GATE

ALL BRANCHES

ENGINEERING MATHEMATICS

LINEAR ALGEBRA

DPP: 2

Q1 Find the rank of the matrix $A=\begin{bmatrix}4 & 2 & 1 & 3\\ 6 & 3 & 4 & 7\\ 2 & 1 & 0 & 1\end{bmatrix}$

Q2 Find the rank of the matrix $A=\begin{bmatrix}-1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 4\end{bmatrix}$

Q3 Find the rank of the matrix $A=\begin{bmatrix}1 & 1 & 1\\ a & a & a\\ a^3 & a^3 & a^3\end{bmatrix}$

Q4 Let $M=\begin{bmatrix}1 & 1 & 0\\ -1 & 1 & 2\\ 2 & 2 & 0\\ -1 & 0 & 1\end{bmatrix}$.Then, the rank of M is-

(A) 3 (B) 4
(C) 2 (D) 1

Q5 If P and Q are non- singular matrices, then for matrix M, which of the following is correct?

(A) Rank (PMQ) > Rank M
(B) Rank (PMQ) = Rank M
(C) Rank (PMQ) < Rank M
(D) Rank (PMQ) = Rank M + Rank (PQ)

Q6 Rank of singular matrix of order 4 can be at most

(A) 1 (B) 2
(C) 3 (D) 4

Q7 The rank of (m × n) matrix (where m< n) cannot be more than

(A) m (B) n
(C) mn (D) Non

Q8 If for a matrix, rank equals both the number of row and number of columns, then the matrix is called.

(A) Non- singular (B) singular
(C) transpose (D) minor

Q9 Determine whether each of the following sets of vectors is a linearly independent subset of V.

V = R^2, {(1, 0,) , (−1, −1)}
V = R^2, {(1, −1,)} , (1, 1), (2, 1)}
V = R^3, {(1,1,0,)} , (−1, 1,1)}
V = R^3, {(1,0,0,)} , (1, 1,0), (1, 1 , 1)}
V =R ^{3}, {(1,0,0,)} , (1, 1,0), (1, 1 , 1)}

Q10 A set of r, n dimensional vector x_1, x_2, x_3 ...... xr is said to be linearly independenly, if every relation of the type $k_1x_1 + k_2x_2 + k_3x_3 + ... + k_rx_r = 0$ implies.

(A) $k_1 + k_2 + k_3 + ... + k_r = 0$
(B) $k_1 = k_2 = k_3 = ... = 0$
(C) $k_1 + k_2 + k_3 + ... + k_r = 0$
(D) None

Q11 If A is matrix of order n × m such that A in singular then column vectors are

(A) LD (B) LI
(C) orthogonal (D) orthonormal

Q12 If there exist no relationship between the column vectors of $A_{m\times n}$ then

(A) ρ(a) < n
(B) ρ(a) = n
(C) ρ(a) < m
(D) ρ(a) ≤ n

Q13 Find λ for which there exits a linear relationship between the vecotr $\hat{i}+2\hat{j}+3\hat{k}$;

GATE

$4\hat{i}+5\hat{j}+6\widehat{k}, \lambda\hat{i}+8\hat{j}+9\widehat{k}.$

(A) λ = 3

(B) λ = 7

(C) λ ± 7

(D) λ = 0

Q14 (a) Show that (2, 1, 1) and (1, -4 , 2) are orthogonal.

(b) Determine which of the following vectors are orthogonal :

$v_1=(-2,6,1), v_2=(9,2,6), v_3=(4,-15,-1).$

Q15 Among the following, the pair of the vector orthogonal to each other is

(A) [3, 4, 7] , [3, 4, 7]

(B) [0, 0, 0], [1, 1, 0]

(C) [1, 0, 2], [0, 5, 0]

(D) [1, 1, 1] , [–, –1, 1]

Q16 If $\bar{a}=3\hat{i}-2\hat{j}+\widehat{k}$ and $\bar{b}=4\hat{i}+3\hat{j}-\lambda\widehat{k}$ are orthogonal then λ = ?

(A) 6 (B) 12

(C) –6 (D) –12

Q17 The vector which is orthogonal is every column vector of A = $\begin{bmatrix}1&1&0\\1&-1&0\\1&0&1\\1&0&-1\end{bmatrix}$ will be ?

(A) $\begin{bmatrix}1\\-1\\0\\1\end{bmatrix}$

(B) $\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$

(C) $\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix}$

(D) $\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

Q18 Norm of vector $[8\ \ 4\ \ 1]^T$ is given as _______?

Q19 The normalized vector of $\begin{bmatrix}2\\1\\-4\end{bmatrix}$ will be ?

Q20 For what values of α and β, the following simultaneous equations have an infinite number of solution?

x + y + z = 5

x + 3y + 3z = 9

x + 2y + az = β

Q21 The following system of equations

$x_1 + x_2 + 2x_3 = 1$

$x_1 + 2x_2 + 3x_3 = 2$

$x_1 + 4x_2 + ax_3 = 4$

has a unique solution. The only possible value of a is/are

(A) 0

(B) either 0 or 1

(C) one of 0, 1 and –1

(D) and real number other than 5

Q22 The solution(s) to the equations 2x + 3y = 1, x – y = 4 , 4x – y = α , will exists if a is equal to

(A) –33 (B) 0

(C) 9 (D) $\frac{59}{5}$

Q23 For the following set of simultaneous equations:

1.5x – 0.5y = 2

4x + 2y + 3z = 9

7x + y + 5z = 10

(A) The solutions is unique

(B) Infinite many solution exists

(C) The equations are incompatible

(D) Finite number of multiple solution exist.

Q24 The conditon for consistency of simultaneous equation AX = B where $C = A : B$

(A) Rank A = Rank C
(B) Rank A ≠ Rank C
(C) Rank A = Rank B
(D) None of these

Q25 In the system of equation AX = B and A, B = C
(a) If the rank of A is not equal to rank of C
(p) consistant with unique solution
(b) If the rank of A is not equal to rank of C
(q) Infinite solutions consistant with
(c) If the rank A = rank of C < No. of unknowns
(r) have a solution
(d) The solution of AX = 0 is always
(s) inconsistant
(A) a->s,b-s,c->q,d->r
(B) a->s,b->p,c->q,d->p
(C) a->s,b->p,c->q,d->q
(D) a->s,b->p,c->q,d->q

Q26 The values of k for which equations x + y + z = 1, x + 2y + 4z = k, x+ 4y + 10z = k^2 have a solution
(A) 1 or 2 (B) 3 or 4
(C) 5 or 6 (D) any values

Q27 For what value of b the following system of equations has hon- trivial solution?
2x +y + 2z = 0
x +y + 3z = 0
4x + 3y + bz = 0

Q28 Let AX = B be a system of linear equations where A is an m × n matrix and B is an n × 1 column matrix which of the following is false?
(A) The system has a solution, it ρA) = r(A/B)
(B) If m = n and B is non-zero vector then the system has a unique solution
(C) If m< n and B is a zero vector then the system has infinitely many solutions
(D) The system will have a trivial solution when m = n, B is the zero vector and rank of A is n.

Q29 Let A be a square matrix of order n, then nullity of A is
(A) n – rank A
(B) rank A – n
(C) n + rank A
(D) None of these

Q30 An n × n homogenous system of equations AX = 0 is given. The rank of A is r < n. Then the system has
(A) n – r independent solutions
(B) r independent solutions
(C) no solution
(D) n – 2r independent solutions

Q31 The symultaneous equation
$a_1x + b_2y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
$\left(i\right)\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\left(p\right)$ $no\ solution$
$\left(ii\right)\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\left(q\right)$ $unique\ solution$
$\left(iii\right)\frac{a_1}{a_2} = \frac{b_1}{b_2} \not\approx \frac{c_1}{c_2}$
$\left(r\right) infinitely\ many\ solutions$
$\left(iv\right)\frac{a_1}{a_2} \not\approx \frac{b_1}{b_2} \not\approx \frac{c_1}{c_2}$ $\left(s\right) None\ of\ these$

(A) a->r
b->q
c->p
d->q
(B) a->p
b->s
c->q
d->r
(C) a->s
b->p
c->r
d->q
(D) None of these

Q32 If x + 2y – 2u = 0, 2x – y – u = 0, x + 2z – u = 0, 4x – y + 3z – u = 0 is a system of equations, then it is
(A) consistant with trivial solution
(B) consistant without trivial solution
(C) inconsistant with trivial solution
(D) inconsistant without trivial solution

Q33 The equations $kx + y + z = 0$, $-x + ky + z = 0$, $-x - y + kz = 0$ will have non-zero soluton, when real k is

(A) 3 (B) zero

(C) 1 (D) $\sqrt{3}$

Q34 For the given set of equations:

$x + y = 1$

$y + z = 1$

$x + z = 1$,

Which one of the following statements is correct?

(A) Equations are inconsistent

(B) Equations are consistent and a single non-trivial solution exists

(C) Equations are consistent and many solutions

(D) Equations are consistent and only a trivial solution exists

GATE

Answer Key

Q1	2
Q2	3
Q3	1
Q4	(C)
Q5	(B)
Q6	(C)
Q7	(A)
Q8	(A)
Q9	The vectors are linearly independent if they cannot be expressed as the linear combination of each others.
Q10	(B)
Q11	(A)
Q12	(B)
Q13	(B)
Q14	v1 and v2 are orthogonal and v2 and v3 are orthogonal.
Q15	(C)
Q16	(A)
Q17	(C, D)
Q18	9
Q19	$\frac{1}{\sqrt{21}}\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$
Q20	$\alpha = 2, \beta = 7$
Q21	(D)
Q22	(D)
Q23	(A)
Q24	(A)
Q25	(A)
Q26	(A)
Q27	8
Q28	(B)
Q29	(A)
Q30	(A)
Q31	(A)
Q32	(A)
Q33	(B)
Q34	(B)

GATE

Hints & Solutions

Q1 Text Solution:

We have, $A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

Performing $R_1 \to R_1 \div 4$

$$A = \begin{bmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Performing $R_2 \to R_2 - 6R_1$, $R3 \to R_3 - 2R_1$

and $A = \begin{bmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 5/2 & 5/2 \\ 0 & 0 & -1/2 & -1/2 \end{bmatrix}$

Now, performing $R_2 \to R_2 \times \left(\frac{2}{5}\right)$, We get

$$A = \begin{bmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1/2 & -1/2 \end{bmatrix}$$

Performing $R_3 \to R_3 + \frac{1}{2}R_2$, we get

$$A = \begin{bmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence, number of non-zero rows = 2.

So, rank of given matrix = 2

Q2 Text Solution:

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$Reaaranging\ the\ rows\ we\ get -$

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$So\ the\ number\ of\ non-zero\ rows\ is\ 3$

Q3 Text Solution:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ a & a & a \\ a^3 & a^3 & a^3 \end{bmatrix}$$

$multiplying\ R1\ with\ a\ and\ subtracting$
$with\ R2\ and\ then\ multiplying\ R1\ with\ a^3$
$and\ subtracting\ with\ R3\ we\ get -$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$Thus\ the\ rank\ is\ 1$

Q4 Text Solution:

We need to find the rank of the matrix,

$$M = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Reduce the matrix to echelon form using the operations

"$R_2 \to R_2 + R_1$", $R_3 \to R_3 - 2R_1$ and $R_4 \to R_4 + R_1$.

Thus we get-

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

and also applying $R_4 \rightarrow 2R_4 - R_2$, we have-

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, rank of M = 2.

Q5 Text Solution:

Rank (PMQ) = Rank M, as P and Q are non singular matrices.

Q6 Text Solution:

So basically, the matrix is of order 4*4. Now as it is a asingular matrix so the rank can never be 4, but if we have to explain for at most number which can be the rank, it will be 3 only.

Q7 Text Solution:

A is any matrix of order m×n then rank or A$\leq$ min {m, n}

rank of A$\leq$ m

$\therefore$ rank of A $\leq$m $\quad (\because \ m < n)$

Q8 Text Solution:

Given that both rows and columns are equal. So let us consider $A_{n\times n}$

Also given that ρ(A) = number of rows of A = number of column of A.

So, ρ(A) = n

$\Rightarrow$ |A| $\neq$0

Q9 Text Solution:

The vectors are linearly independent if they cannot be expressed as the linear combination of each others.Thus after making linear combinations and having the system of solutions we have a trivial solution then the vectors will be independent of each other.

Q10 Text Solution:

The vectors are linearly independent if they cannot be expressed as the linear combination of each others.Thus after making linear combinations and having the system of solutions we have a trivial solution then the vectors will be independent of each other.

Q11 Text Solution:

If A is matrix of order n × m such that A in singular then column vectors are linearly dependent.

As the matrix is singular thus the determinant will be 0, hence the columns are LD.

Q12 Text Solution:

If there exist no relationship between the column vectors of $A_{m\times n}$ then they all are independent of each other thus the rank will be n.

Q13 Text Solution:

So we have 3 vectors as -

$\hat{\text{i}} + 2\hat{\text{j}} + 3\widehat{\text{k}}, 4\hat{\text{i}} + 5\hat{\text{j}} + 6\widehat{\text{k}}, \lambda\hat{\text{i}} + 8\hat{\text{j}} + 9\widehat{\text{k}}$

We have to express them as linear combination thus-

$$\alpha\left(\hat{\text{i}} + 2\hat{\text{j}} + \widehat{3j}\right) + \beta\left(4\hat{\text{i}} + 5\hat{\text{j}} + 6\widehat{\text{k}}\right)$$

$$= \lambda\hat{\text{i}} + 8\hat{\text{j}} + 9\widehat{\text{k}}$$

$$\alpha + 4\beta = \lambda ----\left(1\right)$$

$$2\alpha + 5\beta = 8 ----\left(2\right)$$

$$3\alpha + 6\beta = 9 -----\left(3\right)$$

$\textit{solving } 3 \textit{ and } 4 \textit{ we get} -$

$\alpha = -1, \beta = 2$

$\textit{thus putting it in eqn } 1 \textit{ we get}$

$\lambda = \ -1 + 8 = 7$

Q14 Text Solution:

a) The two vectors are said to be orthogonal if the dot product of two vectors are 0.

Considering the first vector and multiplying the consecutive elements we get-

2*1-4*1+2*1=0

Thus they are orthogonal.

b) Similarly moving to the next question we get-

$v_1 . v_2 = -2 * 9 + 6 * 2 + 6 * 1 = 0,\ \textit{thus orthogonal}$

$v_2 . v_3 = 9 * 4 - 15 * 2 - 1 * 6 = 0, \textit{thus orthogonal}$

GATE

Q15 Text Solution:

The two vectors X_1 & X_2 are said to be orthogonal if $X_1 \cdot X_2 = 0$

Let $X_1 = [1\ 0\ 2]^T$ and $X_2 = [0\ 5\ 0]^T$

So, $X_1 \cdot X_2 = \begin{bmatrix}1\\0\\2\end{bmatrix} \cdot \begin{bmatrix}0\\5\\0\end{bmatrix} = 1 \times 0 + 0 \times 5 + 2 \times 0 = 0$

Method II:

For orthogonal vector: $\vec{a} \cdot \vec{b} = 0$

$i.e.$ if $\vec{a} = x_1 i + y_1 j + z_1 k$

& $\vec{b} = x_2 i + y_2 j + z_2 k$

Then: $x_1 x_2 + y_1 y_2 + z_1 z_2 = 0$

Option 'c' satisfies the condtion.

Q16 Text Solution:

$\bar{a} = 3\hat{i} - 2\hat{j} + \hat{k}, \bar{b} = 4\hat{i} + 3\hat{j} - \lambda\hat{k}$

Here as they are orthogonal thus, the dot product will be 0.

$\bar{a}.\bar{b} = 12 - 6 - \lambda = 0$

$\lambda = 6$

Q17 Text Solution:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

So we have the column vectors as –

$(1, 1, 1, 1)^T, (1, -1, 0, 0)^T, (0, 0, 1, -1)^T$

$$Lets\ take\ the\ option \left(c\right)$$

$If\ they\ are\ orthogonl\ ;\ then\ the\ dot\ product\ will\ turn\ out\ to\ be\ 0.$

$$\left(-1, -1, 1, 1\right) . \left(1, 1, 1, 1\right) = 0$$

$$\left(-1, -1, 1, 1\right) . \left(1, -1, 0, 0\right) = 0$$

$$\left(-1, -1, 1, 1\right) . \left(0, 0, 1, -1\right) = 0$$

$$Lets\ take\ the\ option \left(d\right)$$

$$\left(0, 0, 0, 0\right) . \left(1, 1, 1, 1\right) = 0$$

$$\left(0, 0, 0, 0\right) . \left(1, -1, 0, 0\right) = 0$$

$$\left(0, 0, 0, 0\right) . \left(0, 0, 1, -1\right) = 0$$

Q18 Text Solution:

$Norm\ is\ defined\ as\ -$

$lets\ say\ we\ have\ a\ vector\ \ x_1\hat{i} + y_1\hat{i} + z_1\hat{i}$

$then\ norm\ is\ -\sqrt{x_1{}^2 + y_1{}^2 + z_1{}^2}$

$Thus\ here\ -$

$\sqrt{64+16+1} = \sqrt{81} = 9$

Q19 Text Solution:

$$The\ vector\ given\ is\ \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$$

$Norm\ is\ defined\ as\ -$

$lets\ say\ we\ have\ a\ vector\ \ x_1\hat{i} + y_1\hat{i} + z_1\hat{i}$

$then\ norm\ is\ -\sqrt{x_1{}^2 + y_1{}^2 + z_1{}^2}$

$Thus\ here\ -$

$\sqrt{4+1+16} = \sqrt{21}$

$Thus\ the\ normalised\ vector\ will\ be\ -$

$$\frac{1}{\sqrt{21}}\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$$

Q20 Text Solution:

Putting the system of simultaneous equations in the form

AX = B

We get,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 2 & \alpha \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 5 \\ 9 \\ \beta \end{bmatrix}$$

So, the augmented matrix

$$\widetilde{\mathrm{A}} = [\mathrm{A} : \mathrm{B}] = \begin{bmatrix} 1 & 1 & 1 & : & 5 \\ 1 & 3 & 3 & : & 9 \\ 1 & 2 & \alpha & : & \beta \end{bmatrix}$$

Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

$$\text{We get} \sim \begin{bmatrix} 1 & 1 & 1 & : & 5 \\ 0 & 2 & 2 & : & 4 \\ 0 & 1 & \alpha - 1 & : & \beta - 5 \end{bmatrix}$$

Operating $R_2 \to R_2 \div 2$, we get

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 5 \\ 0 & 1 & 1 & : & 2 \\ 0 & 1 & \alpha - 1 & : & \beta - 5 \end{bmatrix}$$

Operating $R_3 \to R_3 - R_2$, we get

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 5 \\ 0 & 1 & 1 & : & 2 \\ 0 & 1 & \alpha - 2 & : & \beta - 7 \end{bmatrix}$$

Now, for ininfinte solution the last row must be zero.

Therefore, $\alpha - 2 = 0$ implies $\alpha = 2$

$\beta - 7 = 0$ implies $\beta = 7$

Q21 Text Solution:

Putting the given linear equations in AX = B form

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & \alpha \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{and } \mathrm{B} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Now, augmented matrix

$$[A : B] = \begin{bmatrix} 1 & 1 & 2 & : & 1 \\ 1 & 2 & 3 & : & 2 \\ 1 & 4 & \alpha & : & 4 \end{bmatrix}$$

Operation $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$= \begin{bmatrix} 1 & 1 & 2 & : & 1 \\ 0 & 1 & 1 & : & 1 \\ 0 & 3 & a - 2 & : & 3 \end{bmatrix}$$

Operating $R_3 \to R_3 - 3R_2$, we get

$$= \begin{bmatrix} 1 & 1 & 2 & : & 1 \\ 0 & 1 & 1 & : & 1 \\ 0 & 0 & a - 5 & : & 0 \end{bmatrix}$$

If $a - 5 \neq 0 \Rightarrow a \neq 5$

Then, rank of [A] = rank of [A : B] =3

Hence, a can take any value expect 5

Q22 Text Solution:

2x + 3y = 1........(i)

x − y = 4............(ii)

4x − y = α.......(iii)

Form equation (i) and equation (ii)

$x = \frac{13}{5}, y = \frac{-7}{5}$

The solution of equations exists

$\Rightarrow \alpha = 4\left(\frac{13}{5}\right) - \left(-\frac{7}{5}\right)$

$\alpha = \frac{59}{5}$

Q23 Text Solution:

$$\left|A\right| = Determinant\ of$$

$$\begin{bmatrix} 1.5 & -0.5 & 0 \\ 4 & 2 & 3 \\ 7 & 1 & 5 \end{bmatrix}$$

= (1.5) (10−3) + (0.5) (20−21)

$= 10.5 - 0.5 = 10 \neq 0$

Q24 Text Solution:

We have to find the conditon for consistency of simultaneous equation AX = B where C=A:B

That C is equal to A augmented B , no in order for the consistency of equations means that the solutions must exist either unique or infinite thus for this the rank of A must be equal to rank of C .

Q25 Text Solution:

(a) If the rank A rank of C

(r) have a solution

(c) If the rank A = rank of C < No. of unknowns

(p) consistant with unique solution

(c) If the rank A = rank of C < No. of unknowns

(q) Infinite solutions consistant with

(d) The solution of AX = 0 is always

(r) have a solution

The correct solutions is a->s,b->p,c->q,d->r

Q26 Text Solution:

The values of k for which equations x + y + z = 1, x + 2y + 4z = k, x+ 4y + 10z = k^2 have a solution

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & k \\ 1 & 4 & 10 & k^2 \end{array}\right]$$

$$R_2 -> R_2 - R_1$$
$$R_3 -> R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & k-1 \\ 0 & 3 & 9 & k^2-1 \end{array}\right]$$

$$R_3 -> R_3 - 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & k-1 \\ 0 & 0 & 0 & k^2-1-3K+3 \end{array}\right]$$

$$K^2 - 3K + 2 = 0$$
$$K - 2 = 0, K - 1 = 0$$
$$K = 1, 2$$

Q27 Text Solution:

Sience the system of homogeneous equations has non-trivial solution

Hence, $|A| = 0$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{vmatrix} = 0$$

$$\Rightarrow 2\begin{vmatrix} 1 & 3 \\ 3 & b \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 4 & b \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2(b - 9) - 1(b-12) + 2(3 - 4) = 0$

$\Rightarrow b - 8 = 0$

$\Rightarrow b = 8$

Q28 Text Solution:

Given, that $A_{m*n} X_{n*1} = B_{m*1}$

According to option (b)

We can take m = n & B = 0

So (1) = $A_{m*n} X_{n*1} = O_{m*1}$

If |A| is not equal to 0, system have unique solution if |A|=0 system have infininte solution.

Hence, option (b) in wrong because condition of unique solution is not mentioned.

Q29 Text Solution:

Let A be a square matrix of order n, then nullity of A is

then the nullity is defined as n-r,

The nullity of a matrix is **the dimension of the null space of A, also called the kernel of A**.

Q30 Text Solution:

An n × n homogenous system of equations AX = 0 is given. The rank of A is r < n. Then the system has will have n-r independent solutions

Q31 Text Solution:

a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (r) Infinity many solutions

b) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (q) unique solution

c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \not\approx \frac{c_1}{c_2}$ (p) No solution

d) $\frac{a_1}{a_2} \not\approx \frac{b_1}{b_2} \not\approx \frac{c_1}{c_2}$ (s) None of these

The correct solutions will be-

a->r

b->q

c->p

d->q

GATE

Q32 Text Solution:

$If\ x + 2y - 2u = 0,\ 2x - y - u = 0,\ x + 2z - u = 0,\ 4x - y + 3z - u = 0$

$Thus\ AX = O$

$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & -1 & -1 & 0 \\ 1 & 0 & -1 & 2 \\ 4 & -1 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ u \\ z \end{bmatrix} = O$$

If u willl calculate the determinant of A its coming out to be non-zero,thus consistant with trivial solution.

Q33 Text Solution:

$The\ equations\ are\ kx + y + z = 0,\ -x + ky + z = 0,\ -x - y + kz = 0$

$$\left[\begin{array}{ccc|c} k & 1 & 1 & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ -1 & k & 1 & \\ -1 & -1 & k & \end{array}\right]$$

$Thus\ the\ determinant\ should\ be\ equal\ to\ 0$

$$\begin{bmatrix} k & 1 & 1 \\ -1 & k & 1 \\ -1 & -1 & k \end{bmatrix} = A$$

$|A| = 0$

$$k\left(k^2 + 1\right) - 1\left(-k + 1\right) + 1\left(1 + k\right) = 0$$

$k^3 + k + k - 1 + 1 + k = 0$

$k^3 + 3k = 0$

$k = 0$

Q34 Text Solution:

$Lets\ make\ the\ augmented\ matrix\ for\ the\ system\ of\ equations -$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ 0 & 1 & 1 & \\ 1 & 0 & 1 & \end{array}\right]$$

$As\ the\ determinant\ of\ A\ is\ not\ 0\ and\ thus\ the\ rank\ of\ A = 3\ thus\ the\ system\ of\ equations\ will\ have\ a\ unique\ solution.$