SYS 660 Decision and Risk Analysis
Homework Assignment 5

Q1.

An investor with utility function $U(x) = \ln(x)$, where x is total wealth, has a choice between the following two alternatives:

- A: Win $10,000 with a probability of 0.2 or win $1,000 with a probability of 0.8
- B: Win $3,000 with a probability of 0.9 or lose $2,000 with a probability of 0.1

a. If the investor currently has $2,500, should they choose A or B?
b. Find the exact Certainty Equivalent for the alternative A and its risk premium.
c. Repeat a, assuming the investor has $5,000.
d. Do you think that this pattern of choices between A and B is reasonable? Why or why not?

Please scroll down for the answer of Q1

a)
If the investor currently has $2,500, they should choose A.
Expected utility of A:
$0.2 \cdot \ln(2,500+10,000) + 0.8 \cdot \ln(2,500+1,000) \approx 9.99$.
Expected utility of B:
$0.9 \cdot \ln(2,500+3,000) + 0.1 \cdot \ln(2,500-2,000) = 9.88$

b)
If an investor were to get a certain amount of money in exchange for winning the lottery, that amount would be the certainty equivalent of A. To calculate the certainty equivalent, we set the expected utility of A equal to the utility of the certainty equivalent:
$0.2 \cdot \ln(2,500+10,000) + 0.8 \cdot \ln(2,500+1,000) = \ln(\text{Certainty Equivalent})$
Solving for the certainty equivalent we get,
$\text{certainty Equivalent} = \exp(0.2 \times \ln(12,500) + 0.8 \times \ln(3,500))$.
$\text{Certainty Equivalent} = \$7,800.65$.

The risk premium is the difference between the expected value of A and its certainty equivalent:

$\text{Risk Premium} = \text{Expected value of A} - \text{certainty Equivalent}$
$= \$10,000 \times 0.2 + \$1,000 \times 0.8 - \$7,800.65$.
$= \$2,200.35$.
Risk Premium $= \$2,200.35$.

c) Explanation:

If the investor currently has $5,000, they should still choose A.

Expected utility of A:

$0.2 \cdot \ln(5,000 + 10,000) + 0.8 \cdot \ln(5,000 + 1,000) = 10.70.$

Expected utility of B:

$0.9 \cdot \ln(5,000 + 3,000) + 0.1 \cdot \ln(5000 - 2,000) = 10.61.$

d)

As a logarithmic function, the investor's utility function, $U(x) = \ln(x)$, indicates that the investor is more sensitive to changes in wealth when they are impoverished than when they are wealthy. This is because a small increase in wealth has a larger proportional impact on utility when the investor is poor.

The investor in this scenario, gets to choose between two lotteries with various expected values and levels of risks. In addition to being riskier, Lottery A has a larger expected value than lottery B. When an investor has less money, they will choose the lower-risk lottery B because they are more susceptible to fluctuations in wealth when they are impoverished. However, as the investor's wealth increases the risk premium of Lottery A becomes less significant, and the investor will eventually prefer lottery.

Investor behaviour that has been seen is consistent with this pattern of choices. When investors are younger and have less wealth, they tend to be more risk averse. They are, nevertheless, more prepared to take on risk in order to generate larger profits as they get older and wealthier.

Q2. Referring to the Texaco-Pennzoil example in Chapter 4 of Making Hard Decisions with Decision Tool (3rd ed., pp. 118-119), Hugh Liedtke, chairman of Pennzoil, is deciding if he should accept $2 billion to settle a lawsuit case between Texaco and Pennzoil, or counteroffer $5 billion to Texaco. If Liedtke refuses the sure $2 billion, Texaco might agree to pay $5 billion, counter with $3 billion, or simply pursue further appeals. Suppose that Liedtke's utility function is given in the following table:

Table 1 Utility function for Liedtke.

Payoff (billions)	Utility
$10.3	1.00
$5.0	0.75
$3.0	0.60
$2.0	0.45
0.0	0.00

a. Graph this utility function. Based on this graph, how would you classify Liedtke's attitude toward risk?
b. Use the utility function with the decision tree shown below to solve Liedtke's problem. With these utilities, what strategy should he pursue? Should he still counteroffer $5 billion? What if Texaco counteroffers $3 billion?
c. Based on this utility function, what is the least amount (approximately) that Liedtke should agree to in a settlement? (Hint: Find a sure amount that gives him the same expected utility he gets for going to court.) What does this suggest regarding plausible counteroffers that Liedtke might make?

Step-1

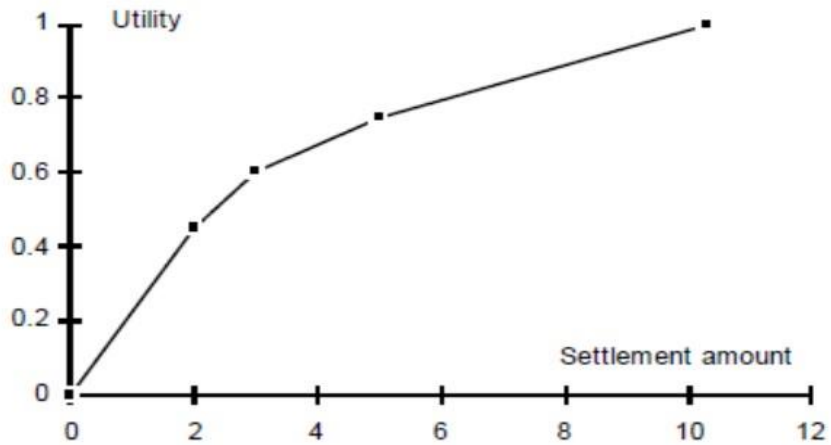
Consider the Li's utility function and his risk attitude.

Step-2

(a)
The objective is to graph the utility function of the Texaco-Pennzoil example and classify the Li's attitude.

Step-3

The utility curve of the problem is plotted based on the given data in the decision tree shown in step-2.



Based on the utility curve,it is clearly visible that Li is risk averse as the curve is concave.

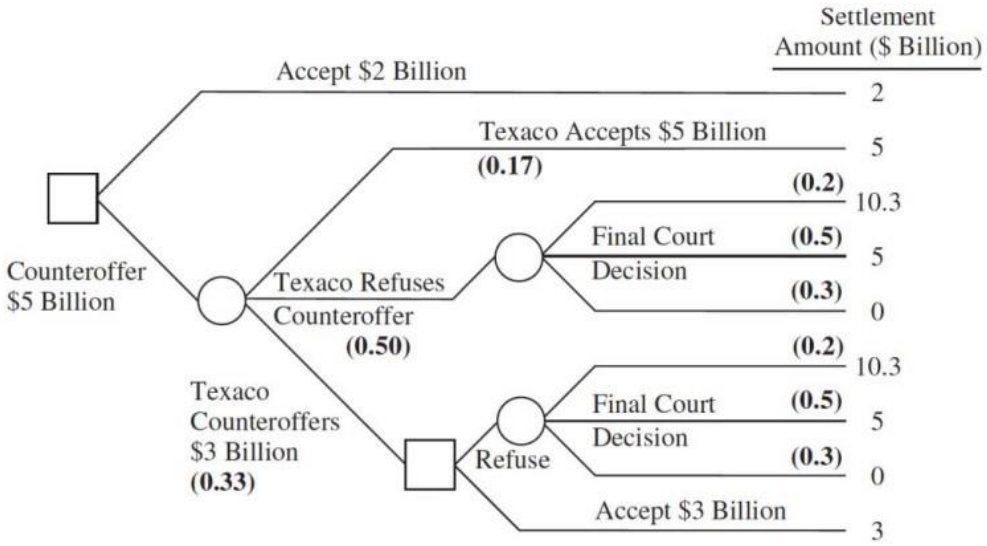
Step-4

Therefore, Li is risk averse as the curve is concave.

Step-5

b.)
The objective is to solve the Li's problem using utility curve and decision tree.

Step-6
The decision tree for Li's decision in the Te-Pe affair is given as below:



Step-7

Li should offer $5 billion because the expected utility for offering the $5 billion is 0.613 that is way far than the utility of $2 billion(0.045).
However, Li should accept the deal of $3 billion if Te offers the deal.
Therefore, he will become slightly more risk-averse than before.

Step-8

Hence, Li should offer $5 billion, yet Li should accept the deal of $3 billion if Te offers the deal.

Step-9

c.)
The objective is to find out the least amount that Li should agree to in a settlement.

Step-10

From the graph,in part (a), Li's certainty equivalent is $2.8 billion for the utility value of 0.575.This CE is less than $3 billion because of utility value for $3 billion is 0.60.

Therefore, he should not make a deal of less than $2.8 billion.

Step-11

Hence, Li should not make a deal of less than $2.8 billion .