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Question 1: M.L.E of μ & σ^2 for Normal distribution

Sol^n: let $X_i \sim N(\mu, \sigma^2)\ \forall\ i = 1, 2, 3, \ldots n$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < x < \infty$$

$$L = \prod_{i=1}^{n} f(x_i, \theta) = f(x_1, \mu, \sigma^2) \cdot f(x_2, \mu, \sigma^2) \cdots\cdots$$

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_1-\mu}{\sigma}\right)^2} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_2-\mu}{\sigma}\right)^2} \cdots$$

$$\Rightarrow \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2}\left(\frac{x_1-\mu}{\sigma}\right)^2 - \frac{1}{2}\left(\frac{x_2-\mu}{\sigma}\right)^2 \cdots\cdots}$$

$$\Rightarrow \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \cdot e^{-\frac{1}{2\sigma^2}\sum(x_i-\mu)^2}$$

$$\Rightarrow \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \cdot e^{-\frac{1}{2\sigma^2}\sum(x_i-\mu)^2}$$

$\Rightarrow$ Taking log both sides

$$\log L = -\frac{n}{2}\log 2\pi - \frac{n}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$$

P.T.O

Now,

$$\frac{\delta \log \alpha}{\delta \mu} = 0$$

$$\Rightarrow 0 - 0 - \frac{1}{\cancel{2}\sigma^2} \sum_{i=1}^{n} \cancel{2}(x_i - \mu)(-1) = 0$$

$$\Rightarrow \sum (x_i - \mu) = 0 \Rightarrow \sum x_i - n\mu = 0$$

$$\Rightarrow \boxed{\mu = \frac{1}{n} \sum x_i}$$

Now,

$$\frac{\delta \log \alpha}{\delta \sigma^2} = 0$$

$$\Rightarrow 0 - \frac{n}{2} \cdot \frac{1}{\sigma^2} + \frac{1}{2\sigma^4} \sum (x_i - \mu)^2 = 0$$

$$\Rightarrow \frac{-n\sigma^2 + \sum (x_i - \mu)^2}{2\sigma^4} = 0$$

$$\Rightarrow \boxed{\sigma^2 = \frac{1}{n} \sum (x_i - \mu)^2}$$

Question 2 : Binomial MLE

Sol^n: $L = \prod f(x_i; n, \theta)$

$\Rightarrow$

$\prod \binom{n}{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$

$\Rightarrow$

$\theta^{\sum x_i} (1-\theta)^{nm - \sum x_i} \prod \binom{n}{x_i}$

Taking log both sides

$\Rightarrow \log L = \log \left[\theta^{\sum x_i} (1-\theta)^{nm-\sum x_i} \prod \binom{n}{x_i} \right]$

$\Rightarrow \log \theta^{\sum x_i} + \log (1-\theta)^{nm-\sum x_i} + \log \prod \binom{n}{x_i}$

Diff. w.r.t ϕ

$\Rightarrow \sum x_i \frac{\partial}{\partial \phi} \log \phi + (nm - \sum x_i) \frac{\partial}{\partial \phi} \log$

$+ \partial [\sum^m \log ($

Equating to 0.

$$\Rightarrow \frac{\sum^m n_i}{\phi} = \left(\frac{nm - \sum n_i}{1-\phi} \right)$$

$\Rightarrow$

$$\sum n_i = nm\phi - \phi \sum^m n_i + \phi \sum^m n_i$$

$$\Rightarrow \sum n_i = mn\phi$$

$$\Rightarrow \phi = \frac{\sum n_i}{mn}$$

$$\Rightarrow \phi = \frac{1}{n} \frac{\sum n_i}{m}$$

$$\Rightarrow \boxed{\phi = \frac{\overline{X}}{n}}$$