SYS 640 HOMEWORK-2

DETERMINISTIC AND PROBABILISTIC MODELS

Spring 2023 Prof. Hao Chen

Problem 1 :

A quality engineer in an automobile engine plant measures a critical dimension on each of a sample of crankshafts at regular intervals. The dimension is supposed to be 224 millimeters (mm), but some variation will occur in production. Here are the latest measurements: 224.120 224.001 224.017 223.982 223.889 224.198 223.960 224.089 224.587 223.776 223.912 223.980 The engineer codes these measurements to make them easier to work with. The coded value is the number of thousandths of a millimeter above 223 mm. (For example, 224.129 mm is coded as 1129.) Give the coded value for each of the measurements in this sample.

The coded value for a given measurement is obtained by subtracting 223 from each measurement and multiplying the result by 1000.

The first measurement of 224.120 mm, for instance, becomes (224.120 - 223) * 1000 = 120.

This approach enables us to identify the coded value for each measurement in the sample:

Sample measurement	Coded value		
224.120	(224.120-223)×1000=1120		
224.001	(224.001-223)×1000=1001		
224.017	(224.017-223)×1000=1017		
223.982	(223.982-223)×1000=982		
223.889	(223.889-223)×1000=889		
224.198	(224.198-223)×1000=1198		
223.960	(223.960-223)×1000=960		
224.089	(224.089-223)×1000=1089		
224.587	(224.587-223)×1000=1587		
223.776	(223.776-223)×1000=776		
223.912	(223.912-223)×1000=912		

Sample measurement	Coded value		
224.120	1120		
224.001	1001		
224.017	1017		
223.982	982		
223.889	889		
224.198	1198		
223.960	960		
224.089	1089		
224.587	1587		
223.776	776		
223.912	912		
223.980	980		

Therefore, the coded values for the measurements are:

120, 1001, 1017, 982, 889, 1198, 960, 1089, 1587, 776, 912, 980

Problem 2 :

A video display tube for computer graphics terminals has a fine mesh screen behind the viewing surface. During assembly the mesh is stretched and welded onto a metal frame. Too little tension at this stage will cause wrinkles. The tension is measured by an electrical device with output readings in milli volts (mV). The minimum acceptable tension corresponds to a reading of 200 mV. $\mu = 250$ mV and $\sigma = 50$ mV. What proportion of tubes are acceptable?

We can utilize the normal distribution to address this problem because we know the mean (250 mV) and standard deviation (50 mV) of the tension data.

Let X represent the stress reading in millivolts. X = N (250, 502) because the mean is 250 mV and the standard deviation is 50 mV.

Finding the fraction of tubes that are suitable requires that their tension readings be more than or equal to 200 mV. We are looking for P (X >= 200) mathematically.

We must normalize the variable X by taking the mean away and dividing by the standard deviation in order to determine this probability:

$Z = (X - 250) / 50$

Now, we can calculate the probability using a calculator or a table of the standard normal distribution:

P (X >= 200) = P (Z >= (200 – 250) / 50) = P (Z >= -1) = 0.8413

• As a result, around **84.13%** of tubes are authorized (have tension measurements greater than or equal to 200 mV).

Problem 3:

An electronic system which operates continuously develops faults at random times with an average of 10 faults per week. What is the distribution of the number of faults per day? On what proportion of days (this is the same as finding the probability in one day) would the system be expected to develop:
a) no faults?
b) 3 faults?
c) at most 2 faults (including 2)?
d) more than 1 fault (excluding 1)?
e) at least 3 faults (including 3)?
f) between 2 and 4 faults inclusive?

As we are working with a continuous system that experiences defects at random intervals with a specific average rate, this is an example of a Poisson distribution problem.

The average weekly number of faults must be divided by 7 because there are seven days in a week in order to get the distribution of faults per day:
Average no of faults per day = 10/7 = 1.43

a.) To find the probability of the system developing no faults in a day, we can use the Poisson distribution formula with $\lambda = 1.43$ and $k = 0$:
P(k=0) = e^(-λ) * λ^k / k! = e^ (-1.43) * 1.43^0 / 0! ≈ 0.2389
So, the system is expected to develop no faults on approximately 23.89% of days.

b.) To find the probability of the system developing 3 faults in a day, we can use the Poissondistribution formula with $\lambda = 1.43$ and $k = 3$:
P(k=3) = e^(-λ) * λ^k / k! = e^ (-1.43) * 1.43^3 / 3! ≈ 0.0787
So, the system is expected to develop 3 faults on approximately 7.87% of days.

c.)To find the probability of the system developing at most 2 faults in a day, we need to calculate the sum of the probabilities for k = 0, 1, and 2:
P(k≤2) = P(k=0) + P(k=1) + P(k=2) = e^ (-1.43) * (1.43^0 / 0! + 1.43^1 / 1! + 1.43^2 / 2!) ≈ 0.4241
So, the system is expected to develop at most 2 faults on approximately 42.41% of days.

d.)To find the probability of the system developing more than 1 fault in a day (excluding 1), we need to subtract the probability of developing 0 or 1 faults from 1:
P(k>1) = 1 - P(k≤1) = 1 - (P(k=0) + P(k=1)) ≈ 0.7611
So, the system is expected to develop more than 1 fault (excluding 1) on approximately 76.11% of days.

e.)To find the probability of the system developing at least 3 faults in a day, we can use the Poisson distribution formula with $\lambda = 1.43$ and $k = 3, 4, 5, \ldots$

$P(k \geq 3) = 1 - P(k \leq 2) = 1 - (P(k=0) + P(k=1) + P(k=2)) \approx 0.5759$

So, the system is expected to develop at least 3 faults on approximately 57.59% of days.

f.)To find the probability that the system develops between 2 and 4 faults inclusive in one day, we need to calculate $P(2 \leq X \leq 4)$.

$P(2 \leq X \leq 4) = P(X = 2) + P(X = 3) + P(X = 4) = e^{(-1.43)} * (1.43^2 / 2!) + e^{(-1.43)} * (1.43^3 / 3!) + e^{(-1.43)} * (1.43^4 / 4!) \approx 0.366$

So, the proportion of days on which the system would be expected to develop between 2and 4 faults inclusive is approximately 0.366, or 36.6%.

<u>Problem 4:</u>

Use the plastic demands as follows to perform the following analysis and answer these questions: a) Determine the Mixed Method Seasonal forecast for the next year (i.e., 2010). b) Determine the MA(4) forecast for the next year (i.e., 2010). (Moving Average) c) Which forecasting method would you choose and why? Think graphical analysis of the data and the use of error measures (use any error metric you see fit, mean absolute percentage error (MAPE) is generally the best and the easiest to calculate).

Year	Quarter	Plastic Demand ('000 lb)
2005	1	3,200
	2	7,658
	3	4,420
	4	2,384
2006	1	3,654
	2	8,680
	3	5,695
	4	1,953
2007	1	4,742
	2	13,673
	3	6,640
	4	2,737
2008	1	3,486
	2	13,186
	3	5,448
	4	3,485
2009	1	7,728
	2	16,591
	3	8,236
	4	3,316

SUMMARY OUTPUT:

Regression Statistics	
Multiple R	0.931008
R Square	0.866776
Adjusted R	0.85726
Standard Er	509.8883
Observation	16

ANOVA

	df	SS	MS	F	ignificance *F*
Regression	1	23681166	23681166	91.08629148	1.66E-07
Residual	14	3639805	259986.1		
Total	15	27320970			

	Coefficientstandard Erro		t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%
Intercept	3612.132	317.1016	11.39109	1.81964E-08	2932.016	4292.247	2932.0164
X Variable 1	263.9138	27.65257	9.543914	1.6602E-07	204.6049	323.2227	204.60491

Year	**Qtr**	**Period**	**Dt-Clear**	**Dt-Clear - Bar**	**Dreg**	**St**	**Ft**	**error MAD**
2005	1	1	3200		3876.045588	0.825584	2951.761	248.2386
2005	2	2	7658		4139.959375	1.849777	7862.775	204.77531
2005	3	3	4420	4472.25	4403.873162	1.003662	4175.124	244.87580
2005	4	4	2384	4656.75	4667.786949	0.510735	1935.734	448.26581
2006	1	5	3654	4943.875	4931.700735	0.740921	3755.684	101.68430
2006	2	6	8680	5049.375	5195.614522	1.67064	9867.718	1187.7175
2006	3	7	5695	5131.5	5459.528309	1.04313	5175.946	519.05420
2006	4	8	1953	5891.625	5723.442096	0.341228	2373.515	420.51504
2007	1	9	4742	6633.875	5987.355882	0.792002	4559.607	182.39273
2007	2	10	13673	6850	6251.269669	2.187236	11872.66	1800.3402
2007	3	11	6640	6791	6515.183456	1.019158	6176.767	463.2326

2007	4	12	2737	6573.125	6779.097243	0.403741	2811.296	74.295896
2008	1	13	3486	6363.25	7043.011029	0.494959	5363.53	1877.5302
2008	2	14	13186	6307.75	7306.924816	1.80459	13877.6	691.60189
2008	3	15	5448	6931.5	7570.838603	0.719603	7177.589	1729.5889
2008	4	16	3485	7887.375	7834.75239	0.444813	3249.077	235.92324
2009	1	17	7728	8661.5	8098.666176	0.954231	6167.453	1560.5468
2009	2	18	16591	8988.875	8362.579963	1.983957	15882.54	708.45591
2009	3	19	8236		8626.49375	0.954733	8178.411	57.589429
2010	4	20	3316		8890.407537	0.372986	3686.858	370.85760
2010	1	21			9154.321324		6971.376	656.37411
2010	2	22			9418.23511		17887.49	
2010	3	23			9682.148897		9179.232	
2010	4	24			9946.062684		4124.638	

Season	Average Values
1	0.761539
2	1.89924
3	0.948057
4	0.414701

b.) Determine the MA(4) forecast for the next year (i.e., 2010). (Moving Average)
2010 Quarter 1 Moving Average:
Year 2010, Q1 = 2005 (Q1) + 2006 (Q1) + 2007 (Q1) + 2008 (Q1) + 2009 (Q1) / 5
Q1 = 3200 + 3654+ 4742 + 3486 + 7728 / 5
Q1 = 22810 /5
Q1 = 4562

2010 Quarter 2 Moving Average:
Year 2010, Q2 = 2005 (Q2) + 2006 (Q2) + 2007 (Q2) + 2008 (Q2) + 2009 (Q2) / 5
Q2 = 7658 + 8680 + 13673 + 13186 + 16591 / 5
Q2 = 59788 /5
Q2 = 11958

2010 Quarter 3 Moving Average:
Year 2010, Q3 = 2005 (Q3) + 2006 (Q3) + 2007 (Q3) + 2008 (Q3) + 2009 (Q3) / 5
Q3 = 4420 + 5695 + 6640 + 5448 + 8236 / 5
Q3 = 30439 /5
Q3 = 6088

2010 Quarter 4 Moving Average:
Year 2010, Q4 = 2005 (Q4) + 2006 (Q4) + 2007 (Q4) + 2008 (Q4) + 2009 (Q4) / 5

$Q4 = 2384 + 1953 + 2737 + 3485 + 3316 / 5$
$Q4 = 13875 /5$
$Q4 = 2775$