Assignment 1

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CE20RESCH13001

PROBLEM II (2I)

Find the distance between points $\begin{pmatrix}7\\6\end{pmatrix}$ and $\begin{pmatrix}4\\5\end{pmatrix}$ with the axes at $60°$

1 SOLUTION

Let the points be

$$\begin{pmatrix}x_1\\y_1\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix} ; \begin{pmatrix}x_2\\y_2\end{pmatrix} = \begin{pmatrix}7\\6\end{pmatrix} \qquad (1.0.1)$$

The problem can be solved by transformation of the given coordinate system to the rectangular coordinate system.

In order to convert to rectangular coordinate system, the y-axis should be rotated by $30°$ in anti-clockwise and x-axis will remain unaltered.

Transformed coordinates of $\begin{pmatrix}x_1\\y_1\end{pmatrix}$ & $\begin{pmatrix}x_2\\y_2\end{pmatrix}$ be $\begin{pmatrix}x_3\\y_3\end{pmatrix}$ & $\begin{pmatrix}x_4\\y_4\end{pmatrix}$ respectively.

$x_3 = OX_1 + X_1X_3 = x_1 + y_1\cos 60°$
$y_3 = OY_1\cos 30° = y_1\cos 30°$

$$\begin{pmatrix}x_3\\y_3\end{pmatrix} = \begin{pmatrix}1 & \cos 60°\\0 & \cos 30°\end{pmatrix}\begin{pmatrix}x_1\\y_1\end{pmatrix} \qquad (1.0.2)$$

Similarly,
$x_4 = OX_2 + X_2X_4 = x_2 + y_2\cos 60°$
$y_4 = OY_2\cos 30° = y_2\cos 30°$

$$\begin{pmatrix}x_4\\y_4\end{pmatrix} = \begin{pmatrix}1 & \cos 60°\\0 & \cos 30°\end{pmatrix}\begin{pmatrix}x_2\\y_2\end{pmatrix} \qquad (1.0.3)$$

Substituting (1.0.1) in (1.0.2) & (1.0.3)

$$\begin{pmatrix}x_3\\y_3\end{pmatrix} = \begin{pmatrix}\frac{13}{2}\\\frac{5\sqrt{3}}{2}\end{pmatrix}; \begin{pmatrix}x_4\\y_4\end{pmatrix} = \begin{pmatrix}10\\3\sqrt{3}\end{pmatrix} \qquad (1.0.4)$$

Obtained points are in the rectangular coordinate system and the distance between points is

$$\left\|\begin{pmatrix}x_3\\y_3\end{pmatrix} - \begin{pmatrix}x_4\\y_4\end{pmatrix}\right\| = \left\|\begin{pmatrix}10\\3\sqrt{3}\end{pmatrix} - \begin{pmatrix}\frac{13}{2}\\\frac{5\sqrt{3}}{2}\end{pmatrix}\right\| = \sqrt{13}\ units$$
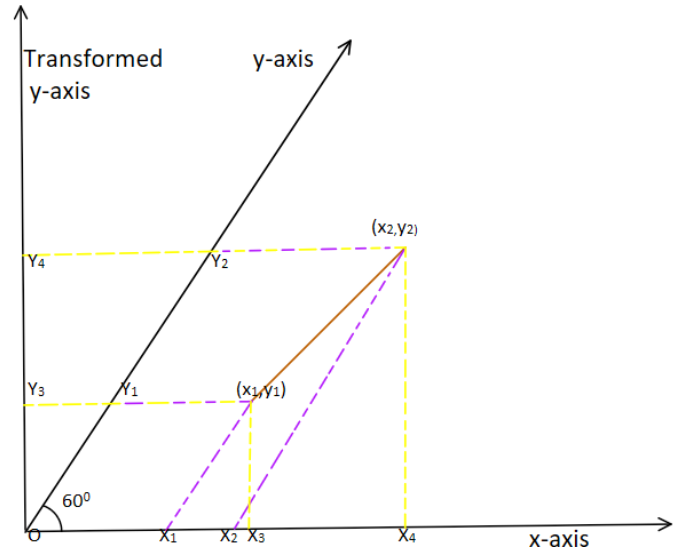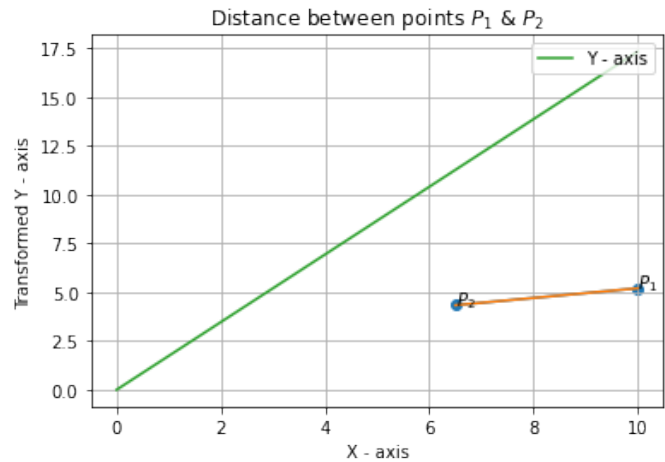

Fig1: Points defined on angular & rectangular axes

Fig2: Points plotted in Python