Assignment 1

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CE20RESCH13001

Problem II (2I)

Find the distance between points $\begin{pmatrix}7\\6\end{pmatrix}$ and $\begin{pmatrix}4\\5\end{pmatrix}$ with the axes at $60°$

1 Solution

Let the points be

$$\begin{pmatrix}x_1\\y_1\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix} ; \begin{pmatrix}x_2\\y_2\end{pmatrix} = \begin{pmatrix}7\\6\end{pmatrix} \tag{1.0.1}$$

In order to convert to rectangular coordinate system, the y-axis should be rotated by $30°$ in anti-clockwise. Transformed coordinates of $\begin{pmatrix}x_1\\y_1\end{pmatrix}$ & $\begin{pmatrix}x_2\\y_2\end{pmatrix}$ be $\begin{pmatrix}x_3\\y_3\end{pmatrix}$ & $\begin{pmatrix}x_4\\y_4\end{pmatrix}$ respectively.

$x_3 = OX_1 + X_1X_3 = x_1 + y_1 \cos 60°$

$y_3 = OY_1 \cos 30° = y_1 \cos 30°$

$$\begin{pmatrix}x_3\\y_3\end{pmatrix} = \begin{pmatrix}1 & \cos 60°\\0 & \cos 30°\end{pmatrix}\begin{pmatrix}x_1\\y_1\end{pmatrix} \tag{1.0.2}$$

Similarly,

$x_4 = OX_2 + X_2X_4 = x_2 + y_2 \cos 60°$

$y_4 = OY_2 \cos 30° = y_2 \cos 30°$

$$\begin{pmatrix}x_4\\y_4\end{pmatrix} = \begin{pmatrix}1 & \cos 60°\\0 & \cos 30°\end{pmatrix}\begin{pmatrix}x_2\\y_2\end{pmatrix} \tag{1.0.3}$$

The generalised equation for transformed coordinates $\begin{pmatrix}x_t\\y_t\end{pmatrix}$ when the angle between axes 'θ is,

$$\boxed{\begin{pmatrix}x_t\\y_t\end{pmatrix} = \begin{pmatrix}1 & \cos(\theta)\\0 & \sin(\theta)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}} \tag{1.0.4}$$

Substituting (1.0.1) in (1.0.2) & (1.0.3)

$$\begin{pmatrix}x_3\\y_3\end{pmatrix} = \begin{pmatrix}\frac{13}{2}\\\frac{5\sqrt{3}}{2}\end{pmatrix}; \begin{pmatrix}x_4\\y_4\end{pmatrix} = \begin{pmatrix}10\\3\sqrt{3}\end{pmatrix} \tag{1.0.5}$$

The distance between points is a norm of the distance vector,

$$\left\|\begin{pmatrix}x_3\\y_3\end{pmatrix} - \begin{pmatrix}x_4\\y_4\end{pmatrix}\right\| = \left\|\begin{pmatrix}10\\3\sqrt{3}\end{pmatrix} - \begin{pmatrix}\frac{13}{2}\\\frac{5\sqrt{3}}{2}\end{pmatrix}\right\| = \sqrt{13}\ units$$

The distance, d can be measured in angular axes directly by following equation,

$$d = \sqrt{\left\|\begin{pmatrix}x_2\\y_2\end{pmatrix} - \begin{pmatrix}x_1\\y_1\end{pmatrix}\right\|^2 + 2\begin{pmatrix}x_2 - x_1\\0\end{pmatrix}^\top \begin{pmatrix}y_2 - y_1\\0\end{pmatrix}\cos\theta}$$

$$d = \sqrt{\left\|\begin{pmatrix}7\\6\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix}\right\|^2 + 2\begin{pmatrix}3 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\cos 60°}$$

$$d = \sqrt{13}\ units \tag{1.0.6}$$

Above results shows that the distance remains constant between the points irrespective of coordinate system and by (1.0.1) & (1.0.5) only the position vector of the point changes with the transformation of coordinate system

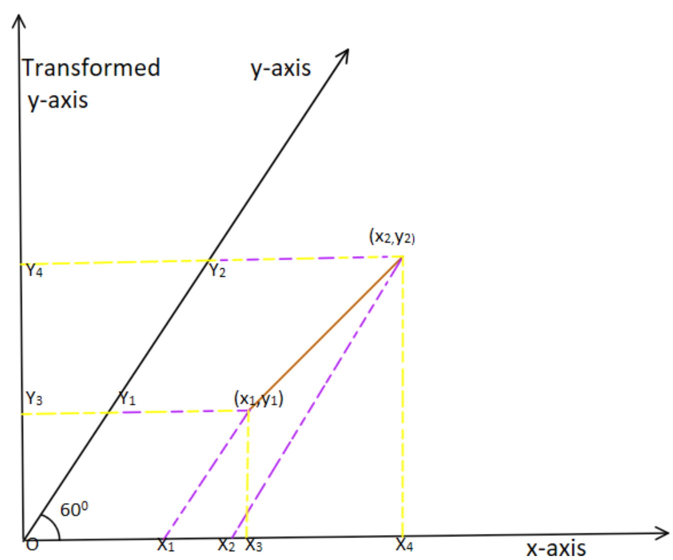

Fig1: Points defined on angular & rectangular axes

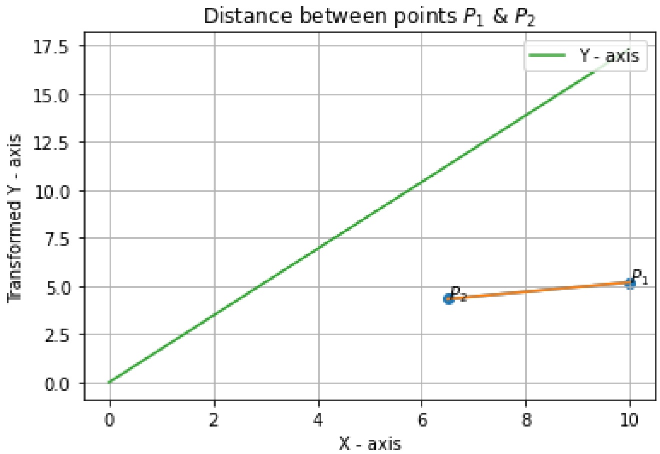

Fig2: Points plotted in Python