CS 5333.001
ASSIGNMENT 1

Discrete Structures

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Exercise 1.1

(8) Let p and q be the propositions.

p: I bought a lottery ticket this week

q: I won the million dollar jackpocket

Express each of these propositions as an English sentence.

ⓐ $\neg p$ I did not buy a lottery ticket this week.

ⓑ $p \vee q$ Either I bought a lottery ticket this week or I won the million dollar jackpocket.

ⓒ $p \longrightarrow q$ If I bought a lottery ticket this week, then I won the million dollar jackpocket

ⓓ $p \wedge q \rightarrow$ I bought a lottery ticket this week and I won the million dollar jackpocket

ⓔ $p \leftrightarrow q$ I bought a lottery ticket if and only if I won the million dollar jackpocket.

ⓕ $\neg p \rightarrow \neg q$ If I did not buy a lottery ticket this week then I wouldn't win the million dollar jackpocket-

(g) $\neg p \wedge \neg q$ I did not buy a lottery ticket this week, and I did not win the million dollar jackpocket.

(h) $\neg p \vee (p \wedge q)$ Either I did not buy a lottery ticket this week, or else I did buy ~~a ticket and~~ a lottery ticket and won the million dollar jackpocket.

(18) Determine whether each of these conditional statements is true or false

(a) If $1+1=3$, then unicorns exist. False

(b) If $1+1=3$, then dogs can fly. False

(c) If $1+1=2$, then dogs can fly. True

(d) If $2+2=4$, then $1+2=3$ True

Exercise 1.2

(19) Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users can not save new files, then the system software is not being upgraded?

* This sentence here have 3 propositions:—

A= the software is upgraded

B= users can access the file system

C= users can save new files

* The system specifications can be translated as the following:—

$A \Rightarrow \neg B$

$B \Rightarrow C$

$\neg C \Rightarrow \neg A$

* Let's draw a truth table:—

A	B	C	$\neg A$	$\neg B$	$\neg C$	$A \Rightarrow \neg B$	$B \Rightarrow C$	$\neg C \Rightarrow \neg A$
T	T	T	F	F	F	F	T	T
T	T	F	F	F	T	F	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	T	T	T	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	T	T	F	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

There is atleast one instance in the truth table where all the 3 conditions becomes true.

Ans:— The system is consistent.

Exercise 1.3

(14) Determine whether $(\neg p \land (p \to q)) \to \neg q$ is a tautology

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \land (p \to q)$	$(\neg p \land (p \to q)) \to q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	~~F~~ T	~~T~~ F
F	F	T	T	T	T	T

No ~~Yes~~, it is not Tautology

(20) Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent

Lets prove by constructing truth table

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Thus through Truth Tables we can draw this conclusion

$\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent

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Exercise 1.4

(6) Let $N(x)$ be the statement "x has visited North Dakota", where the domain consists of the students in your school. Express each of these quatifications in English.

(6) (a) $\exists x P(x)$ There is a student in your school who has visited North Dakota

(b) $\forall x N(x)$ Every student in your school have visited North Dakota

~~(c) / ¬∃x ¬N~~

(c) $\neg \exists x N(x)$ No students in your school have visited North Dakota.

(d) $\exists x \neg N(x)$ There is a student in your school who hasn't visited North Dakota

(e) $\neg \forall x N(x)$ There is a student in your school who hasn't visited North Dakota.

(f) $\forall x N(x)$ No student in your school have visited North Dakota.

(14) Determine the truth ~~table~~ value of each of these statements if the domain for all real numbers.

(a) $\exists x (x^3 = -1)$ This is true because if we consider ~~x/ / Aker~~ "$x = -1$" then "$x^3 = -1$".

(b) $\exists x (x^4 < x^2)$ This is true. Consider the value of $x = \frac{1}{2}$

then $x^2 = \frac{1}{4}$ and $x^4 = \frac{1}{16}$ and "$\frac{1}{16}$" is smaller than the value "$\frac{1}{4}$".

ⓒ $\forall x\left((-x)^2 = x^2\right)$ Yes, this is true. Consider "$x=-2$",

then, $(-x)^2 \Rightarrow (-[-2])^2 \Rightarrow 4$

Consider "$x=2$",

then, $(-x)^2 \Rightarrow (-[2])^2 \Rightarrow 4$

ⓓ $\forall x\,(2x > x)$ This is not true, for example take "$x=\frac{1}{2}$"

Then:- $2x = \frac{1}{2} \times 2 = 1$

For this:- $2x > x$ condition prevails.

But now consider "$x = -\frac{1}{2}$"

Then:-

$$2x = 2 \times \left(-\frac{1}{2}\right) = -1$$

In this scenario $2x > x$ can't be true.

section 1.5

(30) Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quatifier or an expression involving logical connections)

(a) $\neg \exists y \exists x \, P(x, y)$

By using DeMorgan's Law for quantifiers in negation:—

$$\neg \exists y \exists x \, P(x, y) \equiv \forall y \neg \exists x \, P(x, y)$$

$$\equiv \forall y \forall x \neg P(x, y)$$

(b) $\neg \forall x \exists y \, P(x, y)$

Using De Morgan's Law for quantifiers in negation:—

$$\neg \forall x \exists y \, P(x, y) \equiv \exists x \neg \exists y \, P(x, y)$$

$$\equiv \exists x \forall y \neg P(x, y)$$

c $\neg \exists y (Q(y) \wedge \forall x \neg R(x,y))$

Using DeMorgan's law for quantifiers:—

$$\neg \exists y (Q(y) \wedge \forall x \neg R(x,y)) \equiv \forall y \neg (Q(y) \wedge \forall x \neg R(x,y))$$

$$\equiv \forall y (\neg Q(y) \vee \neg \forall x \neg R(x,y))$$

$$\equiv \forall y (\neg Q(y) \vee \exists x \neg\neg R(x,y))$$

$$\equiv \forall y (\neg Q(y) \vee \exists x R(x,y))$$

d $\neg \exists y (\exists x R(x,y) \vee \forall x S(x,y))$

Using DeMorgan's Law for quantifiers:—

$$\neg \exists y (\exists x R(x,y) \vee \forall x S(x,y)) \equiv \forall y \neg (\exists x R(x,y) \vee \forall x S(x,y))$$

$$\equiv \forall y (\neg \exists x R(x,y) \wedge \neg \forall x S(x,y))$$

$$\equiv \forall y (\forall x \neg R(x,y) \wedge \exists x \neg S(x,y))$$

e $\neg \exists y (\forall x \exists z T(x,y,z) \vee \exists x \forall z U(x,y,z))$

Using DeMorgan's Law for Quantifiers:-

$$\neg\exists y\,(\forall x\,\exists z\,T(x,y,z) \vee \exists x\,\forall z\,U(x,y,z))$$

$$\equiv \forall y\,\neg(\forall x\,\exists z\,T(x,y,z) \vee \exists x\,\forall z\,U(x,y,z))$$

$$\equiv \forall y\,(\neg\forall x\,\exists z\,T(x,y,z) \wedge \neg\exists x\forall z\,U(x,y,z))$$

$$\equiv \forall y\,(\exists x\,\neg\exists z\,T(x,y,z) \wedge \forall x\,\neg\forall z\,U(x,y,z))$$

$$\equiv \forall y\,(\exists x\,\forall z\,\neg T(x,y,z) \wedge \forall x\,\exists z\,\neg U(x,y,z))$$

Section 1.6

(24) Identify the errors in this argument that supposedly shows that $\forall x\,(P(x) \vee Q(x))$ is true then $\forall x\,P(x) \vee \forall x\,Q(x)$ is true.

→ "Step 3" and "Step 5" are the errors.

→ Simplification requires "$\wedge$" but those steps are using "$\vee$"

Section 1.7

(32) Show that these statements about the real number x are equivalent: (i) x is rational (ii) $x/2$ is rational (iii) $3x-1$ is rational.

Lets prove this,

$$(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$$

① $(i) \Rightarrow (ii)$

Let $x = \frac{p}{q}$, where $p, q \in \mathbb{Z}$. We have,

$$\frac{x}{2} = \frac{p}{2q}$$

and we have $p, 2q \in \mathbb{Z}$. Hence,

$$(i) \Rightarrow (ii)$$

② $(ii) \Rightarrow (iii)$.

Let $\frac{x}{2} = \frac{p}{q}$, where $p, q \in \mathbb{Z}$. This gives $x = \frac{2p}{p}$ which turn gives,

$$3x-1 = \frac{6p}{q} - 1 = \frac{6p-q}{q}$$

Since, $p, q \in \mathbb{Z}$, we have $q, 6p-q \in \mathbb{Z}$. Hence,

$$(ii) \Rightarrow (iii)$$

(3) (iii) $\Rightarrow$ (i),

Let $3x-1 = \frac{p}{q}$, where $p, q \in \mathbb{Z}$. This gives

$$3x = \frac{p}{q} + 1 \Rightarrow x = \frac{p+q}{3q}$$

Since $p, q \in \mathbb{Z}$, we have $3q$, $p+q \in \mathbb{Z}$, Hence,
(iii) $\Rightarrow$ (i)

Exercise 1.8

(18) Show that if r is an irrational number, there is an unique integer n such that the distance between r and n is less than $1/2$.

Let's prove this,

We know that an irrational number r lies in between two consecutive integers (for example: $\sqrt{2}$ lies between 1 and 2), thus there exist an integer x such that:-

$$x < r < x+1$$

Since r is irrational, r is also not equal to 0.5 and thus the distance from r to either x or $x+1$ is less than $\frac{1}{2} = 0.5$

So,

$$|x-r| < \frac{1}{2} \text{ or } |(x+1)-r| < \frac{1}{2}$$

If,

$|x-r| < \frac{1}{2}$, then we choose $n = x$

If,

$|(x+1)-r| < \frac{1}{2}$, then we choose $n = x+1$

This value of n is then a value for which the distance between r and n is less than $1/2$.

Now we will prove that such ~~a~~ a value of n is a unique integer for which the statements holds. Let y be an integer for which the statement holds:—

$$|r-y| < \frac{1}{2}$$

Let us then estimate the distance between the ~~values~~ two values n and y:—

$$|n-y| \leq |n-r| + |r-y| = |n-r| + |y-r| = \frac{1}{2} + \frac{1}{2} = 1$$

Since an integer is always at least a distance of 1 from another integer, we then obtain that n and y need to be the same integer.

$$n = y$$

We then have derived that n for which the statement hold is also unique.

Final Verdict:-

There exist a unique integer n such that the distance between r and n is less than 1/2.

Chapter 2

Section 2.1

(2) Use set builder notation to give a description of each of these sets:

(a) $\{0, 3, 6, 9, 12\}$

This set contains natural numbers (nonnegative integers) that are multiples of 3 and at most 12.

$$\{x \in \mathbb{N} \mid x \text{ is a multiple of 3 and } x \leq 12\}$$

(b) $\{-3, -2, -1, 0, 1, 2, 3\}$

The set contains integers that are between -3 to +3.

$$\{x \in \mathbb{Z} \mid -3 \leq x \leq 3\}$$

(c) $\{m, n, o, p\}$

This set has english letters from m to p.

$\{x \mid x$ is a letter in the alphabets from m to p$\}$

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(10) Determine whether these statements are true or false.

(a) $\emptyset \in \{\emptyset\}$

$\emptyset$ = means empty set

cardinality of empty set = 0

So, this is True.

(b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

$\emptyset$ = empty set

$\emptyset \in \{\emptyset, \{\emptyset\}\} \Rightarrow$ Empty set is an element from the set that contains the "empty set $\emptyset$" and the "subset of $\emptyset$".

So, this is true

(c) $\{\emptyset\} \in \{\emptyset\}$

This is false, the right hand side does not contain any sets and so they won't have any elements.

~~$\{2\} \in \{2\}$~~

ⓓ $\{\emptyset\} \in \{\{\emptyset\}\}$

This is True because $\{\emptyset\}$ is a set contained in the set $\{\{\emptyset\}\}$

ⓔ $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

This is True. The $\{\emptyset\}$ is an element inside $\{\emptyset, \{\emptyset\}\}$

ⓕ $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

This is True. The set $\{\{\emptyset\}\}$ is a subset of the set containing elements $\emptyset$ and $\{\emptyset\}$.

ⓖ $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

This is false. Both the sides are the same.

(18) Find two sets A and B such that $A \in B$ and $A \subseteq B$

Let, $A = \emptyset$ and $B = \{\emptyset, \{x y, \{y\}, \{z\}\}$

~~It's prov~~ We can notice that both conditions are satisfied where $A \in B$ and $A \subseteq B$

Section 2.2

(16) Let A and B be sets. Show that:—

(a) $(A \cap B) \subseteq A$

Let there be an x which belongs in A intersection B

$\rightarrow \quad x \in A \cap B$

Intersection mean,

x is a subset of A

Thus, $(A \cap B) \subseteq A$ is proved.

(b) $A \subseteq (A \cup B)$

Let $x \in A$,

Its also true that,

$$x \in A \vee x \in B$$

By Union Definition,

$$x \in A \cup B$$

By definiton of subset,

$$A \subseteq (A \cup B)$$

(c) $A - B \subseteq A$

Let $x \in A - B$,

By Definition of Difference,

$$x \in A \wedge x \notin B$$

Using Simplification of propositions,

$$x \in A$$

Thus by definition of subsets,

$$A - B \subseteq A$$

(d) $A \cap (B-A) = \emptyset$

Let $x \in A \cap (B-A)$,

We know by intersection rule,

$$x \in A \wedge x \in B-A)$$

But "B-A" will contain parts that doesn't have any elements of A in it according to difference rule:-

$$x \in A \wedge (x \in B \wedge \neg(x \in A))$$

Using Commutative Law,

$$x \in A \wedge (\neg(x \in A) \wedge x \in B))$$

Using Associative Law,

$$\underbrace{(x \in A \wedge - (x \in A))}_{\text{This is false}} \wedge x \in B$$

Thus by using Domination Law,

$$F \wedge x \in B \Rightarrow F$$

Thus the set doesn't have any elements and is an empty set.

(e) $A \cup (B - A) = A \cup B$

Lets prove the Left Hand Side: $\underline{A \cup (B - A)}$

Let $x \in A \cup (B - A)$

By Union Law,

$$x \in A \vee (x \in B \wedge \neg (x \in A))$$

By Commutative Law,

$$x \in A \vee \underbrace{(\neg (x \in A) \wedge x \in B)}_{\text{This is same as "} x \in B \text{"}}$$

Thus we get,

$$x \in A \vee (x \in B)$$

By Union,

$$x \in A \cup B$$

Thus L.H.S = R.H.S and proved.

(30) Can you conclude that $A=B$ if A, B, C are sets such that :—

ⓐ $A \cup C = B \cup C$?

Lets assume,

$A = \{P, R, I\}$
$B = \{T, O, M\}$
$C = \{P, R, I, T, O, M\}$

$A \cup C = \{P, R, I, T, O, M\}$

$B \cup C = \{P, R, I, T, O, M\}$

$\Rightarrow$ Although $A \cup C = B \cup C$ is satisfied but sets A and B are different

$\Rightarrow$ It is False (So, $A \neq B$)

ⓑ $A \cap C = B \cap C$?

Lets assume the same values of A, B, C from above.

$A \cap C = \{P, R, I, T, O, M\}$

$B \cap C = \{P, R, I, T, O, M\}$

But still "A" and "B" are sets containing different element

$\Rightarrow$ It is false (So, $A \neq B$)

(30) (c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$

Lets see,

$A \cup C = \{P, R, I, T, O, M\}$

$B \cup C = \{P, R, I, T, O, M\}$

$A \cap C = \{P, R, I\}$

$B \cap C = \{T, O, M\}$

So, $A \cup C = B \cup C \implies$ let it be case 1

$A \cap C = B \cap C \implies$ let it be case 2

$\implies$ case 1 and case 2

$\implies$ True and False

$\implies$ False

Thus again $(A \neq B)$.

Section 2.3

(10) Determine whether each of these functions from $\{a,b,c,d\}$ to itself is one-to-one

(a) $f(a)=b$, $f(b)=a$, $f(c)=c$, $f(d)=d$

This is true as every output is unique.

(b) $f(a)=b$, $f(b)=b$, $f(c)=d$, $f(d)=c$

This is false as "b" repeats when $f(a)$ and $f(b)$.

(c) $f(a)=d$, $f(b)=b$, $f(c)=c$, $f(d)=d$

This is also false as "d" repeats when $f(a)$ and $f(d)$.

(22) Determine whether each of these functions is a bijection from R to R.

(a) $f(x)=-3x+4$

Yes

(b) $f(x)=-3x^2+7$

NO

(c) $f(x)=(x+1)/(x+2)$

NO

(d) $f(x)=x^5+1$

NO

bijection meaning