DS & AI
CS & IT

Probability & Statistics -I

Probability
Lecture - 04

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Recap of previous lecture

Topic

BASICS OF PROBABILITY

(Part-3)

Topics to be Covered

Topic

BASICS of PROBABILITY

(Part- 4)

* (Conditional Probability)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your $\boxed{\text{little mind}}$ until you have a complete understanding of the chapter) & try to solve the Quest.

Short RECAP

Operation	P&C	Prob	Formula	ME	Ind.
Either or	Plus	union	Addition Th	$P(A\cup B)=P(A)+P(B)$	*
AND	Multiply	Intersection	Multi Th	$P(A\cap B)=0$	$P(A\cap B)=P(A)\cdot P(B)$

Addition Th: $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

* for Independency: $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

for ME: $P(A\cup B)=P(A)+P(B)-0$

Independent Events → If occurrence or non occurrence of one event does not alter the occurrence or non occurrence of other Event then Events are called Independent events

$\Longleftrightarrow$

Mathematically: If A & B are Ind Events then $\Longleftrightarrow$ $\boxed{P(A\cap B) = P(A)\cdot P(B)}$

eg; $S_{Coin} = \{H, T\}$, $S_{Die} = \{1, 2, 3, 4, 5, 6\}$

$A = \{H\} \Rightarrow P(A) = \frac{1}{2}$ $\quad B = \{1, 2, 3, 4\} \Rightarrow P(B) = \frac{4}{6}$

then $\underbrace{P(A\cap B) = P(A)\cdot P(B)}_{\because \text{ A \& B are Ind. Events.}} = \frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$

RECAP

X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^c) = 0.7$. Which one of the following is the value of $P(X \cup Y)$?

(a) 0.7
(b) 0.5
(c) 0.4
(d) 0.3

$P(X \cup \bar{Y}) = 0.7$

$P(X) + P(\bar{Y}) - P(X \cap \bar{Y}) = 0.7$

$0.4 + (1 - P(Y)) - P(X) \cdot P(\bar{Y}) = 0.7$

$-P(Y) - P(X)P(\bar{Y}) = -0.7$

$-P(Y) - P(X)[1 - P(Y)] = -0.7$

$-P(Y) - 0.4 + 0.4P(Y) = -0.7$

$(-1 + 0.4)P(Y) = -0.7 + 0.4$

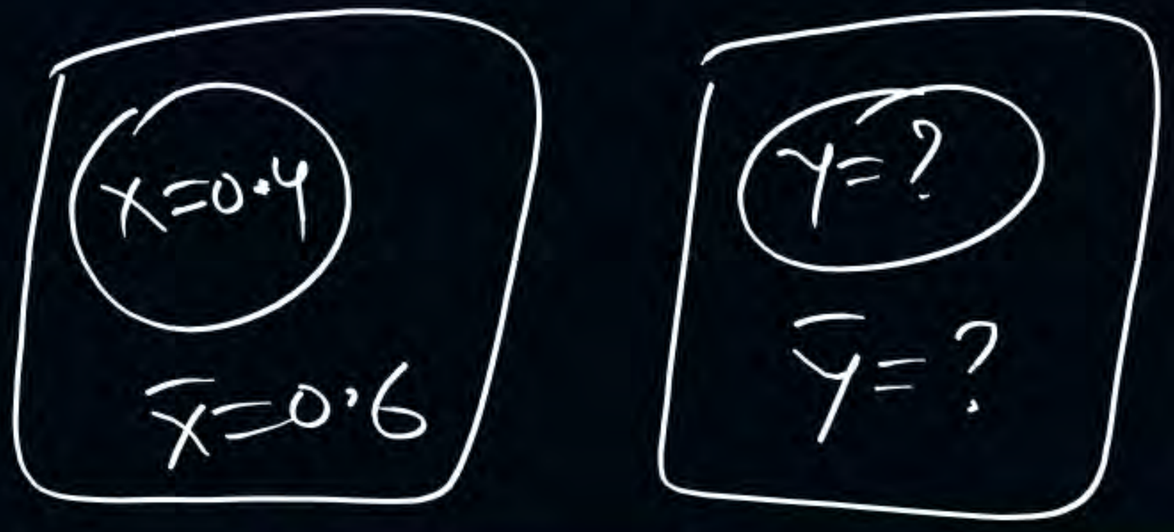
$-0.6P(Y) = -0.3 \Rightarrow P(Y) = \frac{1}{2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= 0.4 + 0.5 - (0.4)(0.5)$

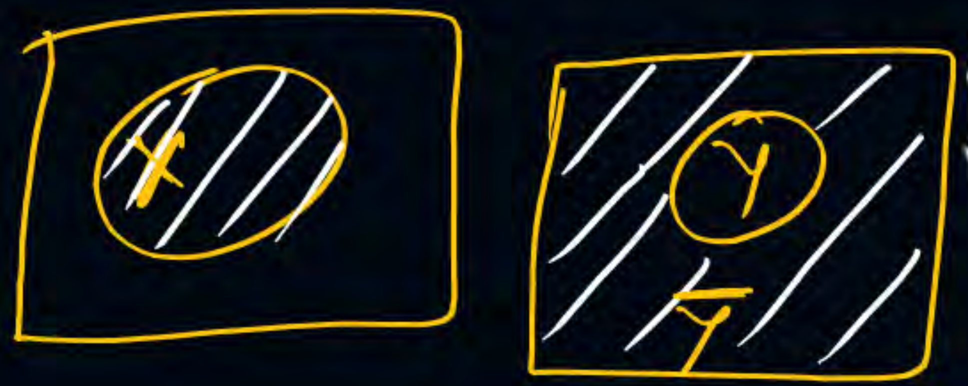
$= 0.9 - 0.20 = 0.7$

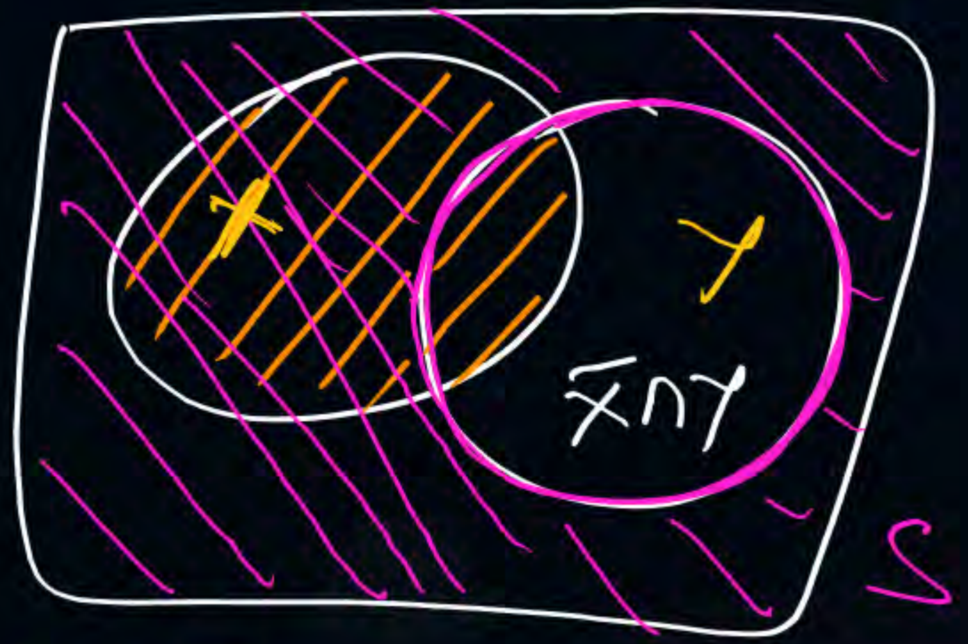
M-II



$$X \cup \bar{Y} = 0.7$$

$$X \cap \bar{Y} + \bar{X} \cap \bar{Y} = 0.7$$





$$X \cup \bar{Y} = S - \bar{X} \cap Y$$

$$\bar{X} \cap Y = S - X \cup \bar{Y} = 1 - 0.7 = 0.3$$

So $X \cup Y = X + \bar{X} \cap Y$

$$= 0.4 + 0.3 = 0.7$$

Mutually Exclusive Events → RECAP

If, two events can't occur simultaneously, then these are called M.E. Events

OR

If occurrence of one event prevents the occurrence of other event & vice versa than events are called ME Events. i.e

If A & B are ME then only one can occur at a time

Mathematically: if E_1 & E_2 are ME events then $E_1 \cap E_2 = \phi$

Conclusion: if E_1 & E_2 are ME then

→ $P(E_1 \cap E_2) = 0$

→ $P(E_1 \cup E_2) = P(E_1) + P(E_2) - 0$

eg $S_D = \{1, 2, 3, 4, 5, 6\}$ & let us consider following events

$E_1 = \{1, 3, 5\}$ $\because E_1 \cap E_2 = \phi \Rightarrow E_1$ & E_2 are M.E & $P(E_1 \cap E_2) = 0$

$E_2 = \{2, 4, 6\}$ $\because E_2 \cap E_3 \neq \phi \Rightarrow E_1$ & E_2 are Not M.E

$E_3 = \{1, 2, 3, 4\}$ $\because E_1 \cap E_3 \neq \phi \Rightarrow E_1$ & E_3 are Not ME

$E_4 = \{2, 4\}$, $\because E_1 \cap E_4 = \phi \Rightarrow E_1$ & E_4 are also ME But $E_1 \cup E_4 \neq S$

i.e it is not Necessary that, in Case of ME Events, you will get their union as S. Space

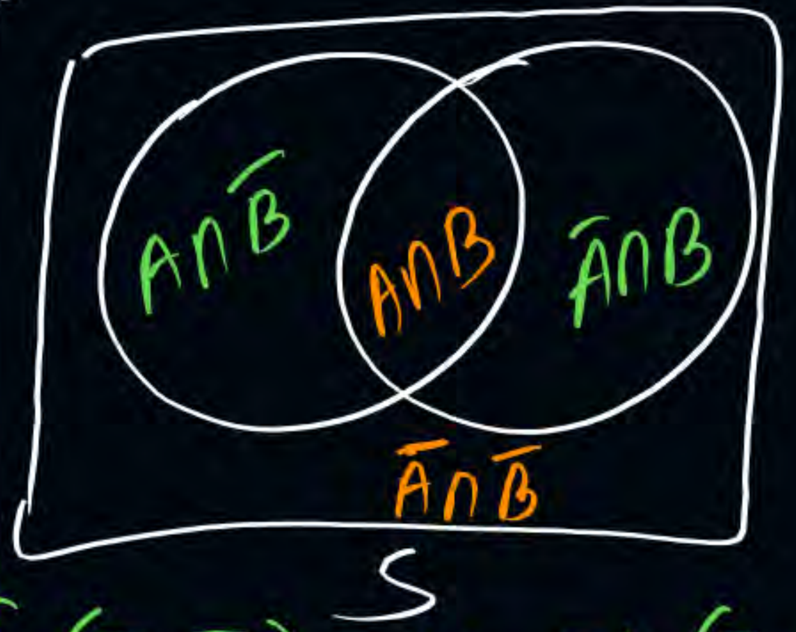
$E_4 = \{x : 1 < x < 5$ & x is divisible by $2\}$

RECAP

Concept of M.E and Independency in a Single Question →

Consider A & B are firing at the Target once then various possibilities are as follows,

$\Longrightarrow$



RECAP

A & B are Ind

A & $\bar{B}$ " "

$\bar{A}$ & B " "

$\bar{A}$ & $\bar{B}$ " "

$$S = \{\underbrace{(\bar{A} \cap \bar{B})}_{=E_1}, \underbrace{(A \cap \bar{B})}_{=E_2}, \underbrace{(\bar{A} \cap B)}_{=E_3}, \underbrace{(A \cap B)}_{=E_4}\}$$

$\because E_1, E_2, E_3, E_4$ are Individual elements of this New S. Space

so these 4 events are ME.

F.Q3 →

RECAP

There are two Gangsters Munna Mobile (A) & Pappu Pazer (B), They both fire at the target once with probability of their hitting is $\frac{4}{5}$ & $\frac{3}{4}$ resp.

Sol: $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$, A & B are Ind & $S = \{\underbrace{(\bar{A}\cap\bar{B})}_{=E_1}, \underbrace{(A\cap\bar{B})}_{=E_2}, \underbrace{(\bar{A}\cap B)}_{=E_3}, \underbrace{(A\cap B)}_{=E_4}\}$

$P(\bar{A}) = \frac{1}{5}$, $P(\bar{B}) = \frac{1}{4}$,

(1) Find the prob that Both will hit = ? $= P(E_4) = P(A\cap B) = \frac{4}{5}\times\frac{3}{4} = \frac{3}{5}$

(2) " " " " None will hit = ? $= P(E_1) = P(\bar{A}\cap\bar{B}) = \frac{1}{5}\times\frac{1}{4} = \frac{1}{20}$

(3) " " " " Atleast one will hit = ? $= 1 - P(\text{None will hit}) = 1 - \frac{1}{20} = \frac{19}{20}$

(4) " " " " target will be hit = ? $= P(\text{at least one will hit}) = \frac{19}{20}$.

(5) Find the prob that either of them will hit $= ? = P(A \cup B)$

(either A or B or Both)

$= P(A) + P(B) - P(A \cap B)$

$= \frac{4}{5} + \frac{3}{4} - \frac{3}{5} = \frac{19}{20}$ (same as (4))

RECAP

(6) Find the prob that $\underbrace{\text{A hit \& B missed}}_{\text{only A will hit}} = ? = P(E_2) = P(A \cap \bar{B}) = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$

(7) Find the prob that $\underbrace{\text{only one will hit}}_{\text{(only A or only B)}} = ? = P(E_2 \cup E_3)$

$E_2 \cup E_3$

$\{\because E_2 \,\&\, E_3 \text{ are ME}\}$

$= P(E_2) + P(E_3) - 0$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$

$\{\because A \,\&\, B \text{ are Ind}\}$

(8) If $\underbrace{\text{exactly one person hit}}_{\text{Condition}}$ then Find the prob that A hit & B missed = ?

i.e if $\underbrace{\text{only one will hit}}$ then find the prob that only A will hit = ?

Sol: original Prob = $P(S) = 1$

Reduced Prob = $P(\text{Condition}) = P(\text{only one will hit}) = \frac{7}{20}$ (using Part 7)

Fav Prob = $P(\text{only A will hit}) = P(A \cap \bar{B}) = \frac{4}{20} = \frac{1}{5} = \frac{1/5}{1}$ (using Part 6)

So Conditional Prob = $\frac{\text{Fav Prob}}{\text{R-Prob}} = \frac{1/5}{7/20} = \frac{4}{7}$

Conditional Probability

(just reduce the S.Space according to Condition)

eg: A Couple has 3 kids & 1st child is known to be a Boy then Find the prob of having exactly 2 Boys? Condition

Sol: original S.Sp = { (GGG), (GGB), (GBG), (GBB), (BBB), (BBG), (BGB), (BGG) } = 8

Reduced S.Sp = { 1st child is a Boy } = { (BBB), (BBG), (BGB), (BGG) } = 4 Triplets

fav Cases = { exactly 2B } = { (BBG), (BGB) } = 2 Triplets

$$\text{Conditional Prob} = \frac{\text{fav Cases}}{\text{R.S.Sp}} = \frac{2}{4} = \frac{1}{2} \neq \frac{4}{8}$$

App III Req Prob $= P\{(BBG) \text{ or } (BGB)\}$ ME

$= P(BBG) + P(BGB)$

$= 1 \times \frac{1}{2} \times \frac{1}{2} + 1 \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq \frac{4}{8}$

Note: $\frac{1}{2} \neq \frac{4}{8}$

But $\frac{4}{8} = \frac{1}{2}$ ✓

Sp Note: In a Questions Based on Conditional Prob, $\underbrace{\text{we should not generalize}}$ the final Ans.

Note: Had the Condition was not there then answer would have been $= ? = \frac{^3C_2}{2^3} = \frac{3}{8}$

A Couple has 3 kids then Find OR the Prob of having exactly 2 Boys $= ? = \frac{3}{8}$

eg: A Die is throw twice. If $\underbrace{\text{sum of the outcomes is 6}}_{\text{condition}}$, then Find the prob that number 4 has appeared at least once?

Sol: original S.Sp = $\left\{\begin{matrix}(11),(12)\ldots(16),(21),(22)\ldots(26)\\(31),(32)\ldots\ldots(66)\end{matrix}\right\} \Rightarrow n(S) = 6^2 = 36$ pair

R.S.Sp = {sum of the outcomes is 6} = {(15), (51), (24), (42), (33)} = 5 pair

fav cases = {'4' appeared at least once} = {(42), (24)} = 2 pair

Hence Conditional Prob = $\frac{\text{fav Cases}}{\text{R.SSp}} = \frac{2}{5}$

Note: A Die is thrown twice then Find the prob that 4 has appeared at least once = ? $= \frac{11}{36}$

fav = $\left\{\begin{matrix}(41)(42)(43)(44)(45),(46)\\(14)(24)(34)\cancel{(44)}(54)(64)\end{matrix}\right\}$ = 11 pair

Eg: A coin is tossed 6 times & 1st three outcomes are H H H then find the prob of occurring Tail in remaining tosses ?

App III: Req Prob $= P\left[\left(HHH\ \underline{T}\ \underline{T}\ \underline{T}\right)\right] = 1^3 \times \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

(SALMAAN KHAN)

App II: original S.Sp $= \{(HHHHHH), (HHHHHT), \ldots, (TTTTTT)\} = 2^6 = 64$ tuples.

(AAMIR KHAN)

Reduced S.Sp $= \{(1^{st}$ three outcomes are $HHH)\} = \{(\underline{H}\ \underline{H}\ \underline{H}\ \underline{H/T}\ \underline{H/T}\ \underline{H/T})\}$

$= 1\times1\times1\times2\times2\times2 = 8$ tuples.

fav Cases $= \{(HHH\ \underline{T}\ \underline{T}\ \underline{T})\} = 1$ tuple

So Conditional Prob $= \frac{fav}{RSSp} = \frac{1}{8} \neq \frac{8}{64}$

eg: Two Integers are to be Selected from integers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

If their sum is Even then find the prob that both the selected integers are odd? Condition

Sol: App II

$$\underbrace{1,2,3,4,5,6,7,8,9,10,11}_{{}^{11}C_2 = 55 \text{ ways}}$$

$2, 4, 6, 8, 10 \longrightarrow {}^{5}C_2 = 10$ ways

$1, 3, 5, 7, 9, 11 \longrightarrow {}^{6}C_2 = 15$ ways

Total ways of Selecting 2 integers $= {}^{11}C_2 = 55$ ways.

Reduced " " " " = {Sum is Even}

= {Either (Both are even) or (both are odd)}

using F.P of addition

$= {}^{5}C_2 + {}^{6}C_2 = 10 + 15 = 25$ ways

Fav ways of " " " = {Both are odd} $= {}^{6}C_2 = 15$ ways.

So Conditional Prob $= \dfrac{\text{fav cases}}{\text{R. cases}} = \dfrac{{}^6C_2}{{}^6C_2 + {}^5C_2} = \dfrac{15}{25} = \dfrac{3}{5}$

Note: Had the condition were not there then answer would have been $= ? = \dfrac{\text{fav}}{\text{s.sp}} = \dfrac{15}{55}$

App I   See Next Slide

$\underbrace{1, 2, 3, 4, 5, 6, 7, 8, 9, 10,}_{\text{RNA}}$ 11 (Toy)

RNA if only selection is Required

i.e there should not be any arrangement

Appr: original S.Sp = $\left\{\begin{array}{l} \cancel{(11)}\ (12)\ (13)\ (14) \ldots (1,11) \\ \cancel{(21)}\ \cancel{(22)}\ (23)\ (24) \ldots (2,11) \\ \cancel{(31)}\ \cancel{(32)}\ \cancel{(33)} \ldots (3\ 11) \\ \cancel{(41)} \ldots (10,11) \end{array}\right\}$ $= 10+9+8+7+6+5+4+3+2+1$

$= 55$ pair

Reduced S.Sp = {Sum is Even} = {(EE) or (OO)}

$= \left\{\begin{array}{l} (2\ 4)\ (2\ 6)\ (2\ 8)\ (2\ 10),\ (4\ 6),\ (4\ 8),\ (4\ 10) \\ (6\ 8),\ (6\ 10),\ (8\ 10) \longrightarrow 10 \text{ pair} \\ (1\ 3)\ (1\ 5)\ (1\ 7)\ (1\ 9)\ (1,11) \\ (3\ 5)\ (3\ 7)\ (3\ 9)\ (3\ 11),\ (5\ 7)\ (5\ 9),\ (5\ 11) \\ (7\ 9),\ (7\ 11),\ (9,11) \longrightarrow 15 \text{ pair} \end{array}\right\}$

10 pair or 15 pair $= 25$ pair

fav Pair = {Both are odd} = 15 pair

So Conditional Prob $= \dfrac{\text{fav Cases}}{\text{R.S.Sp.}} = \dfrac{15}{25}$

Q. Parcels are sending from Sender S to Receiver R sequentially through two post offices. The Probability of loosing an incoming parcel by each P.O is $\frac{1}{5}$ independently of all other parcels.
Given that, $\underbrace{\text{Parcel is lost}}_{\text{Condition}}$ then Find the prob that it was lost by 2nd P.O?

(a) 1/5
(b) 4/5
(c) 4/25
(d) 4/9

Analysis: let No. of parcels sended by sender = 25 P.

ie $P(L) = \frac{1}{5}$

$P(NL) = 1 - \frac{1}{5} = \frac{4}{5}$

$$\underset{(25)}{\text{Sender}} \longrightarrow PO\text{-}1\,(20) \xrightarrow[NL=20]{\text{lost}=5} P.O.2\,(20) \xrightarrow[NL=16]{\text{lost}=4} \text{Receiver}\,(16P)$$

$$\underset{(50)}{\text{Sender}} \longrightarrow \underset{(50)}{PO\text{-}1} \xrightarrow[NL=40]{L=10} \underset{(40)}{PO.2} \xrightarrow[NL=32]{L=8} \underset{(32)}{\text{Receiver}}$$

App III (using Conditional Prob) $\rightarrow$ original Prob $= P(S) = 1$.

Reduced Prob $= P(\text{Condition}) = P(\text{parcel is lost})$ $\rightarrow$ ME

$= P\{\text{either (Lost by } 1^{st}) \text{ or (Lost by } 2^{nd} \text{ P.O)}\}$

$= P(\text{Lost by } 1^{st} \text{ PO}) + P(\text{Lost by } 2^{nd} \text{ P.O})$

$= P(\text{Lost by } 1^{st}) + P\{(\text{NL by } 1^{st}) \,\&\, (\text{Lost by } 2^{nd})\}$

$= \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{9}{25}$.

Fav Prob $= P(\text{Lost by } 2^{nd}) = P(\text{NL by } 1^{st} \,\&\, \text{L by } 2^{nd}) = \frac{4}{5} \times \frac{1}{5} = 4/25 = \frac{4/25}{1}$

Hence Condn Prob $= \frac{\text{Fav Prob}}{\text{R-Prob}} = \frac{4/25}{9/25} = \frac{4}{9}$

M-II (using Baye's Th) $\rightarrow$ Have patience

Formulatic Approach of Conditional Prob →

① $\boxed{P(A/B) = \dfrac{P(A\cap B)}{P(B)}}$ it is the prob of A when B has already occurred.

② $P(B/A) = \dfrac{P(B\cap A)}{P(A)}$ it is the Prob of B when A has already occurred

③ $P(A\cap B/C) = \dfrac{P(A\cap B\cap C)}{P(C)}$ it is the prob of simultaneous occurrence of A & B when C has already occurred.

Sp. Note :- If A & B are Ind then $\to P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{P(A)\cdot P(B)}{P(B)} = P(A)$

$\to P(B/A) = P(B)$

i.e In Case of Independency, Condition has NO significance.

V. V. Special Note :- To check the Independency of Events, we have following three methods;

M-I By defⁿ. (Best Method)

M-II if $[P(A \cap B) = P(A) \cdot P(B)]$ then $\Longleftrightarrow$ [A & B are called Ind]

M-III if $[P(A/B) = P(A)]$ then $\Longleftrightarrow$ [A & B are Ind.]

(*) The Relation of Dependency or Independency is a Vice-Versa Relation.

eg $C=\{H,T\}$, $D=\{1,2,3,4,5,6\}$, Coin & Die are Ind.

$A=\{H\}$, $B=\{\text{No} \le 4\}$

$P(A)=\frac{1}{2}$, $P(B)=\frac{4}{6}$

$P(A/B)=?=P(A)=\frac{1}{2}$ ($\because$ A & B are Ind.)

& $P(B/A)=?=P(B)=\frac{4}{6}$ (" " ")

Q. (ME) An Hydraulic Structure has four GATES which operates Independently. The prob of failure of each GATE is 0.2. Given that, Gate 1 has failed then find the prob that Gate 2 & Gate 3 will also fail?

Condition

(a) 0.2

(b) 0.04

(c) 0

(d) 1

ATQ, $P(G_1) = P(G_2) = P(G_3) = P(G_4) = 0.2$

M-I $P(G_2 \cap G_3) = ? = P(G_2) \cdot P(G_3) = 0.2 \times 0.2 = 0.04$

M-II $P\left[\frac{G_2 \cap G_3}{G_1}\right] = ? = P(G_2 \cap G_3) = 0.2 \times 0.2 = 0.04$

M-III $P[\underset{\text{given}}{G_1} \cap G_2 \cap G_3 \cap S.O] = 1 \times 0.2 \times 0.2 \times 1 = 0.04$

Q. If $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$, $P(A/B)=\frac{1}{6}$ then find the prob of their $\underbrace{\text{simultaneous occurrence}}$?

$P(A \cap B) = ?$

(a) $\frac{1}{24}$

(b) $\frac{1}{12}$

(c) 0

(d) 1

$\because \boxed{P(A/B) \neq P(A)} \Rightarrow$ By M-II, A & B are Not Ind

So we can not multiply individual prob

Hence we will use Multi Theorem

$$\boxed{P(A \cap B) = P(A/B) \cdot P(B)}$$

$$= \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

If P(A) = 1/3, P(B) = 1/4, P(A/B) = 1/6, then what is P(B/A) equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

In previous Q, we have shown that, $P(A \cap B) = \frac{1}{24} = P(B \cap A)$

$\because$ A & B are Not Ind so By standard Result

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1/24}{1/3} = \frac{1}{8}$$

If A and B are events such that

$$P(A \cup B) = 0.5, P(\bar{B}) = 0.8 \text{ and } P(A/B) = 0.4,$$

What is $P(A \cap B)$ equal to?

(a) 0.08 (b) 0.02 (c) 0.8 (d) 0.2

this Info is just to confuse us

$P(\bar{B}) = 0.8$

$1 - P(B) = 0.8$

$P(B) = 0.2$

Now, $P(A/B) = 0.4$

$$\frac{P(A \cap B)}{P(B)} = 0.4 \Rightarrow P(A \cap B) = 0.4 \times 0.2 = 0.08$$

Q. PYQ If $P(A)=1$, $P(B)=\frac{1}{2}$ then find $P(A/B)=?$, $P(B/A)=?$ respectively.

(a) 1, 0

(b) 0, 1

(c) $1, \frac{1}{2}$

(d) Data Inadequate.

M-I $\cancel{P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{?}{1/2}}$ = Data Inadequate

M-II $\because P(A)=1$

$\Downarrow$

A will definitly occur

$\Downarrow$

A is Ind from B (By M-I)

$\Downarrow$

B is also Ind. from A

$P(A/B)=P(A)=1$

$P(B/A)=P(B)=1/2$

A box contains 4 while balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is

(a) 1/3 (b) 3/7

(c) 1/2 (d) 4/7

$$\boxed{\begin{matrix}4W\\3R\end{matrix}} \xrightarrow{W} \boxed{\begin{matrix}3W\\3R\end{matrix}}$$

* "one by one w/o Replacement"

App III Req. Prob = P[W ∩ R]

$= 1 \times \frac{3}{6} = \frac{1}{2}$

HW App I

original SSp = { }

R-SSp = { }

fav Cases = { }

then Ans = ??

Qs (2009/EE) Consider an unbalanced die numbered 1 to 6.

The prob of an odd face is 90% the prob of an Even face. &

The prob of any even numbered face is same

The prob of an Even face given that face Value exceeds 3 is 0.75

then find the prob that face Value exceeds 3 ?

(a) 10/19

(b) 10/57

(c) 8/3

(d) 80/171

NIRMAN
2025

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Thank you

Keep Hustling!