SYS 660 Decision and Risk Analysis
Homework Assignment 3

Q1. $P(A) = 0.10$, $P(B|A) = 0.39$, and $P(B|\bar{A}) = 0.39$.

Find the following:

$P(\bar{A})$, $P(\bar{B}|A)$, $P(\bar{B}|\bar{A})$, $P(B)$, $P(\bar{B})$, $P(A|B)$, $P(\bar{A}|B)$, $P(A|\bar{B})$, $P(\bar{A}|\bar{B})$

Please scroll down for the answers

Q1.

$$P(\bar{A}) = 1 - P(A) = 1 - 0.10 = 0.90.$$

$$\underline{P(\bar{A}) = 0.90.}$$

Using Baye's rule : $P(B) = P(B|A) \times P(A) + P(B|\bar{A}) \times P(\bar{A})$

$$P(B) = 0.39 * 0.10 + 0.39 * 0.90$$
$$P(B) = 0.039 + 0.351$$
$$P(B) = 0.39.$$

$$\underline{P(B) = 0.39.}$$

So $P(\bar{B}) = 1 - P(B) = 1 - 0.39 = 0.61$

$$\underline{P(\bar{B}) = 0.61}$$

$P(B|A) = 0.39 = P(A \cdot B)/P(A)$

So, $P(A \cdot B) = P(B \cdot A) = P(A) \times P(B|A) = 0.10 \times 0.39 = 0.039.$

Use the complementary rule of conditional probability:

$P(\bar{B}|A) = 1 - P(B|A)$

$P(\bar{B}|A) = 1 - 0.39.$

$\underline{P(\bar{B}|A) = 0.61}$

And, $P(\bar{B}|\bar{A}) = 1 - P(B|\bar{A})$

$P(\bar{B}|\bar{A}) = 1 - 0.39.$

$\underline{P(\bar{B}|\bar{A}) = 0.61.}$

$P(A|B) = P(A \cdot B)/P(B)$

$P(A|B) = 0.039 / 0.39.$

$\underline{P(A|B) = 0.10.}$

$\cancel{P(B|\bar{A}) = 0.39 = P(B \cdot \bar{A})/P(A)}$

$P(B|\bar{A}) = 0.39 = P(B \cdot \bar{A})/P(\bar{A})$.

So, $P(B \cdot \bar{A}) = P(\bar{A} \cdot B) = P(B|\bar{A}) \times P(\bar{A}) = 0.39 \times 0.90 = 0.351$

$P(\bar{A}|B) = P(\bar{A} \cdot B)/P(B)$

$P(\bar{A}|B) = 0.351/0.39$.

$\underline{P(\bar{A}|B) = 0.90}$

$P(A|\bar{B}) = P(A \cdot \bar{B})/P(\bar{B}) = P(\bar{B}|A) \times P(A)/P(\bar{B})$

$P(A|\bar{B}) = P(A \cdot \bar{B})/P(\bar{B}) = 0.61 \times 0.10/0.61$

$P(A|\bar{B}) = P(A \cdot \bar{B})/P(\bar{B}) = 0.10$.

$\underline{P(A|\bar{B}) = P(A \cdot \bar{B})/P(\bar{B}) = 0.10}$.

$P(\bar{A}|\bar{B}) = 1 - P(A|\bar{B}) = 1 - 0.10$

$P(\bar{A}|\bar{B}) = 0.90$.

$\underline{P(\bar{A}|\bar{B}) = 0.90}$.

Note that $P(A) \times P(B) = 0.10 \times 0.39 = 0.039$ and again $P(A \cdot B) = 0.029$. So, $P(A \cdot B) = P(A) \times P(B)$. This happens for independent events and hence A and B are independent events.

Q2. Suppose that a company produces three different products. The sales for each product are independent of the sales for the others. The information for these products is given in the table that follows:

Product	Price($)	Expected Unit Sales	Variance of Unit Sales
A	3.50	2,000	1,000
B	2.00	10,000	6,400
C	1.87	8,500	1,150

What are the company's overall expected revenue and variance of its revenue?

Q2.

→ For the expected revenue of product A. It is the expected sales * unit price

= 2000 * 3.5

= $7,000.

Variance of 1000 units can occur either sales

Variance revenue = 1000 × 3.5 = $3,500 variance revenue can occur. i.e the max revenue is 7000 + 3,500 = 10,500 or min revenue of 7000 − 3,500 = 3500 can occur.

⇒ Company's expected revenue is the summation of individual expected revenue. Similarly company's variance is the summation of individual variance revenue of products.

Expected revenue of company = (2000 × 3.5) + (2 × 10,000) + (1.87 × 8500)

= (2000 × 3.5) + (2 × 10,000) + (1.87 × 8500)

= (7000) + (20,000) + 15,895.

= $42,895.

Variance revenue of the company =

= (1000 × 3.5) + (6,400 × 2) + (1150 × 1.87)

= 3500 + 12,800 + 2150.5

= $18,450.5

= $18,450.5 is the variance revenue.

Q3. Using Excel, re-create the Monte Carlo simulation shown in Week 4. Note the instructions on how to do this on Slide 51 and the example we did in class.
Imagine that you have the following payoff function

$$P(X, Y, Z) = 3.2\sqrt{|XY|} - 2.6Z\ 2$$

where X, Y, and Z are all normally distributed random variables

X~N(2, 1.7), Y~N(3.2, 0.5), and Z~N(1.83, 1.12)

You should be able to generate a similar histogram to what is shown on slide 49.
Note: to receive full point, show the steps and final graph in pdf, and attach the Excel file or other source code, as the supplement material.

Please scroll down for the answer image and histogram image

Q3.

1. Open Excel and create a new spreadsheet.
2. In cell A1, label it "Iteration".
3. In cell B1, label it "X".
4. In cell C1, label it "Y".
5. In cell D1, label it "Z".
6. In cell E1, label it "Payoff".
7. In cell A2, enter the number 1 to represent the first iteration.
8. In cell B2, use the following formula to generate a random value for X from the normal distribution:
   =NORM.INV(RAND(), 2, 1.7).
9. In cell C2, use the following formula to represent a random value for Y from the normal distribution:
   = NORM.INV(RAND(), 3.2, 0.5).
10. In cell D2, use the following formula to represent a random value for Z from the normal distribution:
    = NORM.INV(RAND( ), 1.83, 1.12)
11. In cell E2, calculate the payoff using the provided formula:
    =3.2*SQRT(ABS(B2*C2))-2.6*D2^2.
12. Copy cells A2 to E2 and paste them down for as many iterations as you want (eg 1000 iterations for a histogram).
13. You have a column of simulated payoffs in column E, Select a range of values in column F.
14. Explanation to create a histogram:

14 Go to the "insert" tab and click on "Insert Statistic Chart" in the "charts" group.

15. Choose the "histogram" chart type.

16. Adjust the chart as desired, including labeling the axes and adding a chart title.

17. The resulting histogram should resemble the one shown on slide 49 of the presentation.

<u>Note</u>: The Monte Carlo simulation involves generating random numbers from the specified normal distribution for each variable (X, Y and Z), calculating the payoff for each set of random values, and repeating this process multiple times to obtain a distribution of payoffs.

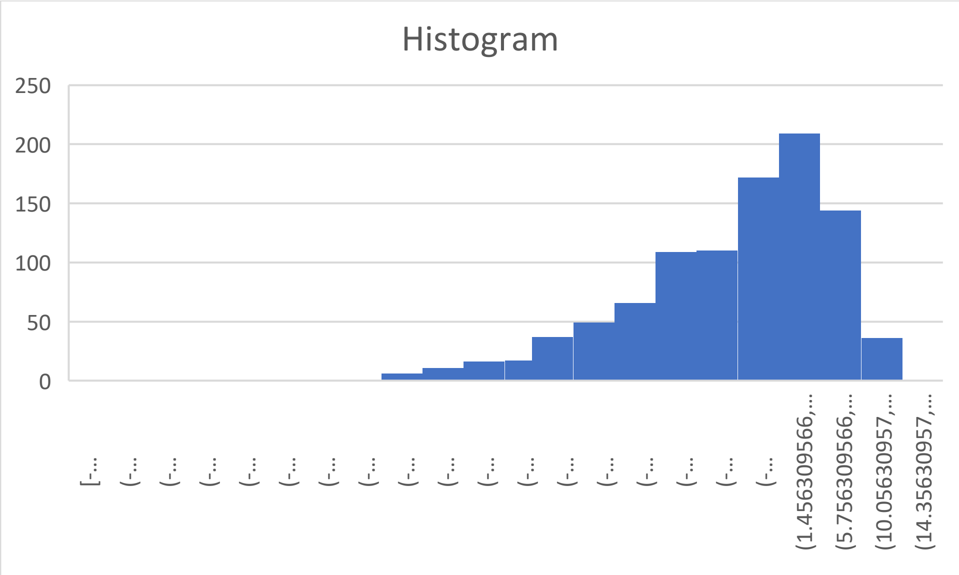
<u>Final answer</u>

To recreate the Monte Carlo simulation in Excel, follow these steps.

1. Create a new excel spreadsheet and enter the headings for X, Y, & Z.
2. Enter the mean and standard deviation values for X, Y, Z in the ~~correct~~ appropriate cells.
3. Use the NORM.INV (RAND(), mean, standard deviation) formula to generate random values for X, Y & Z.
4. Calculate the payoff for each set of random values using the given formula.
5. Repeat steps 3 and 4 multiple times to generate a range of payoffs.
6. Create a histogram using the payoffs to visualize the distribution of outcomes.

Scroll down for the histogram image

After running 1000 trials,



I have generated the histogram at the end of 1000 trials on the excel sheet.Please scroll down.