DS & AI
CS & IT

Statistics -1
(Discrete Random Variable)
Lecture - 02

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Recap of previous lecture

Topic

BASICS of STATISTICS

Topics to be Covered

Topic

1. PROBABILITY DISTRIBUTION
2. Geometric Distribution

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your $\boxed{\text{little mind}}$ until you have a complete understanding of the chapter) & try to solve the Quest.

Random Variable

RECAP

D.R.V (X)

↓

Discrete Prob Distribution

eg (Geometric, Binomial, Poisson)

↓

Prob Mass func^n (p.m.f) $= p_i$

$p_i \geq 0, \; \sum p_i = 1$

C.R.V (x)

↓

Continuous Prob Distribution

eg (Exponential, uniform, Normal)

↓

Prob. Density func^n (p.d.f) $= f(x)$

$f(x) \geq 0, \; \int_{-\infty}^{\infty} f(x)\,dx = 1$

Some More Standard Results $\rightarrow$ Let X & Y are R.V & a, b, c are constants

RECAP

(i) $E(aX \pm bY \pm c) = aE(X) \pm bE(Y) \pm E(c)$

$= aE(X) \pm bE(Y) \pm c$

(ii) $Var(aX + b) = a^2 Var(X) + Var(b)$

$= a^2 Var(X) + 0$

eg $Var(-X+3) = (-1)^2 Var(X) + Var(3)$

$= Var(X) + 0$ //

(iii) $\boxed{Var(aX \pm bY) = a^2 Var(X) + b^2 Var(Y) \pm 2ab\, Cov(X, Y)}$

Learn only w/o Proof.

eg Consider $X = \underbrace{62, 62, 62, 62, 62, 62, 62}_{\text{Constant Data}}$ then $\bar{X} = 62$ kg & $Var = 0$, $SD = 0$

H. Q. If mean & Variance of R.V x is given as μ & σ^2 resp then then mean & Variance of $\frac{x-\mu}{\sigma}$ are, respectively?

(a) $\{0, 1\}$

(b) $\{0, \sigma\}$

(c) $\{\mu, \sigma\}$

(d) $\{\mu, \sigma^2\}$

ATQ, $E(x) = \mu$ & $Var(x) = \sigma^2$ & let $\frac{x-\mu}{\sigma} = z$

So $Mean(z) = E(z) = E\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma}\left[E(x) - E(\mu)\right] = \frac{1}{\sigma}\left[\mu - \mu\right] = 0$

& $Var(z) = Var\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma^2} Var(x-\mu) = \frac{1}{\sigma^2}\left\{Var(x) + Var(-\mu)\right\}$

$= \frac{1}{\sigma^2}\left\{\sigma^2 + 0\right\} = 1.$

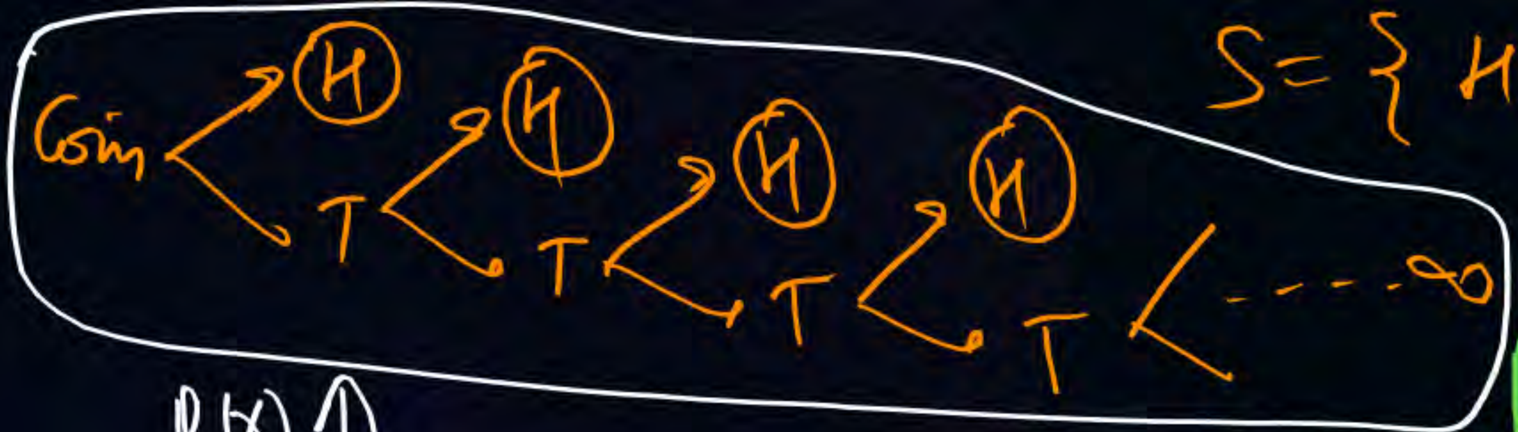
Note: $Var(x-\mu) = (1)^2 Var(x) + (-1)^2 Var(\mu)$

$= Var(x) + (0) = Var(x)$

Probability Dist^n — The Table representing Distribution of probabilities is called Prob Distribution. where X = {which is Required should be assumed as X}

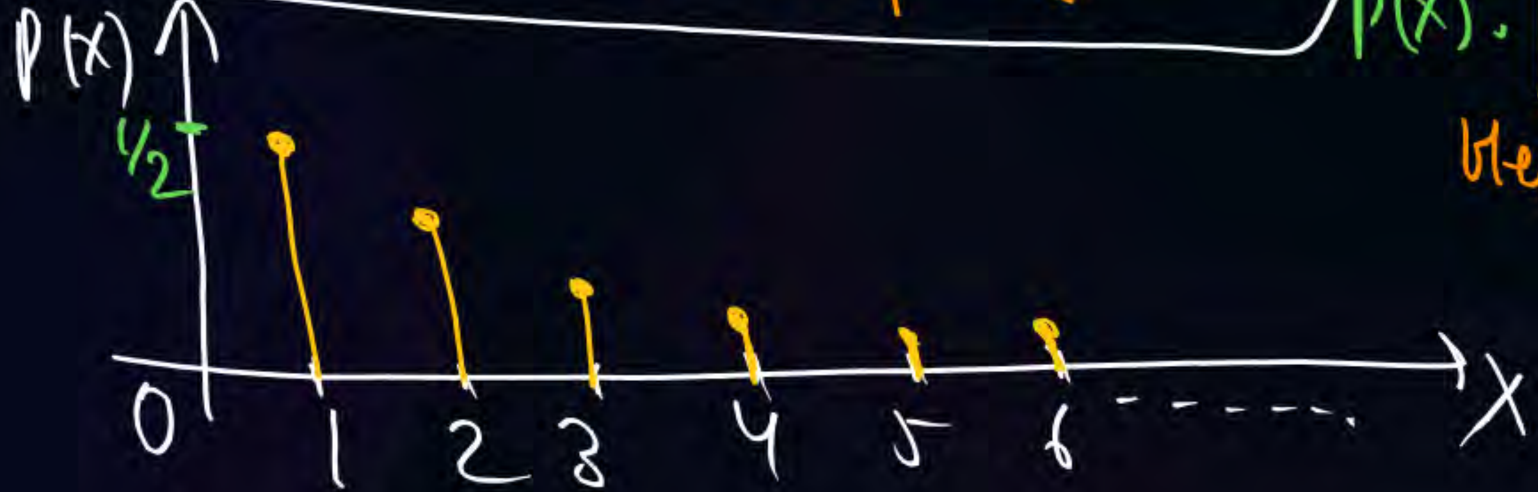
eg A coin is tossed until H appears then find Prob Dist of No. of tosses.

Sol: Let X = {Number of tosses} = {1, 2, 3, 4, ...}



S = {H, TH, TTH, TTTH, ...}

X:	1	2	3	4	5	......
P(X):	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	......



Here we have two conditions

$p_i \geq 0$ & $\sum p_i = \frac{1/2}{1-1/2} = 1$

eg: A coin is tossed thrice then find the prob Dist of Number of Heads?

Sol: $X = \{\text{Number of Heads}\} = \{0, 1, 2, 3\}$

$$S = \{\underset{=3}{(HHH)}, \underset{=2}{(HHT)}, \underset{=2}{(HTH)}, \underset{=1}{(HTT)}, \underset{=2}{(THH)}, \underset{=1}{(THT)}, \underset{=1}{(TTH)}, \underset{=0}{(TTT)}\} = 8 \text{ Triplets}$$

$$p_1 = P(X = 0H) = \frac{1}{8} = \frac{{}^3C_0}{2^3}$$

$$p_2 = P(X = 1H) = \frac{3}{8} = \frac{{}^3C_1}{2^3}$$

$$p_3 = P(X = 2H) = \frac{3}{8} = \frac{{}^3C_2}{2^3}$$

$$p_4 = P(X = 3H) = \frac{1}{8} = \frac{{}^3C_3}{2^3}$$

X:	0	1	2	3
$P(X)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\underbrace{p_i \geq 0 \;\&\; \sum p_i = 1}_{\text{Verified.}}$

Q. A coin is tossed thrice then Find Mean, Variance & S.D of Number of Heads.

Sol: $X = \{\text{Number of Heads}\} = \{0, 1, 2, 3\}$,

x:	0	1	2	3
$p(x)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

M-I $\boxed{E(X) = \sum p_i x_i} = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$

(Mature Method)

$= \frac{1}{8}(0) + \frac{3}{8}(1) + \frac{3}{8}(2) + \frac{1}{8}(3) = \frac{3}{2} = 1.5$

Average Number of Heads $= 1.5$ Ans

Now, $\boxed{E(X^2) = \sum p_i x_i^2} = p_1x_1^2 + p_2x_2^2 + p_3x_3^2 + p_4x_4^2$

$= \frac{1}{8}(0)^2 + \frac{3}{8}(1)^2 + \frac{3}{8}(2)^2 + \frac{1}{8}(3)^2 = \frac{0+3+12+9}{8} = 3$

So $\boxed{Var(X) = E(X^2) - (E(X))^2} = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4} = 0.75$ Ans.

& $S.D(\sigma) = +\sqrt{Var(X)} = \sqrt{0.75} = \frac{\sqrt{3}}{2} = 0.866$

M-II we have $x = \{0, 1, 2, 3\} \Rightarrow \cancel{\bar{x} = \frac{\sum x}{N} = \frac{0+1+2+3}{4}} = 1.5$

& $Var(X) = \frac{\sum (x-\bar{x})^2}{N} \cancel{= \frac{(0-1.5)^2+(1-1.5)^2+(2-1.5)^2+(3-1.5)^2}{4}} = \ldots = 1.25$

WRONG APPROACH.

KAISA LAGA MERA MAZAK.

M-III (Childhood Method) we have $\boxed{X = \{0, 1, 1, 1, 2, 2, 2, 3\}} \Rightarrow \bar{x} = \frac{\sum x}{N} = \frac{0+1+1+1+2+2+2+3}{8} = 1.5$ ✓

& $Variance(X) = \frac{\sum (x-\bar{x})^2}{N} = \frac{(0-1.5)^2 + 3\times(1-1.5)^2 + 3\times(2-1.5)^2 + (3-1.5)^2}{8} = 0.75$ ✓

$S.D(\sigma) = +\sqrt{0.75} = 0.866$ ✓

Doubt:

$$Var(x) = \frac{\sum (x-\bar{x})^2}{N}$$

$$= \frac{(0-1.5)^2+(1-1.5)^2+(1-1.5)^2+(1-1.5)^2+(2-1.5)^2+ \cdots}{8}$$

MAI THAK GAYA/GAYI

for students

$$= \frac{(0-1.5)^2+3\times(1-1.5)^2+3\times(2-1.5)^2+(3-1.5)^2}{8} = 0.75$$

Sp. Note: It is advisable to follow Mature Method further.

PODCAST (Deep thinking) :-

A coin is tossed thrice, Average Number of Heads $= 1.5$, $Var = 0.75$, $S.D = \sqrt{0.75}$

" " " $\boxed{\text{once}}$, Average " " " $= \boxed{0.5}$, $Var = 0.25$, $S.D = \frac{\sqrt{0.75}}{\sqrt{3}} = 0.5$

" " " 10000 times, Average " " Heads $= 5000$, $Var = 2500$, $SD = 0.5 \times \sqrt{10000}$
$= 50$

ie we have a very good chance that No. of Heads lies in b/w 4950 & 5050.

where $\mu - \sigma = 5000 - 50 = 4950$ ie $\boxed{4950 \le \text{No. of Heads} \le 5050}$

$\mu + \sigma = 5000 + 50 = 5050$

Q: A die is thrown $\boxed{\text{Large No. of times}}$ then find Expected Value of the outcome occurs?

Sol: For Die, possible outcomes are 1, 2, 3, 4, 5, 6

Let $X = \{\text{outcome of a Die when it is thrown}\} = \{1, 2, 3, 4, 5, 6\}$

& $\boxed{P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}}$

$\because$ we are throwing die large number of times that's we have taken these Prob.

X:	1	2	3	4	5	6
$P(X)$:	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

So $E(X) = \sum p_i x_i = \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6)$

$= \frac{1+2+3+4+5+6}{6} = 3.5$

Average Value of the outcome occurs $= 3.5$

Analysis:

① $P(H) = ? = \dfrac{5000 \text{ Cr Head}}{10000 \text{ Cr Tosses}} = \dfrac{1}{2}$ & $P(\text{Tail}) = \dfrac{5000 \text{ Cr Tail}}{10000 \text{ Cr toss}} = \dfrac{1}{2}$

② $P(\text{Boy}) = \dfrac{350 \text{ Cr Boys}}{700 \text{ Cr persons}} = \dfrac{1}{2}$ & $P(\text{Girl}) = \dfrac{350 \text{ Cr Girls}}{700 \text{ Cr persons}} = \dfrac{1}{2}$

③ A Couple has 5 Kids in which there are exactly 2 B & 3 G & we are choosing a kid then $P(\text{Boy}) = \dfrac{2}{5}$ & $P(\text{Girl}) = \dfrac{3}{5}$

We have so many kids in a town & we want to choose a kid then $P(\text{Boy}) = \dfrac{1}{2}$, $P(\text{Girl}) = \dfrac{1}{2}$

(4) When sample size is large enough then it is called Population / Universe.

" " " is small then it is simple called SAMPLE.

Deciding Factors of population are called PARAMETERS (μ & σ)

& " " " sample are called STATISTICS ($\bar{x}$ & s)

Chance of getting success in Population is called PROBABILITY

& " " " " Sample " " Proportion

Q. A coin is tossed until Head appears or Tail appears 4 times in succession, then Find the Average Number of tosses Required

(a) 2.50

(b) 1.87 ✓

(c) 4

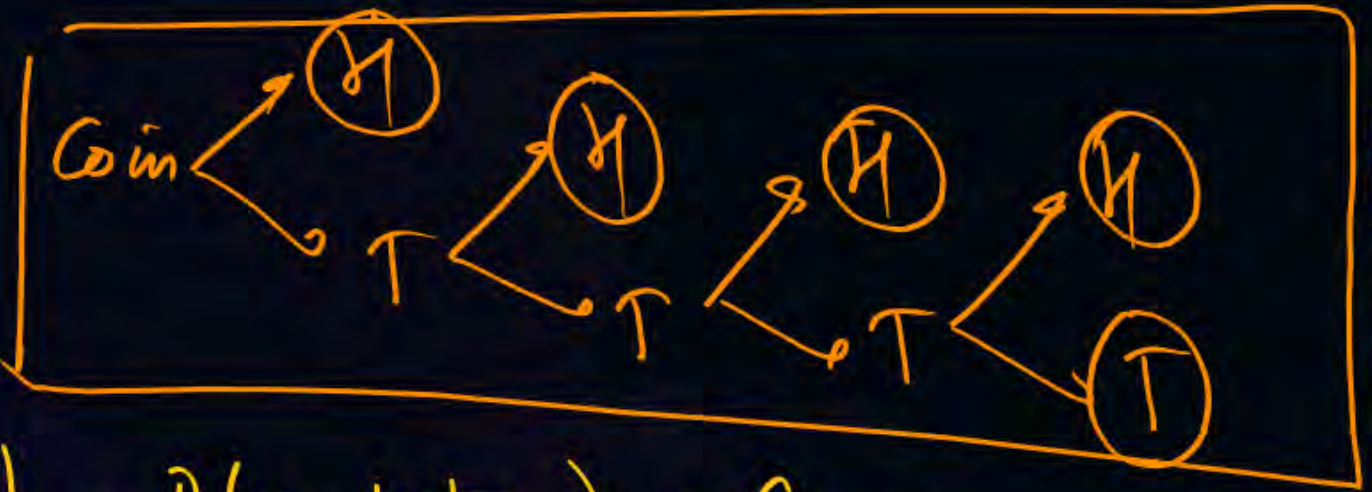
(d) 3.89

$X = \{\text{Number of tosses}\} = \{1, 2, 3, 4\}$

$S = \{\overset{=1}{H}, \overset{=2}{TH}, \overset{=3}{TTH}, \overset{=4}{TTTH}, \overset{=4}{TTTT}\}$

X:	1	2	3	4
$P(X)$:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

$\because \sum p_i = 1$ ✓



$p_1 = P(X = 1\ \text{Toss}) = P(H) = \frac{1}{2}$

$p_2 = P(X = 2\ \text{tosses}) = P(TH) = \frac{1}{4}$

$p_3 = P(X = 3\ \text{tosses}) = P(TTH) = \frac{1}{8}$

$p_4 = P(X = 4\ \text{tosses}) = P(TTTH \text{ or } TTTT) = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$

$$E(X) = \sum p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4$$

$$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{8}(4)$$

$$= \frac{15}{8} = 1.87 \quad (b)$$

A random variate has the following distribution:

x :	0	1	2	3	4	5	6	7
$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

The value of k is ______. (a) 0.5 (b) 0.2 (c) -1 (d) 0.1

[Ans: [redacted]]

w.k.that $\sum p_i = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$

$10k^2 + 10k - k - 1 = 0$

$10k(k+1) - 1(k+1) = 0$

$(10k - 1)(k+1) = 0$

$k = \frac{1}{10}$ & $\underbrace{k = -1}_{\text{N.P.}}$

$k = 0.1$ ✓

Gate 2004 (CS) → 1000 students are appearing in a test in which each student has to solve 150 MCQ'S. Each correct ans gives 1 Marks and Each incorrect ans fetches −0.25 Marks.

If all the students have given all their answers Randomly, then find the sum total of their expected Marks?

No one knows Nothing

Guess Method will follow.

(a) 150000
(b) −37500
(c) 9375
(d) 37500
(e) 1/16

Let $N_1 = 1000$ students, $N_2 = 150$ Questions

For single student in single Question

$X = \{\text{Marks obtained by single student in single Q}\}$
$= \{1, -1/4\}$

X :	1	$-1/4$
$P(x)$:	$1/4$	$3/4$

$p_1 = P(X = 1\text{ Marks}) = P(\text{correct ans}) = \frac{1}{4}$

$p_2 = P(X = -\frac{1}{4}\text{ Marks}) = P(\text{wrong ans}) = \frac{3}{4}$

$E(x) = \sum p_i x_i = p_1 x_1 + p_2 x_2 = \frac{1}{4}(1) + \frac{3}{4}\left(\frac{-1}{4}\right) = \frac{1}{16}$ Marks

Average Marks obtained by single student, in single Quest $= \frac{1}{16}$ Marks.

Average " " " single " in 150 Q $= \frac{1}{16} \times 150$ Marks

Average " " " 1000 " " 150 Q $= \frac{1}{16} \times 150 \times 1000$

$= 9375$ Marks Ans

Note (1) Max Marks that can be obtained $= 1000 \times 150 \times (1) = 150000$ Marks.

(2) Min " " can be " $= 10000 \times 150 \times \left(-\frac{1}{4}\right) = -37500$ "

(3) Average Marks obtained by them $= 1000 \times 150 \times \left(\frac{1}{16}\right) = 9375$ Marks.

Q In a Game a person is paid Rs 5 when all Heads or all Tail occurs and he will have to pay Rs 3 if either one or two Head occurs when three Coins are tossed simultaneously then find the expected amount wins or looses by him on an average per Game?

(ii) If above Game is played by 150 persons, 200 times each then find the Expected amount wins or looses by Game organiser

Sol: $N_1 = 150$ persons, $N_2 = 200$ Games

For single person in single Game

$X = \{$ Amount Received by single person in single Game $\} = \{5, -3\}$

$S = \{\underbrace{(HHH)}_{W}, \underbrace{(HHT), (HTH), \ldots\ldots}_{\text{Looses}}, \underbrace{(TTT)}_{W}\}$

$p_1 = P(X = 5\ Rs) = P(W) = 2/8$

$p_2 = P(X = -3\ Rs) = P(L) = 6/8$

$$\boxed{\begin{array}{c|cc} X: & 5 & -3 \\ P(x): & \frac{2}{8} & \frac{6}{8} \end{array}}$$

$$\boxed{\begin{array}{lcc} X: & 5 & -3 \\ P(X): & \frac{2}{8} & \frac{6}{8} \end{array}}$$

So $E(X) = \sum p_i x_i = p_1 x_1 + p_2 x_2 = \frac{1}{4}(5) + \frac{3}{4}(-3) = -1$ Rs

Average Amount Received by single person in single Game $= -1$ Rs

ie He/she will loose 1 Rs on an average per Game.

(ii) Single person in single Game will loose $= 1$ Rs

Single " in 200 Games " " $= 1 \times 200$ Rs

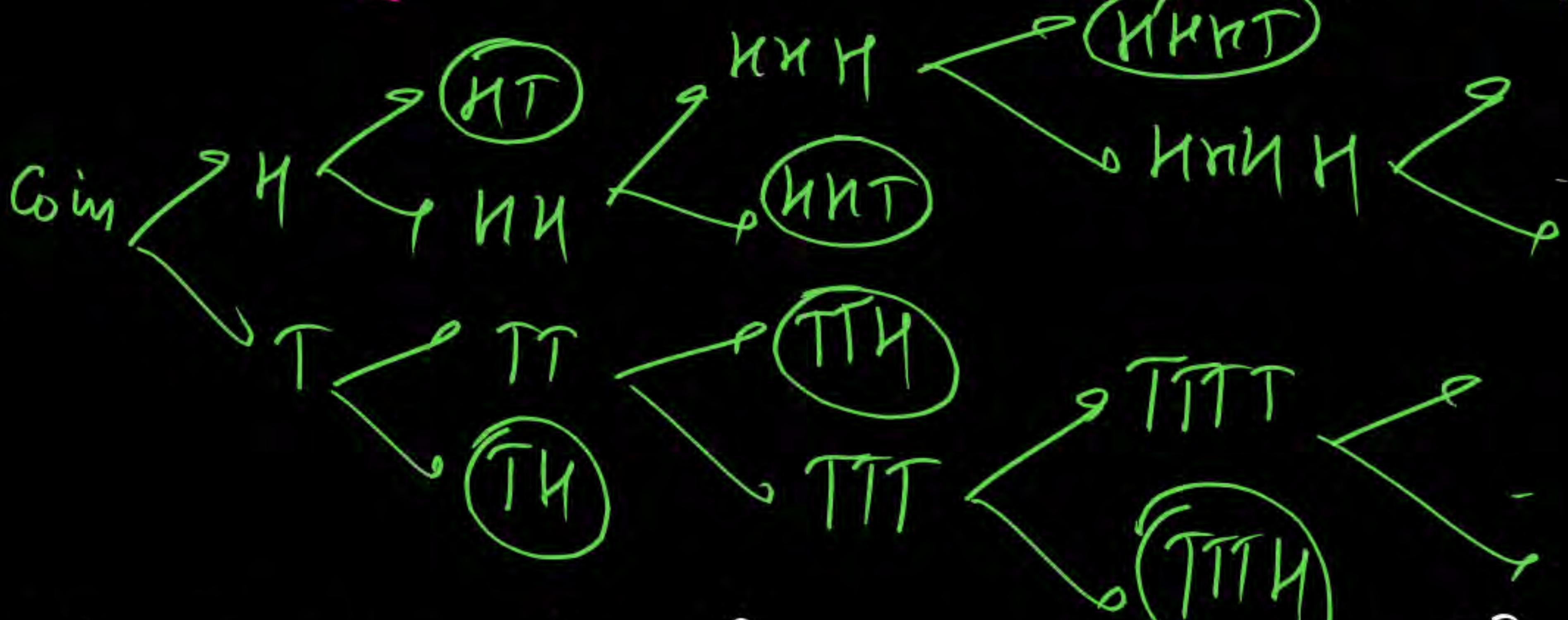
150 " " 200 Games " " $= 1 \times 200 \times 150 = 30000$ Rs.

A fair coin is tossed repeatedly till both head and tail appear at least once. The average number of tosses required is ______.

Ans = 3

Min tosses Req = 2
Max " " = No idea
Av " " = ??

$X = \{\text{No. of tosses Required}\} = \{2, 3, 4, \ldots\ldots\}$



X:	2	3	4	......
$P(X)$:	?	?	?	

HINT: $1-2x+3x^2-4x^3+\ldots\ldots\infty = (1-x)^{-2}$

NIRMAN
2025

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Thank

YOU