Assignment 1

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CE20RESCH13001

Problem II (2I)

Find the distance between points (7,6) and (4,5) with the axes at 60°

1 Solution

Let the points be P_1 (7,6) and P_2 (4,5) and also the angle between axes is 60°

$$\mathbf{P_1} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \; ; \; \mathbf{P_2} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \tag{1.0.1}$$

The problem can be solved by transformation of the given coordinate system to the rectangular coordinate system.

In order to convert to rectangular coordinate system, the y-axis should be rotated by 30° in anti-clockwise and x-axis will remain unaltered.

The transformation matrix should be computed for transforming the given $\mathbf{P_1}, \mathbf{P_2}$ into the rectangular coordinate system.

The angle between the transformed x-axis and given x-axis be $\theta_{11} = 0°$

The angle between the transformed x-axis and given y-axis be $\theta_{12} = 60°$

Likewise, $\theta_{21} = 90°$; $\theta_{22} = 30°$

Transformed matrix $\mathbf{T}$ will be the cosines of the above the angles

$$\mathbf{T} = \begin{pmatrix} \cos(\theta_{11}) & \cos(\theta_{12}) \\ \cos(\theta_{21}) & \cos(\theta_{22}) \end{pmatrix}$$
$$= \begin{pmatrix} \cos(0°) & \cos(60°) \\ \cos(90°) & \cos(30°) \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \tag{1.0.2}$$

From equations (1.0.1) and (1.0.2), the transformed vector corresponding to $\mathbf{P_1}$ be $\mathbf{P_{1T}} = (\mathbf{T})\ (\mathbf{P_1})$

Transformed vector corresponding to $\mathbf{P_2}$ be $\mathbf{P_{2T}} = (\mathbf{T})\ (\mathbf{P_2})$

$$\mathbf{P_{1T}} = \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 7 \\ 6 \end{pmatrix} ; \mathbf{P_{2T}} = \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$\implies \mathbf{P_{1T}} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} ; \mathbf{P_{2T}} = \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix}$$

Now, obtained points are in the rectangular coordinate system and the distance vector between points will be

$$\mathbf{P_{12T}} = \mathbf{P_{1T}} - \mathbf{P_{2T}} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} - \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{7}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$$

and the magnitude will be $\|\mathbf{P_{1T}} - \mathbf{P_{2T}}\|$

Therefore, the distance between the points is equal to $\sqrt{13}$ *units*

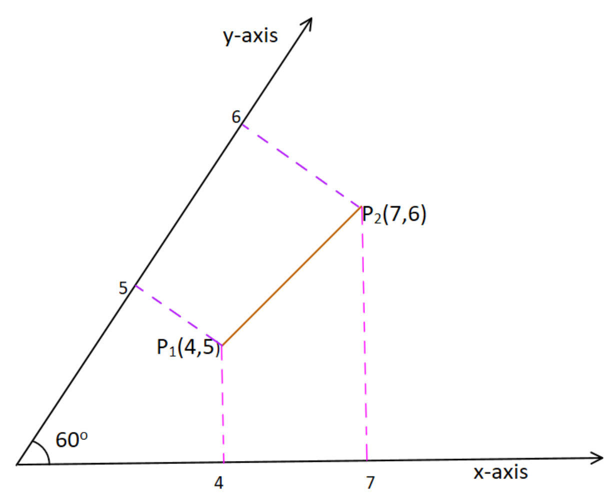

Fig1: Points defined on the angular axis

The above figure shown is not to the scale.

The blue dotted lines indicates the perpendicular bisector to the xy axis.

The same points are now transformed as explained in the solution and plotted using python

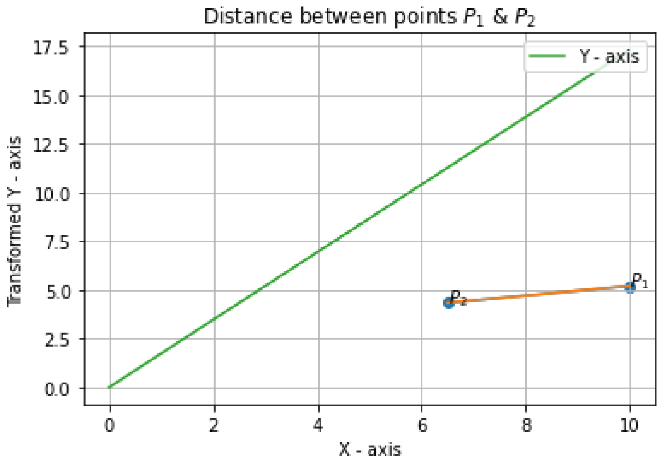

Fig2: Points plotted in Python