

Assignment 1

S Prithvi
CE20RESCH13001

PROBLEM II (2I)

Find the distance between points $\begin{pmatrix} 7 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ with the axes at 60°

1 SOLUTION

Let the points be

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}; \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \quad (1.0.1)$$

In order to convert to rectangular coordinate system, the y-axis should be rotated by 30° in anti-clockwise. Transformed coordinates of $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ & $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

be $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ & $\begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$ respectively.

$$x_3 = OX_1 + X_1X_3 = x_1 + y_1 \cos 60^\circ$$

$$y_3 = OY_1 \cos 30^\circ = y_1 \cos 30^\circ$$

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & \cos 60^\circ \\ 0 & \cos 30^\circ \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad (1.0.2)$$

Similarly,

$$x_4 = OX_2 + X_2X_4 = x_2 + y_2 \cos 60^\circ$$

$$y_4 = OY_2 \cos 30^\circ = y_2 \cos 30^\circ$$

$$\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & \cos 60^\circ \\ 0 & \cos 30^\circ \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad (1.0.3)$$

The generalised equation for transformed coordinates $\begin{pmatrix} x_t \\ y_t \end{pmatrix}$ when the angle between axes ' θ ' is,

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 & \cos(\theta) \\ 0 & \sin(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.0.4)$$

Substituting (1.0.1) in (1.0.2) & (1.0.3)

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix}; \begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} \quad (1.0.5)$$

The distance between points is a norm of the

distance vector,

$$\left\| \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} - \begin{pmatrix} x_4 \\ y_4 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} - \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix} \right\| = \sqrt{13} \text{ units}$$

The distance, d can be measured in angular axes directly by following equation,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + 2(x_2 - x_1)(y_2 - y_1) \cos \theta}$$

$$d = \sqrt{(7 - 6)^2 + (6 - 5)^2 + 2(7 - 6)(6 - 5) \cos 60^\circ}$$

$$d = \sqrt{13} \text{ units} \quad (1.0.6)$$

Above results shows that the distance remains constant between the points irrespective of coordinate system and by (1.0.1) & (1.0.5) only the position vector of the point changes with the transformation of coordinate system

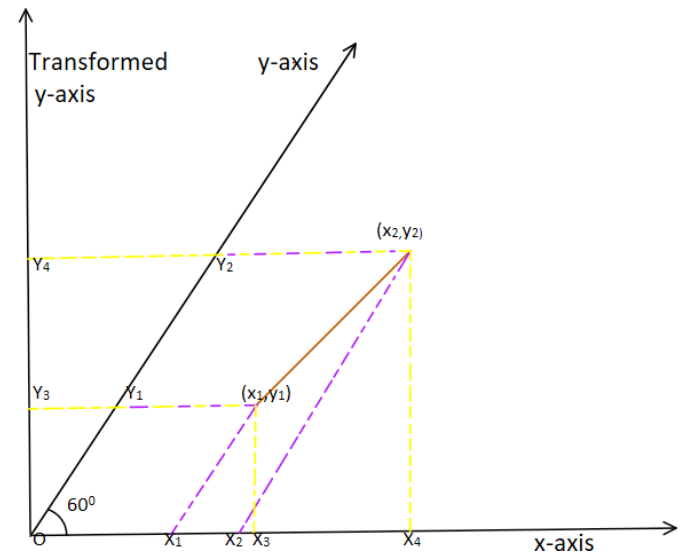


Fig1: Points defined on angular & rectangular axes

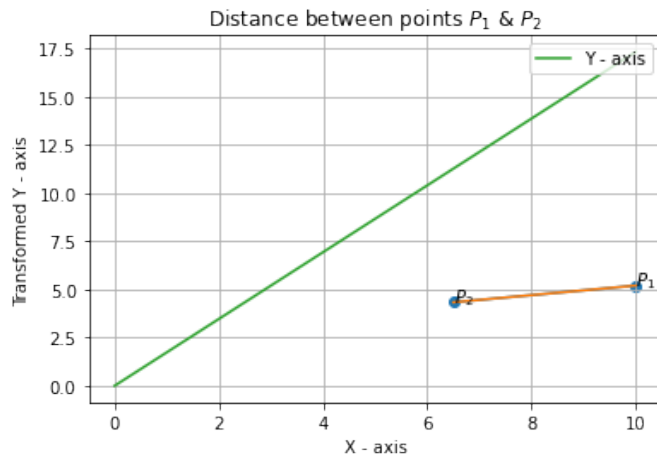


Fig2: Points plotted in Python