

Left Recursion – Test Yourself

Basic rule:

$$A \rightarrow Aa \mid b$$

can be replaced by the pair of rules

$$A \rightarrow bA' \text{ and } A' \rightarrow aA' \mid \varepsilon$$

If you have something more complicated, e.g.

$$X \rightarrow Yx$$
$$Y \rightarrow Zy$$
$$\mid x$$
$$Z \rightarrow Xy$$
$$\mid z$$

you first need to “massage” it so that it matches the above form. Typically this involves re-writing rules. So for the above grammar, we get:

$$X \rightarrow Zyx$$
$$\mid xx$$
$$Z \rightarrow Xy$$
$$\mid z$$

which in turn becomes:

$$X \rightarrow Xyyx$$
$$\mid zyx$$
$$\mid xx$$

Now, relating that back to the original form $A \rightarrow Aa \mid b$, we have $A = X$, $a = yyx$, $b = zyx \mid xx$. So the pair of rules $A \rightarrow bA'$ and $A' \rightarrow aA' \mid \varepsilon$ become:

$$X \rightarrow (zyx \mid xx)X' \text{ and } X' \rightarrow yyxX' \mid \varepsilon$$

which can also be written as:

$$X \rightarrow zyxX' \mid xxX' \text{ and } X' \rightarrow yyxX' \mid \varepsilon$$

Ok, now that you've learnt the rule, try removing left recursion from these grammars:

1.

$$\begin{array}{lcl} S & \rightarrow & Aa \\ & | & b \\ A & \rightarrow & Ac \\ & | & dS \\ & | & \epsilon \end{array}$$

2.

$$\begin{array}{lcl} A & \rightarrow & Br \\ B & \rightarrow & Cs \\ C & \rightarrow & At \end{array}$$

3.

$$\begin{array}{lcl} S & \rightarrow & AA \\ & | & 0 \\ A & \rightarrow & SS \\ & | & 1 \end{array}$$

4. $S \rightarrow Ra \mid Aa \mid a$
 $R \rightarrow ab$
 $A \rightarrow AR \mid AT \mid b$
 $T \rightarrow Tb \mid a$

5. $A \rightarrow B \mid a \mid CBD$
 $B \rightarrow C \mid b$
 $C \rightarrow A \mid c$
 $D \rightarrow d$

6. $Q \rightarrow QED \mid q$
 $E \rightarrow e$
 $D \rightarrow NFA \mid d$
 $N \rightarrow DFA \mid n$
 $F \rightarrow f$
 $A \rightarrow a$