Assignment 2

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CE20RESCH13001

1 Chapter III, Example III, Q.2

In what ratio is the join of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ is divided by the join of $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

1.1 Solution

Let the given points be

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} ; \mathbf{B} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} ; \mathbf{C} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} ; \mathbf{D} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \tag{1.1.1}$$

Let $\mathbf{X}$ be the point of intersection of lines joining $\mathbf{A}$,$\mathbf{B}$ & $\mathbf{C}$,$\mathbf{D}$. From the definition of the slopes,

$$\mathbf{B} - \mathbf{X} = k_1(\mathbf{B} - \mathbf{A}) \tag{1.1.2}$$
$$\mathbf{D} - \mathbf{X} = k_2(\mathbf{D} - \mathbf{C}) \tag{1.1.3}$$

Subtracting (1.1.3) from (1.1.2), we get

$$\mathbf{B} - \mathbf{D} = k_1(\mathbf{B} - \mathbf{A}) - k_2(\mathbf{D} - \mathbf{C}) \tag{1.1.4}$$

Substituting (1.1.1) in (1.1.4)

$$\begin{pmatrix} -2 \\ 6 \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} - k_2 \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{1.1.5}$$
$$\begin{pmatrix} 1 & -2 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix} \tag{1.1.6}$$
$$\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & 5 \end{pmatrix}^{-1} \begin{pmatrix} -2 \\ 6 \end{pmatrix} \tag{1.1.7}$$
$$\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{7} \\ \frac{8}{7} \end{pmatrix} \tag{1.1.8}$$

Substituting (1.1.8) in (1.1.2),

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \mathbf{X} = \frac{2}{7} \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \frac{2}{7} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.9}$$
$$\mathbf{X} = \begin{pmatrix} \frac{19}{7} \\ \frac{26}{7} \end{pmatrix} \tag{1.1.10}$$

ratio (r) by which the intersection point divides the line joining points $\mathbf{A}$ & $\mathbf{B}$ is given as

$$r = \frac{\|\mathbf{B} - \mathbf{X}\|}{\|\mathbf{A} - \mathbf{X}\|} \tag{1.1.11}$$
$$r = \frac{\left\| \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} \frac{19}{7} \\ \frac{26}{7} \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} \frac{19}{7} \\ \frac{26}{7} \end{pmatrix} \right\|} \tag{1.1.12}$$
$$r = \frac{2}{5} \tag{1.1.13}$$

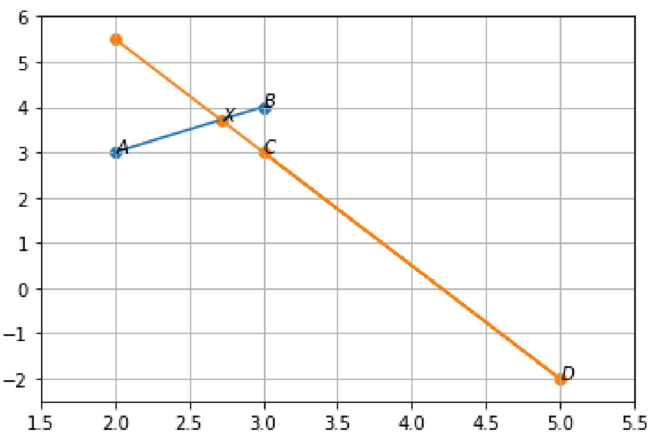

Fig. 1.1: Points plotted on xy plane