Running Time of Insertion Sort on an Array with Three Distinct Values

Analysis

Suppose we use **insertion sort** on a randomly ordered array where elements have only one of three possible values. The running time of the algorithm in this case would likely be **closer to linear** rather than quadratic, though it may exhibit characteristics somewhere between linear and quadratic depending on the distribution of values.

1. Fewer Unique Elements (Three Distinct Values)

When sorting, the number of comparisons and shifts performed by insertion sort depends on how out-of-order the elements are. Since there are only three distinct values, the number of comparisons and shifts is reduced because many elements will already be close to their final position. Moreover, there are fewer possible incorrect comparisons.

2. Expected Behavior for Random Distribution

Insertion sort generally has a best-case running time of $O(n)$ (linear) when the array is already sorted or nearly sorted. With only three distinct values randomly distributed, many elements are likely to be either already near their correct position or in small groups, requiring fewer shifts to achieve a sorted order.

In this case, elements with the same value do not need to be shifted relative to each other, further reducing the number of shifts and comparisons compared to an array with more distinct values.

3. Amortized Shifts

While the worst-case time complexity of insertion sort is $O(n^2)$, this worst-case scenario occurs when every element needs to be shifted for each insertion. In the case of three distinct values, the probability of requiring n shifts for each insertion diminishes significantly. As a result, the number of shifts is expected to be much smaller than $O(n^2)$.

4. Probabilistic Analysis

In a probabilistic analysis, most elements are likely to be near their sorted position after each pass, which makes the average number of shifts much smaller. Therefore, the overall running time is expected to behave more like $O(n)$ than $O(n^2)$.

Conclusion

The running time of insertion sort on an array with only three distinct values and a random distribution will likely be closer to **linear** ($O(n)$) than **quadratic** ($O(n^2)$). It could exhibit behavior between linear and quadratic time complexities, but it tends to lean toward linear performance due to the restricted number of distinct values.