M-II only (d) has orthogonal vectors when A = Symm Mat so (d) ✓

∵ $AX_2 = \lambda X_2$

$\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \lambda \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 22 \\ 26 \end{bmatrix} = \begin{bmatrix} 3\lambda \\ 5\lambda \end{bmatrix} \Rightarrow \underbrace{\lambda = \frac{22}{3},\ \lambda = \frac{26}{5}}_{\text{Not unique}}$

Q: Find the E. Values & E. Vectors of $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Sol: $\because$ A is U.T.M so $\lambda = 3, 2, 2$ are the E Values.

E. Vector $(\lambda = 3) \rightarrow$ Consider $AX = \lambda X$

or $(A - \lambda I)X = 0$

$(A - 3I)X = 0$

$$\begin{bmatrix} (2-3) & 1 & 0 \\ 0 & (2-3) & 0 \\ 0 & 0 & (3-3) \end{bmatrix} X = 0$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -x_1 + x_2 \\ -x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} -x_1 + x_2 = 0 \\ -x_2 = 0 \\ 0 = 0 \end{matrix}\right\} \Rightarrow \begin{matrix} x_1 = x_2 = 0 \\ x_2 = 0 \end{matrix}$$

Let $x_3 = K$ then $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} = K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

ie for $\lambda = 3$, $X = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

E.Vector for $(\lambda=2) \rightarrow$

$$Ax=\lambda X$$

$$(A-\lambda I)X=0$$

$$(A-2I)X=0$$

$$\begin{bmatrix} (2-2) & 1 & 0 \\ 0 & (2-2) & 0 \\ 0 & 0 & (3-2) \end{bmatrix} X = 0$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = 0$$

$R_2 \leftrightarrow R_3$
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x_2=0,\ x_3=0,\ x_1=K \text{ (let)}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K \\ 0 \\ 0 \end{bmatrix} = K \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Hence for $\lambda=2$, $X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$(\lambda = 2, 2)$

PODCAST :→

(1) $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $(\lambda = 3, 2, 2)$

$\lambda = 3,\ X_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \ldots \infty$ E.V. exist

But all are LD on X_1

$\lambda = 2,\ X_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2/3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \ldots \infty$ Evectors exist.

But all are LD on X_2

(2) Here X_1 & X_2 are LI (By observation)

(3) Here we are getting two Ind families of Evectors, one for $\lambda = 3$ & one for $\lambda = 2$

(4) $P = [X_1\ \underbrace{X_2\ X_3}]$ where X_2 & X_3 are LD i.e $|P| = 0$.

(5) AM of 3 = one, AM of 2 = two, GM of 3 = one, GM of 2 = one

M-II Find AM of λ for $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Sol: $\lambda = 3, 2, 2$ i.e. AM of 3 = one, GM of 3 = one

AM of 2 = two, GM of 2 = ? = one

GM of $(\lambda = 2) = \text{order} - \rho(A - 2I)$

$= 3 - \rho \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3 - 2 = \text{one}$

Conclusion: No. of L.I. E. vectors for A are two

one for $(\lambda = 3)$ & one for $(\lambda = 2)$

Q3 Find the E-Values & E-Vectors of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Sol: $\because$ A is U.T.M so $\lambda = 3, 2, 2$

E. Vector for ($\lambda = 3$) $\rightarrow$

$$AX = \lambda X$$

$$(A - \lambda I)X = 0$$

$$(A - 3I)X = 0$$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -x_1 + x_3 \\ -x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -x_1 + x_3 &= 0 \\ -x_2 &= 0 \\ 0 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} & x_1 = x_3 = K \text{ (let)} \\ & x_2 = 0 \end{aligned}$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K \\ 0 \\ K \end{bmatrix} = K \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

So E Vector for $\lambda = 3$, $X = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

E.vector for $\lambda=2$ $\rightarrow$

$$AX=\lambda X$$

$$(A-\lambda I)X=0$$

$$(A-2I)X=0$$

$$\begin{bmatrix}0&0&1\\0&0&0\\0&0&1\end{bmatrix}X=0$$

R_3-R_1 $\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$$\begin{bmatrix}x_3\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$x_3=0$, let $x_1=k_1$, $x_2=k_2$

$$X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}k_1\\k_2\\0\end{bmatrix}=\begin{bmatrix}k_1\\0\\0\end{bmatrix}+\begin{bmatrix}0\\k_2\\0\end{bmatrix}=k_1\begin{bmatrix}1\\0\\0\end{bmatrix}+k_2\begin{bmatrix}0\\1\\0\end{bmatrix}$$

ie for $\lambda=2$ $\rightarrow$ $X_2=\begin{bmatrix}1\\0\\0\end{bmatrix}$ $\because AX_2=2X_2$

$\rightarrow$ $X_3=\begin{bmatrix}0\\1\\0\end{bmatrix}$ $\because AX_3=2X_3$

$\lambda=2,2$

PODCAST :→

① $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $(\lambda = 3, 2, 2)$

→ $\lambda = 3$, $X_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ → ∞ E. vector exist.

→ $\lambda = 2$ → $X_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ → ∞ E. Vectors exist

→ $X_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ → ∞ E. vectors exist.

② Here X_1, X_2, X_3 are LI ∵ $P = [X_1 \; X_2 \; X_3]$ = Modal Mat. & $|P| \neq 0$ so By, Tricky Method X_1, X_2, X_3 are LI

③ In fact, we are getting three LI families of E vectors, one for $\lambda = 3$ & two for $\lambda = 2$

④ AM of 3 = one, GM of 3 = one
AM of 2 = two, GM of 2 = two

(M-II) Find AM & GM of λ for $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\because$ A is U.T.M so $\lambda = 2, 2, 3$ i.e. AM of 3 = one, GM of 3 = one

AM of 2 = two, GM of 2 = ? = two.

$$GM \text{ of } (\lambda = 2) = \boxed{\text{order} - \rho(A - 2I)}$$

$$= 3 - \rho \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3 - \rho \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 3 - 1 = 2$$

Q. Find E. Values & E. Vectors of $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ & Hence Find AM, GM

Sol: $\because$ A is U.T.M so $\lambda = 0, 0, 0$ Hence AM of $(\lambda = 0)$ = three

E. Vector for $\lambda = 0$:→

$$AX = \lambda X$$
$$(A - \lambda I)X = 0$$
$$(A - 0I)X = 0$$
$$AX = 0$$
$$\begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 5x_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

i.e $x_3 = 0$, $x_1 = k_1$ & $x_2 = k_2$ then

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ 0 \end{bmatrix} = \begin{bmatrix} k_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ k_2 \\ 0 \end{bmatrix} = k_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + k_2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

i.e for $\lambda = 0$ → $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ → family 1

→ $X_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ → family 2

Hence GM of $(\lambda = 0)$ = two

M-II GM of $(\lambda = 0)$ = order $- \rho(A - 0.I)$

$= 3 - \rho(A) = 3 - 1 =$ two

Q. one of the E Vector of $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$ will be ?

(a) $\begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$, (c) $\begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix}$, (d) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}$

(ii) Also find Corresponding E. Value.

Taking (a) $AX = \lambda X$

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ 5 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} \lambda \\ -2\lambda \\ 3\lambda \\ 0 \end{bmatrix} \Rightarrow \underbrace{\lambda = 5, \lambda = \frac{-5}{2}, \lambda = 2}_{\lambda = \text{Not Unique}}, \underbrace{9 = 0}_{\text{Invalid}}$$

i.e (a) Cannot be E. Vector.

Taking (b) $AX = \lambda X \Rightarrow \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 5 \\ -10 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda \\ -2\lambda \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \underbrace{\lambda = 5, \lambda = 5}_{\text{Same/Unique}}, \underbrace{0 = 0, 0 = 0}_{\text{Valid}}$$

(ii) Corresponding E. Value is 5 ($\because AX = 5X$)

(1993) The E.Vectors of $A=\begin{bmatrix}0&0&\alpha\\0&0&0\\0&0&0\end{bmatrix}$, $\alpha\neq0$ is/are

MSQ

✓ (a) $(\alpha\ 0\ 0)'$ (b) × $(0\ 0\ 1)'$ ✓ (c) $(0\ \alpha\ 0)'$ (d) × $(0\ 0\ \alpha)'$

Taking (a): $AX=\lambda X$

$$\begin{bmatrix}0&0&\alpha\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}\alpha\\0\\0\end{bmatrix}=\lambda\begin{bmatrix}\alpha\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}\lambda\alpha\\0\\0\end{bmatrix}$$

$$\lambda\alpha=0,\ \underbrace{0=0,\ 0=0}_{\text{Valid.}}$$

$\because \alpha\neq0,\ \lambda=0$

unique/same/one value exist

Taking (b) $AX=\lambda X$

$$\begin{bmatrix}0&0&\alpha\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}=\lambda\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}\alpha\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\\lambda\end{bmatrix}$$

$\alpha=0$ (Invalid), $\underbrace{0=0}_{\text{valid}}$, $\underbrace{\lambda=0}_{\text{unique}}$

Taking (c) (HW) ✓

Taking (d) (HW) ×

For the matrix $A = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \lambda = 3, -2, 1$ one of the eigen values is equal to -2 which of the following is an eigen vector?

(a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ ✗

(b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$ ✗

(c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ ✗

(d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ ✓

(a) $AX = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \\ 1 \end{bmatrix} \neq -2X$

(b) $AX = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -15 \\ -5 \\ -1 \end{bmatrix} \neq -2X$

(c) $AX = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 16 \\ \\ \end{bmatrix} \neq -2X$

(d) $AX = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ -10 \\ 0 \end{bmatrix} = -2 \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} = -2X$

The vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of

$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then corresponding eigen value of A is

(a) 1 (b) 2

(c) 5 (d) –1

$$AX = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ -5 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$= 5X$$

HW8 Gate + Gate + ESE

Find the E Values & E Vectors of $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

OR

If one pair of LI E.vectors of $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1 \\ b \end{bmatrix}$ then $a+b = ?$

0.5 Ans

OR

one pair of L.I E.Vectors for $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ will be?

(a) $[1 \;\; 0]'$, $[-2 \;\; 0]'$

(b) $[2 \;\; 1]'$, $[-2 \;\; -1]'$

(c) $[4 \;\; 0]'$, $[2 \;\; 3]'$

(d) $[1 \;\; 0]'$, $[2 \;\; 1]'$

Some Confusions: → (M. Imp points)

(1) Number of L.I Row vectors of $A = \rho(A)$

(2) No. of L.I Column vectors of $A = \rho(A)$

(3) No. of LI Solutions of Homog system $(AX = 0)$ = $\boxed{\text{No. of Columns} - \rho(A)}$

Nullity (A)

(4) No. of LI E. Vectors for Repeated E. Value (λ) = $\boxed{\text{order} - \rho(A - \lambda I)}$

(GM of λ)

THANK - YOU

Tel : $\boxed{\text{dr puneet sir pw}}$