



PES University, Bangalore
(Established under Karnataka Act No. 16 of 2013)
APRIL 2021: IN SEMESTER ASSESSMENT (ISA) B.TECH. IV SEMESTER
UE19MA251- LINEAR ALGEBRA
Mathematics Lab
Session: Jan-May 2021

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SRN : PES2UG19CS301

BRANCH : COMPUTER SCIENCE

SEMESTER & SECTION : SEMESTER IV SECTION E

FOR OFFICE USE ONLY:

Marks : /05

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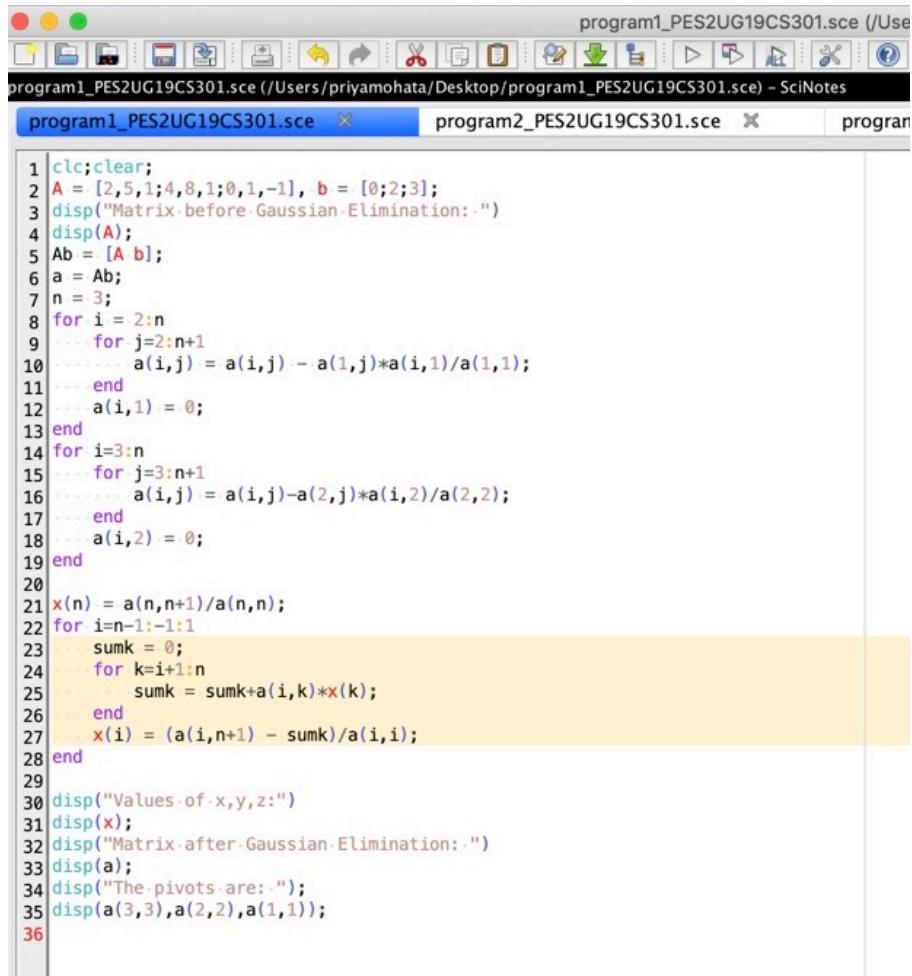
Signature of the Course Instructor :

Class Number	Topic
1	Gaussian Elimination
2	The LU Decomposition
3	Inverse of a Matrix by the Gauss- Jordan Method
4	The Span of Column Space of a Matrix
5	The Four Fundamental Subspaces
6	Projections by Least Squares
7	The Gram-Schmidt Orthogonalization
8	Eigen values and Eigen Vectors of a Matrix

PROBLEM 1: Solve the following system of equations by Gaussian Elimination. Identify the pivots.

$$2x + 5y + z = 0, \quad 4x + 8y + z = 2, \quad y - z = 3$$

PROGRAM CODE

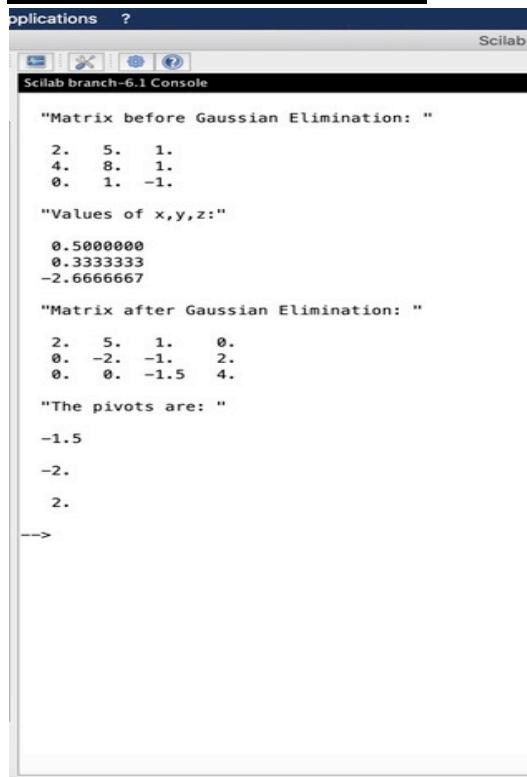


```

1 clc;clear;
2 A = [2,5,1;4,8,1;0,1,-1];
3 disp('Matrix-before-Gaussian-Elimination:');
4 disp(A);
5 Ab = [A b];
6 a = Ab;
7 n = 3;
8 for i = 2:n
9     for j=2:n+1
10        a(i,j) = a(i,j) - a(1,j)*a(i,1)/a(1,1);
11    end
12    a(i,1) = 0;
13 end
14 for i=3:n
15     for j=3:n+1
16        a(i,j) = a(i,j)-a(2,j)*a(i,2)/a(2,2);
17    end
18    a(i,2) = 0;
19 end
20
21 x(n) = a(n,n+1)/a(n,n);
22 for i=n-1:-1:1
23     sumk = 0;
24     for k=i+1:n
25         sumk = sumk+a(i,k)*x(k);
26     end
27     x(i) = (a(i,n+1) - sumk)/a(i,i);
28 end
29
30 disp("Values-of-x,y,z:")
31 disp(x);
32 disp("Matrix-after-Gaussian-Elimination:");
33 disp(a);
34 disp("The-pivots-are:");
35 disp(a(3,3),a(2,2),a(1,1));
36

```

OUTPUT SCREENSHOT



The screenshot shows the Scilab 6.1 Console window. The title bar says "Scilab" and the window title is "Scilab branch-6.1 Console". The console output is as follows:

```
Matrix before Gaussian Elimination:
2. 5. 1.
4. 8. 1.
0. 1. -1.

Values of x,y,z:
0.5000000
0.3333333
-2.6666667

Matrix after Gaussian Elimination:
2. 5. 1. 0.
0. -2. -1. 2.
0. 0. -1.5 4.

The pivots are:
-1.5
-2.
2.

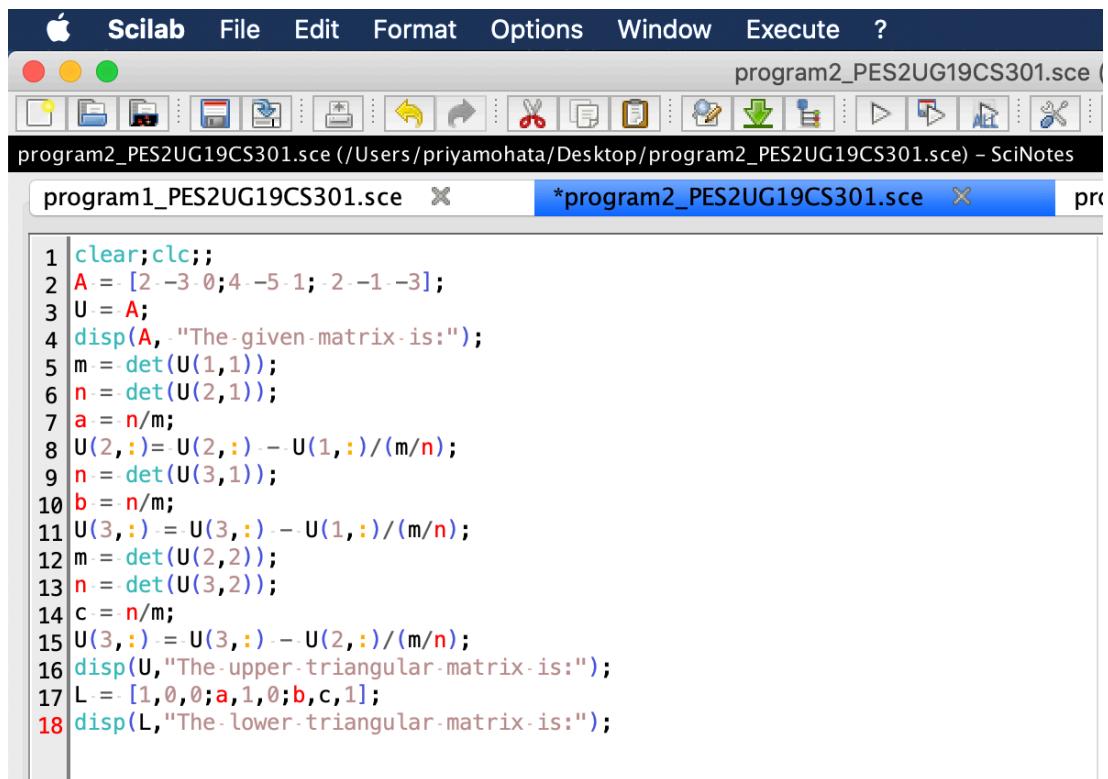
-->
```

Problem 2:

Factorize the following matrices as $A = LU$

$$A = \{[2, -3, 0], [4, -5, 1], [2, -1, -3]\}$$

PROGRAM CODE



```

1 clear;clc;
2 A = [2 -3 0;4 -5 1;2 -1 -3];
3 U = A;
4 disp(A, "The given matrix is:");
5 m = det(U(1,1));
6 n = det(U(2,1));
7 a = n/m;
8 U(2,:) = U(2,:)-U(1,:)/(m/n);
9 b = n/m;
10 U(3,:) = U(3,:)-U(1,:)/(m/n);
11 m = det(U(2,2));
12 n = det(U(3,2));
13 c = n/m;
14 U(3,:) = U(3,:)-U(2,:)/(m/n);
15 disp(U,"The upper triangular matrix is:");
16 L = [1,0,0;a,1,0;b,c,1];
17 disp(L,"The lower triangular matrix is:");

```

OUTPUT SCREENSHOT



Scilab branch-6.1 Console

```
2. -3.  0.
4. -5.  1.
2. -1. -3.

"The given matrix is:"
```

```
2. -3.  0.
0.  1.  1.
0.  0. -5.

"The upper triangular matrix is:"
```

```
1.  0.  0.
2.  1.  0.
1.  2.  1.

"The lower triangular matrix is:"
```

```
--> |
```

The screenshot shows a Scilab console window titled "Scilab branch-6.1 Console". The window displays the following text output:

```
2. -3.  0.
4. -5.  1.
2. -1. -3.

"The given matrix is:"
```

```
2. -3.  0.
0.  1.  1.
0.  0. -5.

"The upper triangular matrix is:"
```

```
1.  0.  0.
2.  1.  0.
1.  2.  1.

"The lower triangular matrix is:"
```

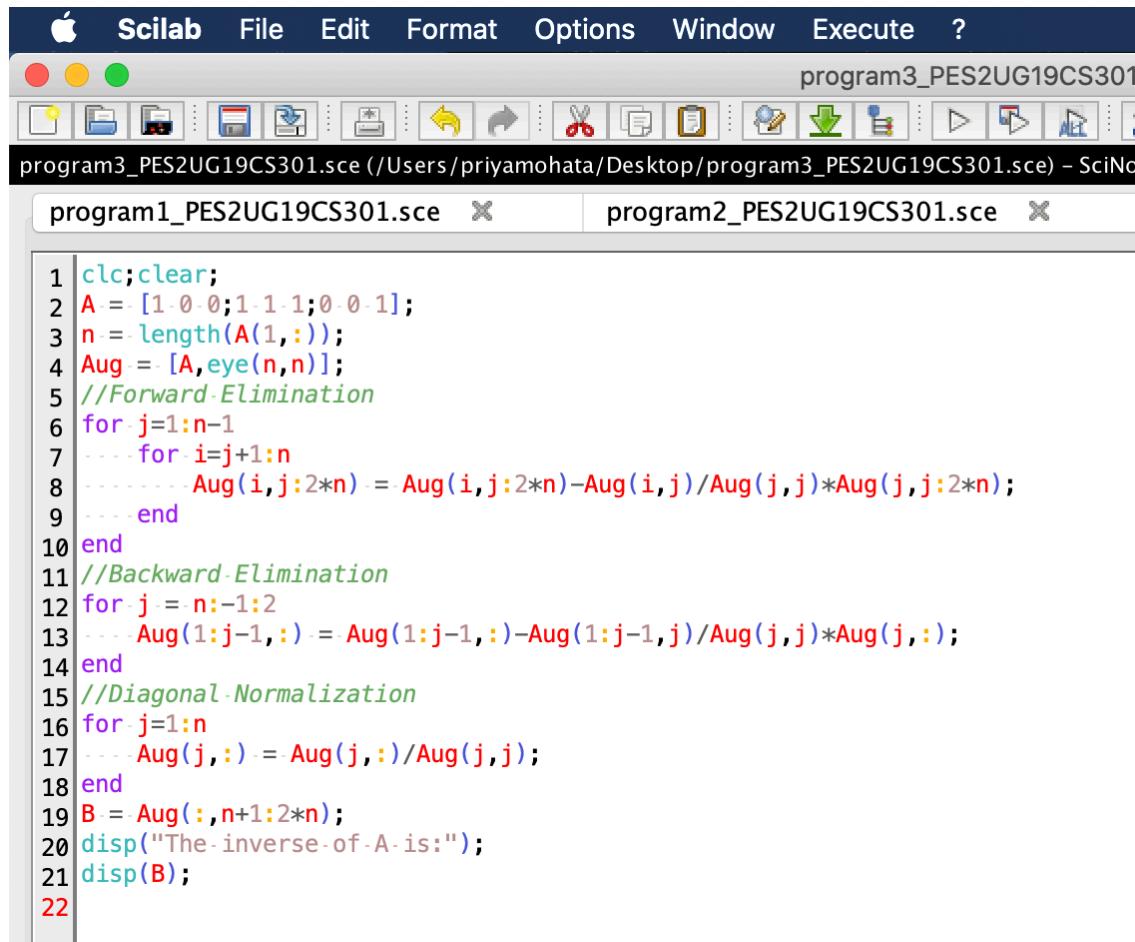
```
--> |
```

PROBLEM-3

Find the inverse of the following matrix

$$A = \{[1,0,0], [1,1,1], [0,0,1]\}$$

PROGRAM CODE



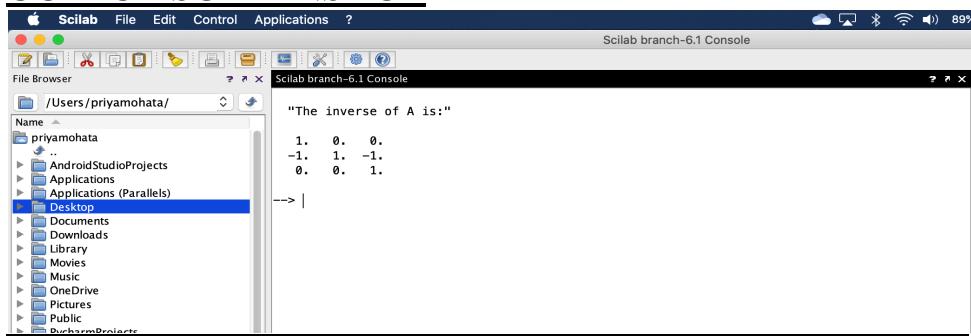
The screenshot shows the Scilab software interface. The menu bar includes Scilab, File, Edit, Format, Options, Window, Execute, and ?. The toolbar below the menu bar contains various icons for file operations like open, save, and print. The window title is "program3_PES2UG19CS301". Below the title, there are tabs for "program1_PES2UG19CS301.sce" and "program2_PES2UG19CS301.sce". The main code area contains the following Scilab script:

```

1 clc;clear;
2 A=[1 0 0;1 1 1;0 0 1];
3 n=length(A(:,1));
4 Aug=[A,eye(n,n)];
5 //Forward-Elimination
6 for j=1:n-1
7   for i=j+1:n
8     Aug(i,j:2*n)=Aug(i,j:2*n)-Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
9   end
10 end
11 //Backward-Elimination
12 for j=n:-1:2
13   Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
14 end
15 //Diagonal-Normalization
16 for j=1:n
17   Aug(j,:)=Aug(j,:)/Aug(j,j);
18 end
19 B=Aug(:,n+1:2*n);
20 disp("The inverse of A is:");
21 disp(B);
22

```

OUTPUT SCREENSHOT



The screenshot shows the Scilab 6.1 Console window. The title bar reads "Scilab branch-6.1 Console". The console window displays the following text:

```
"The inverse of A is:"  
1. 0. 0.  
-1. 1. -1.  
0. 0. 1.
```

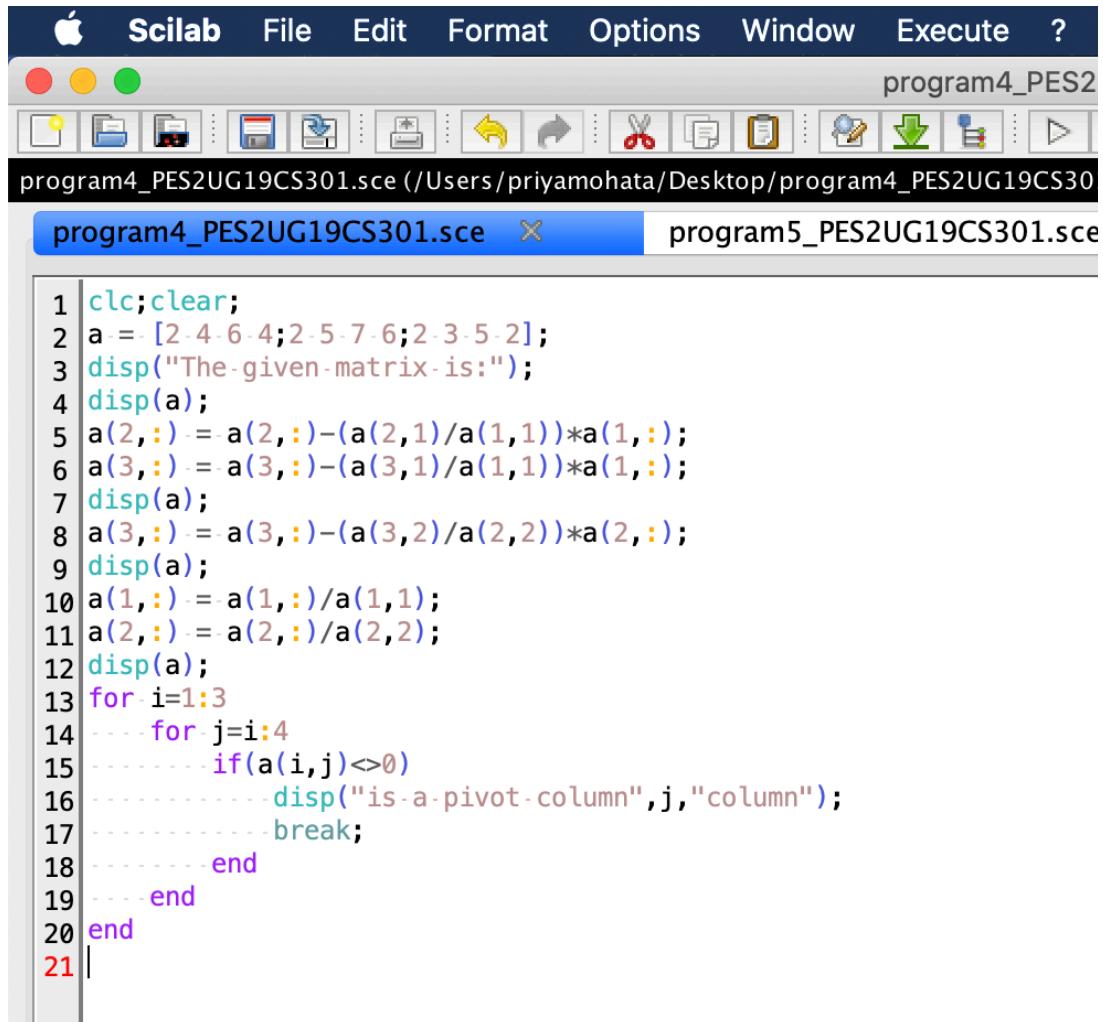
On the left, there is a "File Browser" window showing the directory structure of the user's home directory. The "Desktop" folder is selected.

PROBLEM 4

Identify the columns that are in the column space of A where

$$A = \{[2,4,6,4], [2,5,7,6], [2,3,5,2]\}$$

PROGRAM CODE



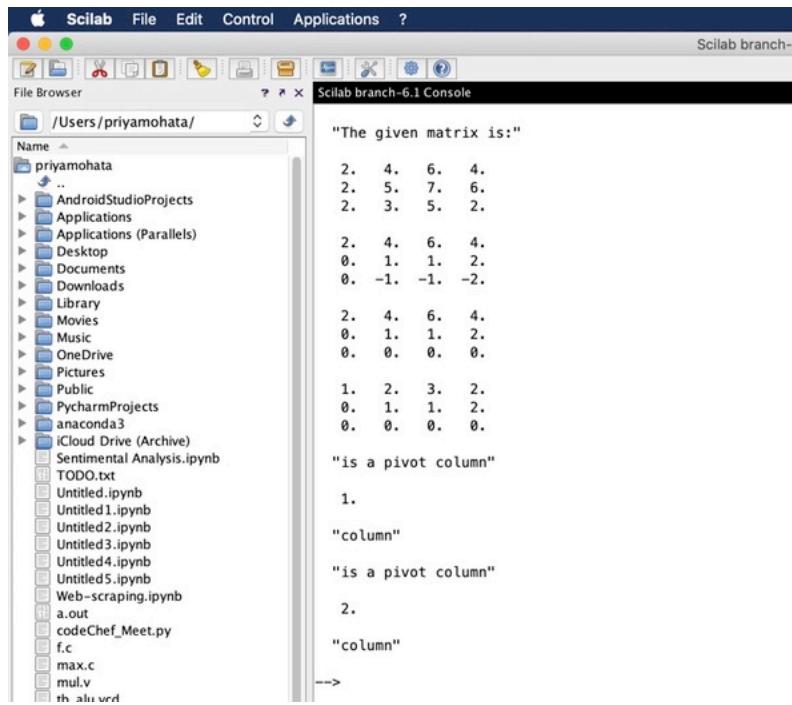
The screenshot shows the Scilab software interface. The menu bar includes Scilab, File, Edit, Format, Options, Window, Execute, and ?. The toolbar below the menu bar contains various icons for file operations like Open, Save, Print, and Copy. The title bar shows 'program4_PES2' and the current file path: 'program4_PES2UG19CS301.sce (/Users/priyamohata/Desktop/program4_PES2UG19CS301.sce)'. Below the title bar, there are two tabs: 'program4_PES2UG19CS301.sce' (active) and 'program5_PES2UG19CS301.sce'. The main workspace displays the following Scilab code:

```

1 clc;clear;
2 a = [2 4 6 4;2 5 7 6;2 3 5 2];
3 disp("The given matrix is:");
4 disp(a);
5 a(2,:) = a(2,:)-(a(2,1)/a(1,1))*a(1,:);
6 a(3,:) = a(3,:)-(a(3,1)/a(1,1))*a(1,:);
7 disp(a);
8 a(3,:) = a(3,:)-(a(3,2)/a(2,2))*a(2,:);
9 disp(a);
10 a(1,:) = a(1,:)/a(1,1);
11 a(2,:) = a(2,:)/a(2,2);
12 disp(a);
13 for i=1:3
14   for j=i:4
15     if(a(i,j)<>0)
16       disp("is a pivot column",j,"column");
17       break;
18     end
19   end
20 end
21

```

OUTPUT SCREENSHOT



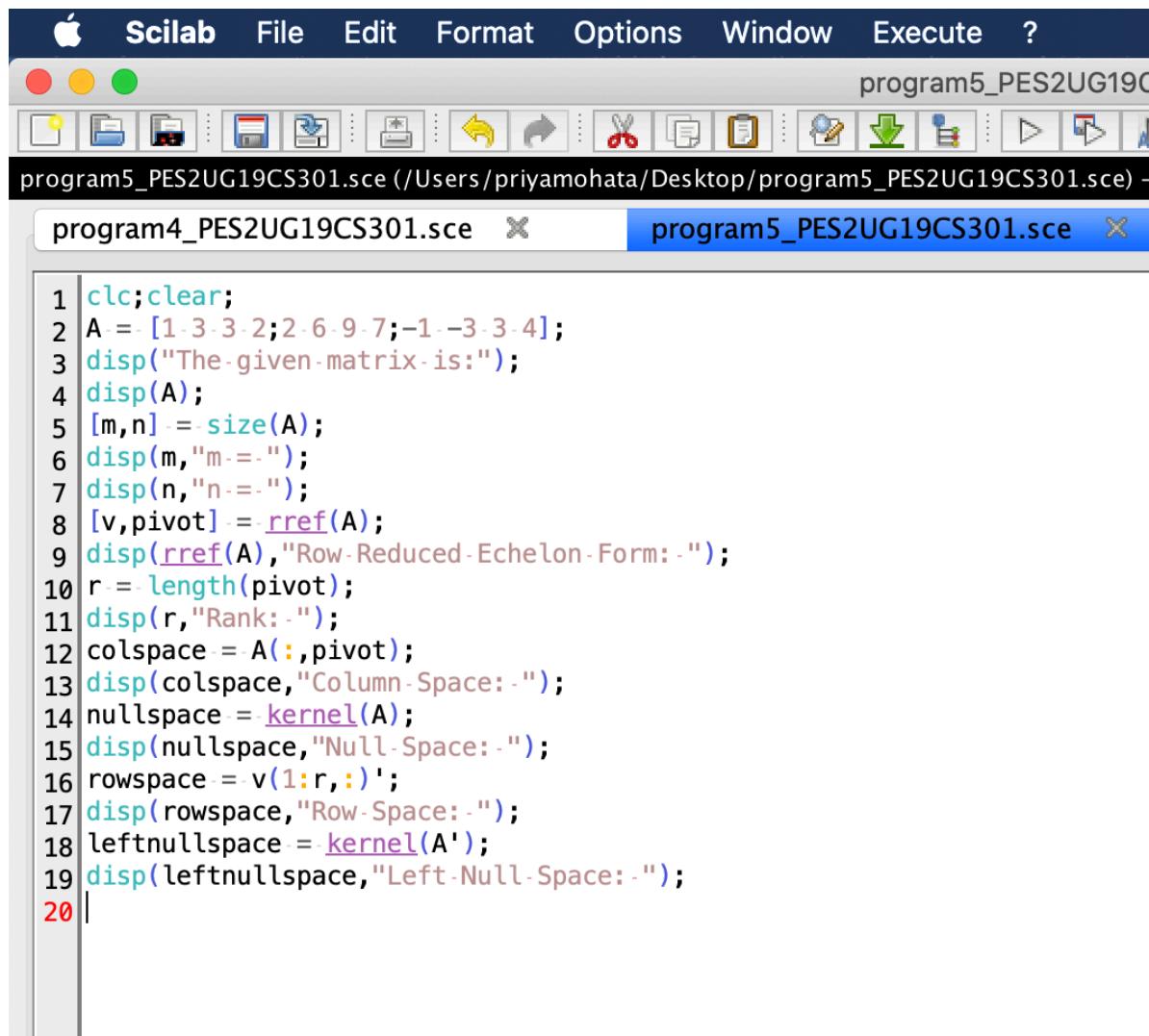
The screenshot shows the Scilab 6.1 Console window. The file browser on the left shows the directory `/Users/priyamohata/` with various files and folders. The console window displays the following text:

```
"The given matrix is:"  
2. 4. 6. 4.  
2. 5. 7. 6.  
2. 3. 5. 2.  
2. 4. 6. 4.  
0. 1. 1. 2.  
0. -1. -1. -2.  
2. 4. 6. 4.  
0. 1. 1. 2.  
0. 0. 0. 0.  
1. 2. 3. 2.  
0. 1. 1. 2.  
0. 0. 0. 0.  
"is a pivot column"  
1.  
"column"  
"is a pivot column"  
2.  
"column"  
-->
```

PROGRAM-5

*Find the four fundamental subspaces of
 $A = \{[1,3,3,2], [2,6,9,7], [-1,-3,3,4]\}$*

PROGRAM CODE



The screenshot shows the Scilab interface with the following details:

- Menu Bar:** Scilab, File, Edit, Format, Options, Window, Execute, ?
- Toolbar:** Standard Mac-style toolbar with icons for file operations.
- Current File:** program5_PES2UG19CS301.sce
- Open Files:** program4_PES2UG19CS301.sce (closed), program5_PES2UG19CS301.sce (active).
- Code Editor:**

```

1 clc;clear;
2 A=[1 3 3 2;2 6 9 7;-1 -3 3 4];
3 disp("The given matrix is:");
4 disp(A);
5 [m,n]=size(A);
6 disp(m,"m=");
7 disp(n,"n=");
8 [v,pivot]=rref(A);
9 disp(rref(A),"Row Reduced Echelon Form:");
10 r=length(pivot);
11 disp(r,"Rank:");
12 colspace=A(:,pivot);
13 disp(colspace,"Column Space:");
14 nullspace=kernel(A);
15 disp(nullspace,"Null Space:");
16 rowspace=v(1:r,:);
17 disp(rowspace,"Row Space:");
18 leftnullspace=kernel(A');
19 disp(leftnullspace,"Left Null Space:");
20

```

OUTPUT SCREENSHOT

The figure shows the Scilab interface with the following details:

- File Browser:** Shows a list of files and folders including `/Users/priyamohata/`, `Scilab branch-6.1 Console`, `Scilab branch-6.1 Console`, `Variable Browser`, `Command History`, and `News feed`.
- Scilab branch-6.1 Console:** Displays the following code and matrix A:

```
"The given matrix is:"  
1. 3. 3. 2.  
2. 6. 9. 7.  
-1. -3. 3. 4.  
  
3.  
  
"m = "  
4.  
  
"n = "  
1. 3. 0. -1.  
0. 0. 1. 1.  
0. 0. 0. 0.  
  
"Row Reduced Echelon Form: "  
2.  
  
"Rank: "  
1. 3.  
2. 9.  
-1. 3.  
  
"Column Space: "  
-0.2016536 0.9295686  
0.287881 -0.2449115  
-0.6619896 -0.194834  
0.6619896 0.194834  
  
"Null Space: "  
1. 0.  
3. 0.  
0. 1.  
-1. 1.  
  
"Left Null Space: "  
0.9128709  
-0.3651484  
0.1825742
```

- Variable Browser:** Shows the matrix `A` as a 3×4 Double type variable.
- Command History:** Displays the Gaussian elimination code:

```
a(i,j) = a(i,j)-a(2,j)*a(i,2)/a(2,2);  
end  
a(1,2) = 0;  
end  
x(n) = a(n,n+1)/a(n,n);  
for i=n-1:-1:1  
sumk = 0;  
for k=i+1:n  
sumk = sumk+a(i,k)*x(k);  
end  
x(i) = (a(i,n+1) - sumk)/a(i,i);  
end  
disp("Values of x,y,z:");  
disp(x);  
disp("Matrix after Gaussian Elimination: ");  
disp(A);  
disp("The pivots are: ");  
disp(p(3,3),p(2,2),p(1,1));  
// -- 20/04/2021 19:49:05 -- //
```

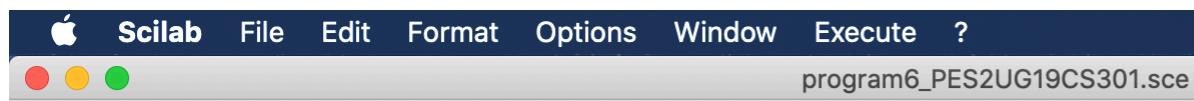
- News feed:** Shows a message for the `March 2021: IIT Bombay Workshop`.
- Scilab branch-6.1 Console (Bottom):** Displays the same Gaussian elimination code and the message `March 2021: IIT Bombay Workshop`.
- Scilab branch-6.1 Console (Bottom Left):** Shows the same matrix A and code as the top console.

PROGRAM-6

Find the solution $x = (C, D)$ of the system $AX = b$ and the line of best fit $C + Dt = b$ given

$A = \{[1,0], [0,1], [1,1]\}$ and $b = \{[1,1,0]\}$

PROGRAM CODE



program6_PES2UG19CS301.sce

program6_PES2UG19CS301.sce (/Users/priyamohata/Desktop/program6_PES2UG19CS301.sce) – SciNotes

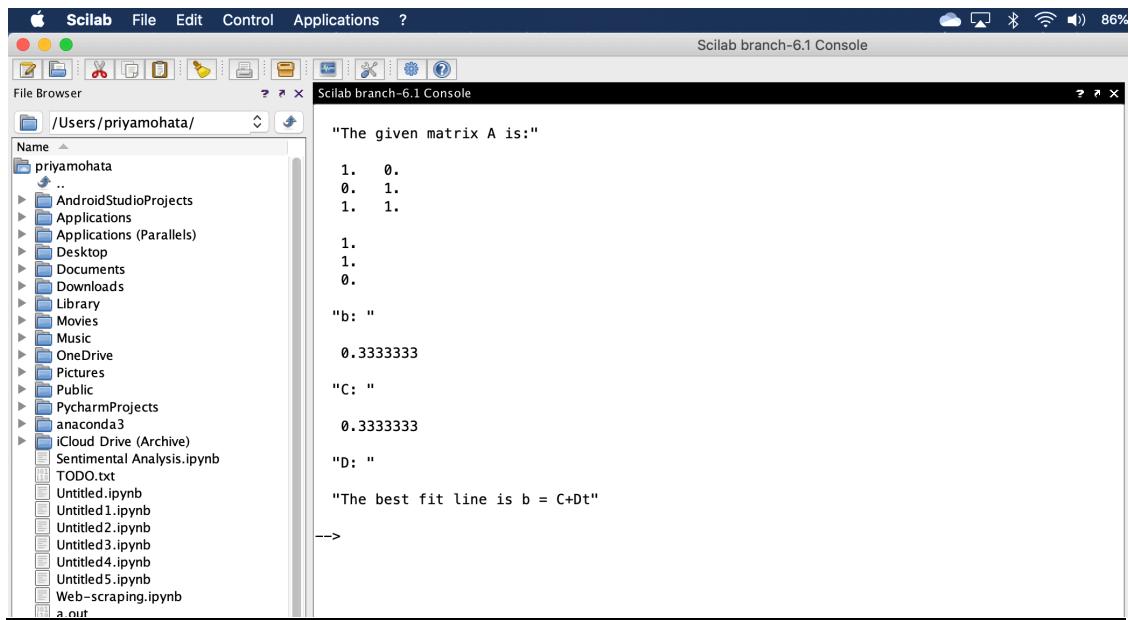
program4_PES2UG19CS301.sce program5_PES2UG19CS301.sce

```

1 clc;clear;
2 A = [1 0; 0 1; 1 1];
3 b = [1; 1; 0];
4 disp("The given matrix A is:")
5 disp(A);
6 disp(b, "b:");
7 x = (A'*A)\(A'*b)
8 C = x(1,1);
9 D = x(2,1);
10 disp(C, "C:");
11 disp(D, "D:");
12 disp("The best fit line is b = C + Dt")
13

```

OUTPUT SCREENSHOT



The given matrix A is:

1.	0.
0.	1.
1.	1.

1.

1.

0.

"b: "

0.3333333

"C: "

0.3333333

"D: "

"The best fit line is b = C+Dt"

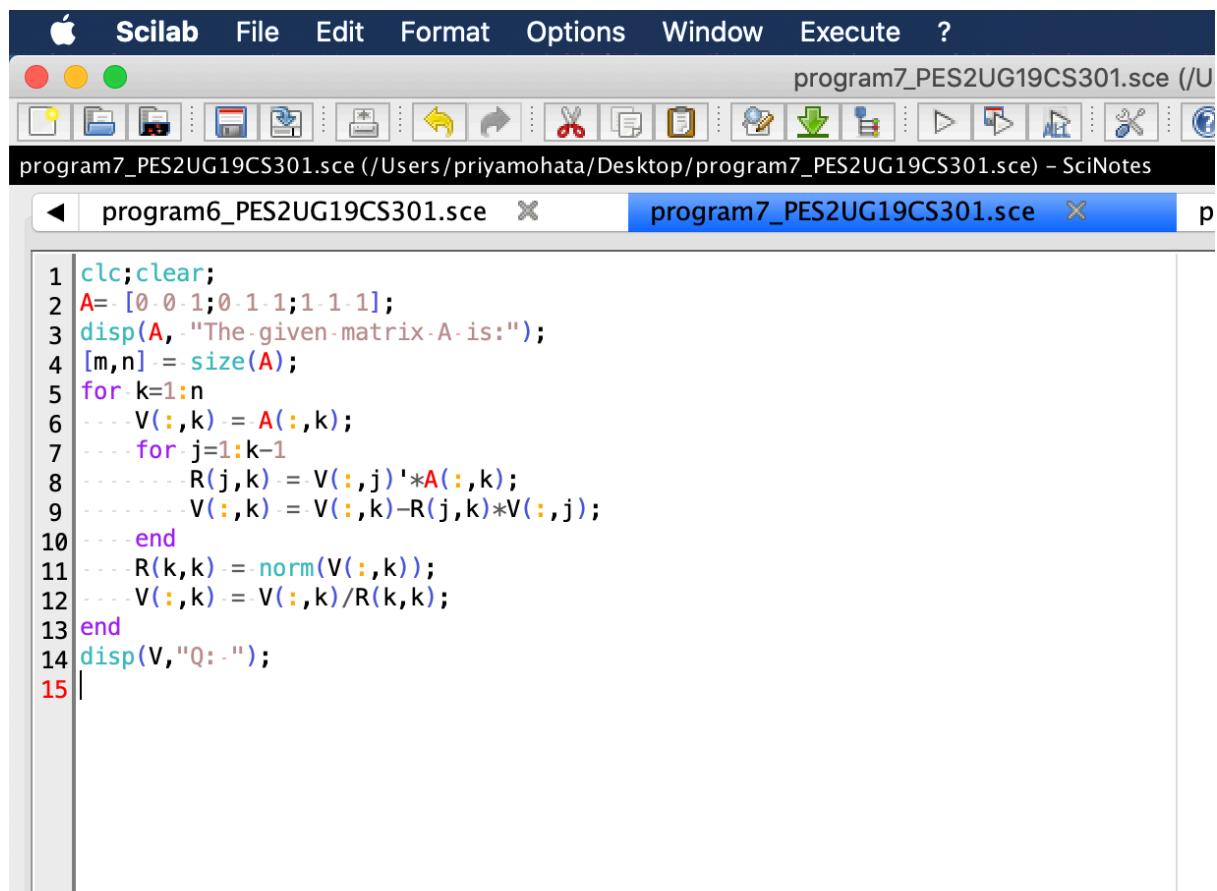
-->

PROGRAM-7

Apply the Gram – Schmidt process to the following set of vectors and find the orthogonal matrix:

$(0, 0, 1), (0, 1, 1), (1, 1, 1)$

PROGRAM CODE



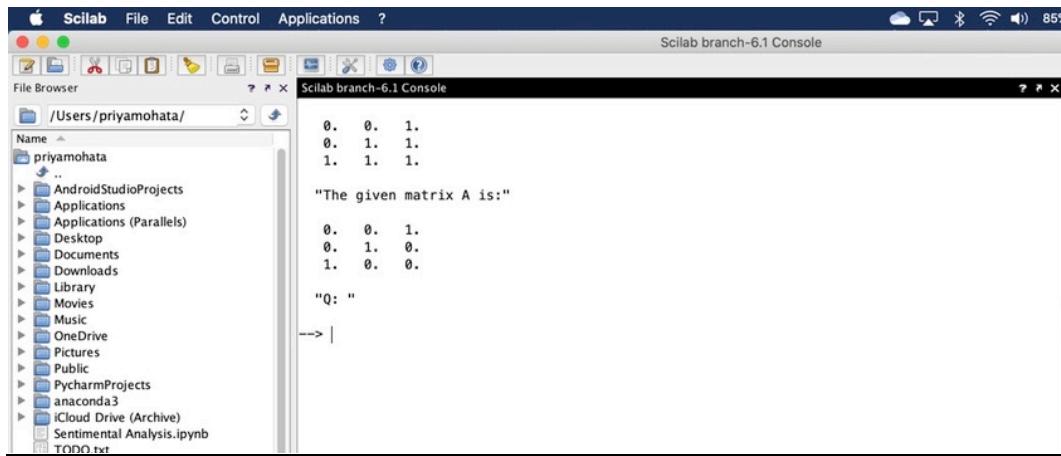
The screenshot shows the Scilab software interface. The menu bar includes Scilab, File, Edit, Format, Options, Window, Execute, and ?. The toolbar below the menu bar contains various icons for file operations. The main window shows two tabs: 'program6_PES2UG19CS301.sce' and 'program7_PES2UG19CS301.sce'. The 'program7_PES2UG19CS301.sce' tab is active, displaying the following Scilab script:

```

1 clc;clear;
2 A=[0 0 1;0 1 1;1 1 1];
3 disp(A, "The given matrix A is:");
4 [m,n]=size(A);
5 for k=1:n
6     V(:,k)=A(:,k);
7     for j=1:k-1
8         R(j,k)=V(:,j)'*A(:,k);
9         V(:,k)=V(:,k)-R(j,k)*V(:,j);
10    end
11    R(k,k)=norm(V(:,k));
12    V(:,k)=V(:,k)/R(k,k);
13 end
14 disp(V,"Q:");
15

```

OUTPUT SCREENSHOT



The screenshot shows the Scilab 6.1 branch console window. The console displays the following text:

```
Scilab branch-6.1 Console
0.  0.  1.
0.  1.  1.
1.  1.  1.

"The given matrix A is:"
0.  0.  1.
0.  1.  0.
1.  0.  0.

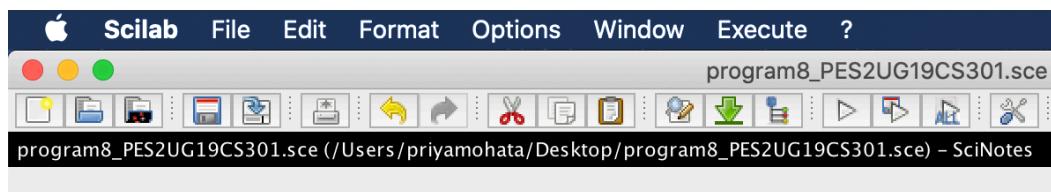
"Q: "
--> |
```

The console window is titled "Scilab branch-6.1 Console". The left side of the window shows a "File Browser" with a list of files and folders in the "/Users/priyamohata/" directory. The list includes "priyamohata", "..", "AndroidStudioProjects", "Applications", "Applications (Parallels)", "Desktop", "Documents", "Downloads", "Library", "Movies", "Music", "OneDrive", "Pictures", "Public", "PycharmProjects", "anaconda3", "iCloud Drive (Archive)", "Sentimental Analysis.ipynb", and "TODO.txt".

PROGRAM-8

*Find the Eigen values and the corresponding Eigen vectors of
 $A=\{[2,2,1],[1,3,1],[1,2,2]\}$*

PROGRAM CODE

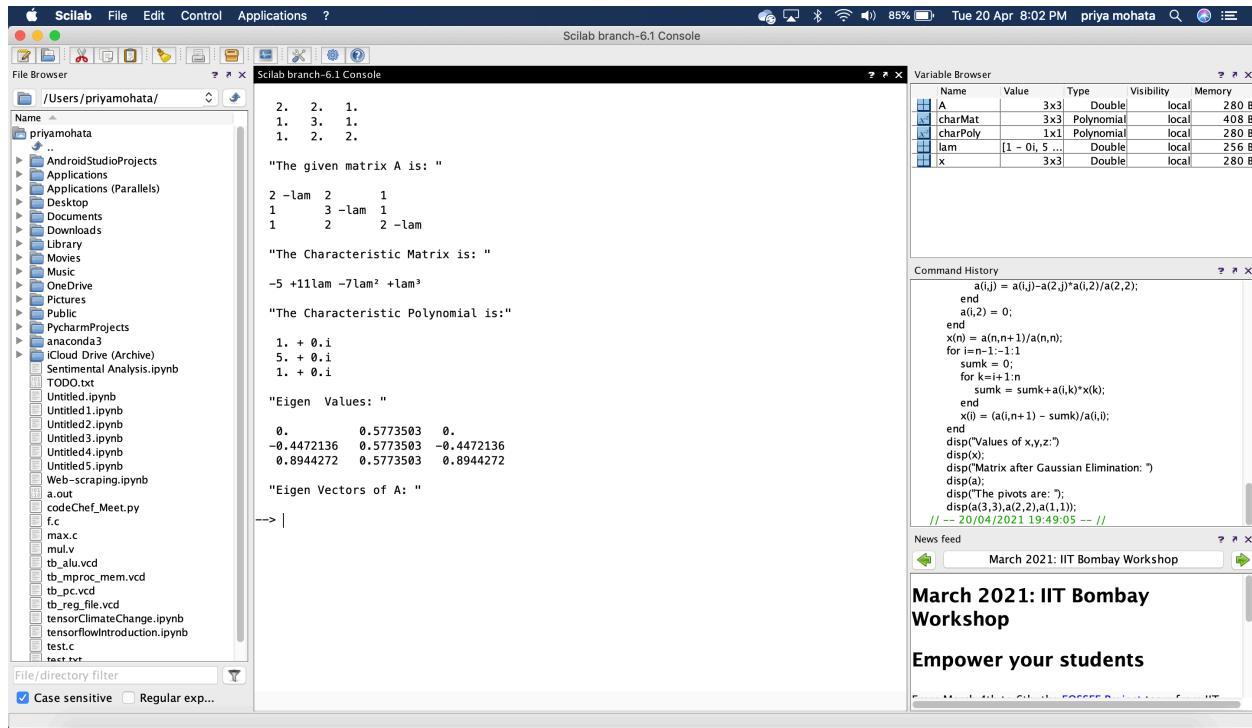


```

1 clc;clear;
2 A = [2 2 1;1 3 1;1 2 2];
3 disp(A,"The-given-matrix-A-is:-");
4 lam = poly(0,"lam");
5 charMat = A-lam*eye(3,3);
6 disp(charMat,"The-Characteristic-Matrix-is:-");
7 charPoly = poly(A,"lam");
8 disp(charPoly,"The-Characteristic-Polynomial-is:");
9 lam = spec(A);
10 disp(lam,"Eigen--Values:-");
11 function[x, lam] = eigenvectors(A)
12 [n,m] = size(A);
13 lam = spec(A)';
14 x = [];
15 for k=1:3
16 B = A-lam(k)*eye(3,3);
17 C = B(1:n-1,1:n-1);
18 b = -B(1:n-1,n);
19 y = C\b;
20 y = [y;1];
21 y = y/norm(y);
22 x = [x y];
23 end
24 endfunction
25 [x, lam] = eigenvectors(A);
26 disp(x,"Eigen-Vectors-of-A:-");
27

```

OUTPUT SCREENSHOT



The screenshot shows the Scilab 6.1 Console interface. The left pane is the File Browser showing a list of files in the directory `/Users/priyamohata/`. The right pane is the Scilab branch-6.1 Console. The Command History window displays the following Scilab code and its execution results:

```

2. 2. 1.
1. 3. 1.
1. 2. 2

"The given matrix A is: "
2 -lam 2 1
1 3 -lam 1
1 2 2 -lam

"The Characteristic Matrix is: "
-5 +11lam -7lam^2 +lam^3

"The Characteristic Polynomial is: "
1. + 0.i
5. + 0.i
1. + 0.i

Eigen Values:
0. 0.5773503 0.
-0.4472136 0.5773503 -0.4472136
0.8944272 0.5773503 0.8944272

Eigen Vectors of A:
--> |

```

The Variable Browser window shows the following variables:

Name	Value	Type	Visibility	Memory
A	3x3	Double	local	280 B
charMat	3x3	Polynomial	local	408 B
charPoly	1x1	Polynomial	local	280 B
lam	[1 - 0i, 5 ...	Double	local	256 B
x	3x3	Double	local	280 B

The Command History window also contains the following code and output:

```

a(i,j) = a(i,j)-a(2,j)*a(i,2)/a(2,2);
end
a(1,2) = 0;
end
x(n) = a(n,n-1)/a(n,n);
for i=n-1:-1:1
    sumk = 0;
    for k=i+1:n
        sumk = sumk+a(i,k)*x(k);
    end
    x(i) = (a(i,n+1) - sumk)/a(i,i);
end
disp("Values of x,y,z:")
disp(x);
disp("Matrix after Gaussian Elimination:")
disp(a);
disp("The pivots are:");
disp(a(3,3),a(2,2),a(1,1));
// -- 20/04/2021 19:49:05 -- //

```

The right side of the interface shows a news feed window with the following content:

March 2021: IIT Bombay Workshop

Empower your students

THANK YOU