DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE

& also for CS/IT

Permutations and Combinations

Lecture No. 03

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Recap of previous lecture

Topic

Permutation & Combination
(Part-2)

Topics to be Covered

Topic

" PERMUTATION & COMBINATION "

(Part-3)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

gmsj → Not Accepted.

COUNTING PRINCIPLE

Fundamental Principle of Addition → If we have to perform only one of the job at a time out of n jobs then use this principle.

Keywords : "Either or, only one, Anyone"

Fundamental Principle of Multiplication → If we have to perform all the jobs at a time out of n jobs then use this principle.

Keywords: "AND, Both, All"

GAZAB KA Conclusion → (M. Imp Slide)

(1) if $n > r$ & RNA, then Multi Rule = Perm. Rule

(2) if $n = r$ & RNA, Then Multi Rule = Perm Rule = Factorial Rule

(3) if RA, then only use Multi Rule.

ie the Concept of ${}^nC_r$, ${}^nP_r$ & $r!$ is applicable only when RNA

Some Useful Information (Based on Experience) →

① Always together / Not separated → Assume them as one unit with in Bracket.

② All Never together / All do not come together → $\boxed{\text{Total} - \text{Always together.}}$

③ No two Girls are together → First arrange Boys.

④ Alternately (Linear Case) → Two Cases will arise.

⑤ Alternately (Circular Case) → only one Case will arise.

⑥ Particular / fix → No Need to select & No Need to arrange

⑦ At least one = Total − None.

⑧ At least = Go up to last Point (using Common Sense)

⑨ At Most = Include None also (if Possible)

FQ 2 :- 4 Boys & 4 Girls are to be seated in a Row in which there are exactly 2 sisters & 1 Brother. Then find the number of seating arrangements if

① There is No Restriction = ? = 8! (Max Ans)
(RNA)

② All Boys are together & All Girls are together = ?

$$\underbrace{(B_1 B_2 B_3 B_4)}_{1^{st}}, \underbrace{(G_1 G_2 G_3 G_4)}_{2^{nd}} = \underset{(P)}{2!} \times \underset{(B)}{4!} \times \underset{(G)}{4!} = 1152$$

(3) All G are together = ? $= \underset{(P)}{5!} \times \underset{(G)}{4!} = 2880$

$\underset{1^{st}}{B_1}, \underset{2^{nd}}{B_2}, \underset{3^{rd}}{B_3}, \underset{4^{th}}{(G_1 G_2 G_3 G_4)}, \underset{5^{th}}{B_4}$

(4) All Girls are Never together = ? = Total − 'G' always together

$= 8! - (5! \times 4!) = 37440$

Explanation :- $(G_1 G_2 G_3 G_4) B_1 B_2 B_3 B_4 = 4! \times (1 \times 4!)$

or $B_1 (G_1 G_2 G_3 G_4) B_2 B_3 B_4 = 4! \times (1 \times 4!)$

or $B_1 B_2 (G_1 G_2 G_3 G_4) B_3 B_4 = 4! \times (1 \times 4!)$

or $B_1 B_2 B_3 (G_1 G_2 G_3 G_4) B_4 = 4! \times (1 \times 4!)$

or $B_1 B_2 B_3 B_4 (G_1 G_2 G_3 G_4) = 4! \times (1 \times 4!)$

$\Rightarrow 5 \times 4! \times 4!$

$= 5! \times 4!$

$= 2880$

(5) No two Girls are together = ? = First arrange Boys.

$\_\ B_1\ \_\ B_2\ \_\ B_3\ \_\ B_4\ \_$

$$= \underset{(B)}{4!} \times \underset{(G)}{{}^5P_4} = 2880$$

Explanation:

B G B G B G B G $= 4! \times 4!$

or

G B G B G B G B $= 4! \times 4!$

or

G B B G B G B G $= 4! \times 4!$

or

G B G B B G B G $= 4! \times 4!$

or

G B G B G B B G $= 4! \times 4!$

Part (5) is a particular case of part (4)

eg (B G B G B G G B) →

Such types of cases are not possible for part (5).

Total cases $= 5 \times 4! \times 4! = 2880$

(6) Boys and Girls are seated alternately = ? (Two cases will arise)

$\underset{1^{st}}{\underline{B}}\ G\ \underset{3^{rd}}{\underline{B}}\ G\ \underset{5^{th}}{\underline{B}}\ G\ \underset{7^{th}}{\underline{B}}\ G \longrightarrow 4! \times 4! = 4! \times {}^4P_4$

OR

OR $G\ \underset{2^{nd}}{\underline{B}}\ G\ \underset{4^{th}}{\underline{B}}\ G\ \underset{6^{th}}{\underline{B}}\ G\ \underset{8^{th}}{\underline{B}} \longrightarrow 4! \times 4! = 4! \times {}^4P_4$

$\Rightarrow$ Req Ans $= 2 \times 4! \times 4!$
$= 1152$

(*) Part (6) is a particular case of part (5)

(7) Two sisters are $\underbrace{\text{not separated}}_{\text{always together}}$ = ? $= \underset{(P)}{7!} \times \underset{(S)}{2!}$

$\underset{1}{\underline{B}}\ \underset{2}{\underline{B}}\ \underset{3^{rd}}{\underline{(S_1 S_2)}}\ \underset{4}{\underline{G}}\ \underset{5}{\underline{B}}\ \underset{6}{\underline{G}}\ \underset{7^{th}}{\underline{B}}$

(8) Two sisters do not come together = ? = Total − Sisters always together

or (Both the sisters are never together) $= 8! - (7! \times 2!)$

(9) There is exactly one Boy b/w two sisters = ?

$$\underset{1^{st}}{\underline{G}}\ \underset{2}{\underline{B}}\ \underset{3^{rd}}{\underline{(S_1\ B\ S_2)}}\ \underset{4}{\underline{B}}\ \underset{5}{\underline{B}}\ \underset{6^{th}}{\underline{G}} = \left(\underset{(Boy)}{{}^4C_1} \times 1\right) \times \underset{(P)}{6!} \times \underset{(S)}{2!}$$

(10) There is exactly one person b/w two sisters = ?

$$\underset{1^{st}}{\underline{P}}\ \underset{2^{nd}}{\underline{(S_1\ P\ S_2)}}\ \underset{3}{\underline{P}}\ \underset{4}{\underline{P}}\ \underset{5}{\underline{P}}\ \underset{6^{th}}{\underline{P}} = \left({}^6C_1 \times 1\right) \times \underset{(P)}{6!} \times \underset{(S)}{2!}$$

(11) Two sisters are always separated by one particular persons

$$\underset{1^{st}}{P}\ \underset{2}{P}\ \underset{3}{P}\ \underbrace{(S_1 P S_2)}_{4^{th}}\ \underset{5}{P}\ \underset{6^{th}}{P} = ({}^1C_1 \times 1) \times \underset{(P)}{6!} \times \underset{(S)}{2!} = 6! \times 2! \text{ Ans}$$

(12) Sisters always want to be seated at the adjacent sides of their BRO

$$\underset{1^{st}}{B}\ \underset{2}{B}\ \underbrace{(S_1 B S_2)}_{3^{rd}}\ \underset{4}{B}\ \underset{5}{G}\ \underset{6^{th}}{G} = ({}^1C_1 \times 1) \times \underset{(P)}{6!} \times \underset{(S)}{2!}$$

(13) Elder & younger sister wants to be seated at 1st & last postion Resp.

$$S_E\ (\underline{G}\,\underline{G}\,\underline{B}\,\underline{B}\,\underline{B}\,\underline{B})\ S_Y = 1 \times 6! \times 1 = 6!\ \text{ways}$$

(14) Elder & younger sister wants to be seated at 1st & last postion.

$$S_E\ (G\,G\,B\,B\,B\,B)\ S_Y = \underset{(P)}{6!} \times \underset{(S)}{2!}$$

(15) Two sisters wants to be seated at extreme Positions but together.

See Next Slide

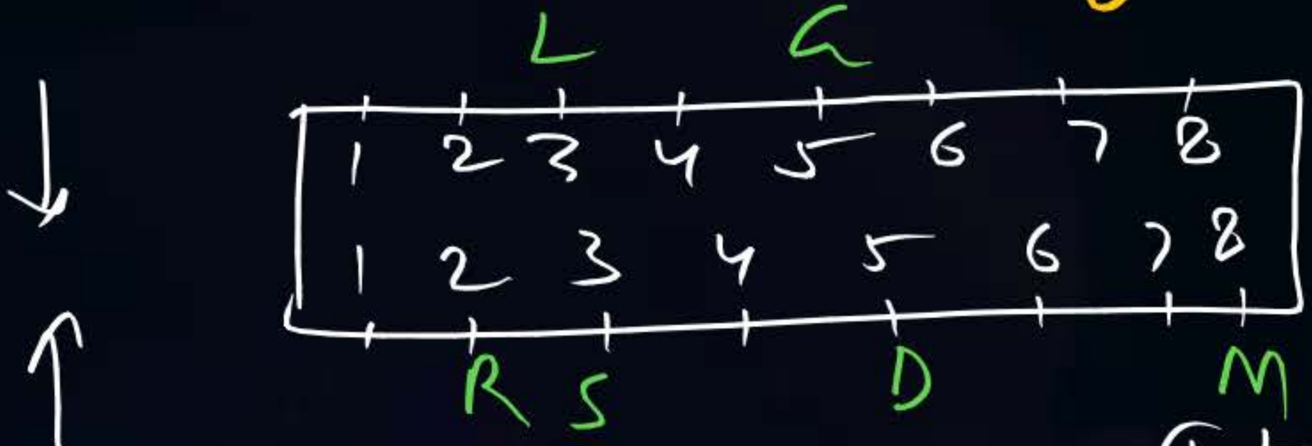
$(S_1 S_2)$ $\underline{B}$ $\underline{B}$ $\underline{B}$ $\underline{B}$ $\underline{G}$ $\underline{G}$ $= 6! \times 2!$

OR OR

$\underline{B}$ $\underline{B}$ $\underline{B}$ $\underline{B}$ $\underline{G}$ $\underline{G}$ $(S_1 S_2)$ $= 6! \times 2!$

Total cases $= 2 \times 6! \times 2!$

Q. 16 persons are to be arranged along a long Table with 8 chairs on each side. Four particular persons wants to be seated on one particular side while two particular persons on the other side. Then find the number of seating Arrangements.



Let Particular persons are R, S, D, M & other two Particular persons are L & G

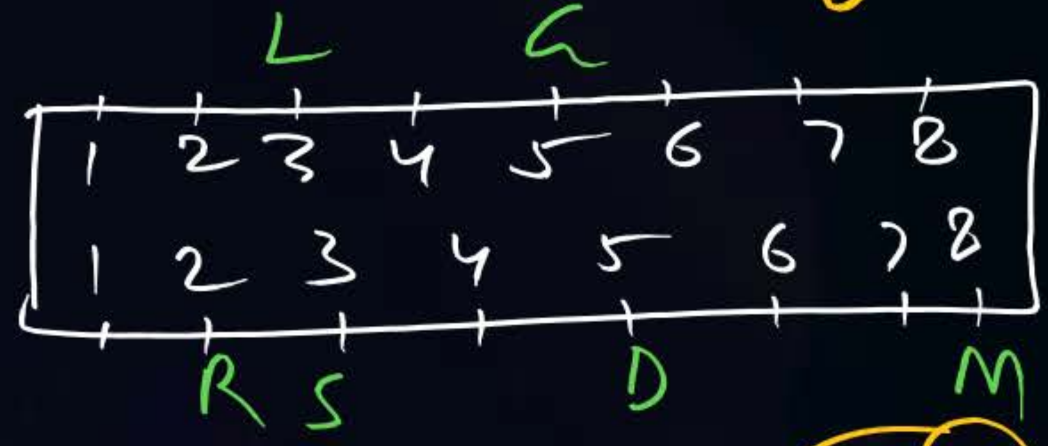
Total arrangements $= \left({}^8C_4 \times 4!\right) \times \left({}^8C_2 \times 2!\right) \times 10!$

$= \left({}^1C_1 \times {}^8P_4\right) \times \left({}^1C_1 \times {}^8P_2\right) \times 10!$

(ii) If side is not particular then

Ans → ? $= \left({}^2C_1 \times {}^8P_4\right) \times \left({}^1C_1 \times {}^8P_2\right) \times 10!$

(ii) 16 persons are to be arranged along a long Table with 8 chairs on each side. Four particular persons wants to be seated on one side while two particular persons on the other side. Then find the number of seating Arrangements.



Let Particular Persons are R, S, D, M & other two particular persons are L & G

Req Ans

M-I $= \left({}^{2}C_{1} \times {}^{8}P_{4}\right) \times \left({}^{1}C_{1} \times {}^{8}P_{2}\right) \times 10!$

M-II $= \left({}^{2}C_{1} \times {}^{8}P_{2}\right) \times \left({}^{1}C_{1} \times {}^{8}P_{4}\right) \times 10!$

CIRCULAR PERMUTATION →

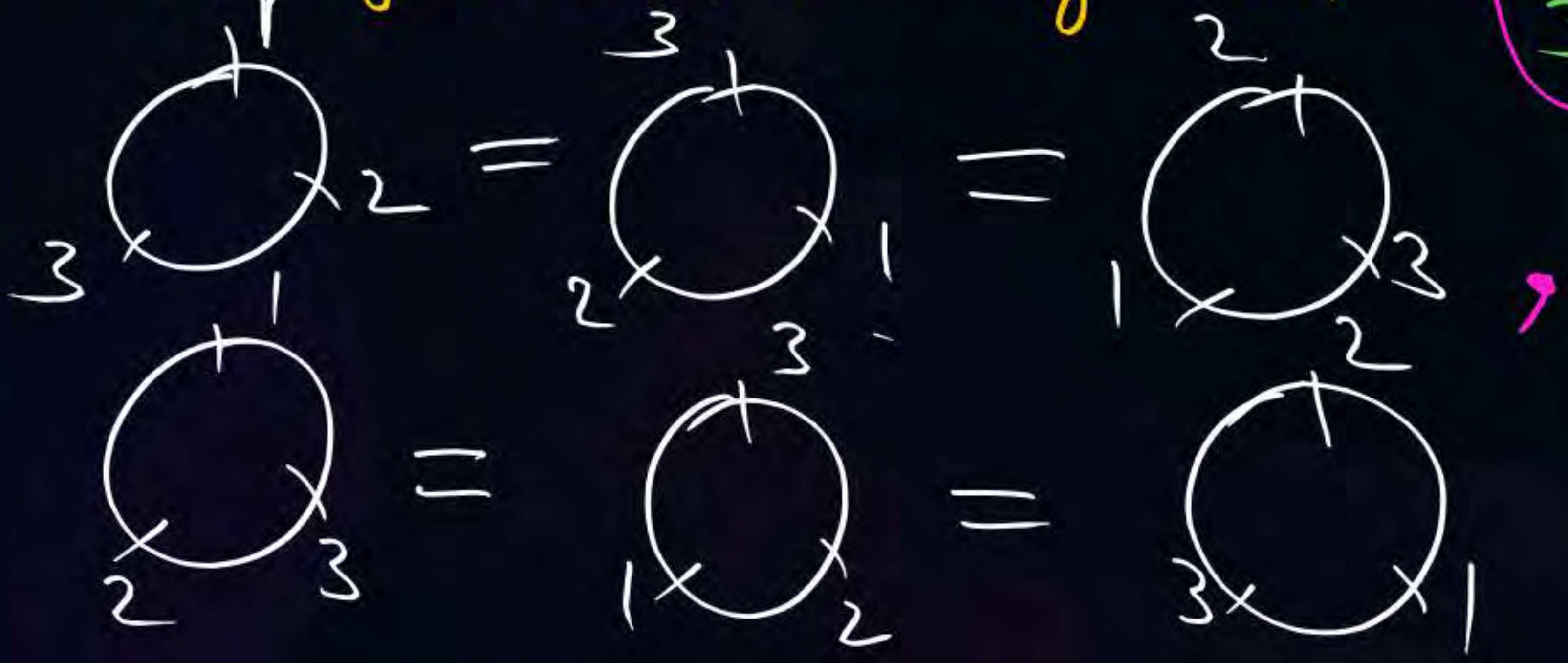
Number of Linear Arrangements of n different things $= n!$

Number of Circular Arrangements of n different things $= \frac{n!}{n} = \boxed{(n-1)!}$

Let $n=3$ then No. of linear arrangements $= 3! = 6$

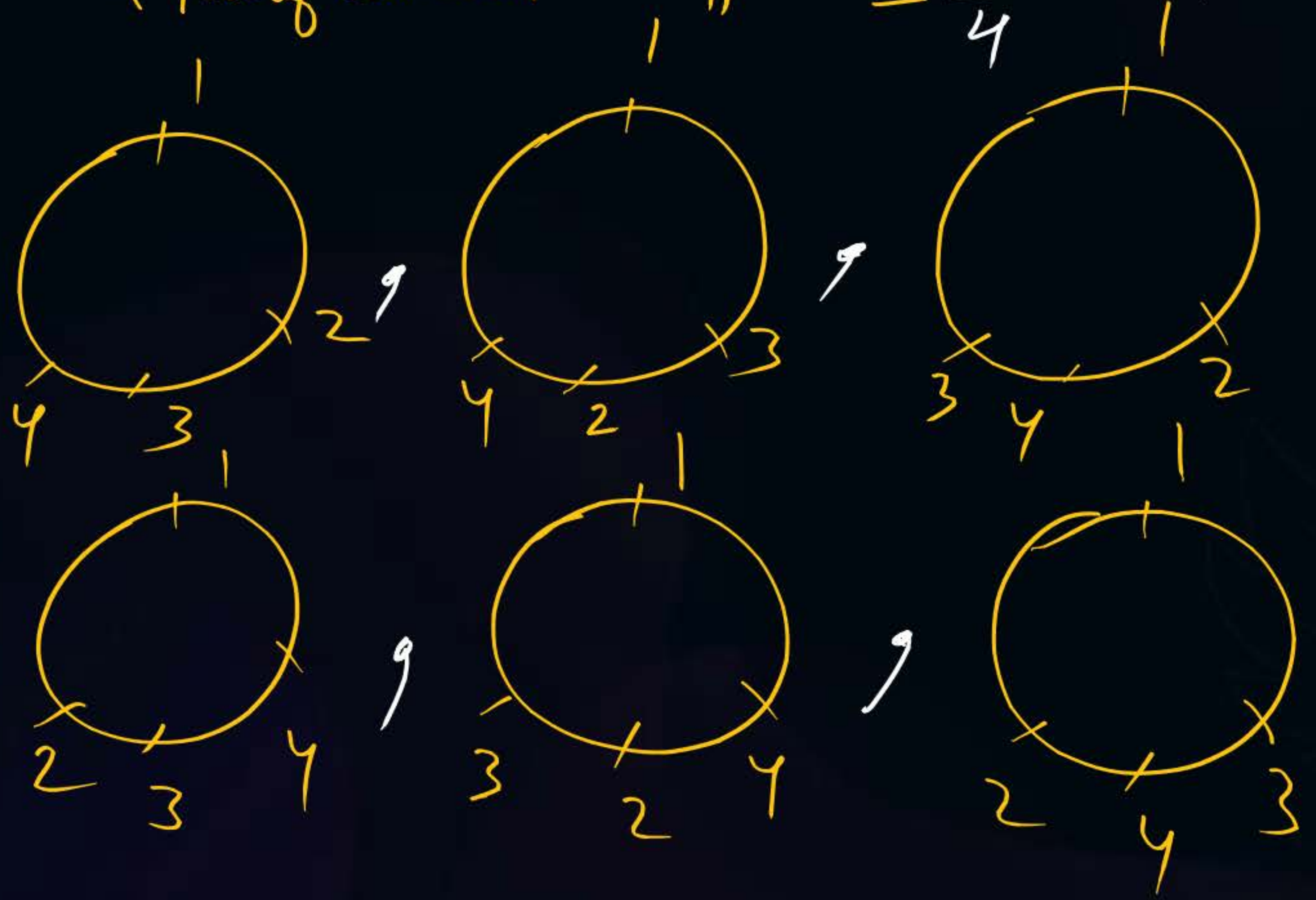
eg $(1\,2\,3), (1\,3\,2), (2\,1\,3), (2\,3\,1), (3\,1\,2), (3\,2\,1)$

& No. of Circular Arrangements $= \frac{3!}{3} = (3-1)! = 2! = 2$



For $n=4$, No. of linear arrangements $= 4! = 24$

& No. of Circular " $= \frac{4!}{4} = (4-1)! = 3! = 6$



FQ 3 : 4 Boys & 4 Girls are to be seated around circular Table for Tea Party in which there are two sisters & 1 Brother. then find the number of possible arrangements if.

(1) there is No Restriction = ? $= (8-1)! = 7!$ (Max Ans).

(RNA)

(2) All Boys are together & All Girls are together = ?

$= (2-1)! \times 4! \times 4!$

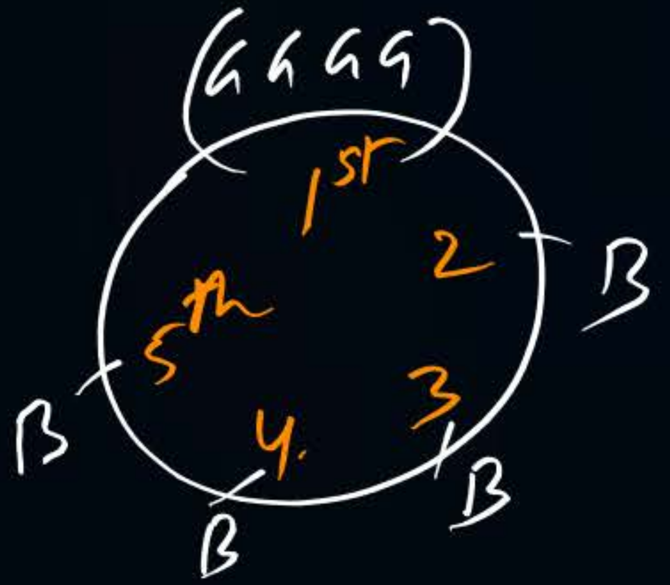
(P) (B) (G)

(=CA) (LA) (LA)

(3) All Girls are not separated = ?

OR

Girls are always together = ?



$= (5-1)! \times 4!$

(P) (G)

($\cong$ CA) ($\cong$ L.A)

Doubt: why Not Ans $= \underset{(P)}{(5-1)!} \times \underset{(G)}{(4-1)!}$

$\boxed{\text{it dosen't seem good.}}$

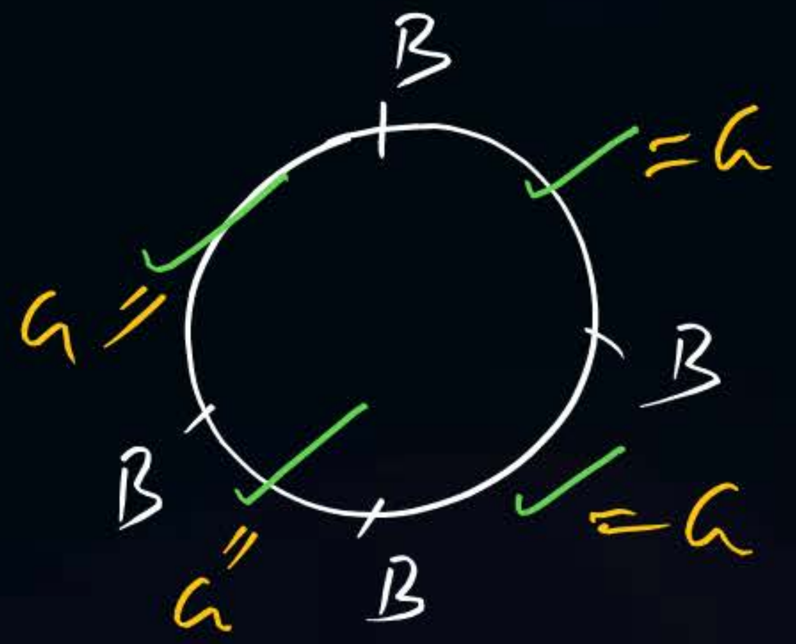
Reason:

Rule: form circle only once in a given question.

(4) All G $\underbrace{\text{do not}}_{\text{never}}$ come together = ? = Total − All G are together

$= (8-1)! - (5-1)! \times 4! = 7! - 4! \times 4!$ Ans

(5) No two Girls are together = ? = First arrange Boys in a Circle.



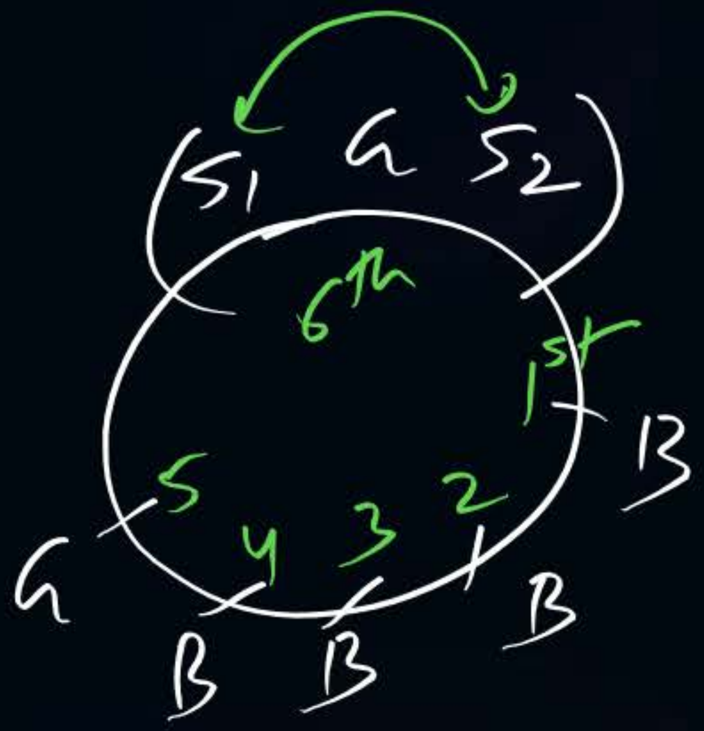
$$= (4-1)! \times (4!)$$

$\underset{(= CA)}{B}$ $\quad$ $\underset{(= LA)}{G}$

(6) Boys & Girls are seated alternately = ? = Same as above.

$$= 3! \times 4!$$

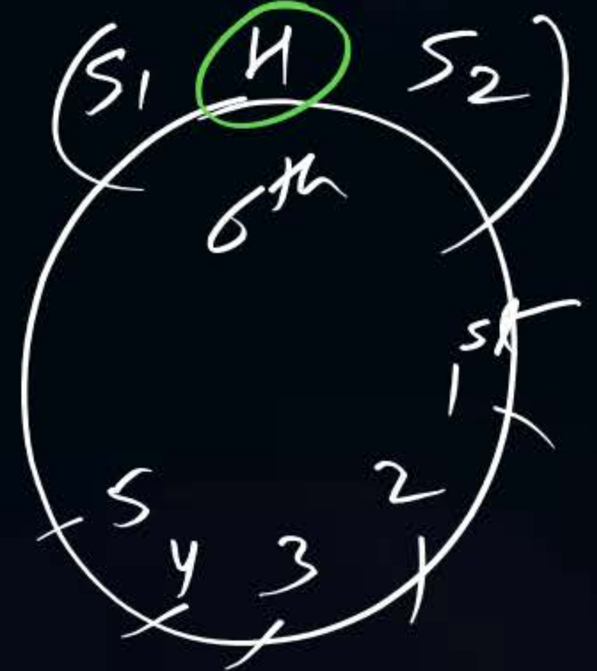
Sol:



$$= \underbrace{\left({}^{2}C_{1} \times 1\right)}_{G} \times \underbrace{(6-1)!}_{P\ (\equiv C.A)} \times \underbrace{2!}_{S\ (\equiv L.A)}$$

(7) there is exactly one G b/n two sisters = ?

(8) Two sisters wants to be seated at the adjancent sides of Host ?



$$=({}^1C_1 \times 1) \times (6-1)! \times 2!$$

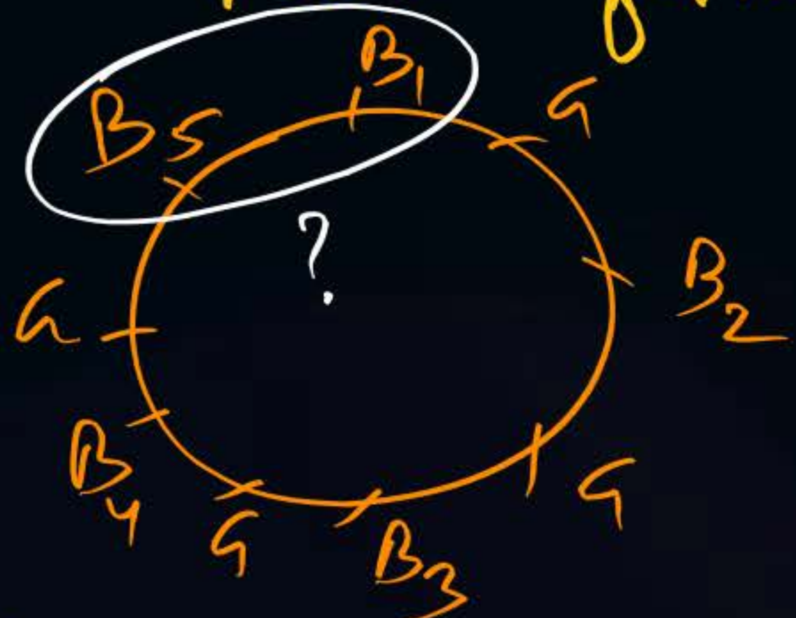
P (=CA)   S (=LA)

No need to select Host ∵ Host is a particular Person & He/She knows it.

(9) Two sisters are always separated by Ashutosh ?

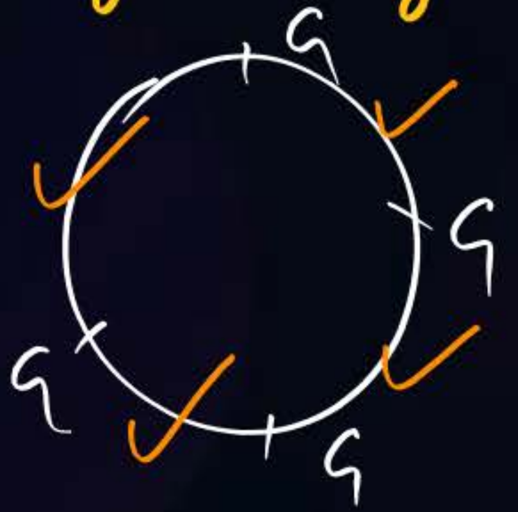
$$=({}^1C_1 \times 1) \times (6-1)! \times 2!$$

Q. If there are 5B & 4G are to be seated around a circular Table then find No. of arrangements (1) in which B & G are seated alternately?



ie Senseless Question so Ans = 0

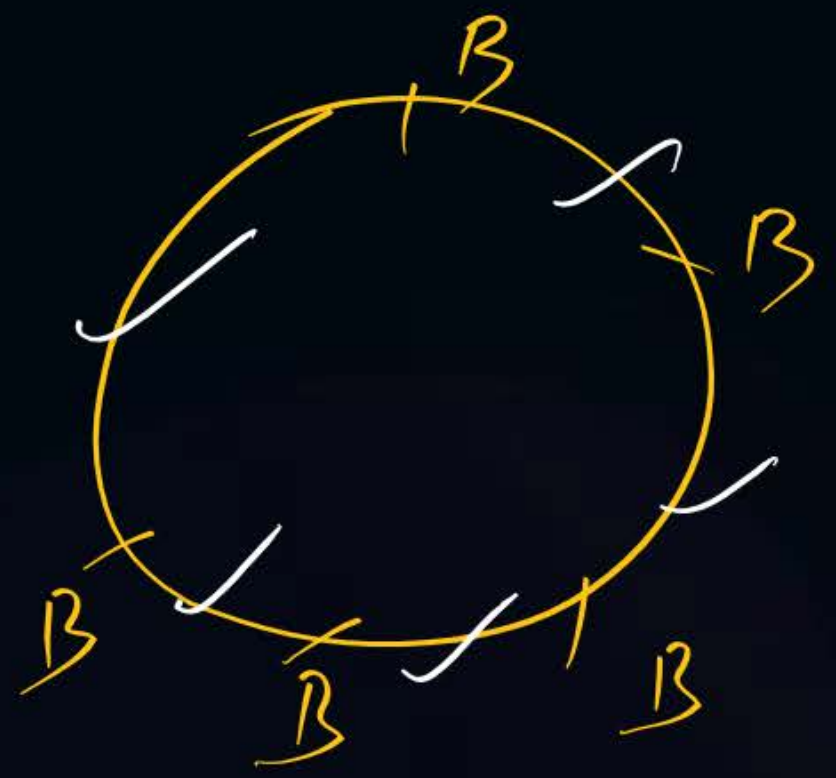
(2) No two Boys are together = ? = first arrange Girls in a circle (~~GEN Z girls~~)



= (4-1)! × (Not possible to arrange 5 Boys at 4 places)

= Senseless Question ie Ans = 0

(3) No two Girls are together = ? = First arrange Boys in circle.



$$= (5-1)! \times {}^5P_4$$

Boys = C.A

Girls = L.A

Thank you

Keep Hustling!