## Systemic Risk and Stability in Financial Networks

Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi

February 9, 2021

#### Classical Theories of Banking (usually) study only one bank

Banks are connected in real life

The GFC revealed some banks are not only too big to fail ...

... but also too inter-connected to fail

Need to investigate the role of bank network structures ...

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#### "Interconnectedness"

A network is more inter-connected ...

... if one bank has higher debts with another bank

... if each bank borrows from/lends to more banks

NB: Different from "connectedness" in the network literature

#### Questions

How "inter-connectedness" affects bank failures?

Do banks fail more if their mutual debt levels high?

Do banks fail more if they borrow from/lend to more banks?

# This paper (and the next)

Model N banks with with long-term assets y, short-term liquidity risk L

Assumption 1: Pledgeability is limited

Can't borrow against full asset value to meet liquidity shock

Assumption 2: Liquidity shocks are non-contractable

Cannot pay premium for contingent insurance to shock

Assumption 3a: Financial networks are short-term debts

Interbank debts mature when the liquidity shocks occur

## Typical Network Structures

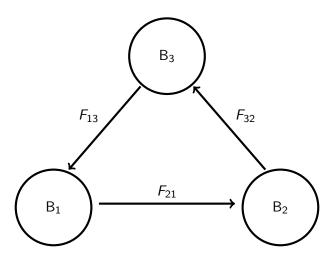


Figure: Ring Network (RN)

#### Typical Network Structures

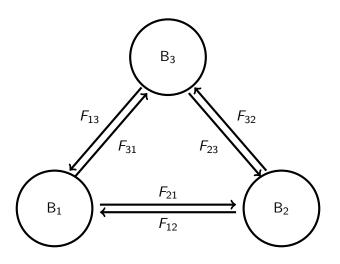


Figure: Complete Network (CN)

#### **Key Results**

R1: Higher debt levels lead to liquidation cascade

R2: Phase transition

Complete network most stable with small shocks

Complete network least stable with large shocks

#### The Model

#### Date 0:

> Bank j with existing short-term interbank debts of total face value  $F_j$ 

#### Date 1:

- > M liquidity shocks are realized and interbank debts mature
- > Banks receive payment from/repay other banks
- > Banks borrow at most  $\theta y$  from the market
- > Bank are liquidated if they cannot repay in full

#### Date 2:

Asset y is realized if not liquidated

#### Default and Liquidation

When a bank defaults on any liabilities

A fraction  $(1 - \theta)y$  is liquidated

The rest  $\theta y$  is realized and paid out

The liquidated bank cannot borrow senior

#### Liquidation Rule

Bank j has two types of obligations

- 1 Liquidity shock  $L_j \in \{0, L\}$ , which is senior (e.g. deposits)
- 2 Liabilities to other banks  $F_j$

And he has two sources of liquidity

- 1 Payment from other banks  $X^{j}$
- 2 Additional  $\theta y$ , from borrowing or liquidation

Thus, bank j is liquidated if

$$X^j + \theta y \le L_j + F_j$$



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## Payment to Other Banks

Let  $X_j$  be the payment of bank j to other banks

When  $X^j + \theta y \ge L_j + F_j$ , no default

$$X_j = F_j$$

When  $X^{j} + \theta y \leq L_{j}$ , full default on the interbank debts

$$X_j = 0$$

When  $L_j + F_j \ge X^j + \theta y \ge L_j$ , partial default on the interbank debts

$$X_j = X^j + \theta y - L_j$$



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$$X_j = X^j + \theta y - L_j$$

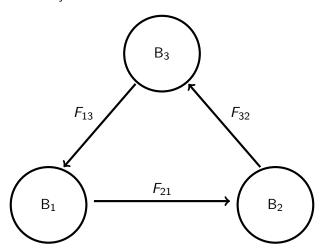
where network kicks in



# R1: Higher debt levels lead to more liquidation

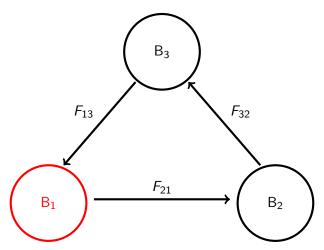
#### An Example of Ring Network with Three Banks

Zero-net-positions  $F_{ij} = F$ 



#### An Example of Ring Network with Three Banks

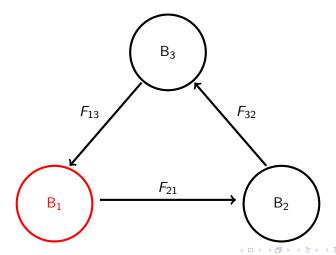
One bank is shocked with  $L = 2.5\theta y$ 



#### An Example of Ring Network with Three Banks

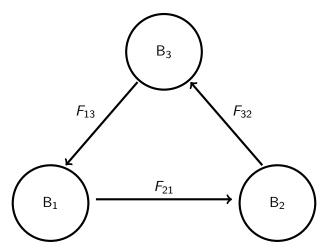
We will show

- > One bank is liquidated when F is small  $(F = \theta y)$
- > Two banks are liquidated when F is large  $(F = 2\theta y)$

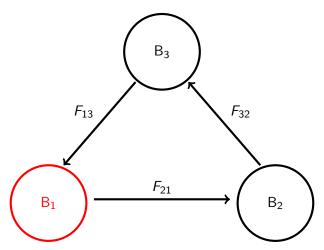


R1a: One Bank Liquidated if Debts Low

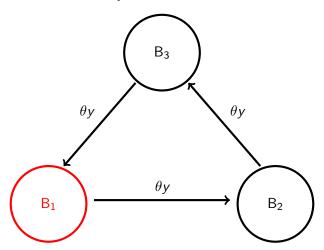
A ring network with 3 banks



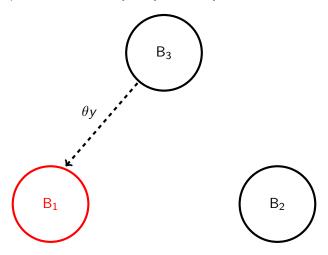
Bank 1 is shocked with  $L = 2.5\theta y$ 



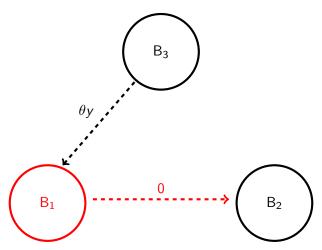
Suppose the face values are  $F_{ij} = \theta y$ 



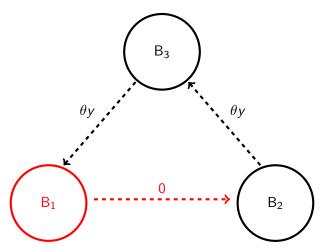
Bank 1 is liquidated because  $\theta y + \theta y < L + \theta y$ 



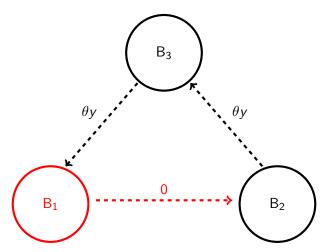
Bank 1 pays out 0



Bank 2 does not default  $\theta y \leq \theta y$ 

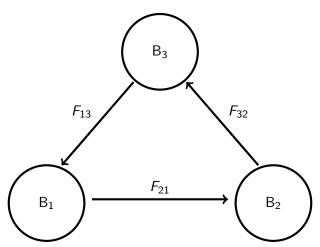


Verify Bank 3 does not default

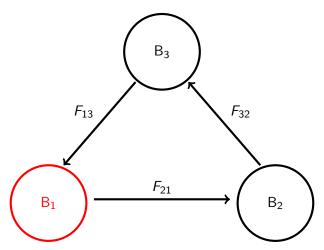


R1b: Two Banks Liquidated if Debts High

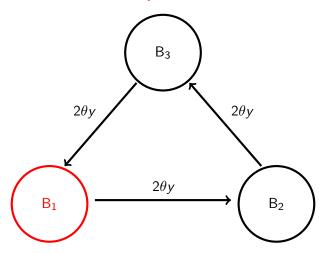
A ring network.



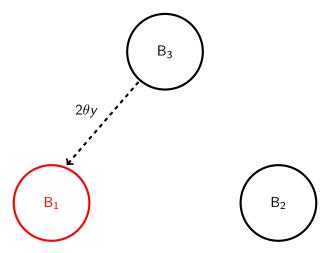
Bank 1 is shocked with  $L = 2.5\theta y$ 



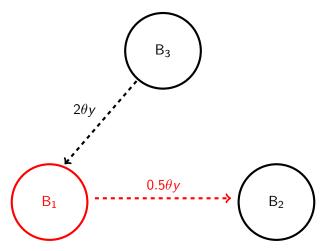
Suppose now the face values are  $F_{ij} = 2\theta y$ 



Suppose bank 3 does not default

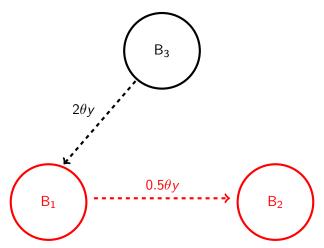


Bank 1 defaults and pays out  $2\theta y + \theta y - 2.5\theta y = 0.5\theta y$ 



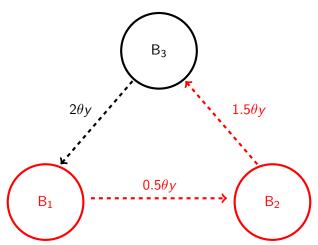
#### Result 1b: Two Banks Liquidated if $F = 2\theta y$

Since  $0.5\theta y + \theta y < 2\theta y$ , Bank 2 defaults ...



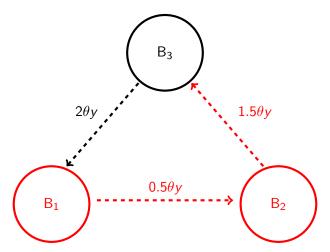
#### Result 1b: Two Banks Liquidated if $F = 2\theta y$

... and pays out  $0.5\theta y + \theta y = 1.5\theta y$ 



#### Result 1b: Two Banks Liquidated if $F = 2\theta y$

Verify bank 3 does not default



#### Intuition

Why higher debts lead to more liquidation?

A not-shocked bank is liquidated if the counter-party risk exceeds  $\theta y$ 

$$\underbrace{\mathcal{F}^{j}}_{\text{what other banks owe you}} - \underbrace{\mathcal{X}^{j}}_{\text{what they actually pay you}} \geq \theta y$$

High debt levels allow transmission of greater counter-party risk

Transmitted  $\theta y$  when  $F = \theta y$ 

Transmitted  $1.5\theta y$  when  $F = 2\theta y$ 

Debt level is an upper bound of largest possible counter-party risk

$$F^j - X^j \leq F^j$$



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## Result 2: Phase Transition

#### Roadmap

We will show that

R2a: Only one bank is liquidated with a small shock

Shocks that can be absorbed by the system  $ML < N\theta y$ 

R2b: Only one bank is liquidated with a small shock and more debts

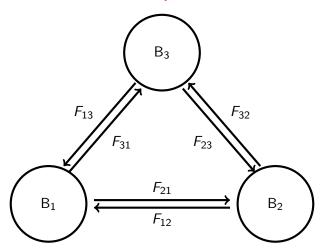
Show R2c is not driven by more debts (not result 1)

R2c: Two banks are liquidated with large small shock

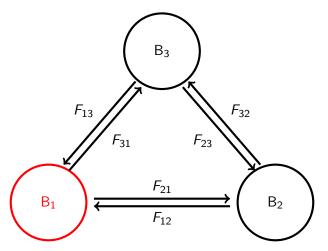
Complete network achieves the lower/upper bound of # bank failures

Show it's indeed the most/least stable network

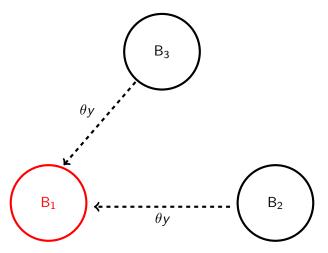
The banks,  $L = 2.5\theta y$ ,  $F = 2\theta y$  but  $F_{ij} = \theta y$ 



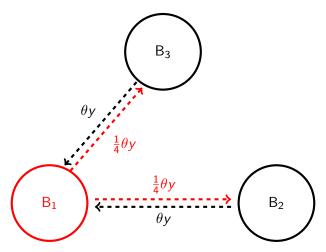
Bank 1 is shocked again (Sorry, Bank 1!)



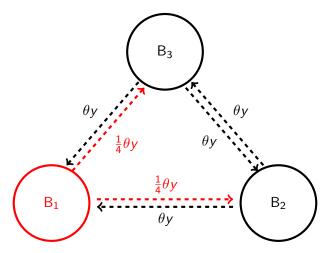
Suppose bank 2 and 3 do not default



Bank 1 pays out  $2\theta y + \theta y - 2.5\theta y = 0.5\theta y$ 



Verify bank 2 and 3 don't default.



#### Intuition

With small shocks, connected networks diversify the liquidity risk

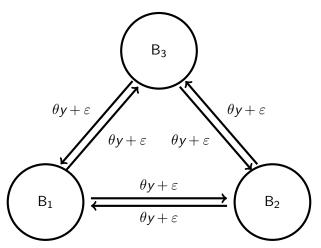
The default of shocked bank is borne by two banks, each only a half  $1.5\theta y$  in the ring network but  $0.75\theta y$  in the complete network.

Bank 2 also receives payment from not-shocked bank  $\theta y$  in the ring network but 0 in the complete network.

# R2b: One Bank Liquidated with Higher Debt Level

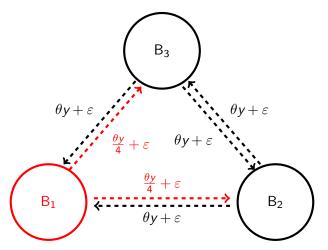
#### R2b: One Bank Liquidated with Higher Debt Level

Suppose the face value of each bank's liability increases by  $2\varepsilon$ 



#### R2b: One Bank Liquidated with Higher Debt Level

Equilibrium: The payment is also increased by 2 arepsilon



#### Intuition

The liquidity shock is completely absorbed

The shocked bank absorbs  $\theta y$ 

Each not-shocked bank absorbs  $\frac{3\theta y}{4}$ 

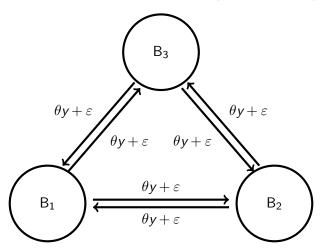
Small shocks can be absorbed by all banks, without full liquidation

Complete network ensures enough diversification

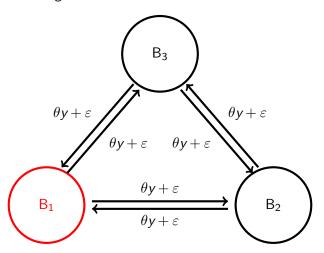
Liquidity shock fully transmitted when F small

No more transmission of liquidity shock with higher debt level

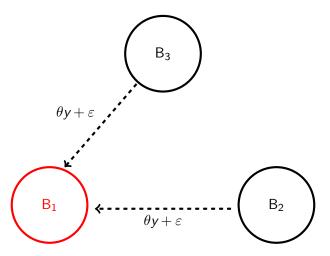
Again, 3 banks,  $F = 2\theta y + 2\varepsilon$  but  $L = 3.5\theta y$ .  $(0 < 2\varepsilon < 0.5\theta y)$ 



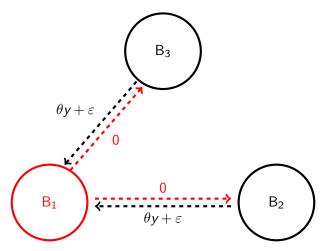
#### Bank 1 is shocked again



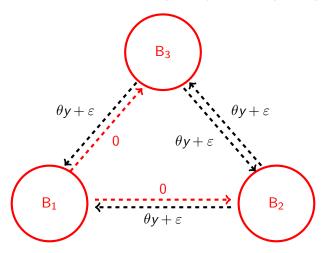
Suppose bank 2 and 3 do not default



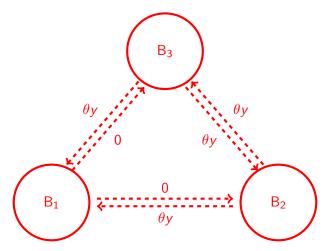
Bank 1 pays out  $\max\{2\theta y + 2\varepsilon + \theta y - L, 0\} = 0$ 



Bank 2 and 3 have to be liquidated!  $(\theta y + \varepsilon) + \theta y < 2(\theta y + \varepsilon)$ 



Actual equilibrium payment (for any  $\varepsilon > 0$ )



#### Intuition

Larger shock cannot be absorbed even by all firms

Full liquidation if liquidity risk spread out

Better to concentrate liquidity within the shocked banks

#### Number of Liquidated banks

With ST network, (we will prove) the number of liquidated banks  $|\mathcal{D}|$  is

$$M \leq |\mathcal{D}| \leq \frac{ML}{\theta y}$$

All shocked banks at least default on the junior debt

A not-shocked bank is liquidated if counter-party risk exceeds  $\theta y$ 

#### Implication

Complete Network is the most stable network when shocks small attaining the lower bound

Complete Network is the least stable network when shocks large attaining the upper bound

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#### Implication:

Complete Network is the most stable network when shocks small attaining the lower bound

Complete Network is the least stable network when shocks large attaining the upper bound

If bank j defaults on  $F_j$  but not  $L_j$ 

$$X^j + \theta y = X_j + L_j$$

If bank i defaults on  $L_i$  (and hence all of  $F_i$ )

$$X^i + \theta y < \underbrace{X_i}_{=0} + L_i$$

Let  $\mathcal{D}$  be the defaulting banks

$$\sum_{i \in \mathcal{D}} X^i + |\mathcal{D}|\theta y < \sum_{i \in \mathcal{D}} X_i + ML$$

If bank j defaults on  $F_j$  but not  $L_j$ 

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Let  $\mathcal{D}$  be the defaulting banks

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The rest banks  $k \notin \mathcal{D}$  do not default  $X^k = X_k$ 

$$\sum_{i\in\mathcal{D}} X^i = \sum_{i\in\mathcal{D}} X_i$$



If bank j defaults on  $F_j$  but not  $L_j$ 

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Let  $\mathcal{D}$  be the defaulting banks

$$\sum_{i \in \mathcal{D}} X^i + |\mathcal{D}|\theta y < \sum_{i \in \mathcal{D}} X_i + ML$$

We obtain the upper bound

$$|\mathcal{D}| < \frac{ML}{\theta y}$$



#### Can we avoid the liquidation cascade?

Yes, we can net out the interbank debts

It can be netted out ex ante

Mutual liabilities are netted out before liquidity shocks occur

Equivalent to a null network if no net positions

It can be netted out ex post

Mutual liabilities are netted out when one party defaults

Equivalent to a complete network with lower face value

#### (Ex ante) Netting is good in AOT

At most M banks are liquidated, without a short-term debt network

At least M banks are liquidated, with a short-term debt network

Suggesting (ex ante) netting is good

Null network achieves the lower bound

#### (Ex post) Close-Out Netting is also good in AOT

The liquidation of a not-shocked bank is caused by default of shock banks

Bank i and bank j owe F to each other

Bank i repays only X < F to bank j

Bank j's liability to bank i is also reduced to X

Net-liability of a not-shocked bank is zero, no liquidation.

NB1: Close-out netting changes priority structure, sparking policy debate

NB2: Close-out netting not always working (Recall ring network)

## Netting

Jason R. Donaldson, Giorgia Piacentino, Xiaobo Yu

#### Motivation

Banks have huge gross positions

E.g. HSBC's net position  $|\text{£24B} - \text{£21.5B}| \approx 10\%$  gross

Previous analysis (AOT) shows

Off-setting debts amplify financial risk

Netting mitigate risk transmission

Why not net them out?

Note, these gross positions are not *short-term*!

Average maturity more than a year

Unaccounted by the AOT Model with short-term debt network

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#### Question

How are long-term debt networks different?

Does higher face value exacerbate financial fragility?

Does borrowing from/lending to more banks lead to more liquidation?

#### This paper

Model N banks with with long-term assets y, short-term liquidity risk L

Assumption 1: Pledgeability is limited

Can't borrow against full asset value to meet liquidity shock

Assumption 2: Liquidity shocks are non-contractable

Cannot pay premium for contingent insurance to shock

Assumption 3b: Financial networks are long-term debts

Interbank debts mature when the assets mature

#### **Key Results**

R3: Higher Debt Level leads to Less Liquidation

R4: Phase transition

Complete network most stable with small shocks

Complete network least stable with large shocks

#### Opposite Results

With short-term networks, high debt level leads to more liquidation

More risks are transmitted to healthy banks

With long-term networks, high debt level leads to less liquidation

Shocked banks can dilute not-shocked banks more

#### Same Result but Different Mechanisms

#### In both models

Complete network most stable when shocks small

ST network diversifies the risk

LT network provides sufficient dilution

Complete network least stable when shocks large

ST network transmitted too much risks

LT network cannot provide enough dilution

#### The Model

#### Date 0:

> Bank j with existing long-term interbank debts of total face value  $\mathit{F}_{j}$ 

#### Date 1:

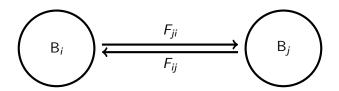
- > M Liquidity shocks are realized
- $>\,$  Banks sell their claims to the competitive market  $\it C$
- > Banks borrow at most  $\theta y$  from the market, diluting the other banks
- > Bank are liquidated if they cannot satisfy the liquidity needs

#### Date 2:

- Asset y is realized if not liquidated
- > Banks decide to default or not

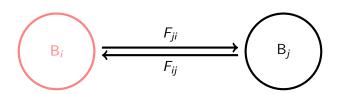
Two banks with mutual liabilities



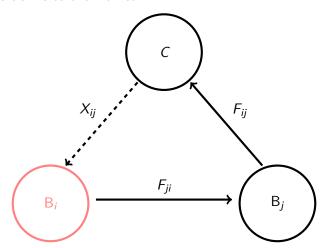


#### Bank i is shocked

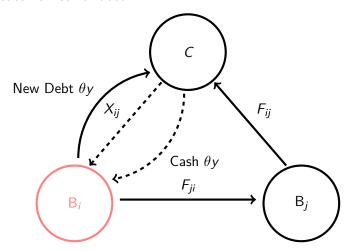


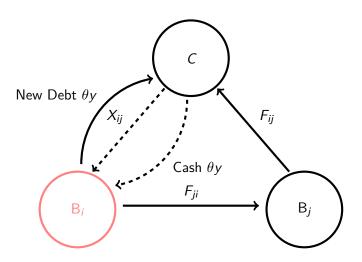


Bank i sells claims to the market



Bank i issues new senior debt





No liquidation if  $X_{ij} + \theta y \ge L$ 

Let  $X^j$  be the amount each bank receives from selling the claims

A shocked bank is liquidated if and only if

$$X^{j} + \theta y < L$$

A not-shocked bank is never liquidated.

A bank can default and divert  $(1 - \theta)y$ . But when?

New senior debt repaid in full

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New senior debt repaid in full

$$y + X^j - \underbrace{(F_j + L_j)}_{\text{repay in full}} < (1 - \theta)y$$

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New senior debt repaid in full

$$X^j + \theta y - L_j$$
  $< F_j$ 
Default Payment  $X_j$ 



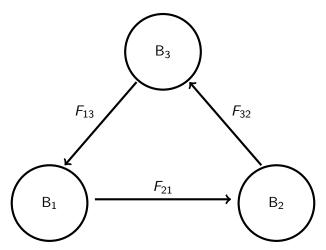
#### Two ways to raise Liquidity

A shocked bank can raise liquidity in two ways

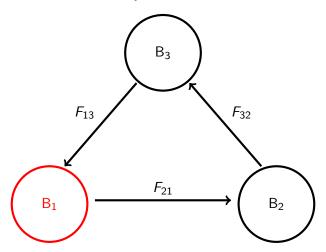
- 1 Selling other banks debt
- 2 Issue new debts, diluting original creditors

# R3: Higher Debt Level Leads to Less Liquidation

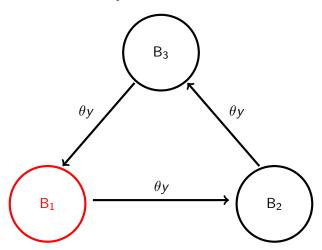
A ring network with 3 banks



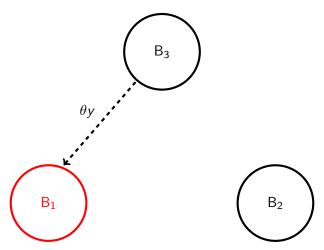
Bank 1 is shocked with  $L = 2.5\theta y$ 



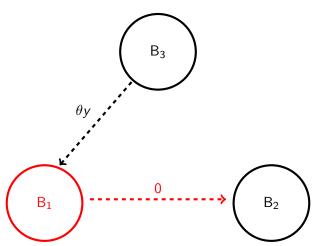
Suppose the face values are  $F_{ij} = \theta y$ 



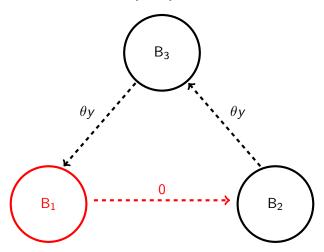
Bank 1 is liquidated because  $\theta y + \theta y < L$ 



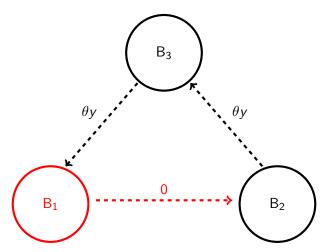
Bank 1 pays out 0



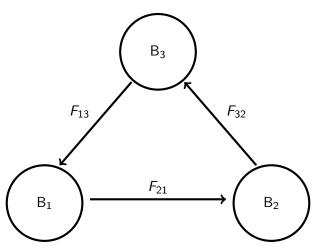
Bank 2 does not default as  $0 + \theta y \ge \theta y$ 



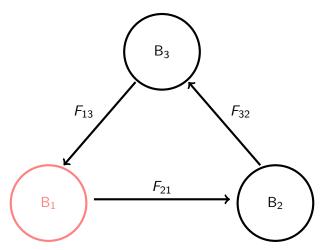
Verify Bank 3 does not default



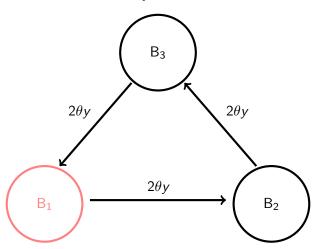
A ring network



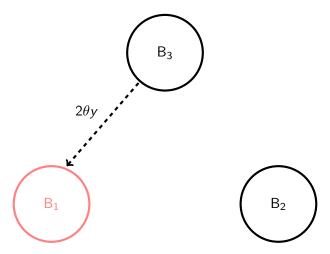
Bank 1 is shocked with  $L = 2.5\theta y$ 



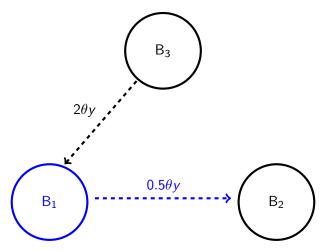
Suppose now the face values are  $F_{ij} = 2\theta y$ 



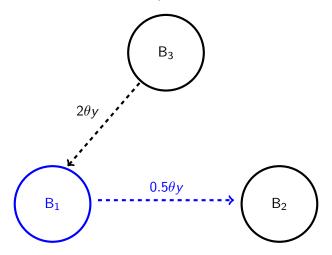
Suppose bank 3 does not default



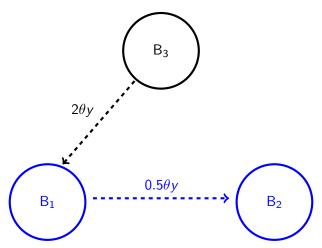
Bank 1 is not liquidated  $2\theta y + \theta y > L$ , and pays out  $0.5\theta y$ 



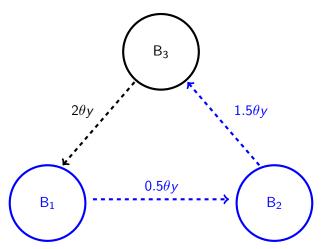
Bank 2 is not shocked, hence not liquidated



Since  $0.5\theta y + \theta y < 2\theta y$ , Bank 2 defaults ...

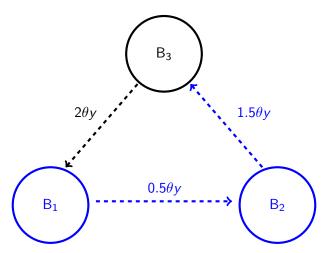


...and pays out  $0.5\theta y + \theta y = 1.5\theta y$ 



## Result 3b: No Banks Liquidated if Debts High

Verify Bank 3 does not default



#### Intuition

With long-term debts, a bank is liquidated only if it is shocked

But a shocked bank can avoid liquidation if it raises enough liquidity

from both issuing new senior debt

and selling the long-term debts of other banks

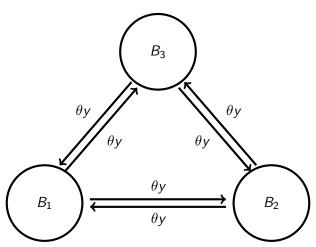
Higher debts level enhances the second channel

# Result 4: Phase Transition

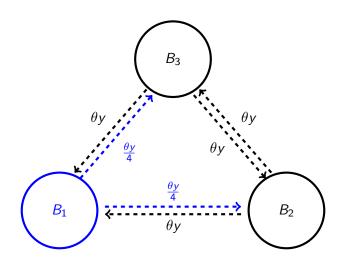
# R4a: With Small Shocks, No Banks Liquidated in Complete Network when Debt Level High

# R4a: No Banks Liquidated in CN when Debt Level High

Suppose  $L = 2.5\theta y$ 



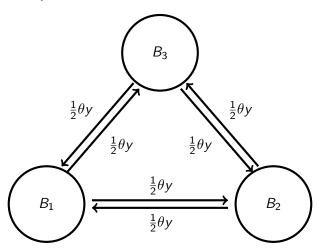
# R4a: No Banks Liquidated in CN when Debt Level High



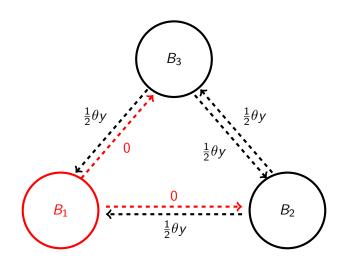
# R4b: With Small Shocks, One Bank Liquidated in Complete Network when Debt Level Low

### R4b: One Bank Liquidated in CN when Debt Level Low

Suppose  $L = 2.5\theta y$ 



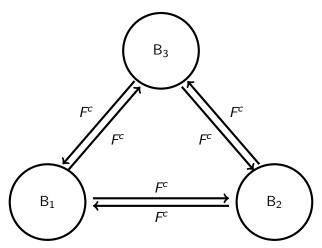
## R4b: One Bank Liquidated in CN when Debt Level Low



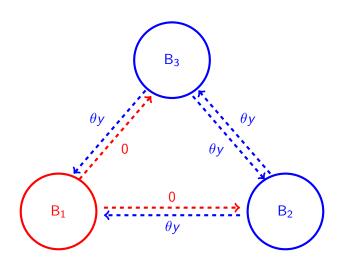
# R4c: With Large Shocks, One Bank Liquidated in Complete Network

# R4c: One Bank Liquidated in CN when Shocks Large

Suppose  $L = 3.5\theta y$ ,  $F^c \ge \theta y$ 



# R4c: One Bank Liquidated in CN when Shocks Large



### Why Higher Debts not Helpful?

Total liquidity in the system not enough

Need to dilute more than  $\theta y$ , for each bank

Net payment from each bank cannot be higher than heta for any F

Bank 1 liquidated for any F

### Number of Liquidated Banks

With LT interbank debts, the number of liquidated banks  $|\mathcal{D}|$  is

$$0 \le |\mathcal{D}| \le M$$

No banks are liquidated with enough good dilution

All shocked banks are liquidated with insufficient dilution

#### Implication:

Complete Network is the most stable network when shocks small attaining the lower bound

Complete Network is the least stable network when shocks large attaining the upper bound

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#### Are the two Phase Transitions Result the Same?

With short-term debt network

Complete network most stable when shocks small

Complete network least stable when shocks large, *only if F large* transmitting more liquidity risk

With long-term debt network

Complete network most stable when shocks small, *only if F large* allows more dilution

Complete network least stable when shocks large

#### Are the two Phase Transitions Result the Same?

With short-term debt network

Complete network most stable when shocks small

Complete network least stable when shocks large, *only if F large* transmitting more liquidity risk

With long-term debt network

Complete network most stable when shocks small, *only if F large* allows more dilution

Complete network least stable when shocks large

#### Conclusion

Short-term debt network

Transmits liquidity risk

Higher debt leads to more liquidation

Netting helps reduce risk transmission

Long-term debt networks

Facilitates dilution

Higher debt leads to less liquidation

Netting prevents good dilution

#### Conclusion

Short-term debt network

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