Assignment 1

S Prithvi CE20RESCH13001

PROBLEM II (2I)

Find the distance between points (7,6) and (4,5)with the axes at 60°

1 Solution

Let the points be P_1 (7,6) and P_2 (4,5) and also the angle between axes is 60°

$$\mathbf{P_1} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \; ; \; \mathbf{P_2} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \tag{1.0.1}$$

The problem can be solved by transformation of the given coordinate system to the rectangular coordinate system.

In order to convert to rectangular coordinate system, the y-axis should be rotated by 30° in anti-clockwise and x-axis will remain unaltered.

Let the projections of points P_1 and P_2 on x-axis and y-axis of angular axes be X_1, X_2 and Y_1, Y_2 respectively.

Let x&y ordinates of the points P_1 & P_2 on rectangular axes be $X_3, X_4 \& Y_3, Y_4$ respectively.

From the Fig1, $\angle P_1X_1X_3 = \angle P_2X_2X_4 = 60^\circ$ and $\angle Y_1 O Y_3 = \angle Y_2 O Y_4 = 30^{\circ}.$

$$OX_3 = OX_1 + X_1X_3 = 4 + X_1P_1\cos 60^\circ = 4 + 5(\frac{1}{2}) = \frac{13}{2}$$

$$OX_4 = OX_2 + X_2X_4 = 7 + X_2P_2\cos 60^\circ = 7 + 6(\frac{1}{2}) = 10$$

$$OY_3 = OY_1\cos 30^\circ = \frac{5\sqrt{3}}{2}$$

$$OY_4 = OY_2 \cos 30^\circ = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$$

 $OY_4 = OY_2 \cos 30^\circ = \frac{6\sqrt[2]{3}}{2} = 3\sqrt{3}$ The transformed coordinates of points P_1, P_2 are as follows,

$$\mathbf{P_1} = \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix}; \mathbf{P_2} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix}$$

Now, obtained points are in the rectangular coordinate system and the distance vector between points will be

$$\mathbf{P_{12}} = \mathbf{P_2} - \mathbf{P_1} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} - \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{7}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$$
and the magnitude will be $\|\mathbf{P_2} - \mathbf{P_1}\|$

Therefore, the distance between the points is equal to $\sqrt{13}$ units

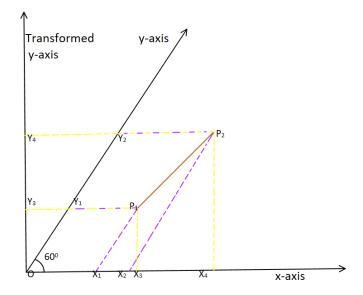


Fig1: Points defined on angular & rectangular axes

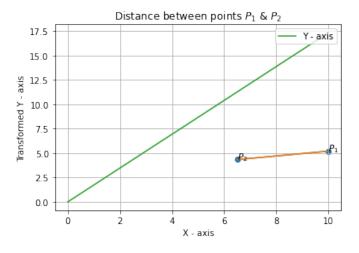


Fig2: Points plotted in Python