Assignment 1

S Prithvi
CE20RESCH13001

Problem II (2I)

Find the distance between points (7,6) and (4,5) with the axes at 60°

1 Solution

Let the points be P_1 (7,6) and P_2 (4,5) and also the angle between axes is 60°

$$\mathbf{P_1} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} ; \mathbf{P_2} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \quad (1.0.1)$$

The problem can be solved by transformation of the given coordinate system to the rectangular coordinate system.
In order to convert to rectangular coordinate system, the y-axis should be rotated by 30° in anti-clockwise and x-axis will remain unaltered.
Let the coordinates of points P_1 and P_2 on x-axis and y-axis of angular axes be (x_1,y_1) & (x_2,y_2) respectively.
Let coordinates of the points P_1 & P_2 on rectangular axes be (x_3,y_3) & (x_4,y_4) respectively.
From the Fig1, $\angle P_1X_1X_3 = \angle P_2X_2X_4 = 60°$ and $\angle Y_1OY_3 = \angle Y_2OY_4 = 30°$.
$x_3 = OX_1 + X_1X_3 = x_1 + y_1 \cos 60°$
$y_3 = OY_1 \cos 30° = y_1 \cos 30°$
In matrix notation, we can write the above equation as,

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & \cos 60° \\ 0 & \cos 30° \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix} \quad (1.0.2)$$

Similarly for the point P_2, we have
$x_4 = OX_2 + X_2X_4 = x_2 + y_2 \cos 60°$
$y_4 = OY_2 \cos 30° = y_2 \cos 30°$
The matrix notation of the above equations is,

$$\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & \cos 60° \\ 0 & \cos 30° \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} \quad (1.0.3)$$

From the equations (1.0.2) & (1.0.3), the transformed coordinates of points P_1,P_2 in vectorial representation are,

$$\mathbf{P_1} = \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix}; \mathbf{P_2} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix}$$

Now, obtained points are in the rectangular coordinate system and the distance vector between points will be

$$\mathbf{P_{12}} = \mathbf{P_2} - \mathbf{P_1} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} - \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{7}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$$

and the magnitude will be $\|\mathbf{P_2} - \mathbf{P_1}\|$
Therefore, the distance between the points is equal to $\sqrt{13}$ *units*

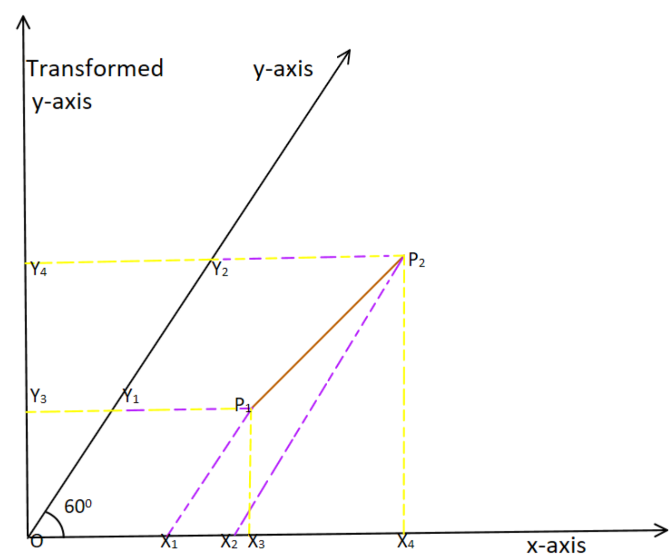

Fig1: Points defined on angular & rectangular axes

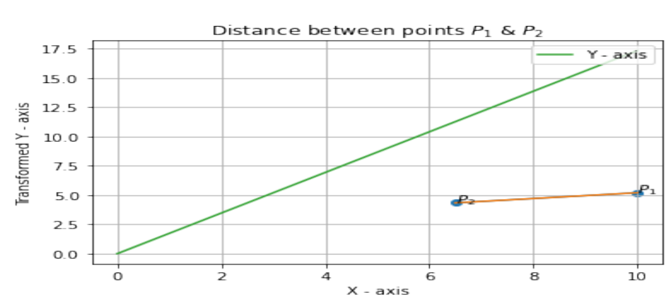

Fig2: Points plotted in Python