Market Liquidity and Funding Liquidity

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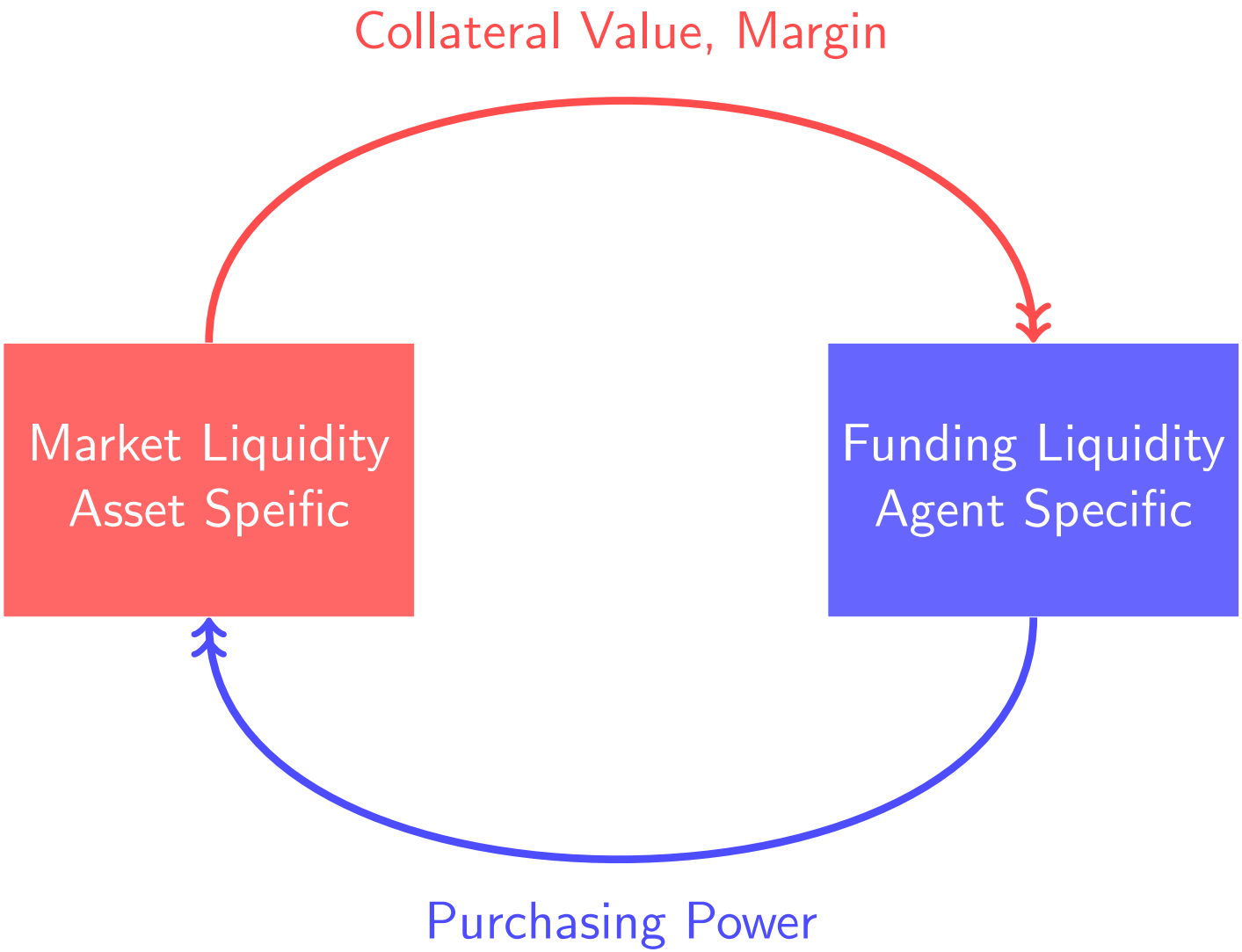
Liquidity

Market Liquidity
Asset Speific

Funding Liquidity
Agent Specific

Liquidity


Collateral Value, Margin
Market Liquidity
Asset Speific
Funding Liquidity
Agent Specific
Purchasing Power

Motivation

Stylized Facts

- Market Liquidity Dry-Ups
- Correlated with Volatility both in time series and cross-sectional
- Commonality within and across asset classes
- Flight to quality
- ...

Questions

- What is market liquidity? Why does it suddenly dry up?
- How does assets price and funding liquidity affects market liquidity?
- How does volatility affects market liquidity?
- Why is market liquidity correlated within and across asset classes
- What's the reason behind flight to quality?
- ...

Results Preview

- Risk-neutral Speculator trades the assets with highest profit/margin
- Margin set by informed financier stabilizes the market
- Margin set by *un*informed financier *de*stabilizes the market
- Switching from a high-liq/low-margin eqm to a low-liq/high-margin eqm leads to liquidity dry-up
- A *margin spiral* emerges if margins are increasing in illiquidity
- A *loss spiral* arise if speculators' position is negatively correlated with demand shock
- Market liquidity is positively correlated with funding liquidity
- Market liquidity is positively correlated across assets
- Market fragility is positively correlated across assets
- Risky assets requires higher margin, leading to illiquidity

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Model Setup

- **Economy** J risky assets, traded at $t = 0, 1, 2, 3$. Payoff realized at $t = 3$. The fundamental value $v_t^j = \mathbb{E}_t\left[v^j\right]$ has an ARCH structure

$$\Delta v_{t+1}^j = \sigma_{t+1}^j \varepsilon_{t+1}^j \text{ where } \sigma_{t+1}^j = \underline{\sigma}^j + \theta^j |\Delta v_t^j|$$

  We will drop subscripts and superscripts whenever no confusion arises.

- **Participants**
  - **Financier** sets margin to limit credit risk (π-VaR) $\pi = \Pr(|\Delta p| > m)$
  - **Speculator** risk-neutral, initial cash W_0 and $\Delta W_t = \Delta p_t' x_{t-1} + \eta_t$
  - **Three Customers** risk averse, with initial cash W_0 and known endowment shock z^k at date 3. They may arrive simultaneously at $t = 0$ or sequentially with proba. a.

Result 1: Market and Funding Liquidity

Result: market liquidity $|\Lambda| = |p - v|$ is bounded by the margin requirement m and the shadow cost of capital ϕ

$$|\Lambda| \leq (\phi - 1)m$$

Reason: risk neutral speculator invests in the asset with the highest profit per unit of margin

Result 1: Market and Funding Liquidity (step-by-step)

Demand of the speculator

1. If an asset has value v but the price is $p < v$
2. The speculator could pay the margin cost m and make a profit of $v - p$
3. The profit per dollar is $\frac{v-p}{m}$ for long position
4. The speculator trades only the most profitable security
5. So the shadow cost of capital is $\phi = 1 + \max \frac{v-p}{m}$
6. It links the market illiquidity $\Lambda = p - v$ to the funding cost ϕ thru

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Result 2: Margin Setting and Cushion Effect

Result:

- Informed financier sets lower long/short margin when the price is lower/higher than the value, stabilizing the market
- Uninformed financier sets higher margin when the price movement is large, destabilizing the market *when the fundamental shock is small*

Reason: Uninformed financier cannot distinguish fundamental shocks from liquidity/demand shocks

Result 2: Margin Setting and Cushion Effect (step-by-step)

The **Informed** Financier who knows v

1. sets the margin m so that the proba. price drop $-\Delta p$ exceeding m is π
2. Price drop $-\Delta p$ consists of value drop $-\Delta v$ and market illiquidity Λ

$$p_1 - p_2 = \underbrace{p_1 - v_1}_{\Lambda} + \underbrace{v_1 - v_2}_{-\Delta v} + \underbrace{v_2 - p_2}_{=0}$$

3. $\pi = \Pr(-\Delta p > m) = 1 - \Phi\left(\frac{m-\Lambda}{\sigma}\right)$ by normality of Δv
4. $m = \Phi^{-1}(1-\pi)\sigma + \Lambda = \bar{\sigma} + \bar{\theta}|\Delta v| + \Lambda$ for long by ARCH
5. Similarly for short margin $m = \bar{\sigma} + \bar{\theta}|\Delta v| - \Lambda$

- When $p < v$, $\Lambda < 0$, long margin is low, speculator buys more
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Eqm with Informed Financier

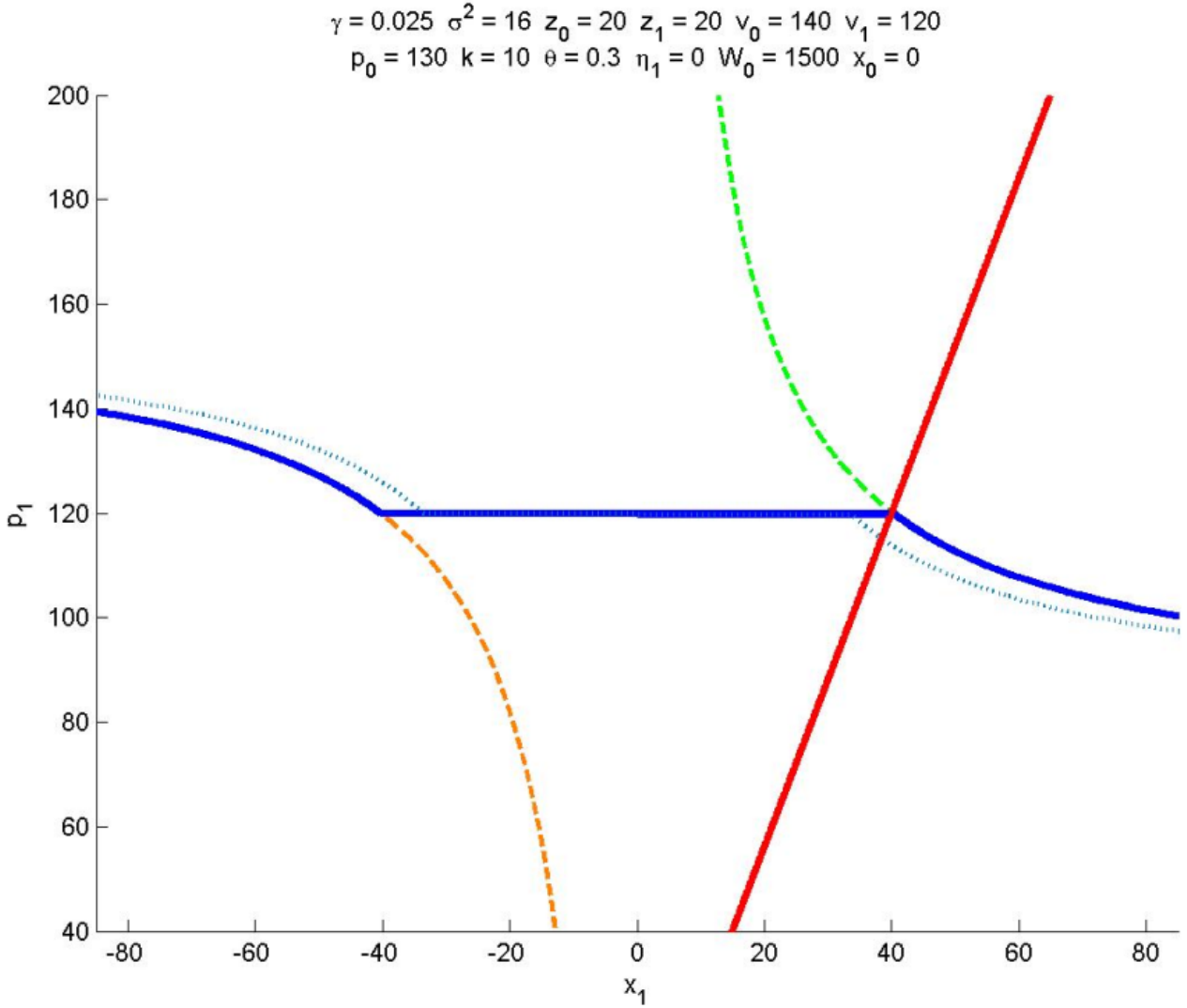

Figure:

Eqm with Informed Financier

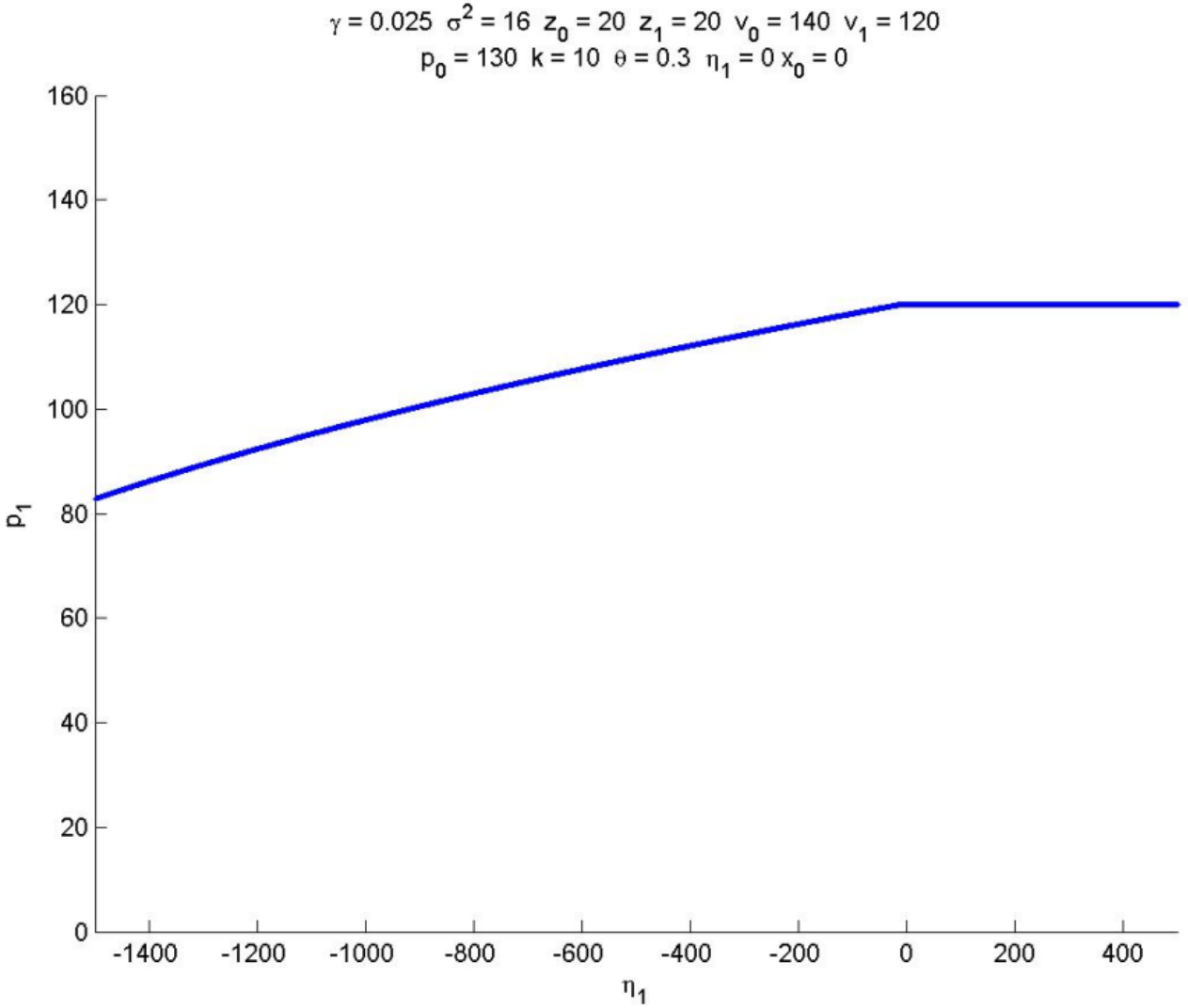

Figure:

Destabilizing Margin

The Uninformed Financier who doesn't know v

1. cannot distinguish the fundamental shock Δv from liquidity shock Λ
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3. and hence set the margin $m = \bar{\sigma} + \bar{\theta}|\underbrace{\Delta v + \Delta\Lambda}_{\Delta p}|$ for both directions

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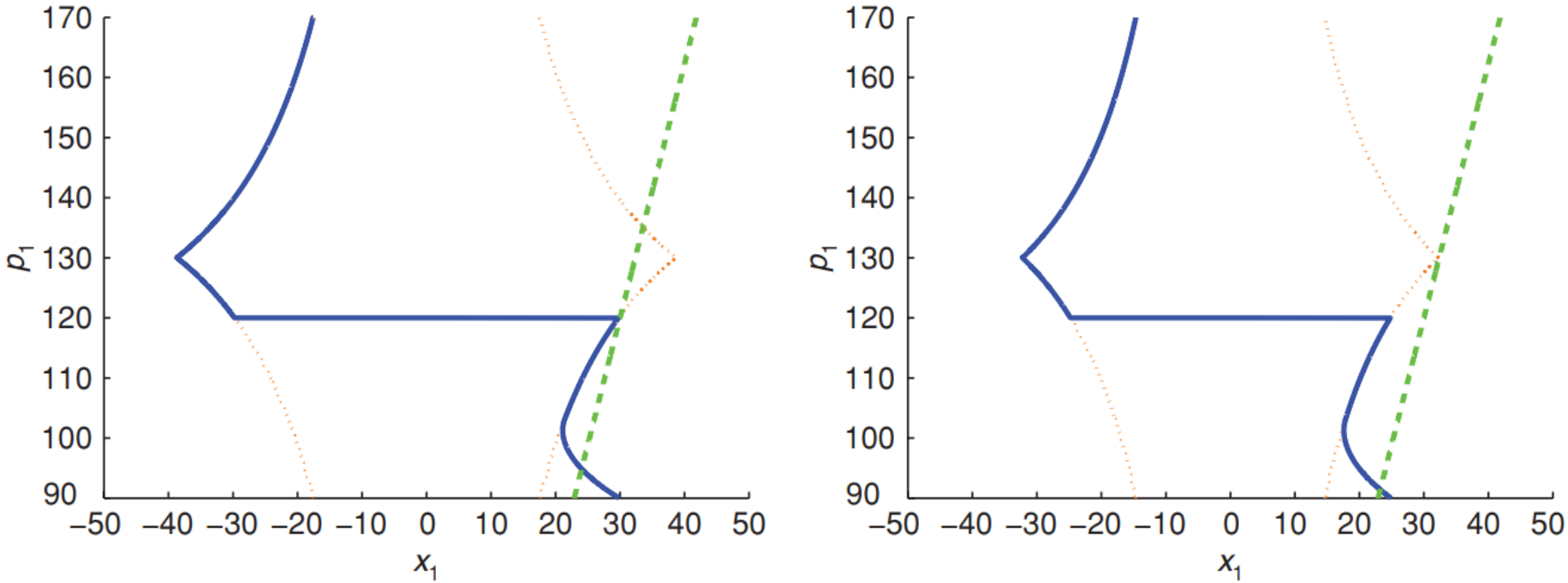
Result 3: Fragility

- Liquidity is *fragile* if eqm. price p not continuous in shocks η and Δv
- Fragility arise when excess demand $x + \sum y$ non-monotonic in price



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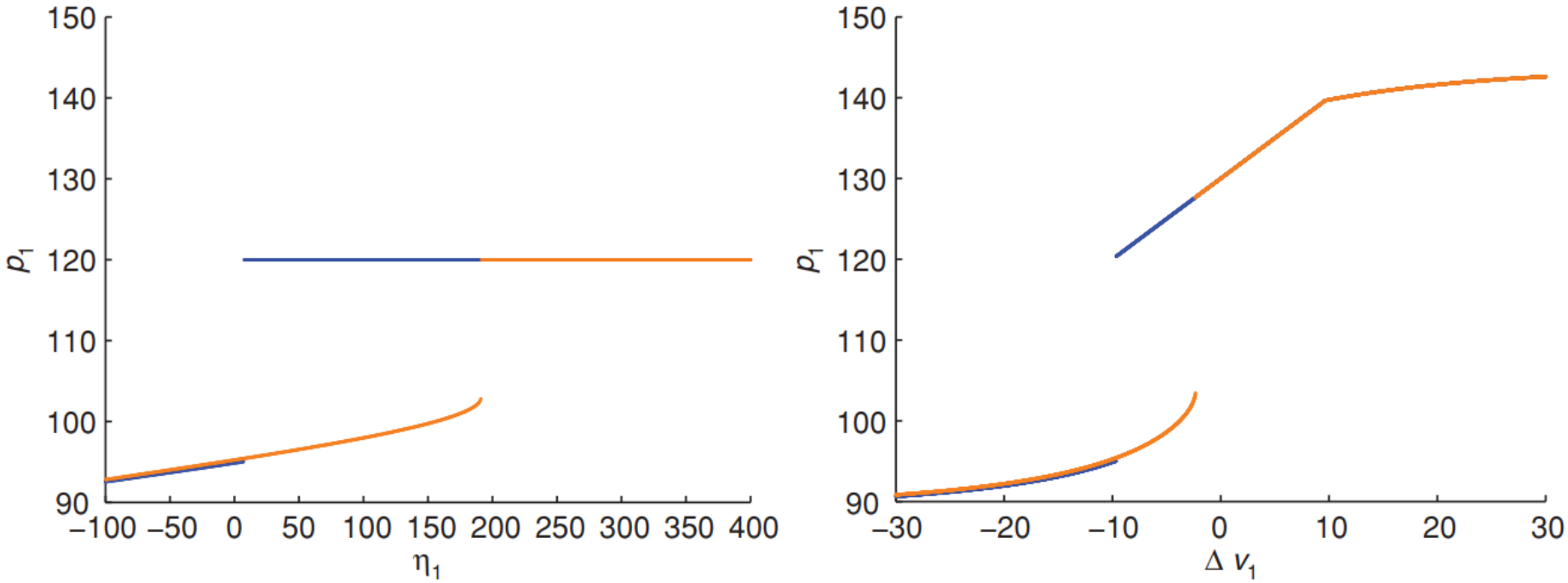

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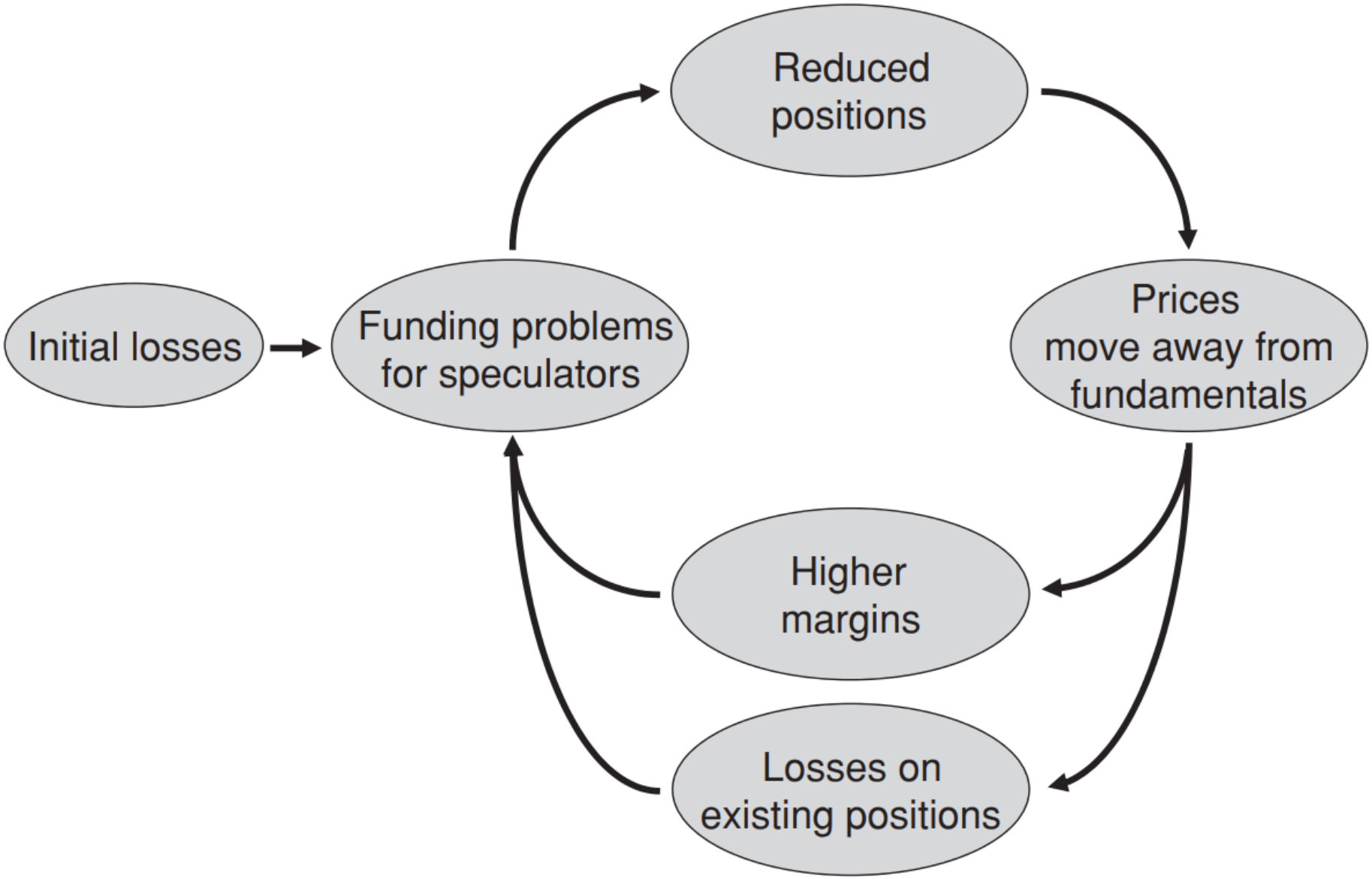
Liquidity Spirals



Result 4a:(Local) Commonality

$$|\Lambda| = \min\{\overbrace{(\phi - 1)m}^{\text{Speculator trades}}, \frac{\gamma^2}{2}Z\} \tag{1}$$

- Market illiquidity co-moves with funding illiquidity
- Market illiquidity co-moves across assets
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Result 4b: Flight to Quality

Result (Flight to Quality):
Risky securities become especially illiquid.
Locally, lower funda. vol. $\sigma^l < \sigma^k$ implies lower market illiq. $|\Lambda^l| \leq |\Lambda^k|$
Reason:

1. Margin is larger for volatile assets with uninformed financier
2. Constrained speculator trades assets with lower margin
3. Volatile assets become illiquid

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Testable Predictions

1. Margin depends on the total volatility and co-moves with illiquidity
2. Exogenous speculator capital shock reduces market liquidity
3. The effect of funding liquidity on market liquidity is non-linear
   - Small when far from being constrained
   - Large when close to the constraint
   - Liquidity can even suddenly jump
4. Speculator capital tightness drives co-movement of market illiquidity
   - Sharp liquidity reductions occur simultaneously
5. Sensitivity of margins and market liq. is large for risk and illiq. asset
   - Spiral effect is stronger for illiquid asset
6. Speculator return is negatively skewed
   - Security prices have conditional skewness and unconditional kurtosis

Related Empirical Research

- Adrian, Etula, Muir 2014 JF:
  - FI leverage and household leverage moves in opposite directions
  - increases in FI leverage are good news
  - leverage shocks carry positive risk price.
- He, Kelly Manela 2017 JFE:
  - shocks to the equity capital ratio of FIs explains cross-sectional variation in expected returns
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Literature Review

- **Limits to Arbitrage** DeLong et al. 1990, Shleifer and Vishny 1997, Grossman and Vila 1992, Liu and Longstaff 2004, Chowdhry and Vayanos 2002, Abreu and Brunnermeier 2002
- **Market Microstructure** Stoll 1978, Ho and Stoll 1981, 1983, Kyle 1985, Glosten and Milgrom 1985, Grossman and Miller 1988
- **Banking** Bryant 1980, Diamond and Dybvig 1983, Allen and Gale 1998, 2004, 2005, 2007, Holmstrom and Tirole 1998, 2001.
- **Collateral Constraint** Aiyagari and Gertler 1999; Bernanke and Gertler 1989; Fisher 1933; Kiyotaki and Moore 1997; Lustig and Chien 2005, Geanakoplos 1997, 2003
- **Constrained Traders** Attari, Mello, and Ruckes 2005; Bernardo and Welch 2004; Brunnermeier and Pedersen 2005; Eisfeldt 2004; Morris and Shin 2004; Weill 2007

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Result 5: Liquidity Risk at $t = 0$

- Funding liquidity risk matters even before margin requirements bind. What about limited liability?
- Speculator's First Order Condition $\mathbb{E}_0\left[\phi_1(p_1^j - p_o^j)\right] = 0 \implies$ Pricing kernel $\frac{\phi_1}{\mathbb{E}_0[\phi_1]}$ depends on future funding liquidity ϕ_1
- Date 1 price p_1 is *conditionally* skewed due to funding constraint
- Date 0 margin m_0 can be positively correlated with Λ_0 even with an *informed* financier