Fundamentals–LinearSearch and Binary Search

COMPSCI 2CO3: Data Structures and Algorithms

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Reflection on Contains

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Contains is correct and has a runtime complexity of $\Theta(|L|) \longrightarrow$ Sounds good to me!

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Critique: CONTAINS is *too specialized* $\longrightarrow$.

We cannot use CONTAINS for anything else than the contains problem!

Example

- Searching in only *part* of the list?
- Finding where v is in the list?

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Algorithm LINEARSEARCH(L, v, o):

Input: L is an *array*, v a value, $0 \leq o \leq |L|$.

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while $r \neq |L|$ **and also** $L[r] \neq v$ **do**
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4: **return** r.

Result: return the first offset r, $o \leq r < |L|$, with $L[r] = v$ or, if no such offset exists, $r = |L|$.

Algorithm LSCONTAINS(L, v):

1: **return** LINEARSEARCH$(L, v, 0) \neq |L|$.

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Assume: $L[i] = v$ and i is the *first* offset after o equivalent to v.
The runtime complexity of LinearSearch is $\Theta(i - o)$.

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Assume: $L[i] = v$ and i is the *first* offset after o equivalent to v.
The runtime complexity of LinearSearch is $\Theta(i - o)$ with $i =$ LinearSearch(L, v, o).

Problem: What if we search often?

LinearSearch(L, v, o) can read all of array L: *potentially-high cost*.

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No: We do not know anything about L to help us!

$\longrightarrow$ we have to look at all elements in L.

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LINEARSEARCH(L, v, o) can read all of array L: *potentially-high cost*.

Can we do better?

No: We do not know anything about L to help us!

$\longrightarrow$ we have to look at all elements in L.

Maybe: If we know more about L.

An example of a list

Consider a list *enrolled* with schema

$$\text{enrolled}(\mathit{dept}, \mathit{code}, \mathit{sid}, \mathit{date})$$

that models a list of all students enrolled for a course.

What if...

We add enrollment data to *the end of the list*.

Question: What do we know about enrolled?

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that models a list of all students enrolled for a course.

What if...

We add enrollment data to *the end of the list*.

Question: What do we know about enrolled? $\longrightarrow$ enrolled is *ordered* on *date*!

Searching in an ordered list

3	14	16	18	30	37	44	49	66	76	84	90

v

Conclusion of comparing $L[i]$ and v?

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$L[i] < v$ As the list L is ordered, every value in $L[0, i]$ is smaller than v.

$\longrightarrow v \in L$ if and only if $v \in L[i+1, |L|)$.

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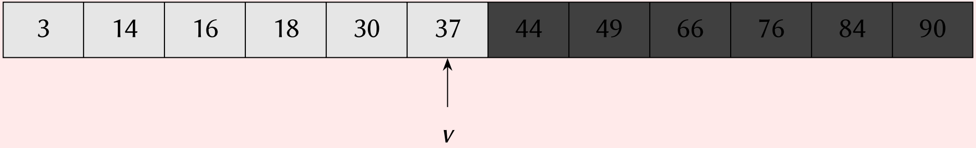

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$L[i] = v$ We found v!

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One comparison can remove a large portion of the array.

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One comparison can remove a large portion of the array.

Binary Search: *Maximize potential* by comparing v with the middle of L.

The *recursive* Binary Search algorithm

Algorithm $\textsc{LowerBoundRec}(L, v, begin, end)$:

Input: L is an ordered *array*, v a value, and $0 \leq begin \leq end \leq |L|$.

if $begin = end$ **then**
    return $begin$.
else
    $mid := (begin + end) \operatorname{div} 2$.
    if $L[mid] < v$ **then**
        return $\textsc{LowerBoundRec}(L, v, mid + 1, end)$.
    else $L[mid] \geq v$
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Result: return the first offset r, $begin \leq r < end$, with $L[r] = v$ or, if no such offset exists, $r = end$.

- Is $\textsc{LowerBoundRec}$ correct?
- What is the runtime and memory complexity of $\textsc{LowerBoundRec}$?

Correctness of LowerBoundRec

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Induction Hypothesis

For any L', v', and $0 \leq begin' \leq end' \leq |L'|$ with $0 \leq end' - begin' < m$, LowerBoundRec(L', v', $begin'$, end') returns the correct result.

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Base case:
Inspecting $end - begin = 0$ elements.

Recursive case:
Inspecting $end - begin > 0$ elements.

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$begin \leq mid < end$

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Termination

Bound function: $end - begin$.

Intermezzo: Runtime complexity of LowerBoundRec

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Complexity of LowerBoundRec with $n = end - begin$

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Base case:
1 operation.

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Base case:
1 operation.

Recursive case:
1 operation and 1 recursive call.

Complexity of LowerBoundRec with $n = end - begin$

$$T(n) = \begin{cases} 1 & \text{if } n = 0; \\ 1 \cdot T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1 & \text{if } n \geq 1. \end{cases}$$

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work $= 1$

$n = 2^x$ ●

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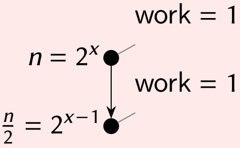

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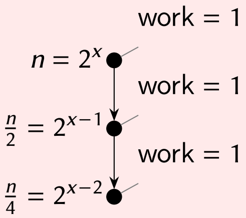

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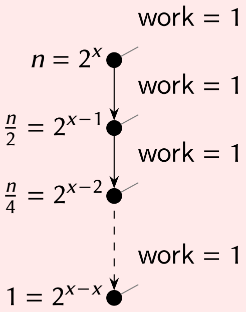

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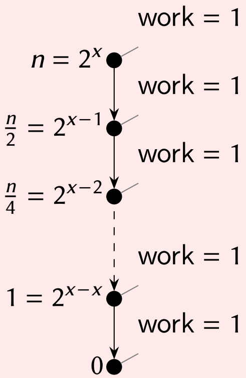

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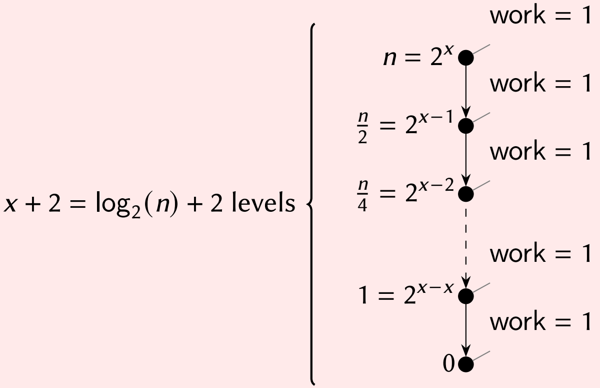

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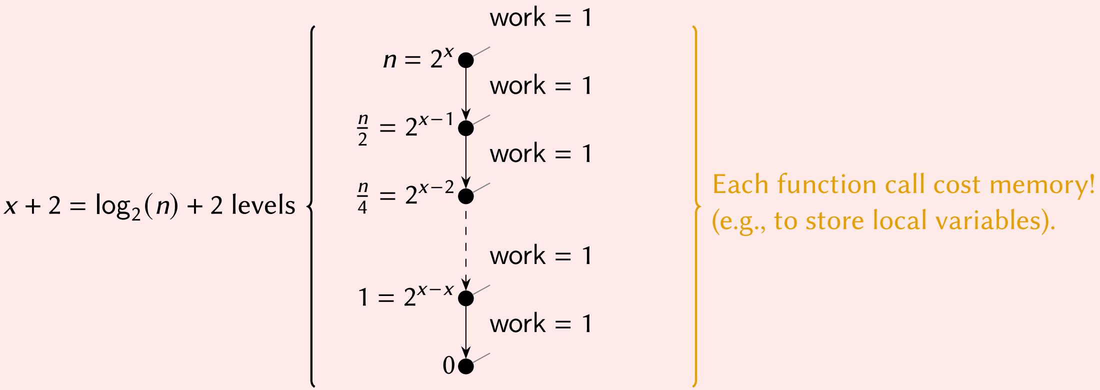
Complexity of LowerBoundRec with $n = end - begin$ (assume: $n = 2^x$)

$$T(n) = 1 \cdot T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1 = \Theta(\log_2(n)).$$



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Result: return the first offset r, $begin \leq r < end$, with $L[r] = v$ or, if no such offset exists, $r = end$.

Theorem

LowerBoundRec is correct and has a runtime and memory complexity of $\Theta(\log_2(|L|))$.

The *non-recursive* Binary Search algorithm

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return $begin$.

Result: return the first offset r, $begin \leq r < end$, with $L[r] = v$ or,
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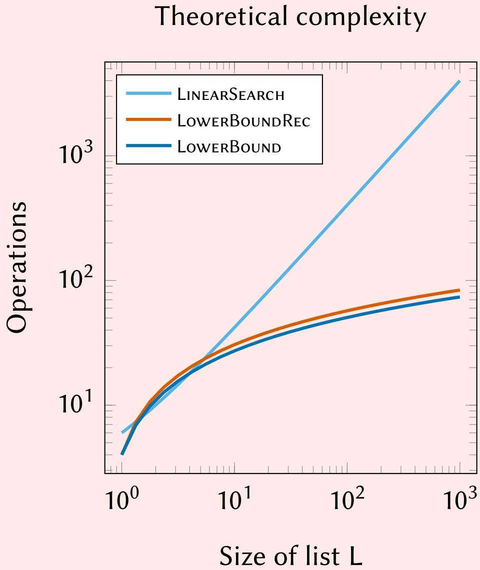
Theorem

LowerBound is correct, has a runtime complexity of $\Theta(\log_2(|L|))$, and a memory complexity of $\Theta(1)$.

Comparing the complexity of searching



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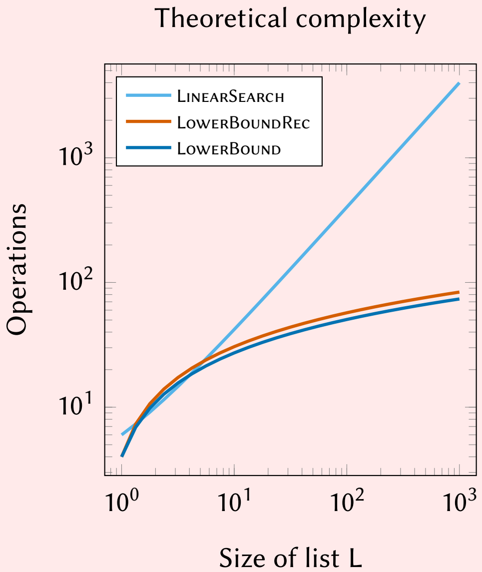

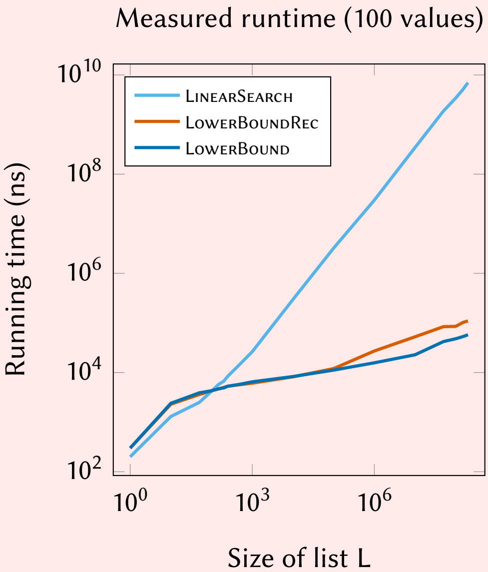

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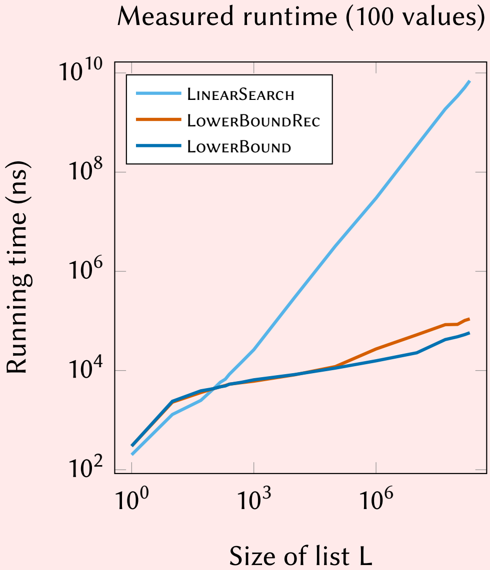

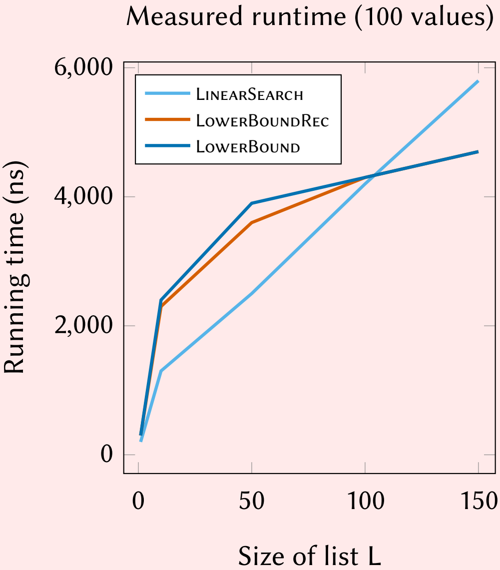

Using Binary Search as a *building block*

Problem

Let L be a list and $[v, w]$ be a range query with $v \leq w$. The solution of the range query problem for L and $[v, w]$ is the list of all values $e \in L$ with $v \leq e \leq w$.

Example

Consider a list *enrolled* with schema enrolled($dept$, $code$, sid, $date$).

Query: All students enrolled in 2023

Range query on *enrolled* with $[(``, ``, -1, 2023), (``, ``, -1, 2024)]$.

Using Binary Search as a *building block*

Problem

Let L be a list and $[v, w]$ be a range query with $v \leq w$. The solution of the range query problem for L and $[v, w]$ is the list of all values $e \in L$ with $v \leq e \leq w$.

Example

Consider a list *enrolled* with schema enrolled(*dept*, *code*, *sid*, *date*).

Query: All students enrolled in 2023

Range query on *enrolled* with $[(`', `', -1, 2023), (`', `', -1, 2024)]$.

We add enrollment data to *the end of the list* $\longrightarrow$ enrolled is *ordered* on *date*!

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Algorithm $\textsc{RangeQuery}(L, [v, w])$:

Input: L is an ordered *array*, v, w are values, and $v \leq w$.

$i := \textsc{LowerBound}(L, v, 0, |L|)$.
$j := i$.
while $j \leq |L|$ **and also** $L[j] \leq w$ **do**
    $j := j + 1$.
return $L[i, j)$.

Result: return the list $L[m, n)$, $0 \leq m \leq n \leq |L|$, such that $L[m, n)$ is the list of all values $e \in L$ with $v \leq e \leq w$.

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return L[i, j).
```

Result: return the list $L[m, n)$, $0 \le m \le n \le |L|$, such that $L[m, n)$ is the list of all values $e \in L$ with $v \le e \le w$.

Theorem

$\textsc{RangeQuery}$ is correct and has worst case runtime complexity $\Theta(|L|)$.

Using Binary Search as a *building block*

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Result: return the list $L[m, n)$, $0 \leq m \leq n \leq |L|$, such that $L[m, n)$ is the list of all values $e \in L$ with $v \leq e \leq w$.

Theorem

RangeQuery is correct and has all case runtime complexity $\Theta(\log_2(|L|) + |result|)$.

Using the *idea* of Binary Search

Problem

Let L be list of values of unknown length and assume we have a function INSPECT*(L, i) that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

The list-length problem is the problem of finding the length of list L.

Using the *idea* of Binary Search

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Example

'apple'	'pear'	'orange'

$$\textsc{Inspect}(L, 0) = \texttt{true} \qquad \textsc{Inspect}(L, 1) = \texttt{true}$$
$$\textsc{Inspect}(L, 2) = \texttt{true} \qquad \textsc{Inspect}(L, 3) = \texttt{false}.$$

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The list-length problem is the problem of finding the length of list L.

Solving the list-length problem

We have ordered list $0, 1, \ldots$ of possible values for $|L|$.

Conclusion of $\textsc{Inspect}(L, i)$?

Using the *idea* of Binary Search

Problem

Let L be list of values of unknown length and assume we have a function $\textsc{Inspect}(L, i)$ that returns `true` *if the list has an i-th value and returns* `false` *otherwise.*

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Conclusion of $\textsc{Inspect}(L, i)$?

$\textsc{Inspect}(L, i) = \texttt{true}$ $|L| > i$ (list L has more than i values).

$\textsc{Inspect}(L, i) = \texttt{false}$ $|L| \leq i$ (list L has at-most i values).

Using the *idea* of Binary Search

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Let L be list of values of unknown length and assume we have a function $\textsc{Inspect}(L, i)$ *that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

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Issue: no upper bound on the ordered list $0, 1, \ldots$ of possible values for $|L|$

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Let L be list of values of unknown length and assume we have a function $\textsc{Inspect}(L, i)$ *that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

The list-length problem is the problem of finding the length of list L.

Issue: no upper bound on the ordered list $0, 1, \ldots$ of possible values for $|L|$

Guess repeatedly with exponentially-growing guesses.

Algorithm $\textsc{ListLengthUB}(L)$**:**

Input: L is an *array* of unknown length.

1: $n := 1$.
2: **while** $\textsc{Inspect}(L, n)$ **do**
3: $\quad n := 2 \cdot n$.
4: **return** n.

Result: return N, $|L| \leq N = 1$ or $|L| \leq N < 2|L|$.

Using the *idea* of Binary Search

Problem

Let L be list of values of unknown length and assume we have a function $\textsc{Inspect}(L, i)$ *that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

The list-length problem is the problem of finding the length of list L.

Algorithm $\textsc{LBListLength}(L, N)$ with $N := \textsc{ListLengthUB}(L)$:

$\mathit{begin}, \mathit{end} := 0, N.$
while $\mathit{begin} \neq \mathit{end}$ **do**
  $\mathit{mid} := (\mathit{begin} + \mathit{end}) \operatorname{div} 2.$
  if $\textsc{Inspect}(L, \mathit{mid})$ **then**
    $\mathit{begin} := \mathit{mid} + 1.$
  else
    $\mathit{end} := \mathit{mid}.$
return $\mathit{begin}.$

Result: return the length $|L|$ of array L.

Optimizing joins using range queries

Definition

A join of two lists L and M results in a list A in which each list value is computed from a combination of values $u \in L$ and $v \in M$ according to some *join condition*.

Example (Return pairs (p, r) of product name p and related category r)

products	
name	*category*
Apple	Fruit
Bok choy	Vegetable
Canelé	Pastry
Donut	Pastry

categories	
category	*related*
Fruit	Food
Fruit	Produce
Pastry	Food
Vegetable	Food
Vegetable	Produce

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categories	
category	*related*
Fruit	Food
Fruit	Produce
Pastry	Food
Vegetable	Food
Vegetable	Produce

Join Result	
name	*related*
Apple	Food
Apple	Produce
Bok choy	Food
Bok choy	Produce
Canelé	Food
Donut	Food

Optimizing joins using range queries

Algorithm NestedLoopPC(products, categories):

Input: relations products($name$, $category$) and categories($category$, $related$).

$output := \emptyset$.

for $(p.n, p.c) \in$ products **do**

    for $(c.c, c.r) \in$ categories **do**

        if $p.c = c.c$ **then**

            add $(p.n, c.r)$ to $output$.

Result: return $\{(p.n, c.r) \mid ((p.n, p.c) \in \text{products}) \land ((c.c, c.r) \in \text{categories})\}$.

Optimizing joins using range queries

Algorithm NestedLoopPC(products, categories):

Input: relations products(*name*, *category*) and categories(*category*, *related*).

$output := \emptyset$.
for $(p.n, p.c) \in$ products **do**
    for $(c.c, c.r) \in$ categories **do**
        if $p.c = c.c$ **then** $\Theta(|categories|)$.
            add $(p.n, c.r)$ to *output*.

Result: return $\{(p.n, c.r) \mid ((p.n, p.c) \in \text{products}) \wedge ((c.c, c.r) \in \text{categories})\}$.

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        if $p.c = c.c$ **then**
            add $(p.n, c.r)$ to *output*.

$\Theta(|categories|)$.

$|products|$ *times.*

Result: return $\{(p.n, c.r) \mid ((p.n, p.c) \in \text{products}) \wedge ((c.c, c.r) \in \text{categories})\}$.

Theorem

The NestedLoopPC *algorithm is correct and has a runtime complexity of* $\Theta(|product| \cdot |categories|)$.

Optimizing joins using range queries

Algorithm NestedBinaryPC(products, categories):

Input: relations products(*name*, *category*) and categories(*category*, *related*),
relation categories ordered.

$output := \emptyset$.
for $(p.n, p.c) \in$ products **do**
  $i :=$ LowerBound(categories, $(p.c, `')$, 0, $|\text{categories}|$).
  while $i < |\text{categories}|$ **and also** categories$[i].category = p.c$ **do**
    add $(p.n, \text{categories}[i].related)$ to *output*.
    $i := i + 1$.

Result: return $\{(p.n, c.r) \mid ((p.n, p.c) \in \text{products}) \land ((c.c, c.r) \in \text{categories})\}$.

Theorem

The NestedBinaryPC *algorithm is correct and has a runtime complexity of* $\Theta(|product| \cdot \log_2(|categories|) + |result|)$.

CONTAINS, LINEARSEARCH, and LOWERBOUND in practice

Algorithm	C++	Java
CONTAINS	`std::ranges::contains`	*collection*.`contains`[a]
LINEARSEARCH LINEARPREDSEARCH	`std::find` `std::find_if`	*collection*.`indexOf`[a] `java.util.stream::filter`[b]
LOWERBOUND	`std::lower_bound` `std::upper_bound`[d]	`java.util.Arrays::`[c] `binarySearch`
Related libraries	`<algorithm>`, `<ranges>`	`java.util.Arrays`, `java.util.ArrayList`, ...

[a] Here, *collection* is a standard Java data collection such as `java.util.ArrayList`.

[b] Using the stream library supported by standard Java data collections.

[c] Does not guarantee to return the offset of the *first* occurrence of a value.

[d] Returns the offset of the first element in the list that is strictly larger than the searched-for value.

Print-friendly summary slides

The following slides are the “final” pages of each slide (with intermediate animation steps removed).

Reflection on CONTAINS

Is CONTAINS a *good* algorithm?

CONTAINS is correct and has a runtime complexity of $\Theta(|L|) \longrightarrow$ Sounds good to me!

Critique: CONTAINS is *too specialized* $\longrightarrow$.

We cannot use CONTAINS for anything else than the contains problem!

Example

- Searching in only *part* of the list?
- Finding where v is in the list?

Reflection on Contains

Critique: Contains is *too specialized* $\longrightarrow$.

We cannot use Contains for anything else than the contains problem!

Algorithm LinearSearch(L, v, o):

Input: L is an *array*, v a value, $0 \leq o \leq |L|$.

$r := o$.

/* invariant: "$o \leq r \leq |L|$ and $v \notin L[o, r)$", bound function: $|L| - r$ */

while $r \neq |L|$ **and also** $L[r] \neq v$ **do**

  $r := r + 1$.

return r.

Result: return the first offset r, $o \leq r < |L|$, with $L[r] = v$ or, if no such offset exists, $r = |L|$.

Algorithm LSContains(L, v):

return LinearSearch$(L, v, 0) \neq |L|$.

Reflection on CONTAINS

Algorithm LINEARSEARCH(L, v, o):

Input: L is an *array*, v a value, $0 \leq o \leq |L|$.

```
r := o.
/* invariant: "o ≤ r ≤ |L| and v ∉ L[o, r)", bound function: |L| − r */
while r ≠ |L| and also L[r] ≠ v do
    r := r + 1.
return r.
```

Result: return the first offset r, $o \leq r < |L|$, with $L[r] = v$ or, if no such offset exists, $r = |L|$.

What is the runtime complexity of LINEARSEARCH?

- With respect to worst case inputs ($v \notin L$): $\Theta(|L|)$.
- With respect to best case inputs ($v = L[o]$): $\Theta(1)$.

Problem: Modeling runtime complexity in terms of *only the input* limits us!

Reflection on Contains

Algorithm LinearSearch(L, v, o):
Input: L is an *array*, v a value, $0 \leq o \leq |L|$.

$r := o$.
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return r.

Result: return the first offset r, $o \leq r < |L|$, with $L[r] = v$ or,
if no such offset exists, $r = |L|$.

What is the runtime complexity of LinearSearch?

Problem: Modeling runtime complexity in terms of *only the input* limits us!

Assume: $L[i] = v$ and i is the *first* offset after o equivalent to v.
The runtime complexity of LinearSearch is $\Theta(i - o)$ with $i = \text{LinearSearch}(L, v, o)$.

Problem: What if we search often?

LINEARSEARCH(L, v, o) can read all of array L: *potentially-high cost*.

Can we do better?

No: We do not know anything about L to help us!

$\longrightarrow$ we have to look at all elements in L.

Maybe: If we know more about L.

An example of a list

Consider a list *enrolled* with schema

$$\text{enrolled}(\mathit{dept}, \mathit{code}, \mathit{sid}, \mathit{date})$$

that models a list of all students enrolled for a course.

What if. . .

We add enrollment data to *the end of the list*.

Question: What do we know about enrolled? $\longrightarrow$ enrolled is *ordered* on *date*!

Searching in an ordered list

3	14	16	18	30	37	44	49	66	76	84	90

$\uparrow$

v

Conclusion of comparing $L[i]$ and v?

$L[i] < v$ As the list L is ordered, every value in $L[0, i]$ is smaller than v.
$\longrightarrow v \in L$ if and only if $v \in L[i+1, |L|)$.

$L[i] > v$ As the list L is ordered, every value in $L[i, |L|)$ is larger than v.
$\longrightarrow v \in L$ if and only if $v \in L[0, i)$.

$L[i] = v$ We found v!

One comparison can remove a large portion of the array.

Binary Search: *Maximize potential* by comparing v with the middle of L.

The *recursive* Binary Search algorithm

Algorithm $\textsc{LowerBoundRec}(L, v, \mathit{begin}, \mathit{end})$:

Input: L is an ordered *array*, v a value, and $0 \leq \mathit{begin} \leq \mathit{end} \leq |L|$.

if $\mathit{begin} = \mathit{end}$ **then**
    return begin.
else
    $\mathit{mid} := (\mathit{begin} + \mathit{end}) \operatorname{div} 2$.
    if $L[\mathit{mid}] < v$ **then**
        return $\textsc{LowerBoundRec}(L, v, \mathit{mid} + 1, \mathit{end})$.
    else $L[\mathit{mid}] \geq v$
        return $\textsc{LowerBoundRec}(L, v, \mathit{begin}, \mathit{mid})$.

Result: return the first offset r, $\mathit{begin} \leq r < \mathit{end}$, with $L[r] = v$ or,
if no such offset exists, $r = \mathit{end}$.

- Is $\textsc{LowerBoundRec}$ correct?
- What is the runtime and memory complexity of $\textsc{LowerBoundRec}$?

Correctness of LowerBoundRec

Recursion is repetition $\longrightarrow$ induction.

Algorithm LowerBoundRec(L, v, $begin$, end):

if $begin = end$ **then**
    return $begin$.
else
    $mid := (begin + end) \operatorname{div} 2$.
    if $L[mid] < v$ **then**
        return LowerBoundRec(L, v, $mid + 1$, end).
    else $L[mid] \geq v$
        return LowerBoundRec(L, v, $begin$, mid).

Base case:
Inspecting $end - begin = 0$ elements.

Recursive case:
Inspecting $end - begin > 0$ elements.

Induction Hypothesis

For any L', v', and $0 \leq begin' \leq end' \leq |L'|$ with $0 \leq end' - begin' < m$,
LowerBoundRec(L', v', $begin'$, end') returns the correct result.

Correctness of LowerBoundRec

Recursion is repetition $\longrightarrow$ induction.

Algorithm LowerBoundRec(L, v, $begin$, end):

if $begin = end$ **then**
    return $begin$.
else
    $mid := (begin + end)$ div 2. $\quad begin \leq mid < end$
    if $L[mid] < v$ **then**
        return LowerBoundRec$(L, v, \overbrace{mid + 1, end}^{\text{Induction Hypothesis}})$.
    else $L[mid] \geq v$
        return LowerBoundRec$(L, v, \underbrace{begin, mid}_{\text{Induction Hypothesis}})$.

Induction Hypothesis

For any L', v', and $0 \leq begin' \leq end' \leq |L'|$ with $0 \leq end' - begin' < m$, LowerBoundRec(L', v', $begin'$, end') returns the correct result.

Correctness of LowerBoundRec

Recursion is repetition $\longrightarrow$ induction.

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if $begin = end$ **then**
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    $mid := (begin + end) \operatorname{div} 2$.
    if $L[mid] < v$ **then**
        return $\textsc{LowerBoundRec}(L, v, mid + 1, end)$.
    else $L[mid] \geq v$
        return $\textsc{LowerBoundRec}(L, v, begin, mid)$.

Termination

Bound function: $end - begin$.

Intermezzo: Runtime complexity of LowerBoundRec

Algorithm LowerBoundRec(L, v, $begin$, end):

```
if begin = end then
    return begin.
else
    mid := (begin + end) div 2.
    if L[mid] < v then
        return LowerBoundRec(L, v, mid + 1, end).
    else L[mid] ≥ v
        return LowerBoundRec(L, v, begin, mid).
```

Base case: 1 *operation.*

Recursive case: 1 *operation and* 1 *recursive call.*

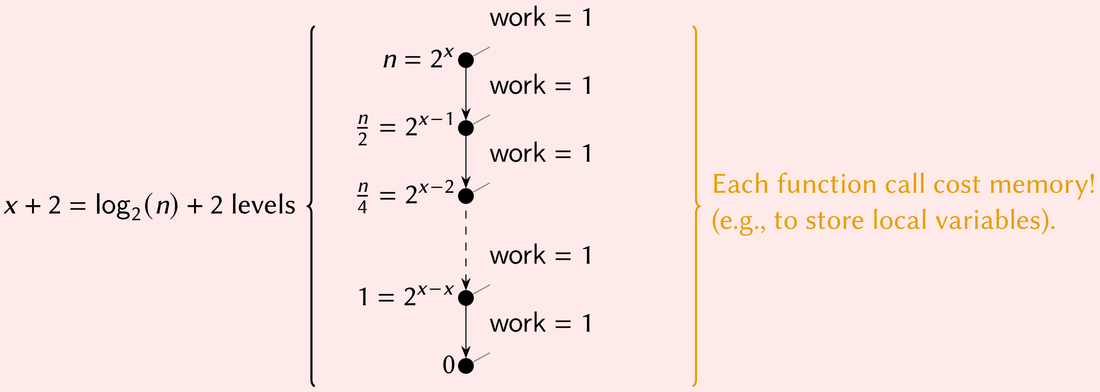
Complexity of LowerBoundRec with $n = end - begin$

$$T(n) = \begin{cases} 1 & \text{if } n = 0; \\ 1 \cdot T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1 & \text{if } n \geq 1. \end{cases}$$

Intermezzo: Runtime complexity of LowerBoundRec

Complexity of LowerBoundRec with $n = \mathit{end} - \mathit{begin}$ (assume: $n = 2^x$)

$$T(n) = 1 \cdot T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1 = \Theta(\log_2(n)).$$



The *recursive* Binary Search algorithm

Algorithm LowerBoundRec(L, v, $begin$, end):

Input: L is an ordered *array*, v a value, and $0 \leq begin \leq end \leq |L|$.

```
if begin = end then
    return begin.
else
    mid := (begin + end) div 2.
    if L[mid] < v then
        return LowerBoundRec(L, v, mid + 1, end).
    else L[mid] ≥ v
        return LowerBoundRec(L, v, begin, mid).
```

Result: return the first offset r, $begin \leq r < end$, with $L[r] = v$ or, if no such offset exists, $r = end$.

Theorem

LowerBoundRec is correct and has a runtime and memory complexity of $\Theta(\log_2(|L|))$.

The *non-recursive* Binary Search algorithm

Algorithm LowerBound(L, v, $begin$, end):

Input: L is an ordered *array*, v a value, and $0 \leq begin \leq end \leq |L|$.

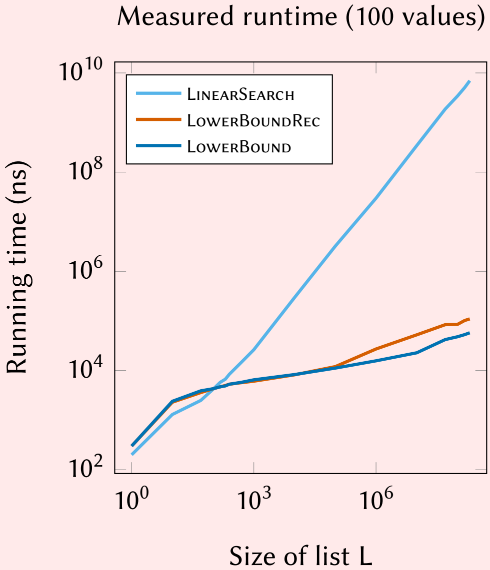
while $begin \neq end$ **do**
    $mid := (begin + end)$ div 2.
    if $L[mid] < v$ **then**
        $begin := mid + 1$.
    else
        $end := mid$.
return $begin$.

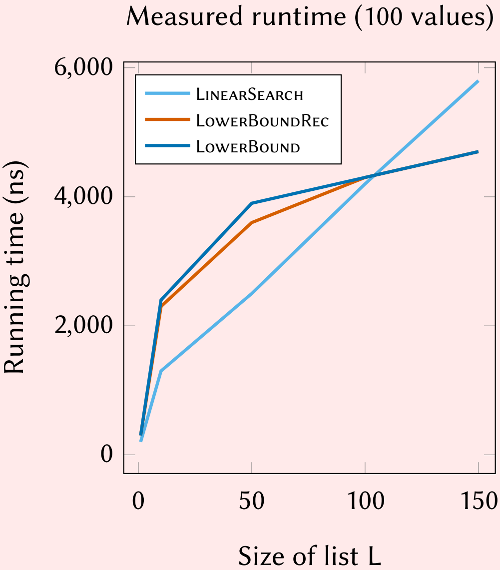
Result: return the first offset r, $begin \leq r < end$, with $L[r] = v$ or,
if no such offset exists, $r = end$.

Theorem

LowerBound is correct, has a runtime complexity of $\Theta(\log_2(|L|))$, *and a memory complexity of* $\Theta(1)$.

Comparing the complexity of searching





Using Binary Search as a *building block*

Problem

Let L be a list and $[v, w]$ be a range query with $v \leq w$. The solution of the range query problem for L and $[v, w]$ is the list of all values $e \in L$ with $v \leq e \leq w$.

Example

Consider a list *enrolled* with schema enrolled(*dept*, *code*, *sid*, *date*).

Query: All students enrolled in 2023

Range query on *enrolled* with $[(``, ``, -1, 2023), (``, ``, -1, 2024)]$.

We add enrollment data to *the end of the list* $\longrightarrow$ enrolled is *ordered* on *date*!

Using Binary Search as a *building block*

Problem

Let L be a list and $[v, w]$ be a range query with $v \leq w$. The solution of the range query problem for L and $[v, w]$ is the list of all values $e \in L$ with $v \leq e \leq w$.

Algorithm RangeQuery(L, $[v, w]$):

Input: L is an ordered *array*, v, w are values, and $v \leq w$.

1: $i :=$ LowerBound$(L, v, 0, |L|)$.
2: $j := i$.
3: **while** $j \leq |L|$ **and also** $L[j] \leq w$ **do**
4: $\quad j := j + 1$.
5: **return** $L[i, j)$.

Result: return the list $L[m, n)$, $0 \leq m \leq n \leq |L|$, such that $L[m, n)$ is the list of all values $e \in L$ with $v \leq e \leq w$.

Theorem

RangeQuery is correct and has all case runtime complexity $\Theta(\log_2(|L|) + |result|)$.

Using the *idea* of Binary Search

Problem

Let L be list of values of unknown length *and assume we have a function* $\textsc{Inspect}(L, i)$ *that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

The list-length problem *is the problem of finding the length of list L.*

Example

'apple'	'pear'	'orange'

$$\textsc{Inspect}(L, 0) = \texttt{true} \qquad \textsc{Inspect}(L, 1) = \texttt{true}$$
$$\textsc{Inspect}(L, 2) = \texttt{true} \qquad \textsc{Inspect}(L, 3) = \texttt{false}.$$

Using the *idea* of Binary Search

Problem

Let L be list of values of unknown length and assume we have a function $\textsc{Inspect}(L, i)$ *that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

The list-length problem is the problem of finding the length of list L.

Solving the list-length problem

We have ordered list $0, 1, \ldots$ of possible values for $|L|$.

Conclusion of $\textsc{Inspect}(L, i)$?

$\textsc{Inspect}(L, i) = \texttt{true}$ $|L| > i$ (list L has more than i values).

$\textsc{Inspect}(L, i) = \texttt{false}$ $|L| \leq i$ (list L has at-most i values).

Issue: no upper bound on the ordered list $0, 1, \ldots$ of possible values for $|L|$

Using the *idea* of Binary Search

Problem

Let L be list of values of unknown length and assume we have a function $\textsc{Inspect}(L, i)$ *that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

The list-length problem is the problem of finding the length of list L.

Issue: no upper bound on the ordered list $0, 1, \ldots$ of possible values for $|L|$

Guess repeatedly with exponentially-growing guesses.

Algorithm $\textsc{ListLengthUB}(L)$:

Input: L is an *array* of unknown length.

1: $n := 1$.
2: **while** $\textsc{Inspect}(L, n)$ **do**
3: $\quad n := 2 \cdot n$.
4: **return** n.

Result: return N, $|L| \leq N = 1$ or $|L| \leq N < 2|L|$.

Using the *idea* of Binary Search

Problem

Let L be list of values of unknown length and assume we have a function $\textsc{Inspect}(L, i)$ *that returns* `true` *if the list has an i-th value and returns* `false` *otherwise.*

The list-length problem is the problem of finding the length of list L.

Algorithm $\text{LB}\textsc{ListLength}(L, N)$ with $N := \textsc{ListLength}\text{UB}(L)$:

$begin, end := 0, N.$

while $begin \neq end$ **do**

    $mid := (begin + end) \operatorname{div} 2.$

    if $\textsc{Inspect}(L, mid)$ **then**

        $begin := mid + 1.$

    else

        $end := mid.$

return $begin.$

Result: return the length $|L|$ of array L.

Optimizing joins using range queries

Definition

A join of two lists L and M results in a list A in which each list value is computed from a combination of values $u \in L$ and $v \in M$ according to some *join condition*.

Example (Return pairs (p, r) of product name p and related category r)

products	
name	*category*
Apple	Fruit
Bok choy	Vegetable
Canelé	Pastry
Donut	Pastry

categories	
category	*related*
Fruit	Food
Fruit	Produce
Pastry	Food
Vegetable	Food
Vegetable	Produce

Join Result	
name	*related*
Apple	Food
Apple	Produce
Bok choy	Food
Bok choy	Produce
Canelé	Food
Donut	Food

Optimizing joins using range queries

Algorithm NestedLoopPC(products, categories):

Input: relations products(*name*, *category*) and categories(*category*, *related*).

$output := \emptyset$.
for $(p.n, p.c) \in$ products **do**
  for $(c.c, c.r) \in$ categories **do**
    if $p.c = c.c$ **then**
      add $(p.n, c.r)$ to *output*.

$\Theta(|categories|)$.

$|products|$ *times.*

Result: return $\{(p.n, c.r) \mid ((p.n, p.c) \in \text{products}) \wedge ((c.c, c.r) \in \text{categories})\}$.

Theorem

The NestedLoopPC *algorithm is correct and has a runtime complexity of* $\Theta(|product| \cdot |categories|)$.

Optimizing joins using range queries

Algorithm $\textsc{NestedBinaryPC}$(products, categories)**:**

Input: relations products(*name*, *category*) and categories(*category*, *related*), relation categories ordered.

$output := \emptyset$.
for $(p.n, p.c) \in$ products **do**
  $i := \textsc{LowerBound}(\text{categories}, (p.c, \text{`'}), 0, |\text{categories}|)$.
  while $i <$ |categories| **and also** categories$[i]$.$category = p.c$ **do**
    add $(p.n, \text{categories}[i].related)$ to *output*.
    $i := i + 1$.

Result: return $\{(p.n, c.r) \mid ((p.n, p.c) \in \text{products}) \wedge ((c.c, c.r) \in \text{categories})\}$.

Theorem

The $\textsc{NestedBinaryPC}$ algorithm is correct and has a runtime complexity of $\Theta(|product| \cdot \log_2(|categories|) + |result|)$.

CONTAINS, LINEARSEARCH, and LOWERBOUND in practice

Algorithm	C++	Java
CONTAINS	std::ranges::contains	*collection*.contains[a]
LINEARSEARCH LINEARPREDSEARCH	std::find std::find_if	*collection*.indexOf[a] java.util.stream::filter[b]
LOWERBOUND	std::lower_bound std::upper_bound[d]	java.util.Arrays::[c] binarySearch
Related libraries	<algorithm>, <ranges>	java.util.Arrays, java.util.ArrayList, ...

[a] Here, *collection* is a standard Java data collection such as java.util.ArrayList.

[b] Using the stream library supported by standard Java data collections.

[c] Does not guarantee to return the offset of the *first* occurrence of a value.

[d] Returns the offset of the first element in the list that is strictly larger than the searched-for value.

Contains, LinearSearch, and LowerBound in practice

Algorithm	C++	Java
Contains	`std::ranges::contains`	*collection*`.contains`[a]
LinearSearch LinearPredSearch	`std::find` `std::find_if`	*collection*`.indexOf`[a] `java.util.stream::filter`[b]
LowerBound	`std::lower_bound` `std::upper_bound`[d]	`java.util.Arrays::`[c] `binarySearch`
Related libraries	`<algorithm>`, `<ranges>`	`java.util.Arrays`, `java.util.ArrayList`, ...

[a]Here, *collection* is a standard Java data collection such as `java.util.ArrayList`.

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