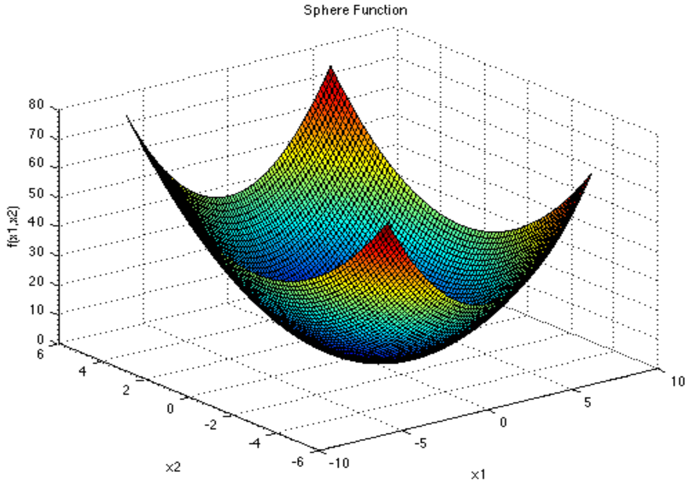
Sphere Function




## Description:

*Dimensions: d*

The Sphere function has d local minima except for the global one. It is continuous, convex and unimodal. The plot shows its two-dimensional form.

## Input Domain:

The function is usually evaluated on the hypercube $x_i \in [-5.12, 5.12]$, for all $i = 1, \ldots, d$.

## Global Minimum:

$$f(\mathbf{x}^*) = 0, \text{ at } \mathbf{x}^* = (0, \ldots, 0)$$

**Illustration:**

$$f(X) = \sum_{i=0}^{d} x_i^2$$

Where d =No of variables i.e. Dimension

Upper Bound $= +5.12$

Lower Bound $= -5.12$

No of variables $= 2 \quad x_1, x_2$

No of candidates = 3 $c_1,\ c_2, c_3$

Reduction factor $r$ = 0.9

## Learning Attempt 1:

$$\begin{array}{c c} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & \begin{bmatrix} 0.5 & 1 \\ -1 & 2 \\ 1 & -2 \end{bmatrix} \end{array}$$

$$f(X)_{c_1} = (0.5)^2 + (1)^2 = 1.25$$

$$f(X)_{c_2} = (-1)^2 + (2)^2 = 5$$

$$f(X)_{c_3} = (1)^2 + (-2)^2 = 5$$

Probability Calculation:

$$P_{c_1} = \frac{{}^{1}/_{f(X)_{c_1}}}{{}^{1}/_{f(X)_{c_1}} + {}^{1}/_{f(X)_{c_2}} + {}^{1}/_{f(X)_{c_3}}}$$

$$P_{c_1} = \frac{0.8}{0.8 + 0.2 + 0.2} = 0.66$$

$$P_{c_2} = \frac{1/f(X)_{c_2}}{1/f(X)_{c_1} + 1/f(X)_{c_2} + 1/f(X)_{c_3}}$$

$$P_{c_2} = \frac{0.2}{0.8 + 0.2 + 0.2} = 0.16$$

$$P_{c_3} = \frac{1/f(X)_{c_3}}{1/f(X)_{c_1} + 1/f(X)_{c_2} + 1/f(X)_{c_3}}$$

$$P_{c_3} = \frac{0.2}{0.8 + 0.2 + 0.2} = 0.16$$

Roulette Wheel Approach:


0 0.66 0.82 0.99

Assume

$c_1$ generated 0.69 and Follows $c_2$

$c_2$ generated 0.77 and Follows $c_2$

$c_3$ generated 0.29 and Follows $c_1$

∴ Matrix for applying reduction factor is given below:

$$\begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{bmatrix} -1 & 2 \\ -1 & 2 \\ 0.5 & 1 \end{bmatrix} \end{array}$$

$Range = Upper\ Bound - Lower\ Bound = 5.12 - (-5.12) = 10.24$

Sampling interval reduction by reduction factor $r = 0.9$

$Range = 10.24 \times 0.9 = 9.216$ (A)

$\frac{9.216}{2} = 4.608$

New Upper Bound $= +4.608$

New Lower Bound $= -4.608$

If new upper bound exceeds original upper bound make it upper bound

If new lower bound exceeds original lower bound make it lower bound

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For $c_1$ : $c_1$ Follows $c_2$

-1

$-4.608$ $+4.608$

$-5.608$ $3.608$

$-5.12$ $3.608$ $-5.608 < -5.12 \therefore round\ off\ to - 5.12$

New bound for $x_1$ are $-5.12\ \ to\ \ 3.608$

2

$-4.608$ $+4.608$

$-2.608$ $6.608$

$-2.608$ $+5.12$ $6.608 > +5.12 \therefore round\ off\ to + 5.12$

New bound for $x_2$ are $-2.608\ \ to\ 5.12$

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For $c_2$: follows $c_2$

so new bounds will be same as new bounds for $c_1$

New bound for $x_1$ are $-5.12\ \ to\ \ 3.608$

New bound for $x_2$ are $-2.608\ \ to\ 5.12$

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For $c_3$: $c_3$ Follows $c_1$

$0.5$

$-4.608 \qquad +4.608$

$-4.108 \qquad\qquad 5.108$

New bound for $x_1$ are $-4.108\ \ to\ \ 5.108$

$1$

$-4.608 \qquad +4.608$

$-3.608 \qquad\qquad 5.608$

$-3.608 \qquad\qquad +5.12 \qquad\qquad 5.608\ > +5.12\ \therefore round\ off\ to + 5.12$

New bound for $x_2$ are $-3.608\ \ to\ 5.12$

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Updated Sampling intervals

$$\begin{array}{cc} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & \begin{bmatrix} -5.12, 3.608 & -2.608, 5.12 \\ -5.12, 3.608 & -2.608, 5.12 \\ -4.108, 5.108 & -3.608, 5.12 \end{bmatrix} \end{array}$$

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## Learning Attempt 2

New matrix randomly obtained from new bounds is given below:

$$\begin{array}{cc} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & \begin{bmatrix} 0.7 & -1 \\ 0.5 & 1.2 \\ 0.3 & 0.9 \end{bmatrix} \end{array}$$

$f(X)_{c_1} = (0.7)^2 + (-1)^2 = 1.49$

$f(X)_{c_2} = (0.5)^2 + (1.2)^2 = 1.69$

$f(X)_{c_3} = (0.3)^2 + (0.9)^2 = 0.9$

Probability Calculation:

$$P_{c_1} = \frac{1/f(X)_{c_1}}{1/f(X)_{c_1} + 1/f(X)_{c_2} + 1/f(X)_{c_3}}$$

$P_{c_1} = 0.2826$

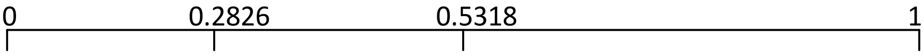
$$P_{c_2} = \frac{1/f(X)_{c_2}}{1/f(X)_{c_1} + 1/f(X)_{c_2} + 1/f(X)_{c_3}}$$

$P_{c_2} = 0.2492$

$$P_{c_3} = \frac{1/f(X)_{c_3}}{1/f(X)_{c_1} + 1/f(X)_{c_2} + 1/f(X)_{c_3}}$$

$P_{c_3} = 0.4680$

Roulette wheel




Assume

$c_1$ generated 0.11 and Follows $c_1$

$c_2$ generated 0.17 and Follows $c_1$

$c_3$ generated 0.45 and Follows $c_2$

∴ Matrix for applying reduction factor is given below:

$$\begin{array}{cc} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & \begin{bmatrix} 0.7 & -1 \\ 0.7 & -1 \\ 0.5 & 1.2 \end{bmatrix} \end{array}$$

$Range = 9.216$ **from A**

Sampling interval reduction by reduction factor $r = 0.9$

$9.216 \times 0.9 = 8.2944$

$\frac{8.2944}{2} = 4.14$

New Upper Bound $= +4.14$

New Lower Bound $= -4.14$

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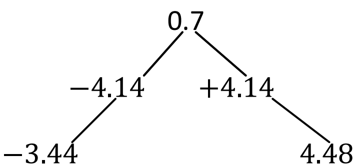
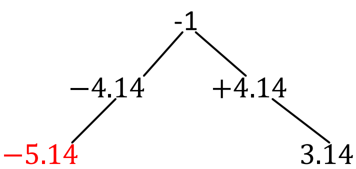
For $c_1$: $c_1$ Follows $c_1$




$-5.12$ $\quad$ $3.14$ $\quad\quad$ $-5.14 < -5.12 \therefore round\ off\ to -5.12$

New bound for $x_1$ are $-3.44\ to\ 4.48$

New bound for $x_2$ are $-5.12 \ \ to\ 3.14$

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For $c_2$: $c_2$ Follows $c_1$

As $c_2$ follows $c_1$ bounds will be same as calculated for $c_1$

New bound for $x_1$ are $-3.44 \ \ to\ \ 4.48$

New bound for $x_2$ are $-5.12 \ \ to\ 3.14$

For $c_3$: $c_3$ Follows $c_2$

$0.5$

$-4.14 \qquad +4.14$

$-3.64 \qquad\qquad\qquad 4.64$

$-1$

$-4.14 \qquad +4.14$

$-2.94 \qquad\qquad\qquad 5.34$

$-2.94 \qquad\qquad\qquad 5.12$ $\qquad\qquad 5.34\ > +5.12\ \ \therefore round\ off\ to + 5.12$

New bound for $x_1$ are $-3.64 \ \ to\ \ 4.64$

New bound for $x_2$ are $-2.94 \ \ to\ 5.12$

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Updated Sampling intervals

$$\begin{array}{cc} & \begin{array}{cc} \quad x_1 \quad\quad & \quad\quad x_2 \end{array} \\ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & \begin{bmatrix} -3.44, 4.48 & -5.12, 3.14 \\ -3.44, 4.48 & -5.12, 3.14 \\ -3.64, 4.64 & -2.94, 5.12 \end{bmatrix} \end{array}$$

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## Learning Attempt 3

New matrix randomly obtained from new bounds is given below:

$$\begin{array}{c c} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & \begin{bmatrix} 0.2 & -0.6 \\ -0.7 & 0.3 \\ -1.1 & -0.01 \end{bmatrix} \end{array}$$

$f(X)_{c_1} = 0.4$

$f(X)_{c_2} = 0.58$

$f(X)_{c_3} = 1.210$