Unbiasing Review Ratings with Tendency based Collaborative Filtering Appendix

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A Notation and Definition

A.1 Known Parameters

1. User Mean: $\overline{r_u} = \frac{1}{n_i}\sum_{i=i}^{n_i} r_{ui}$

2. Product Mean: $\overline{r_i} = \frac{1}{n_u}\sum_{u=1}^{n_u} r_{ui}$

3. User Tendency: $\tau_u = \frac{1}{n_i}\sum_{i=i}^{n_i}(r_{ui} - \overline{r_i})$

4. Item Tendency: $\tau_i = \frac{1}{n_u}\sum_{u=1}^{n_u}(r_{ui} - \overline{r_u})$

   Where,
   n_i- number of items rated by the user (u).
   n_u- number of user who have rated item (i).
   r_{ui}- rating by user (u) to the item (i).

Enough Rating Assumption: To calculate adequate invariant estimates of user/item means and tendencies from the rated data, we need to make the following assumption: The prediction example is based in an online setting. Thus, the distribution means and tendencies (both users/items) will not change significantly, when we predict ratings for new user-item pairs. This helps us in deciding the case on the estimated mean and tendency from the given labeled ratings. Similarly, the case for reverse estimation function will be decided on the prior data mean and tendency.

A.2 Derived Parameters

1. $\sum_{i=1}^{n_i} r_{ui} = \overline{r_u} * n_i$: Sum of all the rating rated by the user (u) for all items.

2. $\sum_{u=1}^{n_u} r_{ui} = \overline{r_i} * n_u$: Sum of all the rating rated by user to an item (i) for all users.

3. $\sum_{i=1}^{n_i} \overline{r_i} = n_i(\overline{r_u} - \tau_u)$: Sum of all item mean (rated by user (u)).

4. $\sum_{u=1}^{n_u} \overline{r_u} = n_u(\overline{r_i} - \tau_i)$: Sum of all user mean (rated for item (i)).

A.3 Derivation of Derived Parameters

1. $\overline{r_u} = \frac{1}{n_i}\sum^{items} r_{ui}$
   $\sum^{items} r_{ui} = \overline{r_u} * n_i$

2. $\overline{r_i} = \frac{1}{n_u}\sum^{users} r_{ui}$
   $\sum^{users} r_{ui} = \overline{r_i} * n_u$

3. $\tau_u = \frac{\sum^{items} r_{ui} - \sum^{items} \overline{r_i}}{n_i}$
   $\tau_u * n_i = \sum^{items} r_{ui} - \sum^{items} \overline{r_i}$
   $\sum^{items} \overline{r_i} = \sum^{items} r_{ui} - (\tau_u * n_i)$
   $\sum^{items} \overline{r_i} = (\overline{r_u} * n_i) - (\tau_u * n_i)$ – From eq. 1
   $\sum^{items} \overline{r_i} = n_i(\overline{r_u} - \tau_u)$

4. $\tau_i = \frac{\sum^{users} r_{ui} - \sum^{users} \overline{r_u}}{n_u}$
   $\tau_i * n_u = \sum^{users} r_{ui} - \sum^{users} \overline{r_u}$
   $\sum^{users} \overline{r_u} = \sum^{users} r_{ui} - (\tau_i * n_u)$
   $\sum^{users} \overline{r_u} = (\overline{r_i} * n_u) - (\tau_i * n_u)$ – From eq. 2
   $\sum^{users} \overline{r_u} = n_u(\overline{r_i} - \tau_i)$

B Unbiased Review Scores

Case	UnBias Function ($\widehat{r_{ui}}$)
1. $\tau_u > 0$, $\tau_i > 0$	max($\overline{r_u} + \tau_i, \overline{r_i} + \tau_u$)
2. $\tau_u < 0$, $\tau_i < 0$	min($\overline{r_u} + \tau_i, \overline{r_i} + \tau_u$)
3. $\tau_u < 0$, $\tau_i > 0$, $\overline{r_u} < \overline{r_i}$	min(max($\overline{r_u}, (\overline{r_i} + \tau_u)\beta$ + $(\overline{r_u} + \tau_i)(1-\beta)), \overline{r_i}$)
4. $\tau_u < 0$, $\tau_i > 0$, $\overline{r_u} > \overline{r_i}$	$\overline{r_i}\beta + \overline{r_u}(1-\beta)$
5. $\tau_u > 0$, $\tau_i < 0$, $\overline{r_u} > \overline{r_i}$	min(max($\overline{r_i}, (\overline{r_u} + \tau_i)\beta$ + $(\overline{r_i} + \tau_u)(1-\beta)), \overline{r_u}$)
6. $\tau_u > 0, \tau_i < 0, \overline{r_u} < \overline{r_i}$	$\overline{r_u}\beta + \overline{r_i}(1-\beta)$

Table 1: Unbias Function

C Derivation of Reverse functions

This sections contains the details about the derivations of the reverse functions. The reverse tendency estimated functions are listed in Table 2. Here, (train) in subscript mean using example from training set, (new) in subscript mean a new example from test/valid test, and (final) in subscript represent the final prediction.

Case	Sub-case	Reverse tendency estimation function
1.τ_u >0, τ_i >0	I. $\widehat{r_{ui}} = \overline{r_u} + \tau_i(new)$ II. $\widehat{r_{ui}} = \overline{r_i} + \tau_u(new)$	$r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1)(n_i+1) - (n_i+1)(n_u \times \tau_i) - (n_i \times n_u \times \overline{r_u})]$ $r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1)(n_i+1) - (n_u+1)(n_i \times \tau_u) - (n_i \times n_u \times \overline{r_i})]$
2.τ_u <0, τ_i <0	I. $\widehat{r_{ui}} = \overline{r_u} + \tau_i(new)$ II.$\widehat{r_{ui}} = \overline{r_i} + \tau_u(new)$	$r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1)(n_i+1) - (n_i+1)(n_u \times \tau_i) - (n_i \times n_u \times \overline{r_u})]$ $r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1)(n_i+1) - (n_u+1)(n_i \times \tau_u) - (n_i \times n_u \times \overline{r_i})]$
3.τ_u <0, τ_i >0, $\overline{r_u} < \overline{r_i}$	I. $\widehat{r_{ui}} = \overline{r_{u(final)}}$ II. $\widehat{r_{ui}} = (\overline{r_i} + \tau_u)\beta + (\overline{r_u} + \tau_i)(1-\beta)$ III. $\widehat{r_{ui}} = \overline{r_{i(final)}}$	$r_{ui(new)} = (n_i+1)\widehat{r_{ui}} - (\overline{r_u} \times n_i)$ $r_{ui(new)} = (\frac{1}{n_i+n_u+1})[(n_u+1)(n_i+1)\widehat{r_{ui}} - (n_u)(\overline{r_u} \times n_i)(1-\beta) - (n_i+1)(1-\beta)(n_u \times \tau_i) - \beta(n_i)(\overline{r_i} \times n_u) - \beta(n_u+1)(n_i \times \tau_u)]$ $r_{ui(new)} = (n_u+1)\widehat{r_{ui}} - (\overline{r_i} \times n_u)$
4.τ_u <0, τ_i >0, $\overline{r_u} > \overline{r_i}$	$\widehat{r_{ui}} = \overline{r_i}\beta + \overline{r_u}(1-\beta)$	$r_{ui(new)} = [\frac{1}{n_u(1-\beta)+1+\beta n_i}] \times [(n_u+1)(n_i+1)\widehat{r_{ui}} - \beta(n_i+1)(\overline{r_i} \times n_u) - (1-\beta)(n_u+1)(\overline{r_u} \times n_i)]$
5.τ_u >0, τ_i <0, $\overline{r_u} > \overline{r_i}$	I. $\widehat{r_{ui}} = \overline{r_i(final)}$ II.$\widehat{r_{ui}} = (\overline{r_u} + \tau_i)\beta + (\overline{r_i} + \tau_u)(1-\beta)$ III. $\widehat{r_{ui}} = \overline{r_u(final)}$	$r_{ui(new)} = (n_u+1)\widehat{r_{ui}} - (\overline{r_i} \times n_u)$ $r_{ui(new)} = (\frac{1}{n_i+n_u+1})[(n_u+1)(n_i+1)\widehat{r_{ui}} - (n_i)(\overline{r_i} \times n_u)(1-\beta) - (n_u+1)(1-\beta)(n_i \times \tau_u) - \beta(n_u)(\overline{r_u} \times n_i) - \beta(n_i+1)(n_u \times \tau_i)]$ $r_{ui(new)} = (n_i+1)\widehat{r_{ui}} - (\overline{r_u} \times n_i)$
6.τ_u >0, τ_i <0, $\overline{r_u}$ < $\overline{r_i}$	$\widehat{r_{ui}} = \overline{r_u}\beta + \overline{r_i}(1-\beta)$	$r_{ui(new)} = [\frac{1}{n_i(1-\beta)+1+\beta n_u}] \times [(n_u+1)(n_i+1)\widehat{r_{ui}} - \beta(n_u+1)(\overline{r_u} \times n_i) - (1-\beta)(n_i+1)(\overline{r_i} \times n_u)]$

Table 2: Reverse Estimation Functions to predict review rating (with bias) for new user-item pair

C.1 Case 1: User Tendency is positive and Product Tendency is positive ($\tau_u > 0, \tau_i > 0$)

Unbias Rating : $\widehat{r_{ui}}$ = max$(\overline{r_u} + \tau_i, \overline{r_i} + \tau_u)$

Reverse Function

1. If $\widehat{r_{ui}} = \overline{r_u} + \tau_i$ (new)
   $\tau_i = \widehat{r_{ui}} - \overline{r_u}$ ——————- -1
   And
   $\tau_i = \frac{\sum_{u \in U_i}(r_{ui} - \overline{r_u})}{n_{u+1}}$ —————- -2
   Now, equating 1 and 2
   $\widehat{r_{ui}} - \overline{r_u} = \frac{\sum_{u \in U_i}(r_{ui} - \overline{r_u})}{n_{u+1}}$
   $\widehat{r_{ui}} - \frac{\sum^{items} r_{ui(final)}}{n_{i+1}} = \frac{\sum^{users} r_{ui(final)} - \sum^{users} \overline{r_{u(final)}}}{n_{u+1}}$
   Where,
   n_{u+1}- number of users who have rated item (including the final user)
   n_{i+1}- numbers of items rated by the user (including the final item)
   $\sum^{items} r_{ui(final)}$ - sum of all the scores rated by the user u (final)
   $\sum^{users} r_{ui(final)}$ - sum of the scores rated by the users to the item i (final)
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_{u(final)}}}{n_u+1}$
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - [\sum^{users} \overline{r_{u(train)}} + \overline{r_{u(new)}}]}{n_u+1}$
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_{u(train)}} - \frac{\sum^{items} r_{ui(final)}}{n_i+1}}{n_u+1}$
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_{u(train)}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1}}{n_u+1}$
   $\Rightarrow \widehat{r_{ui}}(n_u+1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(n_u+1) r_{ui(new)}}{n_i+1} = \sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_{u(train)}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1}$

$\Rightarrow \widehat{r_{ui}}(n_u + 1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(n_u+1)r_{ui(new)}}{n_i+1} = \sum^{users} r_{ui(train)} - \sum^{users} \overline{r_{u(train)}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} + [\frac{(n_i+1)r_{ui(new)} - r_{ui(new)}}{n_i+1}]$

$\Rightarrow \widehat{r_{ui}}(n_u + 1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(n_u+1)r_{ui(new)}}{n_i+1} = \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) - \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{(n_i)r_{ui(new)}}{n_i+1}$

Reshuffling,

$\Rightarrow \frac{(n_i)r_{ui(new)}}{n_i+1} + \frac{(n_u+1)r_{ui(new)}}{n_i+1} = \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) + \widehat{r_{ui}}(n_u + 1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1}$

$\Rightarrow \frac{(n_i+n_u+1)r_{ui(new)}}{n_i+1} = \widehat{r_{ui}}(n_u + 1) - \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) - \frac{(n_u)\sum^{items} r_{ui(train)}}{n_i+1}$

$\Rightarrow r_{ui(new)} = (\frac{n_i+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u + 1) - \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) - \frac{(n_u)\sum^{items} r_{ui(train)}}{n_i+1}]$

$\Rightarrow r_{ui(new)} = (\frac{n_i+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u + 1) - (\overline{r_i} * n_u) + (\overline{r_i} * n_u) - (n_u * \tau_i) - \frac{(n_u)(\overline{r_u} * n_i)}{n_i+1}]$

$\Rightarrow r_{ui(new)} = (\frac{n_i+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u + 1) - (n_u * \tau_i) - \frac{(n_u)(\overline{r_u} * n_i)}{n_i+1}]$

$\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u + 1)(n_i + 1) - (n_i + 1)(n_u * \tau_i) - (n_i * n_u * \overline{r_u})]$

2. If $\widehat{r_{ui}} = \overline{r_i} + \tau_u$ (new)
$\tau_u = \widehat{r_{ui}} - \overline{r_i}$ ————————- -1
And
$\tau_u = \frac{\sum_{u \in I}(r_{ui} - \overline{r_i})}{n_{i+1}}$ —————- -2
Now, equating 1 and 2

$\widehat{r_{ui}} - \overline{r_i} = \frac{\sum_{u \in I}(r_{ui} - \overline{r_i})}{n_{i+1}}$

$\widehat{r_{ui}} - \frac{\sum^{users} r_{ui(final)}}{n_{u+1}} = \frac{\sum^{items} r_{ui(final)} - \sum^{items} \overline{r_{i(final)}}}{n_{i+1}}$

Where,
n_{u+1}- number of users who have rated item (including the final user)
n_{i+1}- numbers of items rated by the user (including the final item)
$\sum^{items} r_{ui(final)}$ - sum of all the scores rated by the user u (final)
$\sum^{users} r_{ui(final)}$ - sum of the scores rated by the users to the item i (final)

$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(final)}}}{n_i+1}$

$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - [\sum^{items} \overline{r_{i(train)}} + \overline{r_{i(new)}}]}{n_i+1}$

$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(final)}}{n_u+1}}{n_i+1}$

$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_u i(new)}{n_u+1}}{n_i+1}$

$\Rightarrow \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1}$

$\Rightarrow \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \sum^{items} r_{ui(train)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} + [\frac{(n_u+1)r_{ui(new)} - r_{ui(new)}}{n_u+1}]$

$\Rightarrow \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) - \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{(n_u)r_{ui(new)}}{n_u+1}$

Reshuffling,

$\Rightarrow \frac{(n_u)r_{ui(new)}}{n_u+1} + \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) + \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1}$

$\Rightarrow \frac{(n_i+n_u+1)r_{ui(new)}}{n_u+1} = \widehat{r_{ui}}(n_i + 1) - \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) - \frac{(n_i)\sum^{users} r_{ui(train)}}{n_u+1}$

$\Rightarrow r_{ui(new)} = (\frac{n_u+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_i+1) - \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) - \frac{(n_i)\sum^{users} r_{ui(train)}}{n_u+1}]$
$\Rightarrow r_{ui(new)} = (\frac{n_u+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_i+1) - (\overline{r_u} * n_i) + (\overline{r_u} * n_i) - (n_i * \tau_u) - \frac{(n_i)(\overline{r_i} * n_u)}{n_u+1}]$
$\Rightarrow r_{ui(new)} = (\frac{n_u+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_i+1) - (n_i * \tau_u) - \frac{(n_i)(\overline{r_i} * n_u)}{n_u+1}]$
$\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1)(n_i+1) - (n_u+1)(n_i * \tau_u) - (n_i * n_u * \overline{r_i})]$

C.2 Case 2: User Tendency is negative and Product Tendency is negative ($\tau_u < 0$, $\tau_i < 0$)

Unbias Rating : $\widehat{r_{ui}}$ = min($\overline{r_u} + \tau_i, \overline{r_i} + \tau_u$)

Reverse Function

1. If $\widehat{r_{ui}} = \overline{r_u} + \tau_i$ (new)
$\tau_i = \widehat{r_{ui}} - \overline{r_u}$ ————————- -1
And
$\tau_i = \frac{\sum_{u \in U_i}(r_{ui} - \overline{r_u})}{n_{u+1}}$ ——————- -2
Now, equating 1 and 2
$\widehat{r_{ui}} - \overline{r_u} = \frac{\sum_{u \in U_i}(r_{ui} - \overline{r_u})}{n_{u+1}}$
$\widehat{r_{ui}} - \frac{\sum^{items} r_{ui(final)}}{n_{i+1}} = \frac{\sum^{users} r_{ui(final)} - \sum^{users} \overline{r_{u(final)}}}{n_{u+1}}$
Where,
n_{u+1}- number of users who have rated item (including the final user)
n_{i+1}- numbers of items rated by the user (including the final item)
$\sum^{items} r_{ui(final)}$ - sum of all the scores rated by the user u (final)
$\sum^{users} r_{ui(final)}$ - sum of the scores rated by the users to the item i (final)
$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_{u(final)}}}{n_u+1}$
$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - [\sum^{users} \overline{r_u(train)} + \overline{r_{u(new)}}]}{n_u+1}$
$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_u(train)} - \frac{\sum^{items} r_{ui(final)}}{n_i+1}}{n_u+1}$
$\Rightarrow \widehat{r_{ui}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_u(train)} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1}}{n_u+1}$
$\Rightarrow \widehat{r_{ui}}(n_u+1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(n_u+1) r_{ui(new)}}{n_i+1} = \sum^{users} r_{ui(train)} + r_{ui(new)} - \sum^{users} \overline{r_{u(train)}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{r_{ui(new)}}{n_i+1}$
$\Rightarrow \widehat{r_{ui}}(n_u+1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(n_u+1) r_{ui(new)}}{n_i+1} = \sum^{users} r_{ui(train)} - \sum^{users} \overline{r_{u(train)}} - \frac{\sum^{items} r_{ui(train)}}{n_i+1} + [\frac{(n_i+1) r_{ui(new)} - r_{ui(new)}}{n_i+1}]$
$\Rightarrow \widehat{r_{ui}}(n_u+1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(n_u+1) r_{ui(new)}}{n_i+1} = \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) - \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{(n_i) r_{ui(new)}}{n_i+1}$
Reshuffling,
$\Rightarrow \frac{(n_i) r_{ui(new)}}{n_i+1} + \frac{(n_u+1) r_{ui(new)}}{n_i+1} = \frac{\sum^{items} r_{ui(train)}}{n_i+1} - \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) + \widehat{r_{ui}}(n_u+1) - \frac{(n_u+1)\sum^{items} r_{ui(train)}}{n_i+1}$
$\Rightarrow \frac{(n_i+n_u+1) r_{ui(new)}}{n_i+1} = \widehat{r_{ui}}(n_u+1) - \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) - \frac{(n_u)\sum^{items} r_{ui(train)}}{n_i+1}$
$\Rightarrow r_{ui(new)} = (\frac{n_i+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1) - \sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}) - \frac{(n_u)\sum^{items} r_{ui(train)}}{n_i+1}]$
$\Rightarrow r_{ui(new)} = (\frac{n_i+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1) - (\overline{r_i} * n_u) + (\overline{r_i} * n_u) - (n_u * \tau_i) - \frac{(n_u)(\overline{r_u} * n_i)}{n_i+1}]$
$\Rightarrow r_{ui(new)} = (\frac{n_i+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1) - (n_u * \tau_i) - \frac{(n_u)(\overline{r_u} * n_i)}{n_i+1}]$
$\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u+1)(n_i+1) - (n_i+1)(n_u * \tau_i) - (n_i * n_u * \overline{r_u})]$

2. If $\widehat{r_{ui}} = \overline{r_i} + \tau_u$ (new)
   $\tau_u = \widehat{r_{ui}} - \overline{r_i}$ ——————- -1
   And
   $\tau_u = \frac{\sum_{u \in I}(r_{ui} - \overline{r_i})}{n_{i+1}}$ —————- -2
   Now, equating 1 and 2
   $\widehat{r_{ui}} - \overline{r_i} = \frac{\sum_{u \in I}(r_{ui} - \overline{r_i})}{n_{i+1}}$
   $\widehat{r_{ui}} - \frac{\sum^{users} r_{ui(final)}}{n_{u+1}} = \frac{\sum^{items} r_{ui(final)} - \sum^{items} \overline{r_{i(final)}}}{n_{i+1}}$
   Where,
   n_{u+1}- number of users who have rated item (including the final user)
   n_{i+1}- numbers of items rated by the user (including the final item)
   $\sum^{items} r_{ui(final)}$ - sum of all the scores rated by the user u (final)
   $\sum^{users} r_{ui(final)}$ - sum of the scores rated by the users to the item i (final)
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(final)}}}{n_i+1}$
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - [\sum^{items} \overline{r_{i(train)}} + \overline{r_{i(new)}}]}{n_i+1}$
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(final)}}{n_u+1}}{n_i+1}$
   $\Rightarrow \widehat{r_{ui}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_u i(new)}{n_u+1}}{n_i+1}$
   $\Rightarrow \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \sum^{items} r_{ui(train)} + r_{ui(new)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{r_{ui(new)}}{n_u+1}$
   $\Rightarrow \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \sum^{items} r_{ui(train)} - \sum^{items} \overline{r_{i(train)}} - \frac{\sum^{users} r_{ui(train)}}{n_u+1} + [\frac{(n_u+1)r_{ui(new)} - r_{ui(new)}}{n_u+1}]$
   $\Rightarrow \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) - \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{(n_u)r_{ui(new)}}{n_u+1}$
   Reshuffling,
   $\Rightarrow \frac{(n_u)r_{ui(new)}}{n_u+1} + \frac{(n_i+1)r_{ui(new)}}{n_u+1} = \frac{\sum^{users} r_{ui(train)}}{n_u+1} - \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) + \widehat{r_{ui}}(n_i + 1) - \frac{(n_i+1)\sum^{users} r_{ui(train)}}{n_u+1}$
   $\Rightarrow \frac{(n_i+n_u+1)r_{ui(new)}}{n_u+1} = \widehat{r_{ui}}(n_i + 1) - \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) - \frac{(n_i)\sum^{users} r_{ui(train)}}{n_u+1}$
   $\Rightarrow r_{ui(new)} = (\frac{n_u+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_i + 1) - \sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}) - \frac{(n_i)\sum^{users} r_{ui(train)}}{n_u+1}]$
   $\Rightarrow r_{ui(new)} = (\frac{n_u+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_i + 1) - (\overline{r_u} * n_i) + (\overline{r_u} * n_i) - (n_i * \tau_u) - \frac{(n_i)(\overline{r_i} * n_u)}{n_u+1}]$
   $\Rightarrow r_{ui(new)} = (\frac{n_u+1}{n_i+n_u+1})[\widehat{r_{ui}}(n_i + 1) - (n_i * \tau_u) - \frac{(n_i)(\overline{r_i} * n_u)}{n_u+1}]$
   $\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[\widehat{r_{ui}}(n_u + 1)(n_i + 1) - (n_u + 1)(n_i * \tau_u) - (n_i * n_u * \overline{r_i})]$

C.3 Case 3: User Tendency is negative and Product Tendency is Positive; User mean $<$ Item mean ($\tau_u < 0, \tau_i > 0, \overline{r_u} < \overline{r_i}\ \widehat{r_{ui}}$)

Unbias Rating : $\widehat{r_{ui}}$ = min(max($\overline{r_u}, (\overline{r_i} + \tau_u)\beta + (\overline{r_u} + \tau_i)(1 - \beta)), \overline{r_i}$)

Reverse Function

1. If $\widehat{r_{ui}} = \overline{r_{u(final)}}$
   $\Rightarrow \widehat{r_{ui}} = \frac{\sum^{items} r_{ui(final)}}{n_i+1}$
   $\Rightarrow \widehat{r_{ui}} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1}$
   $\Rightarrow (n_i + 1)\widehat{r_{ui}} = \sum^{items} r_{ui(train)} + r_{ui(new)}$

$\Rightarrow r_{ui(new)} = (n_i + 1)\widehat{r_{ui}} - \sum^{items} r_{ui(train)}$

$\Rightarrow r_{ui(new)} = (n_i + 1)\widehat{r_{ui}} - (\overline{r_u} * n_i)$

2. If $\widehat{r_{ui}} = (\overline{r_i} + \tau_u)\beta + (\overline{r_u} + \tau_i)(1 - \beta)$

$$\widehat{r_{ui}} = (\frac{\sum^{users} r_{ui(final)}}{n_u+1} + \frac{\sum^{items} r_{ui(final)} - \sum^{items} \overline{r_{i(final)}}}{n_i+1})\beta + (\frac{\sum^{items} r_{ui(final)}}{n_i+1} + \frac{\sum^{users} r_{ui(final)} - \sum^{users} \overline{r_{u(finsl)}}}{n_u+1})(1 - \beta)$$

Break it into 2 parts

I. $(\frac{\sum^{users} r_{ui(final)}}{n_u+1} + \frac{\sum^{items} r_{ui(final)} - \sum^{items} \overline{r_{i(final)}}}{n_i+1})\beta$

$= (\frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(final)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(final)}}}{n_i+1})\beta$

$= (\frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(train)}}}{n_i+1} - \frac{\overline{r_{i(new)}}}{n_i+1})\beta$

$= (\frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(train)}}}{n_i+1} - \frac{\frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1}}{n_i+1})\beta$

$= (\frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(train)}}}{n_i+1} - \frac{\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

$= ([\frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)}] + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

$= (\frac{(n_i) \sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{((n_i+1)+(n_u+1)-1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

$= (\frac{(n_i) \sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

II. $(\frac{\sum^{items} r_{ui(final)}}{n_i+1} + \frac{\sum^{users} r_{ui(final)} - \sum^{users} \overline{r_{u(final)}}}{n_u+1})(1 - \beta)$

$= (\frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(final)}}{n_u+1} - \frac{\sum^{users} \overline{r_{u(final)}}}{n_u+1})(1 - \beta)$

$= (\frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{\sum^{users} \overline{r_{u(train)}}}{n_u+1} - \frac{\overline{r_{u(new)}}}{n_u+1})(1 - \beta)$

$= (\frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{\sum^{users} \overline{r_{u(train)}}}{n_u+1} - \frac{\frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1}}{n_u+1})(1 - \beta)$

$= (\frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{\sum^{users} \overline{r_{u(train)}}}{n_u+1} - \frac{\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})(1 - \beta)$

$= ([\frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)}] + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})(1 - \beta)$

$= (\frac{(n_u) \sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{((n_i+1)+(n_u+1)-1) r_{ui(new)}}{(n_u+1)(n_i+1)})(1 - \beta)$

$= (\frac{(n_u) \sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})(1 - \beta)$

Now,

$$\Rightarrow \widehat{r_{ui}} = (\frac{(n_i) \sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta + (\frac{(n_u) \sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})(1 - \beta)$$

$$\Rightarrow \widehat{r_{ui}} = (\frac{(n_i) \sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta + (\frac{(n_u) \sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)}) - (\frac{(n_u) \sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$$

Reshuffling

$\Rightarrow \frac{(n_i+n_u+1)r_{ui(new)}}{(n_u+1)(n_i+1)} = \widehat{r_{ui}} - \frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)}(1-\beta) - \frac{\sum^{users}(r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1}(1-\beta) - \frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)}\beta - \frac{\sum^{items}(r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1}\beta$

$\Rightarrow r_{ui(new)} = (\frac{(n_i+1)(n_u+1)}{n_i+n_u+1})[\widehat{r_{ui}} - \frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)}(1-\beta) - \frac{\sum^{users}(r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1}(1-\beta) - \frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)}\beta - \frac{\sum^{items}(r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1}\beta]$

$\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[(n_u+1)(n_i+1)\widehat{r_{ui}} - ((n_u)(1-\beta)\sum^{items} r_{ui(train)}) - ((n_i+1)(1-\beta)\sum^{users}(r_{ui(train)} - \overline{r_{u(train)}})) - \beta(n_i)\sum^{users} r_{ui(train)} - (\beta(n_u+1)\sum^{items}(r_{ui(train)} - \overline{r_{i(train)}}))]$

$\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[(n_u+1)(n_i+1)\widehat{r_{ui}} - ((n_u)(\overline{r_u} * n_i)(1-\beta)) - ((n_i+1)(1-\beta)(n_u * \tau_i)) - \beta(n_i)(\overline{r_i} * n_u) - (\beta(n_u+1)(n_i * \tau_u))]$

3. If $\widehat{r_{ui}} = \overline{r_{i(final)}}$

$\Rightarrow \widehat{r_{ui}} = \frac{\sum^{users} r_{ui(final)}}{n_u+1}$

$\Rightarrow \widehat{r_{ui}} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1}$

$\Rightarrow (n_u+1)\widehat{r_{ui}} = \sum^{users} r_{ui(train)} + r_{ui(new)}$

$\Rightarrow r_{ui(new)} = (n_u+1)\widehat{r_{ui}} - \sum^{users} r_{ui(train)}$

$\Rightarrow r_{ui(new)} = (n_u+1)\widehat{r_{ui}} - (\overline{r_i} * n_u)$

C.4 Case 4: User Tendency is negative and Product Tendency is Positive; User mean > Item mean ($\tau_u < 0$, $\tau_i > 0$, $\overline{r_u} > \overline{r_i}\ \widehat{r_{ui}}$)

Unbias Rating : $\widehat{r_{ui}} = \overline{r_i}\beta + \overline{r_u}(1-\beta)$

Reverse functions

$\widehat{r_{ui}} = \overline{r_i}\beta + \overline{r_u}(1-\beta)$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{users} r_{ui(final)}}{n_u+1} + \frac{(1-\beta)\sum^{items} r_{ui(final)}}{n_i+1}$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{users} r_{ui(train)}}{n_u+1} + \frac{(\beta)r_{ui(new)}}{n_u+1} + \frac{(1-\beta)\sum^{items} r_{ui(train)}}{n_i+1} + \frac{(1-\beta)r_{ui(new)}}{n_i+1}$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{users} r_{ui(train)}}{n_u+1} + \frac{(1-\beta)\sum^{items} r_{ui(train)}}{n_i+1} + r_{ui(new)}[\frac{\beta}{n_u+1} + \frac{1-\beta}{n_i+1}]$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{users} r_{ui(train)}}{n_u+1} + \frac{(1-\beta)\sum^{items} r_{ui(train)}}{n_i+1} + r_{ui(new)}[\frac{\beta n_i+\beta+n_u+1-\beta n_u-\beta}{(n_u+1)(n_i+1)}]$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{users} r_{ui(train)}}{n_u+1} + \frac{(1-\beta)\sum^{items} r_{ui(train)}}{n_i+1} + r_{ui(new)}[\frac{n_u(1-\beta)+(1+\beta n_i)}{(n_u+1)(n_i+1)}]$

Reshuffling,

$\Rightarrow r_{ui(new)}[\frac{n_u(1-\beta)+1+\beta n_i}{(n_u+1)(n_i+1)}] = \widehat{r_{ui}} - \frac{\beta\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(1-\beta)\sum^{items} r_{ui(train)}}{n_i+1}$

$\Rightarrow r_{ui(new)} = [\frac{(n_u+1)(n_i+1)}{n_u(1-\beta)+(1+\beta n_i)}](\widehat{r_{ui}} - \frac{\beta\sum^{users} r_{ui(train)}}{n_u+1} - \frac{(1-\beta)\sum^{items} r_{ui(train)}}{n_i+1})$

$\Rightarrow r_{ui(new)} = [\frac{1}{n_u(1-\beta)+(1+\beta n_i)}]((n_u+1)(n_i+1)\widehat{r_{ui}} - \beta(n_i+1)\sum^{users} r_{ui(train)} - (1-\beta)(n_u+1)\sum^{items} r_{ui(train)})$

$\Rightarrow r_{ui(new)} = [\frac{1}{n_u(1-\beta)+(1+\beta n_i)}]((n_u+1)(n_i+1)\widehat{r_{ui}} - \beta(n_i+1)(\overline{r_i} * n_u) - (1-\beta)(n_u+1)(\overline{r_u} * n_i))$

C.5 Case 5: User Tendency is Positive and Product Tendency is Negative; User mean > Item mean ($\tau_u > 0$, $\tau_i < 0$, $\overline{r_u} > \overline{r_i}\ \widehat{r_{ui}}$)

Unbias Rating : $\widehat{r_{ui}}$ = min(max($\overline{r_i}, (\overline{r_u} + \tau_i)\beta + (\overline{r_i} + \tau_u)(1-\beta)), \overline{r_u}$)

Reverse Functions

1. If $\widehat{r_{ui}} = \overline{r_{i(final)}}$

$\Rightarrow \widehat{r_{ui}} = \frac{\sum^{users} r_{ui(final)}}{n_u+1}$

$\Rightarrow \widehat{r_{ui}} = \frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1}$

$\Rightarrow (n_u+1)\widehat{r_{ui}} = \sum^{users} r_{ui(train)} + r_{ui(new)}$

$\Rightarrow r_{ui(new)} = (n_u+1)\widehat{r_{ui}} - \sum^{users} r_{ui(train)}$

$\Rightarrow r_{ui(new)} = (n_i+1)\widehat{r_{ui}} - (\overline{r_i} * n_u)$

2. If $\widehat{r_{ui}} = (\overline{r_u} + \tau_i)\beta + (\overline{r_i} + \tau_u)(1-\beta)$

$\widehat{r_{ui}} = (\frac{\sum^{items} r_{ui(final)}}{n_i+1} + \frac{\sum^{users} r_{ui(final)} - \sum^{users} \overline{r_{u(final)}}}{n_u+1})\beta + (\frac{\sum^{users} r_{ui(final)}}{n_u+1} + \frac{\sum^{items} r_{ui(final)} - \sum^{items} \overline{r_{i(final)}}}{n_i+1})(1-\beta)$

Break it into 2 parts

I. $(\frac{\sum^{items} r_{ui(final)}}{n_i+1} + \frac{\sum^{users} r_{ui(final)} - \sum^{users} \overline{r_{u(final)}}}{n_u+1})\beta$

$= (\frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(final)}}{n_u+1} - \frac{\sum^{users} \overline{r_{u(final)}}}{n_u+1})\beta$

$= (\frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{\sum^{users} \overline{r_{u(train)}}}{n_u+1} - \frac{\overline{r_{u(new)}}}{n_u+1})\beta$

$= (\frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{\sum^{items} \overline{r_{u(train)}}}{n_u+1} - \frac{\frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1}}{n_u+1})\beta$

$= (\frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{\sum^{users} \overline{r_{u(train)}}}{n_u+1} - \frac{\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

$= ([\frac{\sum^{items} r_{ui(train)}}{n_i+1} - \frac{\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)}] + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{r_{ui(new)}}{n_i+1} + \frac{r_{ui(new)}}{n_u+1} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

$= (\frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{((n_u+1)+(n_i+1)-1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

$= (\frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

II. $(\frac{\sum^{users} r_{ui(final)}}{n_u+1} + \frac{\sum^{items} r_{ui(final)} - \sum^{items} \overline{r_{i(final)}}}{n_i+1})\beta$

$= (\frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(final)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(final)}}}{n_i+1})(1-\beta)$

$= (\frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(train)}}}{n_i+1} - \frac{\overline{r_{i(new)}}}{n_i+1})(1-\beta)$

$= (\frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(train)}}}{n_i+1} - \frac{\frac{\sum^{users} r_{ui(train)} + r_{ui(new)}}{n_u+1}}{n_i+1})(1-\beta)$

$= (\frac{\sum^{users} r_{ui(train)}}{n_u+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{\sum^{items} r_{ui(train)}}{n_i+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{\sum^{items} \overline{r_{i(train)}}}{n_i+1} - \frac{\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})(1-\beta)$

$= ([\frac{\sum^{users} r_{ui(train)}}{n_u+1} - \frac{\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)}] + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{r_{ui(new)}}{n_u+1} + \frac{r_{ui(new)}}{n_i+1} - \frac{r_{ui(new)}}{(n_u+1)(n_i+1)})(1-\beta)$

$= (\frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{((n_i+1)+(n_u+1)-1) r_{ui(new)}}{(n_u+1)(n_i+1)})(1-\beta)$

$= (\frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})(1-\beta)$

Now,

$\Rightarrow \widehat{r_{ui}} = (\frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta + (\frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})(1-\beta)$

$\Rightarrow \widehat{r_{ui}} = (\frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{users} (r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)})\beta + (\frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} + \frac{(n_i+n_u+1) r_{ui(new)}}{(n_u+1)(n_i+1)}) - (\frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)} + \frac{\sum^{items} (r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1} +$

$\frac{(n_i+n_u+1)r_{ui(new)}}{(n_u+1)(n_i+1)})\beta$

Reshuffling

$\Rightarrow \frac{(n_i+n_u+1)r_{ui(new)}}{(n_u+1)(n_i+1)} = \widehat{r_{ui}} - \frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)}(1-\beta) - \frac{\sum^{items}(r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1}(1-\beta) - \frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)}\beta - \frac{\sum^{users}(r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1}\beta$

$\Rightarrow r_{ui(new)} = (\frac{(n_i+1)(n_u+1)}{n_i+n_u+1})[\widehat{r_{ui}} - \frac{(n_i)\sum^{users} r_{ui(train)}}{(n_u+1)(n_i+1)}(1-\beta) - \frac{\sum^{items}(r_{ui(train)} - \overline{r_{i(train)}})}{n_i+1}(1-\beta) - \frac{(n_u)\sum^{items} r_{ui(train)}}{(n_u+1)(n_i+1)}\beta - \frac{\sum^{users}(r_{ui(train)} - \overline{r_{u(train)}})}{n_u+1}\beta]$

$\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[(n_u+1)(n_i+1)\widehat{r_{ui}} - ((n_i)(1-\beta)\sum^{users} r_{ui(train)}) - ((n_u+1)(1-\beta)\sum^{items}(r_{ui(train)} - \overline{r_{i(train)}})) - \beta(n_u)\sum^{items} r_{ui(train)} - (\beta(n_i+1)\sum^{users}(r_{ui(train)} - \overline{r_{u(train)}}))]$

$\Rightarrow r_{ui(new)} = (\frac{1}{n_i+n_u+1})[(n_u+1)(n_i+1)\widehat{r_{ui}} - ((n_i)(\overline{r_i} * n_u)(1-\beta)) - ((n_u+1)(1-\beta)(n_i * \tau_u)) - \beta(n_u)(\overline{r_u} * n_i) - (\beta(n_i+1)(n_u * \tau_i))]$

3. If $\widehat{r_{ui}} = \overline{r_{u(final)}}$

$\Rightarrow \widehat{r_{ui}} = \frac{\sum^{items} r_{ui(final)}}{n_i+1}$

$\Rightarrow \widehat{r_{ui}} = \frac{\sum^{items} r_{ui(train)} + r_{ui(new)}}{n_i+1}$

$\Rightarrow (n_i+1)\widehat{r_{ui}} = \sum^{items} r_{ui(train)} + r_{ui(new)}$

$\Rightarrow r_{ui(new)} = (n_i+1)\widehat{r_{ui}} - \sum^{items} r_{ui(train)}$

$\Rightarrow r_{ui(new)} = (n_i+1)\widehat{r_{ui}} - (\overline{r_u} * n_i)$

C.6 Case 6: User Tendency is Positive and Product Tendency is Negative; User mean < Item mean ($\tau_u > 0$, $\tau_i < 0$, $\overline{r_u} < \overline{r_i}$ $\widehat{r_{ui}}$)

Unbias Rating : $\widehat{r_{ui}} = \overline{r_u}\beta + \overline{r_i}(1-\beta)$

Reverse functions

$\widehat{r_{ui}} = \overline{r_u}\beta + \overline{r_i}(1-\beta)$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{items} r_{ui(final)}}{n_i+1} + \frac{(1-\beta)\sum^{users} r_{ui(final)}}{n_u+1}$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{items} r_{ui(train)}}{n_i+1} + \frac{(\beta)r_{ui(new)}}{n_i+1} + \frac{(1-\beta)\sum^{users} r_{ui(train)}}{n_u+1} + \frac{(1-\beta)r_{ui(new)}}{n_u+1}$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{items} r_{ui(train)}}{n_i+1} + \frac{(1-\beta)\sum^{users} r_{ui(train)}}{n_u+1} + r_{ui(new)}[\frac{\beta}{n_i+1} + \frac{1-\beta}{n_u+1}]$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{items} r_{ui(train)}}{n_i+1} + \frac{(1-\beta)\sum^{users} r_{ui(train)}}{n_u+1} + r_{ui(new)}[\frac{\beta n_u + \beta + n_i + 1 - \beta n_i - \beta}{(n_u+1)(n_i+1)}]$

$\Rightarrow \widehat{r_{ui}} = \frac{\beta\sum^{items} r_{ui(train)}}{n_i+1} + \frac{(1-\beta)\sum^{users} r_{ui(train)}}{n_u+1} + r_{ui(new)}[\frac{n_i(1-\beta)+(1+\beta n_u)}{(n_u+1)(n_i+1)}]$

Reshuffling,

$\Rightarrow r_{ui(new)}[\frac{n_i(1-\beta)+1+\beta n_u}{(n_u+1)(n_i+1)}] = \widehat{r_{ui}} - \frac{\beta\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(1-\beta)\sum^{users} r_{ui(train)}}{n_u+1}$

$\Rightarrow r_{ui(new)} = [\frac{(n_u+1)(n_i+1)}{n_i(1-\beta)+(1+\beta n_u)}](\widehat{r_{ui}} - \frac{\beta\sum^{items} r_{ui(train)}}{n_i+1} - \frac{(1-\beta)\sum^{users} r_{ui(train)}}{n_u+1})$

$\Rightarrow r_{ui(new)} = [\frac{1}{n_i(1-\beta)+(1+\beta n_u)}]((n_u+1)(n_i+1)\widehat{r_{ui}} - \beta(n_u+1)\sum^{items} r_{ui(train)} - (1-\beta)(n_i+1)\sum^{users} r_{ui(train)})$

$\Rightarrow r_{ui(new)} = [\frac{1}{n_i(1-\beta)+(1+\beta n_u)}]((n_u+1)(n_i+1)\widehat{r_{ui}} - \beta(n_u+1)(\overline{r_u} * n_i) - (1-\beta)(n_i+1)(\overline{r_i} * n_u))$