Name : V.S. Pandavi Rajeev Kumar
Regd no : 214101114S

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Week - Assignment - 4

Medical diagnosis problem.

Problem Statement :- A patient tested for a certain disease. The Test is 99% accurate, meaning the probablity of a positive Test given by the patient has disease ($P(T^+|D)$) is 0.99 and the probablity of the negative test is given by $P(T^-|D^c)$ is 0.99. The prevalence of the disease in the general population $P(D)$ is 1%. Then what is the probablity that the person actually has disease.

By Bayes Theorem :-

$$P(D|T^+) = \frac{P(T^+|D) \cdot P(D)}{P(T^+)}$$

$$P(T^+) = P(T^+|D) \cdot P(D) + P(T^+|D^c) \cdot P(D^c)$$

Given :-

$$P(T^+|D) = 0.99$$
$$P(D) = 0.01$$
$$P(T^-|D^c) = 0.99 \text{ so } P(T^+|D^c) = 1 - P(T^-|D^c) = 0.01$$
$$P(D^c) = 1 - P(D) = 0.99.$$

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- 21410111145

Calculating $P(T^+)$:-

$P(T^+) = (0.99 * 0.01) + (0.01 * 0.99)$
$P(T^+) = 0.0099 + 0.0099$
$P(T^+) = 0.0198$.

Now apply Bayes Theorem :-

$P(D|T^+) = \frac{0.99 \times 0.01}{0.0198}$

$P(D|T^+) = \frac{0.0099}{0.0198}$

$P(D|T^+) \approx 0.5$

=> So, the probablity that the patient actually has the disease given a positive test result is approximatly 0.5 or 50%.

(2) Finding Eigan values and Eigan vectors.

Given Matrix :-

$A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

Sol $\det(A - \lambda I) = 0$

$A - \lambda I = \begin{pmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{pmatrix}$.

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- 2141011145

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The determinant is :-

$$\det(A - \lambda I) = (4-\lambda)(3-\lambda) - 2\cdot 1$$
$$= \lambda^2 - 7\lambda + 10$$

So, the determinant to zero :-
$\lambda^2 - 7\lambda + 10 = 0$

So, the quadratic equation is :-

$$\lambda = \frac{7 \pm \sqrt{49-40}}{2}$$

$$\lambda = \frac{7 \pm 3}{2}$$

$\lambda 1 = 5$, $\lambda 2 = 2$.

So, for the eiganvalues 5, 2 :-

For $\lambda_1 = 5$:-

$$A - 5I = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}$$

Solving $(A - 5I)x = 0$

$$\begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

for this the equation simplifies to :-

$-x_1 + x_2 = 0$, and $x_2 = x_1$. Thus the eiganvector corresponding to $\lambda_1 = 5$.

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$$x = k\begin{pmatrix}1\\1\end{pmatrix}$$

For $\lambda_2 = 2$:-

$$A - 2I = \begin{pmatrix}2 & 1\\2 & 1\end{pmatrix}$$

Solving, $(A-2I)x = 0$

$$\begin{pmatrix}2 & 1\\2 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

This simplifies the value to:-

$$x = k\begin{pmatrix}1\\-2\end{pmatrix}.$$

(3) Calculating the determinant of 3x3 matrix and finding its inverse:-

Matrix given:-

$$B = \begin{pmatrix}1 & 2 & 3\\0 & 4 & 5\\1 & 0 & 6\end{pmatrix}$$

Solution:-

$$\det(B) = 1\cdot(4\cdot6 - 5\cdot0) - 2\cdot(0\cdot6 - 5\cdot1) + 3\cdot(0\cdot0 - 4\cdot1)$$
$$\det(B) = 1\cdot24 - 2\cdot(-5) + 3\cdot(-4)$$
$$\det(B) = 24 + 10 - 12$$
$$\det(B) = 22.$$

- V.S. Prudhvi Rajeev Kumar
- 21410111145

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The Inverse of B is found using the formula

$$B^{-1} = \frac{1}{\det(B)} \cdot adj(B)$$

The matrix of cofactors is :-

$$cof(B) = \begin{pmatrix} 24 & 5 & -4 \\ 6 & -3 & -2 \\ -20 & -1 & 4 \end{pmatrix}$$

The adjoint of B :-

$$adj(B) = \begin{pmatrix} 24 & 6 & -20 \\ 5 & -3 & -1 \\ -4 & -2 & 4 \end{pmatrix}$$

The Inverse is :-

$$B^{-1} = \frac{1}{22}\begin{pmatrix} 24 & 6 & -20 \\ 5 & -3 & -1 \\ -4 & -2 & 4 \end{pmatrix} \quad \text{(Ans)}$$

(4) Properties and Applications of Normal Distribution :-

i) Properties :-

- It is symmetric about its mean
- The mean, median and mode are equal.
- It is defined by two parameters.

- V.S. Pendhri Rajeev Kumar
- 2141011145.

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- The Total area under curve is 1.

Applications :-

- Used in hypothesis testing
- Commonly applied in natural and social sciences.
- Assumptions of normality underlies many statistical test and procedures.