Demand-Deposit Contracts and The Probability of Bank Runs

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Outline

Motivation

- Theory
  - Diamond & Dybvig has multiple equilibria
  - They cannot determine the probability of runs
  - ... and hence the welfare having banks
  - Runs not related to fundamentals
- Empirics
  - Bank runs result from coordination failure in theory
  - Bank runs occur following negative shocks in reality
- Solution
  - Global Games

Model Setting

- Project
  - Return 1 if liquidated at $t = 1$ (cf $L < 1$ in D& D)
  - Return R with proba. $p(\theta)$ at $t = 2$ and 0 otherwise
- Investor
  - A fraction λ suffers liquidty shocks $u(c_1)$
  - The rest don't $u(c_1 + c_2)$
  - $-\frac{cu''}{u'} > 1$: to get $c^{FB} > 1$
- Information
  - $\theta \sim Unif[0, 1]$
  - Each investor observes $\theta_i = \theta + \varepsilon_i, \varepsilon_i \sim Unif[-\varepsilon, \varepsilon]$
  - Efficiency: $\mathbb{E}[p(\theta)]u(R) > u(1)$

Autarky

Each investor invests 1 at $t = 1$.

With proba. λ, he is impatient and liquidates.

$$u(1)$$

With proba $1 - \lambda$., he is patient and gets

$$u(R)$$

Total Welfare

$$\lambda u(1) + (1 - \lambda)u(R)$$

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First Best: Risk Sharing

Social Planner chooses c_1 to maximize

$$\max_{c_1} \lambda u(c_1) + (1-\lambda) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) \mathbb{E}[p(\theta)]$$

FOC

$$\lambda u'(c_1) - (1-\lambda)\frac{\lambda R}{1-\lambda} u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right) \mathbb{E}[p(\theta)] = 0$$

At $c_1 = 1$, by $-\frac{cu''}{u'} > 1$

$$1u'(c_1) > Ru'(R) > Ru'(R)\mathbb{E}[p(\theta)]$$

Risk Sharing: $c_1^{FB} > 1$

Note: Only the idiocyncratic liquidity shock can be diversified

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Bank Contracts

- Promise $r_1 > 1$ if withdraw at $t = 1$
- Receive $\tilde{r}_2$ if wait until $t = 1$
- Subject to *sequential service constraint*

Suppose a fraction n choose to withdraw

Withdraw at	$n < 1/r_1$	$n \geq 1/r_1$
1	r_1	r_1 with proba. $\frac{1}{nr_1}$
2	$\frac{1-nr_1}{1-n}R$ with proba. $p(\theta)$	0

Withdraw at $t = 2$ if

$$p(\theta_i)u\left(\frac{1-nr_1}{1-n}R\right) > u(r_1) \text{ and } n < 1/r_1$$

Question: what is n?

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Investor Inference

When $-\varepsilon < \theta_i < \varepsilon$,

$$\mathit{Unif}[0, \theta_i + \varepsilon]$$

When $\varepsilon < \theta_i < 1 - \varepsilon$,

$$\mathit{Unif}[\theta_i - \varepsilon, \theta_i + \varepsilon]$$

When $1 - \varepsilon < \theta_i < 1 + \varepsilon$,

$$\mathit{Unif}[\theta_i - \varepsilon, 1]$$

Note: Though $p(\theta)$ unrestricted, the joint uniformality implies

$$p(\theta_i) \approx p(\theta) + p'(\theta)\varepsilon_i$$

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Adding Global Games: Lower Dominance Region

We want when θ is sufficiently low,
all withdraw at $t = 1$ regardless of others' actions.
Let $\underline{\theta}(r_1)$ solves

$$u(r_1) = p(\theta)u\left(\frac{1-\lambda r_1}{1-\lambda}R\right)$$

When $\theta_i < \underline{\theta}(r_1) - \varepsilon$, she belives $\theta < \underline{\theta}(r_1)$ a.s. and withdraws.
We need it to hold for all $r_1 > 1$ and for all i at $\theta = 0$.
Note $\underline{\theta}(r_1)$ is increasing in r_1 (why?) and $\theta_i < \varepsilon$ at $\theta = 0$
A sufficient condition is

$$\varepsilon < \underline{\theta}(1) - \varepsilon \implies p^{-1}\left(\frac{u(1)}{u(R)}\right) > 2\varepsilon.$$

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Adding Global Games: Upper Lower Region (Deus Ex Machina)

We want when θ is sufficiently high,
all patient investors wait regardless of other's action.
To get this, we assume $p(\theta) = 1$
and when $\theta \in [\overline{\theta}, 1]$, liquidation at $t = 1$ yields R with certainty.
Then, no more than $\frac{nr_1}{R}$ needs to be liquidated.
By waiting, the agent can get

$$\frac{R - nr_1}{1 - n} > r_1$$

And a sufficient condition is

$$1 - \varepsilon > \overline{\theta} + \varepsilon$$

Question: is it really deposit? Or putable equity?

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Theorema Egregium

The game admits a unique equilibrium in which a patient investor i runs if and only if $\theta_i < \theta^*$

How many investors run?

Let $n(\theta, \theta')$ be the fraction of investors who run when

- the state is θ and ...
- under the strategy profile that patient i runs iff $\theta_i < \theta'$.

$$n(\theta, \theta') = \lambda + (1-\lambda)\Pr(\varepsilon_i < \theta' - \theta)$$

$$= \begin{cases} 1 & \theta < \theta' - \varepsilon \\ \lambda & \theta > \theta' + \varepsilon \\ \lambda + (1-\lambda)\left(\frac{1}{2} + \frac{\theta' - \theta}{2\varepsilon}\right) & \text{Otherwise} \end{cases}$$

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Incentive to Run

The incremental utility from running for an informed investor at θ

$$v(\theta, n) = \begin{cases} p(\theta)u\left(\frac{1-nr_1}{1-n}R\right) - u(r_1) & \lambda \leq n \leq 1/r_1 \\ -\frac{1}{nr_1}u(r_1) & 1/r_1 < n < 1 \end{cases}$$

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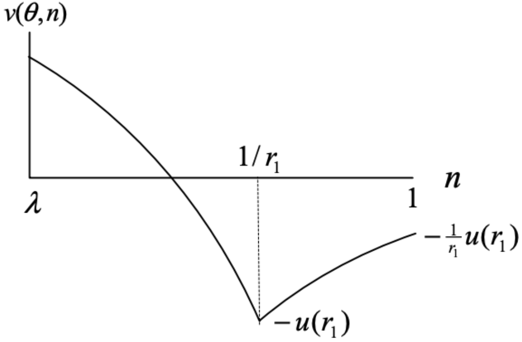

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Let $\Delta^{r_1}(\theta_i, \theta')$ be the expected differential utility when

- the investor observes θ_i and ...
- under the strategy profile that patient i runs iff $\theta_i < \theta'$.

$$\Delta^{r_1}(\theta_i, \theta') = \mathbb{E}[v(\theta, n(\theta, \theta'))|\theta_i]$$

Equilibrium Condition

Necessary condition

$$\Delta^{r_1}(\theta^*, \theta^*) = 0$$

Is it also sufficient?

Not so obvious ...

Higher θ doesn't necessarily implies a higher incentive to run ...

But here it is! Why? Single crossing!

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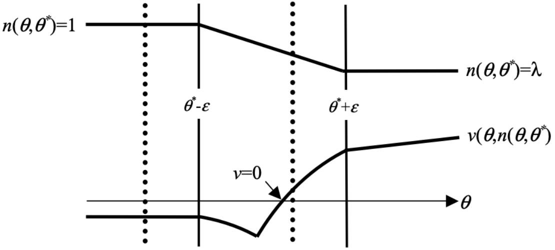

Compute the equilibrium

The investor θ^*'s posterior is

$$Unif\,[\theta^* - \varepsilon, \theta^* + \varepsilon]$$

If $\theta = \theta^* + \varepsilon$, $\theta_i > \theta^*$, all run: $n = 1$.
If $\theta = \theta^* - \varepsilon$, $\theta_i < \theta^*$, only patient ones run: $n = \lambda$.
And n is linear in between.
So posterior of θ^* is

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Computing the equilibrium

Recall

$$v(\theta, n) = \begin{cases} p(\theta) u\left(\frac{1-nr_1}{1-n} R\right) - u(r_1) & \lambda \leq n \leq 1/r_1 \\ -\frac{1}{nr_1} u(r_1) & 1/r_1 < n < 1 \end{cases}$$

and at the limit

$$\lim_{\varepsilon \to 0} \Delta^{r_1}(\theta^*, \theta^*) = v(\theta^*, n(\theta^*, \theta^*))$$

We have

$$p(\theta^*) \int_{\lambda}^{1/r_1} u\left(\frac{1-nr_1}{1-n} R\right) \mathrm{d}n - u(r_1)(1/r_1 - \lambda) - u(r_1) \frac{\ln(r_1)}{r_1} = 0$$

Comparative Statics

$$p(\theta^*)\int_{\lambda}^{1/r_1} u\left(\frac{1-nr_1}{1-n}R\right)\mathrm{d}n = u(r_1)\left(\frac{1+\ln(r_1)}{r_1}-\lambda\right)$$

Thm 2: higher $r_1 \implies$ higher θ^*

Intuition:

- Direct: high payment increases the incentive to run
- Indirect: high payment increases estimation of people running

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Optimal r_1

At the limit $\varepsilon \to 0, \overline{\theta} \to 1$, a benevolent banking system maximize

$$\int_0^{\theta^*} \frac{u(r_1)}{r_1} \mathrm{d}\theta + \int_{\theta^*}^{1} \left[\lambda u(r_1) + (1-\lambda) p(\theta^*) u\left(\frac{1 - n r_1}{1-n} R \right) \right] \mathrm{d}\theta$$

Thm 4&3: $r_1^* < c^{FB}$, and if $\theta(1)$ not too large, $r_1^* > 1$.

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At the limit $\varepsilon \to 0, \overline{\theta} \to 1$, a benevolent banking system maximize

$$\int_0^{\theta^*} \frac{u(r_1)}{r_1} \mathrm{d}\theta + \int_{\theta^*}^1 \left[\lambda u(r_1) + (1-\lambda) p(\theta^*) u\left(\frac{1-nr_1}{1-n} R \right) \right] \mathrm{d}\theta$$

Thm 4&3: $r_1^* < c^{FB}$, and if $\theta(1)$ not too large, $r_1^* > 1$.

Optimal r_1

At the limit $\varepsilon \to 0, \overline{\theta} \to 1$, a benevolent banking system maximize

$$\int_0^{\theta^*} \frac{u(r_1)}{r_1} \mathrm{d}\theta + \int_{\theta^*}^1 \left[\lambda u(r_1) + (1-\lambda) p(\theta^*) u \left(\frac{1 - n r_1}{1-n} R \right) \right] \mathrm{d}\theta$$

Thm 4&3: $r_1^* < c^{FB}$, and if $\theta(1)$ not too large, $r_1^* > 1$.

- r_1^* cannot be larger than c_1^{FB}
  - Loss from imperfect risk sharing
  - Loss from runs
- By deviating from c_1^{FB}
  - Loss from risk sharing is second order
  - Gain from reducing runs is first order

Optimal r_1

At the limit $\varepsilon \to 0, \overline{\theta} \to 1$, a benevolent banking system maximize

$$\int_0^{\theta^*} \frac{u(r_1)}{r_1} \mathrm{d}\theta + \int_{\theta^*}^{1} \left[\lambda u(r_1) + (1-\lambda) p(\theta^*) u \left(\frac{1 - n r_1}{1-n} R \right) \right] \mathrm{d}\theta$$

Thm 4&3: $r_1^* < c^{FB}$, and if $\theta(1)$ not too large, $r_1^* > 1$.

- At $r_1 = 1$, $\theta^* = 0$ no one runs and no risk sharing.
- By increasing r_1 a little bit
  - Most investors benefit from risk sharing: first order
  - Runs occur when $\theta \in [\underline{\theta}(1), \theta^*]$: runs not costly as p small
  - Runs more costly in $[0, \underline{\theta}(1)]$: not too large if $\underline{\theta}(1)$ small

Uniqueness