Assignment 1

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CE20RESCH13001

1 Problem II (2I)

Find the distance between points $\begin{pmatrix}7\\6\end{pmatrix}$ and $\begin{pmatrix}4\\5\end{pmatrix}$ with the axes at 60°

1.1 Solution

Let the points be

$$\begin{pmatrix}x_1\\y_1\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix} ; \begin{pmatrix}x_2\\y_2\end{pmatrix} = \begin{pmatrix}7\\6\end{pmatrix} \tag{1.1.1}$$

In order to convert to rectangular coordinate system, the y-axis should be rotated by 30° in anti-clockwise. Transformed coordinates of $\begin{pmatrix}x_1\\y_1\end{pmatrix}$ & $\begin{pmatrix}x_2\\y_2\end{pmatrix}$ be $\begin{pmatrix}x_3\\y_3\end{pmatrix}$ & $\begin{pmatrix}x_4\\y_4\end{pmatrix}$ respectively.

$x_3 = OX_1 + X_1X_3 = x_1 + y_1\cos 60°$
$y_3 = OY_1\cos 30° = y_1\cos 30°$

$$\begin{pmatrix}x_3\\y_3\end{pmatrix} = \begin{pmatrix}1 & \cos 60°\\0 & \cos 30°\end{pmatrix}\begin{pmatrix}x_1\\y_1\end{pmatrix} \tag{1.1.2}$$

Similarly,
$x_4 = OX_2 + X_2X_4 = x_2 + y_2\cos 60°$
$y_4 = OY_2\cos 30° = y_2\cos 30°$

$$\begin{pmatrix}x_4\\y_4\end{pmatrix} = \begin{pmatrix}1 & \cos 60°\\0 & \cos 30°\end{pmatrix}\begin{pmatrix}x_2\\y_2\end{pmatrix} \tag{1.1.3}$$

The generalised equation for transformed coordinates $\begin{pmatrix}x_t\\y_t\end{pmatrix}$ when the angle between axes 'θ is,

$$\boxed{\begin{pmatrix}x_t\\y_t\end{pmatrix} = \begin{pmatrix}1 & \cos(\theta)\\0 & \sin(\theta)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}} \tag{1.1.4}$$

Let the transformed point be $\mathbf{X_t}$, $\mathbf{T}$ be the transformation matrix and the point in angular axes be $\mathbf{X}$, (1.1.4) can be written as

$$\mathbf{X_t} = \mathbf{T}\ \mathbf{X} \tag{1.1.5}$$

Substituting (1.1.1) in (1.1.2) & (1.1.3)

$$\begin{pmatrix}x_3\\y_3\end{pmatrix} = \begin{pmatrix}\frac{13}{2}\\\frac{5\sqrt{3}}{2}\end{pmatrix}; \begin{pmatrix}x_4\\y_4\end{pmatrix} = \begin{pmatrix}10\\3\sqrt{3}\end{pmatrix} \tag{1.1.6}$$

The distance between points is a norm of the distance vector,

$$d = \|\mathbf{X_{t1}} - \mathbf{X_{t2}}\| \tag{1.1.7}$$

Substituting (1.1.5) in (1.1.7),

$$d = \|\mathbf{TX_1} - \mathbf{TX_2}\| \tag{1.1.8}$$

$$d = \|\mathbf{T}(\mathbf{X_1} - \mathbf{X_2})\| \tag{1.1.9}$$

$$d = (\mathbf{X_1} - \mathbf{X_2})^\top\ \mathbf{T}^\top\ \mathbf{T}\ (\mathbf{X_1} - \mathbf{X_2}) \tag{1.1.10}$$

$$d = \sqrt{13}\ units \tag{1.1.11}$$

Above results shows that the distance remains constant between the points irrespective of coordinate system and by (1.1.1) & (1.1.6) only the position vector of the point changes with the transformation of coordinate system

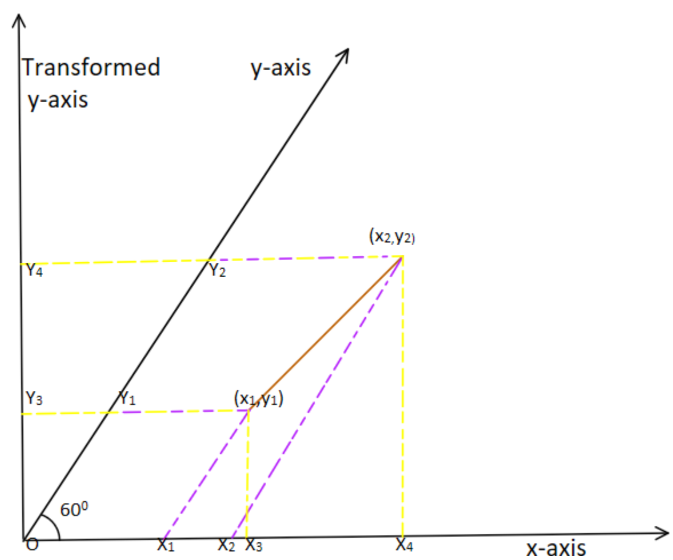

Fig. 1.1: Points defined on angular & rectangular axes

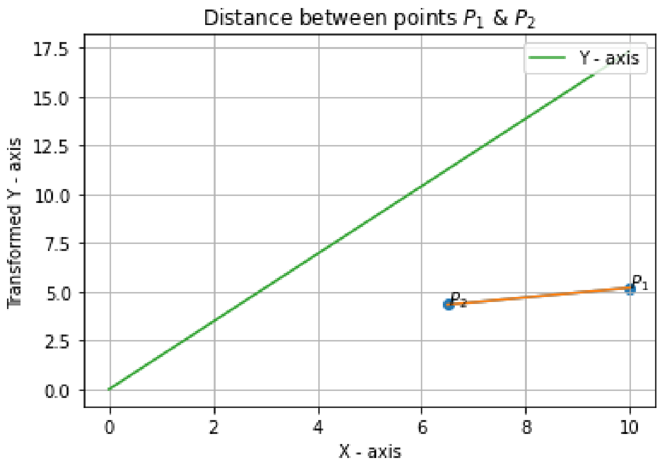

Fig. 1.2: Points plotted in Python