PROJECT TITLE

A Dissertation Submitted
in Partial Fulfilment of the Requirements
for the Degree of

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in

MATHEMATICS

by

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(Roll No. [Roll No.])

to

SCHOOL OF MATHEMATICS
INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH
THIRUVANANTHAPURAM - 695 551, INDIA

April 2023

DECLARATION

I, [**Full Name**] **(Roll No: [Roll Number])**, hereby declare that, this report entitled **"[Project Title]"** submitted to Indian Institute of Science Education and Research Thiruvananthapuram towards the partial requirement of **Master of Science** in [**Department Name**], is an original work carried out by me under the supervision of [**Project Guide(s)**] and has not formed the basis for the award of any degree or diploma, in this or any other institution or university. I have sincerely tried to uphold academic ethics and honesty. Whenever a piece of external information or statement or result is used then, that has been duly acknowledged and cited.

Thiruvananthapuram - 695 551 [**Full Name**]
April 2023

CERTIFICATE

This is to certify that the work contained in this project report entitled **"My Project Title"** submitted by [**Full Name**] (**Roll No: [Roll Number]**) to Indian Institute of Science Education and Research, Thiruvananthapuram towards the partial requirement of [**Master of Science/ Doctor of Philosophy**] in [**Department Name**] has been carried out by [him/her/them] under my supervision and that it has not been submitted elsewhere for the award of any degree.

Thiruvananthapuram - 695 551 | [Project Supervisor]
April 2023 | Project Supervisor

ACKNOWLEDGEMENT

Write about the people and the things you are indebted to in fullfilling this project

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I am thankful to Nikhil Alex Verghese, BS-MS'17, IISER Thiruvananthapuram for creating a 'Official IISER Thiruvananthapuram Thesis/Dissertation Format' in Overleaf.

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ABSTRACT

If you have to structure it as objective, methods, results, and conclusions, do it that way.

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Contents

List of Figures

List of Tables

Chapter 1

Before you copy and paste

1.1 Why this template?

This template is designed with only one thing in mind. The readability of the script. One of the hardest part in debugging any code, not just $LaTeX$ is squashing bugs. With a good structure to your working directory, you'll be able to read and understand what everything means and where to make changes. Without further due let us start

1.2 How much $LaTeX$ you need?

I am under the impression that you might know some $LaTeX$ scripting. Don't worry if you don't, there are plenty of YouTube videos out there. Get just the basics like how to make a simple text document, some math symbols, inline equations, block equations, aligning equations etc. `https://latex.js.org/playground.html` is an online playground which will give you a quick introduction.

1.3 Where to start in this template?

The directory is setup in a way that almost all the settings are inteh `config` directory. If you open `main.tex`, the first thing included is the `config/packages.tex`.

1.3.1 `config/packages.tex`

This file imports some of the essential packages that are required. I've added what some of these packages do as comments next to them. Uncomment/add the required packages here. For more details about these packages go to https://www.ctan.org/pkg/*packagename* and read the documentation there.

1.3.2 config/options.tex

The next thing imported by `main.tex` is `config/options.tex`. This file contains all the settings for the packages imported from `config/packages.tex`. If you want to specify further options, put all of them into this file. Refer the documentation at https://www.ctan.org for specific options of the packages.

1.3.3 config/colors.tex

This file contains color settings.

1.3.4 config/theoremstyle.tex

This file contains the settings to stylize definitions and theorems for your document.

1.3.5 config/mathletters.tex

This file contains some shortcuts which helped me typeset math equations.

1.4 Adding content

The chapters of the report is supposed to be in the folder `01Chapters`. Make a tex file for each chapter as it simplifies the content.

1.5 Front Matter

Edit the files in `00Intro` with your details.

1.6 Citations and References

Cite whatever you want with `autocite` like [Rud87, Theorem 3.14 p. 69] and refer things with `autoref`, its way better than `ref`. You can just refer things like section 1.1 and don't have to do section 1.1(Look at the code to see what I mean)

The bibliography file is set to `02End/math.bib`, using `bibsource` in line 8 of `main.tex`. Edit the `.bib` file adding your citations. The citationstyle can also be changed. Refer `biblatex` documentation for this.

Remember to run `biber` if you are working in your local system and not overleaf.

1.7 Info

Fork me on GitHub and contribute to the project.

Figure 1.1: Poor boy needs money. Help him by sending your charitable donations

Appendices

Appendix A

Results from Measure Theory

Here'll we'll discuss some important results from measure theory which are essential for our subject. We already defined what is an L^p function in a given space at **??**.

Proposition A.0.1. *Continuous functions in $\mathbb{T}$, (refer* **??***) are dense in $L^p(\mathbb{T})$ for $1 \leq p < \infty$.*

Proof. This is a direct consequence of [Rud87, Theorem 3.14 on p. 69]. Since $\mathbb{T}$ is identified with $[0, 1)$, all continuous functions in $\mathbb{T}$ are compactly supported. □

Proposition A.0.2. *Let $C_c(\mathbb{R})$ be the set of all compactly supported continuous functions in $\mathbb{R}$, then $C_c(\mathbb{R})$ is dense in $L^p(\mathbb{R})$. This is [Rud87, Theorem 3.14 p. 69].*

Proposition A.0.3. *$L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ is dense in $L^2(\mathbb{R})$.*

Proof. Let $C_c(\mathbb{R})$ denote the set of compactly supported continuous functions in $\mathbb{R}$. Since every function is continuous and compactly supported, $C_c(\mathbb{R}) \subset L^p(\mathbb{R})$ for all $1 \leq p < \infty$. Therefore $C_c(\mathbb{R}) \subset L^1(\mathbb{R}) \cap L^2(\mathbb{R})$. Then by [Rud87, Theorem 3.14 on p. 69] $C_c(\mathbb{R})$ is dense in $L^2(\mathbb{R})$ and therefore $L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ is dense in $L^2(\mathbb{R})$. □

If you follow the proof of the above theorem close enough, you'll see that we can make a stronger claim. Since, $C_c(\mathbb{R}) \subset L^p(\mathbb{R})$ for all $1 \leq p < \infty$,

$$C_c(\mathbb{R}) \subset \bigcap_{1 \leq p < \infty} L^p(\mathbb{R})$$

and therefore again by [Rud87, Theorem 3.14 on p. 69], $\bigcap_{1 \leq p < \infty} L^p(\mathbb{R})$ is dense in $L^q(\mathbb{R})$ for all $1 \leq q < \infty$. We will state the generalization of this as a seperate result.

Proposition A.0.4. *If $f \in L^p(\mathbb{R})$, then*

$$\lim_{\delta \to 0} \int_{\mathbb{R}} |f(x+\delta) - f(x)|^p \; dx = 0$$

Proof. Since $f \in L^p(\mathbb{R})$, for every $\epsilon > 0$ there exist an X such that

$$\left(\int_{|x|>X-1} |f(x)|^p \; dx \right)^{\frac{1}{p}} < \epsilon$$

Therefore by Minkowski's inequality, for $\delta \leq 1$,

$$\left(\int_{\mathbb{R}} |f(x+\delta) - f(x)|^p \; dx \right)^{\frac{1}{p}} \leq \left(\int_{-X}^{X} |f(x+\delta) - f(x)|^p \; dx \right)^{\frac{1}{p}} + 2\epsilon$$

Now since $C_c(\mathbb{R}) \cap L^p(\mathbb{R})$ are dense in $L^p(\mathbb{R})$ by Proposition A.0.2, there exists a $g \in C([-X-1, X+1]) \cap L^p([-X-1, X+1])$ such that

$$\|f - g\|_{L^p([-X-1,X+1])} = \left(\int_{-X-1}^{X+1} |f(x) - g(x)|^p \; dx \right)^{\frac{1}{p}} < \epsilon$$

Then by Minkowski's inequality

$$\left(\int_{-X}^{X} |f(x+\delta) - f(x)|^p \; dx \right)^{\frac{1}{p}} \leq \left(\int_{-X}^{X} |g(x+\delta) - g(x)|^p \; dx \right)^{\frac{1}{p}} + 2\epsilon$$

Now since g is continuous in a compact space $[-X-1, X+1]$, it is uniformly continuous. and since ϵ does not depend on the therefore as $\delta \to 0$ the above integral tends to 0. Hence the proof. □

Bibliography

[Rud87] Walter Rudin. *Real and complex analysis*. McGraw-Hill, 1987, p. 483. ISBN: 978-0-07-054234-1.