

Assignment 1

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CE20RESCH13001

PROBLEM II (2I)

Find the distance between points (7,6) and (4,5) with the axes at 60°

1 SOLUTION

Let the points be $P_1 (7,6)$ and $P_2 (4,5)$ and also the angle between axes is 60°

$$\mathbf{P}_1 = \begin{pmatrix} 7 \\ 6 \end{pmatrix}; \mathbf{P}_2 = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \quad (1.0.1)$$

The problem can be solved by transformation of the given coordinate system to the rectangular coordinate system.

In order to convert to rectangular coordinate system, the y-axis should be rotated by 30° in anti-clockwise and x-axis will remain unaltered.

The transformation matrix should be computed for transforming the given $\mathbf{P}_1, \mathbf{P}_2$ into the rectangular coordinate system.

The angle between the transformed x-axis and given x-axis be $\theta_{11} = 0^\circ$

The angle between the transformed x-axis and given y-axis be $\theta_{12} = 60^\circ$

Likewise, $\theta_{21} = 90^\circ$; $\theta_{22} = 30^\circ$

Transformed matrix \mathbf{T} will be the cosines of the above the angles

$$\mathbf{T} = \begin{pmatrix} \cos(\theta_{11}) & \cos(\theta_{12}) \\ \cos(\theta_{21}) & \cos(\theta_{22}) \end{pmatrix} = \begin{pmatrix} \cos(0^\circ) & \cos(60^\circ) \\ \cos(90^\circ) & \cos(30^\circ) \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (1.0.2)$$

From equations (1.0.1) and (1.0.2), the transformed vector corresponding to \mathbf{P}_1 be $\mathbf{P}_{1T} = (\mathbf{T}) (\mathbf{P}_1)$

Transformed vector corresponding to \mathbf{P}_2 be $\mathbf{P}_{2T} = (\mathbf{T}) (\mathbf{P}_2)$

$$\mathbf{P}_{1T} = \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 7 \\ 6 \end{pmatrix}; \mathbf{P}_{2T} = \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 5 \end{pmatrix} \\ \Rightarrow \mathbf{P}_{1T} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix}; \mathbf{P}_{2T} = \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix}$$

Now, obtained points are in the rectangular coordinate system and the distance vector between points will be

$$\mathbf{P}_{12T} = \mathbf{P}_{1T} - \mathbf{P}_{2T} = \begin{pmatrix} 10 \\ 3\sqrt{3} \end{pmatrix} - \begin{pmatrix} \frac{13}{2} \\ \frac{5\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{7}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$$

and the magnitude will be $\|\mathbf{P}_{1T} - \mathbf{P}_{2T}\|$
Therefore, the distance between the points is equal to $\sqrt{13}$ units

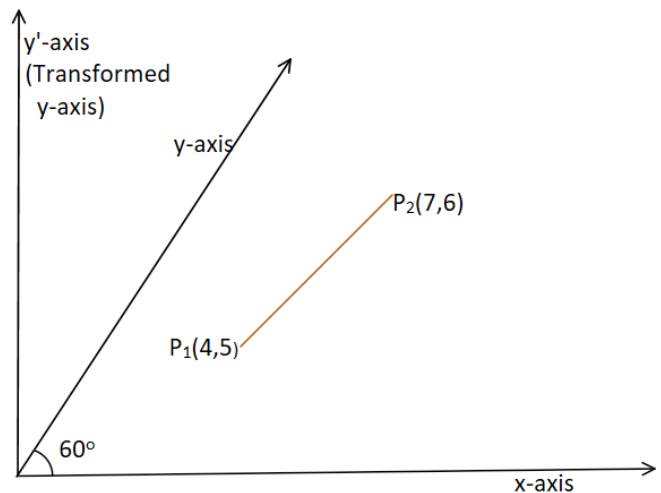


Fig: Distance between two points

*Python code file

https://github.com/Prithvi-Sangani/SM5083_Assignment1/blob/main/Assignment1.ipynb