Assignment 1

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CE20RESCH13001

Problem II (2I)

Find the distance between points (7,6) and (4,5) with the axes at 60°

1 Solution

Let the points be P_1 (7,6) and P_2 (4,5) and also the angle between axes is 60°

$$\mathbf{P_1} = \begin{pmatrix}7\\6\end{pmatrix} \; ; \; \mathbf{P_2} = \begin{pmatrix}4\\5\end{pmatrix} \tag{1.0.1}$$

The problem can be solved by transformation of the given coordinate system to the rectangular coordinate system.
In order to convert to rectangular coordinate system, the y-axis should be rotated by 30° in anti-clockwise and x-axis will remain unaltered.
The transformation matrix should be computed for transforming the given $\mathbf{P_1}, \mathbf{P_2}$ into the rectangular coordinate system.
The angle between the transformed x-axis and given x-axis be $\theta_{11} = 0°$
The angle between the transformed x-axis and given y-axis be $\theta_{12} = 60°$
Likewise, $\theta_{21} = 90° \; ; \; \theta_{22} = 30°$
Transformed matrix $\mathbf{T}$ will be the cosines of the above the angles

$$\mathbf{T} = \begin{pmatrix}\cos(\theta_{11}) & \cos(\theta_{12})\\ \cos(\theta_{21}) & \cos(\theta_{22})\end{pmatrix} = \begin{pmatrix}\cos(0°) & \cos(60°)\\ \cos(90°) & \cos(30°)\end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix}1 & \frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2}\end{pmatrix} \tag{1.0.2}$$

From equations (1.0.1) and (1.0.2), the transformed vector corresponding to $\mathbf{P_1}$ be $\mathbf{P_{1T}} = (\mathbf{T})\ (\mathbf{P_1})$
Transformed vector corresponding to $\mathbf{P_2}$ be $\mathbf{P_{2T}} = (\mathbf{T})\ (\mathbf{P_2})$

$$\mathbf{P_{1T}} = \begin{pmatrix}1 & \frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2}\end{pmatrix}\begin{pmatrix}7\\6\end{pmatrix}; \mathbf{P_{2T}} = \begin{pmatrix}1 & \frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2}\end{pmatrix}\begin{pmatrix}4\\5\end{pmatrix}$$

$$\implies \mathbf{P_{1T}} = \begin{pmatrix}10\\3\sqrt{3}\end{pmatrix}; \mathbf{P_{2T}} = \begin{pmatrix}\frac{13}{2}\\ \frac{5\sqrt{3}}{2}\end{pmatrix}$$

Now, obtained points are in the rectangular coordinate system and the distance vector between points will be

$$\mathbf{P_{12T}} = \mathbf{P_{1T}} - \mathbf{P_{2T}} = \begin{pmatrix}10\\3\sqrt{3}\end{pmatrix} - \begin{pmatrix}\frac{13}{2}\\ \frac{5\sqrt{3}}{2}\end{pmatrix} = \begin{pmatrix}\frac{7}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix}$$

and the magnitude will be $\|\mathbf{P_{1T}} - \mathbf{P_{2T}}\|$
Therefore, the distance between the points is equal to $\sqrt{13}$ *units*

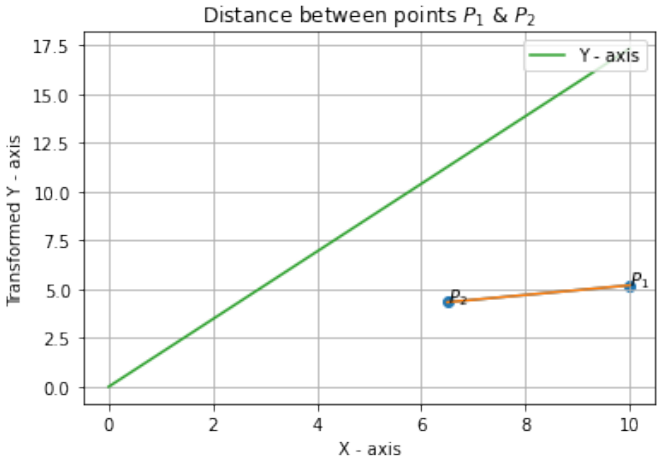

Fig: Distance between two points

*Python code file

https://github.com/Prithvi−Sangani/SM5083_Assignment1/blob/main/Assignment1.ipynb