Name - Priya Reg no. - 11906114
Roll no. - 32 section - K19RC
set - B

1. Form a partial differential equation from
$xyz = f(x+y+z)$ ~~$xyz = f(x+y+z)$~~
where z is a function of x and y

Given,
$f(x+y+z) = xyz$, $z = g(x,y)$

$\therefore$ differentiating function of f with respect to x ~~and y~~.

$f'(x+y+z)(1+Z_x) = yz + xyZ_x$ — (1)

→ differentiating W.R.T to y

— $f'(x+y+z)(1+Z_y) = xz + xyZ_y$ — (2)

By eqn (1) ÷ (2), we get

$$\frac{1+Z_x}{1+Z_y} = \frac{yz+xyZ_x}{xz+xyZ_y}$$

$\therefore$ $Z_x = \frac{\partial z}{\partial x}$, and $Z_y = \frac{\partial z}{\partial y}$

~~$\therefore xz + zx\frac{dz}{dx} + xy\frac{dz}{dy} + xy\frac{dz}{dx}\cdot\frac{dz}{d}$~~

$\therefore xz + xyZ_y + Z_x xz + Z_x Z_y xy = yz + xyZ_x + yzZ_y + xyZ_xZ_y$

$= xz + \frac{\partial z}{\partial y}(xy) + \frac{\partial z}{\partial x}(xz) + \frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}(xy) =$

$yz + xy\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} + \frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}(xy)$

$= (xy - zx)\frac{dz}{dx} + (zy - xy)\frac{dz}{dy} + (xy - xy)\left(\frac{dz}{dx}\cdot\frac{dz}{dy}\right)$

The partial differiant eqn is $=$ $+ (yz - xz) = 0$

$$\therefore = \boxed{(xy - zx)\frac{dz}{dx} + (zy - xy)\frac{dz}{dy} + (yz - xz) = 0}$$

Name - Priya   Section - K19RC
Roll no - 32   Reg no. - 11906117
SET - B

Q2 Find the directional derivative of $F(x,y,z) = x^2y - y^2z - xyz$ at point $(1,-1,0) = x^2y - y^2z - xyz$
$F(x,y,z) = x^2y - y^2z - xyz$ at point $(1,-1,0)$ in the direction
at point $(1,-1,0)$, $(1,-1,0)$ in the direction
$(\hat{i} - \hat{j} + 2\hat{k})$ $(\hat{i} - \hat{j} + 2\hat{k})$

Given.
$F(x,y,z) = x^2y - y^2z - xyz$ at $(1,-1,0)$ in
the direction $(\hat{i} - \hat{j} + 2\hat{k})$
∴ to find direction derivative.

$$D.D = (\nabla F)_p \cdot \hat{b}$$

$$P(1,-1,0), \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\hat{b} = \frac{\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{6}}$$

$$\nabla F = \frac{\partial}{\partial x}(x^2y - y^2z - xyz)\hat{i} + \frac{\partial}{\partial y}(x^2y - y^2z - xyz)\hat{j} +$$

$$\frac{\partial}{\partial z}(x^2y - y^2z - xyz)\hat{k}$$

$$= (2xy - yz)\hat{i} + (x^2 - 2yz - xz)\hat{j} + (-y^2 - xy)\hat{k}$$

$$(\nabla F)_p = -2\hat{i} + \hat{j}$$

$$(\nabla F)_p \cdot \hat{b} = (-2\hat{i} + \hat{j}) \cdot \left(\frac{\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{6}}\right)$$

$$= \frac{-2-1}{\sqrt{6}}$$

$$(\nabla F)_p \cdot \hat{b} = \frac{-3}{\sqrt{6}}$$

$$\boxed{D.D = \frac{-3}{\sqrt{6}}}$$

Name - Priya section - K19RC

Roll no - 32 Reg. no. 11906114

Set - B

3. Show that the vector field
$\vec{V} = e^{xyz}(yz\hat{i} + xz\hat{j} + xy\hat{k})$
$\vec{V} = e^{xyz}(yz\hat{i} + xz\hat{j} + xy\hat{k})$ is irrotational
and find a scalar function $f(x,y,z)$ such that
$\vec{V} = \text{grad} f$, ~~$\vec{V} = \text{grad} f$~~

$\longrightarrow$ Condition of irrotational if curl $\vec{V} = 0$

$$\therefore \underbrace{\nabla \times \vec{V}}_{\text{curl}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xyz} \cdot yz & e^{xyz} \cdot xz & e^{xyz} \cdot xy \end{vmatrix}$$

$$\Rightarrow \left[\frac{\partial}{\partial y}(e^{xyz} \cdot xy) - \frac{\partial}{\partial z}(e^{xyz} \cdot xz)\right]\hat{i} - \left[\frac{\partial}{\partial x}(e^{xyz} \cdot xy) - \frac{\partial}{\partial z} e^{xyz} yz\right]\hat{j}$$
$$+ \left[\frac{\partial}{\partial x}(e^{xyz} xz) - \frac{\partial}{\partial y}(e^{xyz} \cdot yz)\right]\hat{k}$$

$$\Rightarrow \left[\left[x^2yze^{xyz} + xe^{xyz}\right] - \left[x^2yze^{xyz} + xe^{xyz}\right]\right]\hat{i}$$
$$- \left[\left[e^{xyz} \cdot xy^2z + ye^{xyz}\right] - \left[e^{xyz} xyz + e^{xyz} y\right]\right]\hat{j}$$
$$+ \left[\left[e^{xyz} \cdot z + e^{xyz} xyz^2\right] - \left[e^{xyz} z + e^{xyz} xyz^2\right]\right]\hat{k}$$

$$\Rightarrow \left[\cancel{x^2yze^{xyz}} + \cancel{xe^{xyz}} - \cancel{x^2yze^{xyz}} - \cancel{xe^{xyz}}\right]\hat{i}$$
$$- \left[e^{xyz} \cdot xy^2z + ye^{xyz}\right] - e^{xyz} xyz - e^{xyz} y\Big]\hat{j}$$
$$+ \left[\cancel{e^{xyz} \cdot z} + \cancel{e^{xyz} xyz^2} - \cancel{e^{xyz} z} - \cancel{e^{xyz} xyz^2}\right]\hat{k}$$

$$\therefore \boxed{\nabla \times \hat{j} = 0}$$

Name- Priya	Reg no. 11906114
Rollno. 32	Section - K19RC
Set - B

given.

$\vec{v} = e^{xyz}(yz\hat{i} + zx\hat{j} + xy\hat{k})$

$\vec{v} = \nabla f$

$\vec{v} = (e^{xyz}yz\hat{i} + e^{xyz}zx\hat{j} + e^{xyz}xy\hat{k})$ — ①

$f_x = e^{xyz}xz$

$\boxed{f = \int e^{xyz}xz\,dx = e^{xyz} + h(y,z)}$ — ②

~~$f_y = e^{xyz}xzd$~~ Similarly $f_y = e^{xyz}xz$ — ③

$f_z = e^{xyz}xy$ — ④

from equation ②

$f_y = xze^{xyz} + h_y(y,z)$

comparing with equation ③

$h_y(y,z) = 0.$

$\int 0\,dy = g(z)$

So $f = e^{xyz} + g(z)$ — ⑤

diff. W.R.T z

$\boxed{f_z = xye^{xyz} + g'(z)}$

comparing with eqution 4.

$g'(z) = 0.$

$\int 0\,dz = c.$

So $\boxed{f = e^{xyz} + c}$

Name - Priya Reg. no. 11906114

Roll no. 32 Section . K19RC

set - B

4. Solve laplace eq^n $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to conditions

$u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\frac{n\pi x}{l}$

~~$u(0,y) = u(1,y) = u(x,0) = 0$~~

Given eq^n is = $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ — (1)

Trial ~~eu~~ sol^n. $u(x,y) = X(x)\,Y(x)$ — (2)

~~Sub. the va~~

Substitute the value (2) in eq. (1)

$\therefore X''Y + XY'' = 0$

$\frac{X''}{X} = \frac{-Y''}{Y} = \lambda$

$X'' - \lambda X = 0$

$Y'' - \lambda Y = 0$

Case 1 :- If $\lambda = p^2$.

$X'' - p^2 X = 0.$

$X = c_1 e^{px} + c_2 e^{-px}$

y'' to $p^2 y = 0$

$y = c_3 \cos py + \cancel{c_4 \sin p} \; c_4 \sin py$

$\therefore u(x,y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

$u(0,y) = 0$

$(c_1 + c_2)(c_3 \cos py + c_4 \sin py)$

By $c_1 + c_2 \Rightarrow c_1 = -c_2$

$u(1,y) = 0 \Rightarrow (c_1 e^{ip} + c_2 e^{-ip})(c_3 \cos py + c_4 \sin py)$

$0 = c_1(e^{ip} - e^{-ip})\;(c_3 \cos py + c_4 \sin py)$

$c_1 = 0$ and consequently $c_2 = 0$

$u(x,y) = 0$

Name - Priya Reg no. 11906119

Roll no - 32 section - K19RC

SET-B

~~$u(x,y)=0$.~~

Case 2 :- If $\lambda = 0$

$$X'' = 0 \Rightarrow X = c_1 + c_2 x$$

$$Y'' = 0 \Rightarrow Y = c_3 + c_4 y.$$

$$\therefore u(x,y) = (c_1 + c_2 x)(c_3 + c_4 y)$$

$$u(0,y) = 0 \Rightarrow c_1(c_3 + c_4 y)$$

$$c_1 = 0.$$

$$u(l,y) = 0$$

$$c_1 l (c_3 + c_4 y) \Rightarrow c_1 = 0$$

$$u(x,y) = 0$$

Case 3 :- If $\lambda = -p^2$

$$X'' + p^2 X = 0$$

$$X = c_1 \cos px + c_2 \sin px$$

$$\therefore y'' - p^2 y = 0.$$

$$Y = c_3 e^{py} + c_4 e^{-py}$$

$$\therefore u(x,y) = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py})$$

$$u(0,y) = 0$$

$$c_1 (c_3 e^{py} + c_4 e^{-py})$$

$$\Rightarrow c_1 = 0.$$

$$u(l,y) = 0$$

$$c_2 \sin pl \, (c_3 e^{py} + c_4 e^{-py})$$

$$\sin pl = 0$$

$$\sin pl = \sin n\pi$$

$$p = \frac{n\pi}{l}.$$

$$u(x,0) = 0$$

$$c_2 \sin px \, (c_3 + c_4)$$

$$(c_3 + c_4)$$

$$c_3 = -c_4$$

Name - Priya Section - K19RC
Roll - 32 Reg. no. 11906114

Date
Page

SET-B

$$u(x,y) = C_2 \sin \frac{n\pi x}{l}\left[C_3 e^{\cancel{\frac{n\pi x}{l}}} e^{\frac{n\pi y}{l}} - C_4 e^{-\frac{n\pi y}{l}}\right]$$

$$= A \sin \frac{n\pi x}{l}\left[e^{\frac{n\pi y}{l}} - \cancel{C_4}\, e^{-\frac{n\pi y}{l}}\right]$$

$$u(x,a) = \sin \frac{n\pi x}{l}$$

$$A \sin \frac{n\pi x}{l}\left[e^{\frac{n\pi a}{l}} - e^{-\frac{n\pi a}{l}}\right]$$

$$A = \frac{1}{e^{\frac{n\pi a}{l}} - e^{-\frac{n\pi a}{l}}} = \frac{1}{2\sinh\left(\frac{n\pi a}{l}\right)}$$

$$\therefore u(x,y) = \left(\frac{e^{\frac{n\pi y}{l}} - e^{-\frac{n\pi y}{l}}}{2\sinh\left(\frac{n\pi y}{l}\right)}\right) \sin \frac{n\pi x}{l}$$

$$= \left[\frac{2\sinh\left(\frac{n\pi y}{l}\right)}{2\sinh\left(\frac{n\pi y}{l}\right)}\right] \sin \frac{n\pi x}{l}$$

$$\boxed{u(x,y) = \sin \frac{n\pi x}{l}}$$