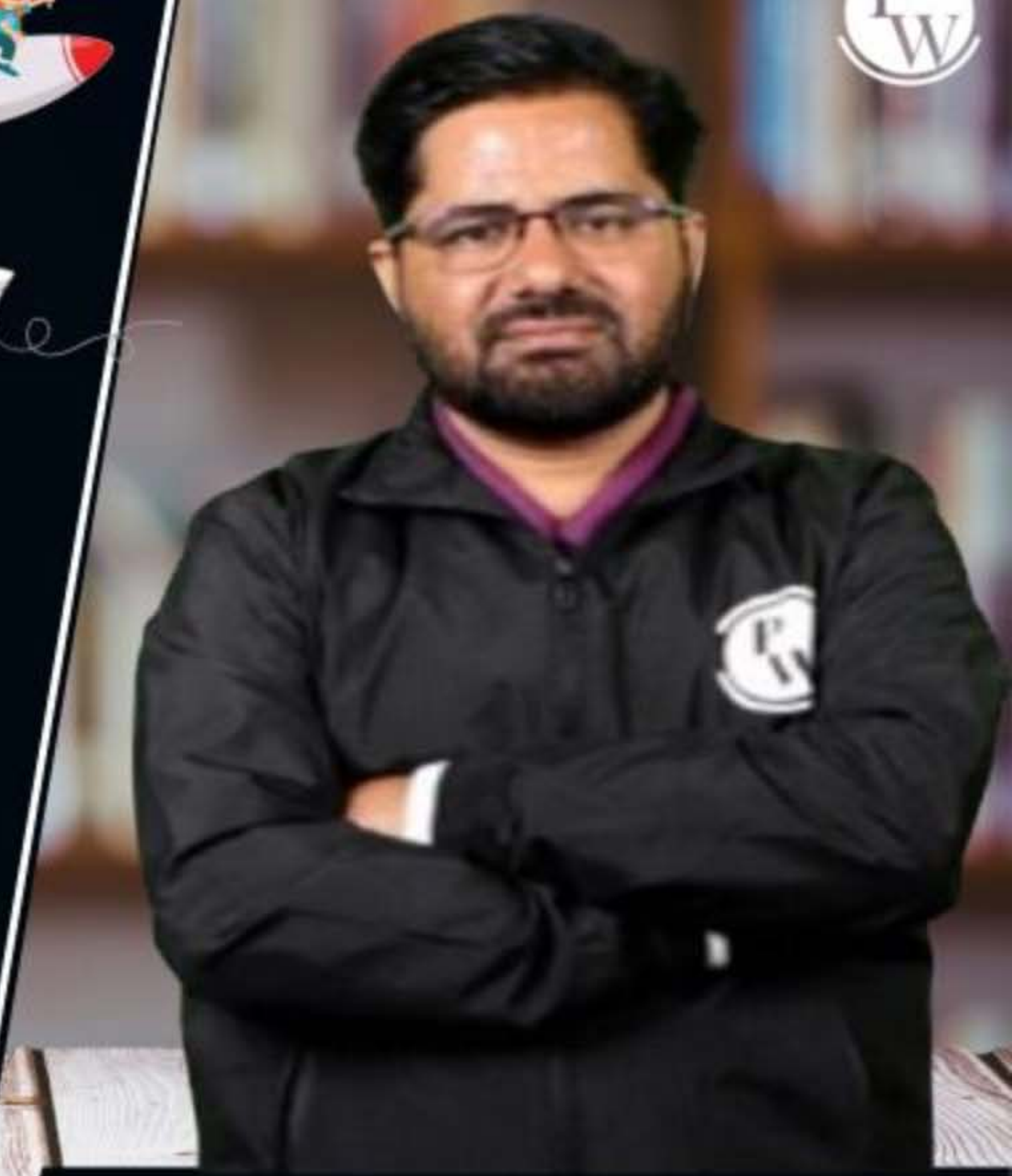
GATE

DS & AI
CS & IT

Linear Algebra

Lecture No. 01



By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

PRE-REQUISITES of ENGG MATHS.

Foundation Series (Maths)

- Determinant
- Matrices.

Topics to be Covered

Topic

Basics of Determinants

Syllabus

(1) Linear Algebra
- 1 (DA & CS Both)
- 2 (only for DA)

(2) Calculus (Common for Both DA & CS)

(3) Prob & Stats
- 1 (DA & CS Both)
- 2 (only for DA)

(x) CS: $\frac{7\text{-}8}{100}$ DA: $\frac{40^+}{100}$

Tel: $\boxed{\text{drpuneetsirpw}}$

Strategy:

1. Live Class.
2. Revision
3. Short Notes (in Later Phase)
4. D.P.P
5. Chapterwise test (Sund)
6. P.Y.Q. → Judge

Book: → No Book is Needed only PYQ Book is required.

eg: L-Algebra: Class (150-200Q) | DPP (70-90) | WT (13Q) | OTS 100Q | PYQ (200Q) | ≈ 700-800 Q

Doubts: → Conceptual Doubts → you can ask anytime.

→ Generic Doubt → will be discussed after class.

Parachute Landing → Conceptual Doubts are also not allowed.

PREREQUISITE of Engg Maths: → ✓ (25 Lectures)

(Foundation Series of Engg Maths)

Engg Maths $\dfrac{\text{CS}}{\frac{7-8}{100}}$, $\dfrac{\text{DS/AI}}{\frac{40+}{100}}$

Maths: is the Language of Engg.

Maths → Language → understanding of Symbols.

Maths → Concept →
- Information Based Concept (No Brain should be used)
- Analysis Based Concept (Pure Maths) (useful to develop Neurons)
- Application Based Concept. (Applied Maths) in LIVE Class.

M.M. Imp Point → Try to have Patience as much as possible

Determinants (Represents any number)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ? = \boxed{(a)(d)-(b)(c)}$$

Eg $|A| = \begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} = ? = (2)(-1)-(4)(-3) = -2+12 = 10$

Eg $|A| = \begin{vmatrix} 3+4i & -i \\ i & 3-4i \end{vmatrix} = ? = (3+4i)(3-4i)-(i)(-i)$

$= (3)^2-(4i)^2+i^2$

where $i = \sqrt{-1}$

$= 9-16i+i^2 = 9-15i^2 = 9-15(-1) = 24$

Sign convention: $\begin{vmatrix} + & - \\ - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix}$, sign of $(a_{ij}) = (-1)^{i+j}$

eg sign of $a_{23} = ? = (-1)^{2+3} = -ve$, sign of $a_{42} = ? = (-1)^{4+2} = +ve$

eg: $\begin{vmatrix} 1 & 3 & -2 \\ 1 & 5 & 0 \\ 1 & 2 & 4 \end{vmatrix} = ? = -(1)\begin{vmatrix} 3 & -2 \\ 2 & 4 \end{vmatrix} + 5\begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix} - (0)\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$

$= -[12+4] + 5[4+2] - 0 = -16 + 30 = 14$

Shortcut (SARRUS Method) →

$$\begin{matrix} 1 & 3 & -2 & 1 & 3 \\ 1 & 5 & 0 & 1 & 5 \\ 1 & 2 & 4 & 1 & 2 \end{matrix}$$

$|A| = (20 + 0 + (-4)) - (-10 + 0 + 12) = 14$

eg: $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{vmatrix} = ?$

$$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & -4 & 1 & 1 & -4 \end{matrix}$$

$|A| = (1 + (-4) + (-24)) - (3 + 8 + 4)$

$= -27 - 15 = -42$

eg: $\begin{vmatrix} 1 & 5 & 25 \\ 1 & 4 & 16 \\ 1 & 2 & 4 \end{vmatrix} = \underline{?}$

M-I SARRUS Method .... $|A| = -6$

M-II $|A| = (5-4)(4-2)(2-5)$

$= 1 \times 2 \times (-3) = -6$ Ans

w.k.that in Class 12^{th}:

① $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)}$

② $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)(a+b+c)}$

eg $|A| = \begin{vmatrix} 1 & 2 & 8 \\ 1 & 3 & 27 \\ 1 & 4 & 64 \end{vmatrix} = ? = (2-3)(3-4)(4-2)(2+3+4) = 2 \times 9 = 18$

Det of order 4×4 →

eg $|A| = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} = ? = -(1)\begin{vmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} + (0)\begin{vmatrix} & ? & \end{vmatrix} - (3)\begin{vmatrix} 0 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 0 & 2 \end{vmatrix} + (0)\begin{vmatrix} & ? & \end{vmatrix}$

$\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix}$

$= -\left[-(3)\{4-3\} + 0 - 1\{1-0\}\right] + 0 - 3\left[0 - 1\{4-3\} + 3\{0-9\}\right]$

$= -\left[-3-1\right] - 3\left[-1-27\right] = 88$

M-II $\quad C_3 \to C_3 - 3C_1$

$|A| = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 0 & 0 \\ 2 & 3 & -6 & 1 \\ 3 & 0 & -8 & 2 \end{vmatrix} = -(1)\begin{vmatrix} 1 & 2 & 3 \\ 3 & -6 & 1 \\ 0 & -8 & 2 \end{vmatrix} = -\left[0 - (-8)\{1-9\} + 2\{-6-6\}\right]$

$= -\left[8(-8) + 2(-12)\right] = -(-64-24)$

$= 88$

Elementary Operations →

E-Row Operations

① $R_i \leftrightarrow R_j$

② $R_i \rightarrow KR_i$

③ $R_i \rightarrow R_i + KR_j$

E-Column Operations

① $C_i \leftrightarrow C_j$

② $C_i \rightarrow KC_i$

③ $C_i \rightarrow C_i + KC_j$

(k = Any Constant or any scalar)

$R_2 \rightarrow R_2 - 5R_1$ (3^{rd} type E-Operation)

$R_2 \rightarrow 5R_2 - R_1$ (No it is Not E-Operation)

it is a Mixed operation of 2^{nd} & 3^{rd} i.e. $\underbrace{R_2 \rightarrow 5R_2 \text{ \& then } R_2 \rightarrow R_2 - R_1}_{R_2 \rightarrow 5R_2 - R_1}$

Note (1) If we interchange any two Rows in a Matrix, Value of it's Determinant Changes by <u>−ve</u> sign.

(2) If multiply any Row by constant K then Value of Det increases by <u>K</u> times.

(3) only 3rd E-operation does not alter the Value of Determinant.

i.e we are free to apply 3rd E-operation in a Mat, while solving Det.

(*) Above properties are also Valid in Case of Columns.

(*) In a single step, use either Row operation or Column operation at a time.

The determinant value of the matrix

$$\begin{vmatrix} 13 & 2 & 1 & 3 \\ 31 & 4 & 5 & 6 \\ 26 & 3 & 7 & 4 \\ 10 & 1 & 3 & 2 \end{vmatrix}$$ is ______.

(a) 55 (b) 101

(c) 126 (d) -10

$$\xrightarrow{C_2 \to C_2 - C_4} \begin{vmatrix} 13 & -1 & 1 & 3 \\ 31 & -2 & 5 & 6 \\ 26 & -1 & 7 & 4 \\ 10 & -1 & 3 & 2 \end{vmatrix} \xrightarrow[\substack{R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1}]{R_2 \to R_2 - 2R_1} \begin{vmatrix} 13 & -1 & 1 & 3 \\ 5 & 0 & 3 & 0 \\ 13 & 0 & 6 & 1 \\ -3 & 0 & 2 & -1 \end{vmatrix}$$

Now Expanding along C_2

$$|A| = -(-1) \begin{vmatrix} 5 & 3 & 0 \\ 13 & 6 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

$$|A| = +5[-6-2] - 3[-13+3] + 0[\ ?\ ]$$

$$= -40 + 30 = -10$$

Q. $|A| = \begin{vmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{vmatrix}_{4\times4} \xrightarrow[R_3 \to R_3 - R_2]{R_4 \to R_4 - R_3} \begin{vmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{vmatrix}$

$\xrightarrow{R_4 \to R_4 - R_3} \begin{vmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 10 & 10 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{vmatrix} = -0+0-0+0 = 0$ Ans

Q. $|A| = \begin{vmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{vmatrix}_{5\times5} = ?$

$\xrightarrow{C_1 \to C_1 + (C_2 + C_3 + C_4 + C_5)} \begin{vmatrix} 0 & - & - & - & - \\ 0 & - & - & - & - \\ 0 & - & - & - & - \\ 0 & - & - & - & - \\ 0 & - & - & - & - \end{vmatrix} = 0$ Ans

PROPERTIES of Det :-

(1) If in a Mat, Any two Rows (or any two Columns) are identical then it's $\text{Det} = 0$

(2) If in a Mat, All the elements in any Row (or in any Column) are all zero then value of it's $\text{Det} = 0$

(3) $|ABC| = |A|.|B|.|C|$

(4) $|A+B+C| \neq |A|+|B|+|C|$

ie $|A+B+C| \leq |A|+|B|+|C|$

(5) $|A^m| = |A|^m$, $m \in N$

(6) $|A^T| = |A|$

(7) $|A|^{-1} = \dfrac{1}{|A|}$ eg If $|A| = 5$ then $|A|^{-1} = 5^{-1} = \dfrac{1}{5}$

(8) $|A^{-1}| = \dfrac{1}{|A|}$ ie $\boxed{\det(A^{-1}) = \dfrac{1}{\det A}}$

where $\cancel{A^{-1} = \dfrac{1}{A}}$; $A^{-1} = \dfrac{\text{adj} A}{|A|} = \dfrac{(\text{Cof } A)^T}{|A|}$ ✓

PAAP

(9) Max Number of terms in the General Expansion of $|A|_{n\times n} = n!$ Learn.

(a) n

(b) n^2

(c) $2n$

(d) $n!$

eg $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ i.e. Max term = 2 term $= 2!$

eg $|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - hf) - b(di - gf) + c(dh - eg)$

i.e. Max terms = 6 terms $= 3!$

eg $|A| = \begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = a\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix}_{3\times3} - b\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix}_{3\times3} + c\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix}_{3\times3} - d\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix}_{3\times3}$

$= (6 \text{ term}) + (6 \text{ term}) + (6 \text{ term}) + (6 \text{ term})$

$= 24$ term $= 4!$ & so on - - - - -

(*) Total Number of term in Mat $A_{n\times n} = n^2$

(10) Area of $\triangle$ formed by $(x_1, y_1), (x_2 y_2), (x_3 y_3)$ is $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

eg Area of $\triangle$ formed by $(2,7), (3,6), (4,5)$ will be? $= \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 3 & 6 & 1 \\ 4 & 5 & 1 \end{vmatrix} = \cdots\cdots = 0$

(ie these points are collinear.)

eg Area of $\triangle$ formed by $(1,0), (2\,2), (4,3)$ will be? $= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix} = -\frac{3}{2}$

$\because$ area can't be $-$ve so Ans $= 1.5$.

(11) $\frac{d}{dx}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a' & b' & c' \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d' & e' & f' \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d & e & f \\ g' & h' & i' \end{vmatrix}$

(12) $$\begin{vmatrix} a+l & b & c \\ d+m & e & f \\ g+n & h & i \end{vmatrix} = ? = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} l & b & c \\ m & e & f \\ n & h & i \end{vmatrix}$$

or $$\begin{vmatrix} a+l & b+m & c+n \\ d & e & f \\ g & h & i \end{vmatrix} = ? = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} l & m & n \\ d & e & f \\ g & h & i \end{vmatrix}$$

eg $$\begin{vmatrix} 1+1 & -3 & 0 \\ 2+3 & +1 & 2 \\ 2+2 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 0 \\ 2 & 1 & 2 \\ 2 & 0 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -3 & 0 \\ 3 & 1 & 2 \\ 2 & 0 & 3 \end{vmatrix}$$

$=27$ $\quad$ $=9$ $\quad$ $=18$

Q. if $A=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$ then $|A^3|=?$

Sol $|A|=(3)(3)-(2)(2)=5$

So $|A^3|=|A|^3=(5)^3=125$.

Q. if $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ & $|A^3|=125$

MSQ

So $\alpha=?$

$|A|^3=5^3$

$|A|=5$

$\alpha^2-4=5$

$\alpha^2=9$

$\alpha=\pm 3$.

~~(a)~~ 3

~~(b)~~ −3

(c) 0

(d) None

Q. If $A=\begin{bmatrix}2 & a & -6 & 0\\ \alpha & 1 & -4 & 2\\ 3 & \tan\pi & 4 & -1\\ 0 & 2 & 1 & 4\end{bmatrix}_{4\times 4}$

then $|A^T \cdot A^{-1}|=?$

$=|A^T|\cdot|A^{-1}|$ $(\because |AB|=|A||B|)$

$=|A|\cdot\dfrac{1}{|A|}$

$=1$

If A and B are matrices of determinant 1 then

(a) Determinant of A + B is 2

(b) Determinant of A + B is 1

(c) Determinant of A + B is 0

(d) Nothing can be concluded about the determinant of A + B

ATQ, $|A|=1, |B|=1$

Now we know that,

$$|A+B| \le |A|+|B|$$
$$\le 1+1$$
$$|A+B| \le 2 \text{ is } |A+B| = 2 \text{ or } 1 \text{ or } 0$$

Thank you

Keep Hustling!