SM5083 - Basics of Programming

Assignment 1

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CE20RESCH13001

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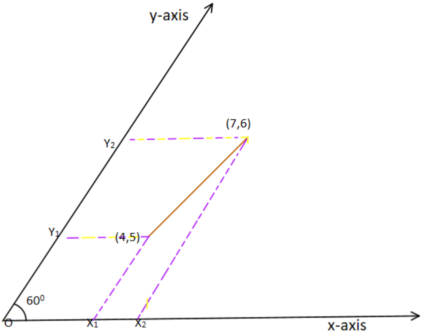
Problem II (2i)

Find the distance between points $\begin{pmatrix} 7 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ with the axes inclined at 60°



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Solution

It can be solved by transforming the axes into rectangular coordinates as shown in figure

Let the transformed coordinates of $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ & $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ be $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ & $\begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$ respectively.

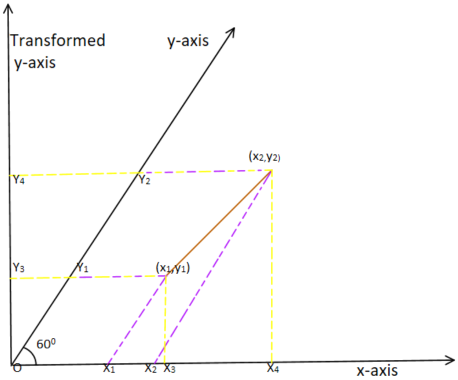

Figure: Points defined on angular and rectangular axes

From the figure,

$x_3 = OX_1 + X_1X_3 = x_1 + y_1 \cos 60^\circ$

$y_3 = OY_1 \cos 30^\circ = y_1 \cos 30^\circ$

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & \cos 60^\circ \\ 0 & \cos 30^\circ \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{1.0.1}$$

Similarly,

$x_4 = OX_2 + X_2X_4 = x_2 + y_2 \cos 60^\circ$

$y_4 = OY_2 \cos 30^\circ = y_2 \cos 30^\circ$

$$\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & \cos 60^\circ \\ 0 & \cos 30^\circ \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \tag{1.0.2}$$

The generalised equation for transformed coordinates $\begin{pmatrix} x_t \\ y_t \end{pmatrix}$ when the angle between axes 'θ' is,

$$\boxed{\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 & \cos(\theta) \\ 0 & \sin(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}} \tag{1.0.3}$$

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Let the transformed point be X_t, T be the transformation matrix and the point in angular axes be X, (1.0.3) can be written as

$$X_t = T\,X \tag{1.0.4}$$

Solving (1.0.1) & (1.0.2) the transformed coordinates are,

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix}; \begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} \tag{1.0.5}$$

The distance between points is a norm of the distance vector,

$$d = \|X_{t1} - X_{t2}\| \tag{1.0.6}$$

$$d = \left\| T(X_1 - X_2) \right\| \tag{1.0.7}$$

$$d = (X_1 - X_2)^\top\ T^\top\ T\,(X_1 - X_2) = \sqrt{13} \tag{1.0.8}$$

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Conclusion

1. From the above solution, it can be concluded that the distance between points remains constant irrespective of coordinate system.
2. It can also be concluded that the position vector of the points change when the axes is transformed.

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