Direct and Indirect Sale of Information

Anat R. Admati and Paul Pleiderer, Econometrica 1990

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Why is information difficult to price?

The value of information depends on how other people processed it

Buyer valuation depends on the content of the information

Dynamism: If you gives away your info, how to make sure they pay

Externality: Information might leak through the equilibrium price.

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Model Setup

Investor demand

$$w = \frac{\mathbb{E}\left[F|\mathcal{I}\right] - R_f P}{\frac{1}{\rho} Var(F|\mathcal{I})}$$

Seller knows F and sells info directly or indirectly thru a fund

- Info cannot be resold: doesn't compete with himself
- Seller doesn't engage in the mkt: no incentive problem

Investor willing to pay (cf. Veldkamp Chapter 7)

$$\phi^v = \frac{\rho}{2} \log \frac{r^v}{r^\emptyset}$$

where $r^\emptyset = Var(F|P)^{-1}$ and $r^v = Var(F|\mathcal{I}^v)^{-1}$

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SBCT S sells the info to all w/o noises

Price is fully revealing $P = F$ (otherwise infinite demand)

Info has no value.

S gets zero profit.

NB: Argument hinges on competitive markets but results holds in Kyle

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Direct Sale-Photocopied

S adds a noise η with variance s_x and sells to λ_x, a fraction of traders.[1]

- The Informed Traders
  - have information $Y = F + \eta$
  - with precision $r(\lambda_x, s_x) = 1 + 1/s_x$
- The Uninformed Traders
  - observe (normalized) price $P = F + \eta - \frac{s_x}{\lambda_x \rho} Z$
  - with precision $r^{\emptyset}(\lambda_x, s_x) = 1 + \frac{1}{s_x + (s_x/\lambda_x \rho)^2 \sigma_Z^2}$

S's problem is

$$\max_{\lambda_x \in [0,1], s_x \geq 0} \lambda_x \rho \log \left(1 + \frac{1}{(\lambda_x \rho / \sigma_Z)^2 (1 + 1/s_x) + s_x}\right)$$

Trade-off 1: Selling to More Dilutes the Value of Info
Trade-off 2: Noises Reduces Responsiveness of Price and Value of Info

[1] The subscript x stands for a well known trade name in the photocopying industry.

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Step 1: For each λ_x

$$\min_{s_x \geq 0} (\lambda_x \rho / \sigma_Z)^2 (1 + 1/s_x) + s_x$$

$$s_x = \frac{\lambda_x \rho}{\sigma_Z}$$

Step 2: Write everything with s_x (instead of λ_x)

$$\max_{s_x \in [0, \rho/\sigma_Z]} s_x \log \left(1 + \frac{1}{s_x(s_x + 2)}\right)$$

$\exists s_x^* \geq 0$ the unique optimal interior solution

S sells to $\lambda_x^0 = \sigma_Z s_x^* / \rho$ with noise s_x^* if dilution strong $\frac{\rho}{\sigma_Z} \geq s_x^*$

S sells to all $\lambda_x^0 = 1$ with noise $s_x = \rho/\sigma_Z$ if dilution weak $\frac{\rho}{\sigma_Z} < s_x^*$

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Direct Sale-Personalized dominates Photocopied

Photocopied

$$P = F + \eta - \frac{s}{\lambda\rho} Z$$

$$r = 1 + \frac{1}{s}$$

$$r^{\emptyset} = 1 + \left(s + \frac{s^2 \sigma_Z^2}{\lambda^2 \rho^2} \right)^{-1}$$

$$\Pi = \frac{\rho}{2} \log \left(1 + \frac{1}{\frac{\lambda^2\rho^2}{s^2\sigma_Z^2}(1 + \frac{1}{s}) + s} \right)$$

Personalized

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Direct Sale-Personalized dominates Photocopied

Price is More Informative with Personalized Info: η cancels out

Individual Info More Valuable with Personalized Info. Why?

Individual Info Contains Info Orthogonal to Price

Selling to more doesn't affect Orthogonal Part

Dilution less pronounced as info sold to more traders

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S Prefers to Sell to All with same precision

Starting from an allocation in which not all precision r^v is the same

Instead, give everyone info with average precision $\bar{r}$

The price responsiveness g depends on $\bar{r}$ only (Difficult!)

$$\rho g \left(1 + \frac{1}{g^2 \sigma_Z^2} - \bar{r}\right) = 1$$

Symmetric precision is better by Jensen

$$\int \log\left(\frac{r^v}{r^\emptyset}\right) \mathrm{d}v \leq \log \int \left(\frac{r^v}{r^\emptyset}\right) \mathrm{d}v = \log \frac{\bar{r}}{r^\emptyset}$$

Implication: no one is uninformed

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What should be surprising but actually not

What remains? Find the optimal level of price responsiveness g (or $\bar{r}$)

Optimal Personalized noise $\bar{r} = \rho/\sigma_Z$ (recall Photocopied case)

Equilibrium Price $P = F + \frac{Z}{\sigma_Z}$

Optimal Profit $\Pi = \frac{\rho}{2} \log\left(1 + \frac{\sigma_Z}{2\rho}\right)$

What if we allow signals to be correlated?

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What is indirect sales of information? And why?

Google promises to not directly sell your privacy

but they can use your info to recommend relevant ads to you

Here, S operates a fund and commits to buy $cF + H(P)$ units of assets

The shares of fund is sold at δ per unit ex ante

Investor can trade on their own account as well

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Result 0: $H(\cdot)$ is 0

Suppose an investor holds x shares of the fund which buys $cF + H(P)$

and the investor buys $G(P)$ shares himself

such that $x(cF + H(P)) + G(P)$ is optimal

For whatever $\tilde{H}$, the investor could buy $\tilde{G}(P) = x(H(P) - \tilde{H}(P)) + G(P)$

Implication: WLOG $H(x) = 0$

NB: Fund's commitment to the purchasing rule is the key

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But it also destroy the value of the signal

Now S has a better way to reduces the info leakage

Increasing δ the price of the fund

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Result 0: S doesn't add noise

Why does S add noise when selling directly?

To reduce the informativeness of the price!

But it also destroy the value of the signal

Now S has a better way to reduces the info leakage

Increasing δ the price of the fund

Investor Direct Demand

Given price $P = aF - bZ$, investor direct demand is linear kP (Wishart!)

Note: investor knows a, b, since fund strategy is known

$$k = \rho + x + \frac{\rho a^2 - \rho a}{b^2 \sigma_Z^2}$$

Implication: high risk tolerant trades $\Longrightarrow$ trades more

Implication: high supply volatility $\Longrightarrow$ less info leakage $\Longrightarrow$ trades less

Implication: indirect demand and direct demand are complimentary (why)

More indirect investment $\Longrightarrow$ more info leaked

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Tedious calculation reveals

$$d(x, a/b) = \frac{\rho^2}{\rho(\rho + 2x) + \frac{a}{b}^2 \frac{\rho}{\sigma_Z}^2}$$

Market clearing $xF - kP = Z$ and $P = aF - bZ$ implies

$$a = x/k, b = 1/k \implies x = a/b$$

Hence aggregate demand function is

$$\delta(x) = d(x, x) = \frac{\rho^2 \sigma_Z^2}{\rho(\rho + 2x)\sigma_Z^2 + \rho^2 x^2}$$

Maximizing $x\delta(x)$ yields $x^* = \sigma_Z$, $\delta^* = \frac{1}{2(1+\sigma_Z/\rho)}$

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$$\Pi = \frac{\rho}{2} \log\left(1 + \frac{\sigma_Z}{2\rho}\right)$$

Maximum profit from indirect sale

$$x^*\delta^* = \frac{1}{2(1/\sigma_Z + 1/\rho)}$$

Can we compare them directly? Why

Consider a hypothetical fund who holds $F + \varepsilon$ where $Var(\varepsilon) = \rho/\sigma_Z$

$$D(x, \rho/\sigma_Z, \sigma_Z) = \frac{\rho\sigma_Z(\sigma_Z - x)}{2\rho\sigma_Z^2 + 2\sigma_Z^2 x - \sigma_Z x^2}$$

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Comparison of Direct and Indirect Sale ($\rho/\sigma_Z = 1$)

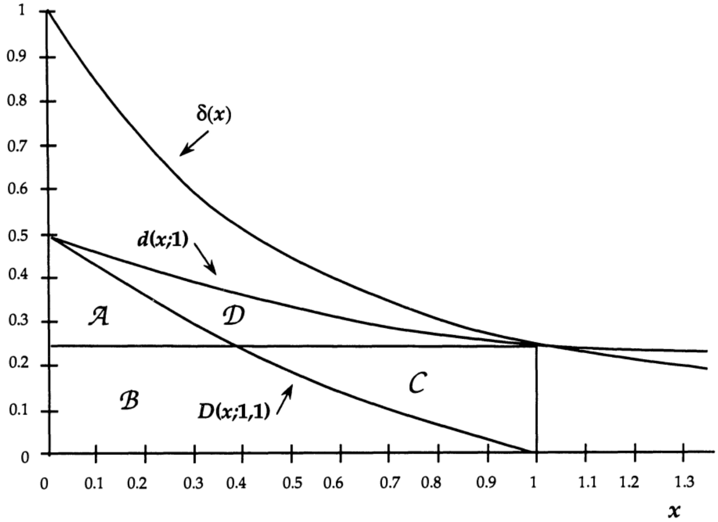


Figure:

Comparison of Direct and Indirect Sale($\rho/\sigma_Z \to \infty$)

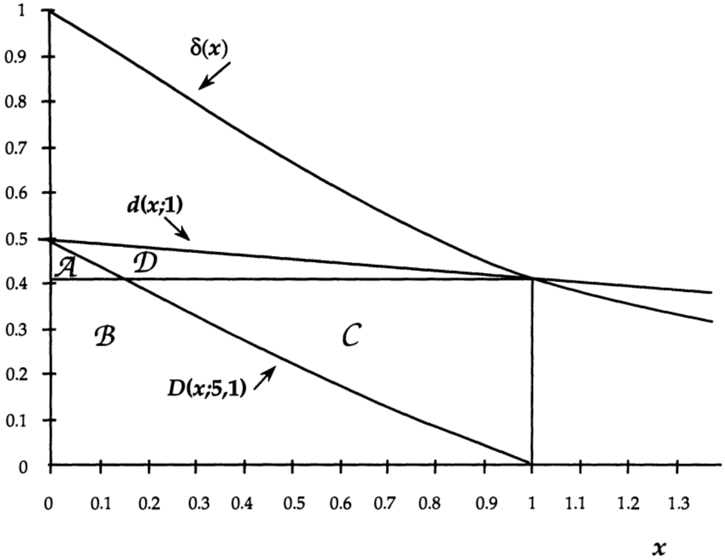
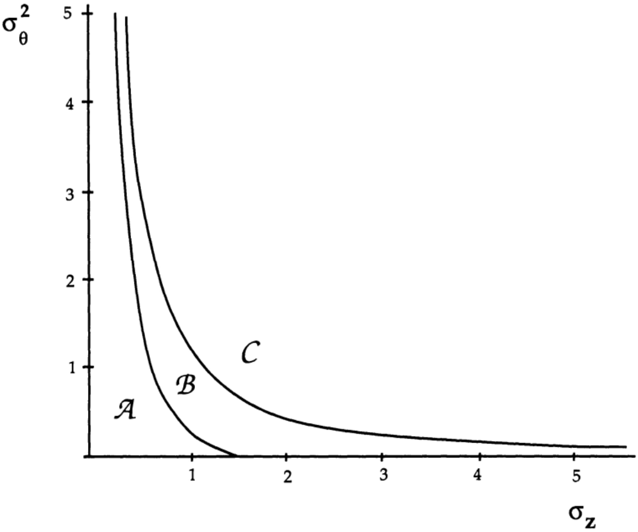


Figure:

Parameter Region



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How about selling both directly and indirectly?

It is never optimal to sell both way. Why?

If a fund exists, S can only sell info at a lower price.

Because the investor can buy the fund instead.

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Conclusion

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Noise is not added when selling indirectly

Selling directly better at extracting surplus

Selling indirectly better at reducing leakage

Selling indirectly when externality is high

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