Homework-1 Applied Machine Learning Fix

part 5 - Added learning rates to SGD and plotted the loss vs epochs graphs

part 6 - added ridge, lasso, etc for polynomial regressor.

We'll work with the Ecommerce Customers data from the company which has customer info such as Email, Address, and their Avatar color and other numeric value columns:

Avg. Session Length: Average session of in-store style advice sessions

Time on App: Average time spent on App in minutes

Time on Website: Average time spent on Website in minutes

Length of Membership: for how long the person has been associated with the company as a customer

Yearly amount spent: Amount spent by the customer per year

In [1]:
```
#importing libraries
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
from sklearn.pipeline import Pipeline
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import SGDRegressor
from sklearn.model_selection import train_test_split
from sklearn.model_selection import StratifiedShuffleSplit
from sklearn.model_selection import KFold
from sklearn.linear_model import Ridge, Lasso, ElasticNet
from sklearn.preprocessing import PolynomialFeatures
from sklearn.metrics import mean_squared_error
```

In [2]:
```
# Load the dataset
ecommerce_df = pd.read_csv('ecommerce.csv')
ecommerce_df.head(10) #print first 10  rows
```

Out[2]:

	Email	Address	Avatar	Avg. Session Length	Time on App	Tim We
0	mstephenson@fernandez.com	835 Frank Tunnel\nWrightmouth, MI 82180-9605	Violet	34.497268	12.655651	39.57
1	hduke@hotmail.com	4547 Archer Common\nDiazchester, CA 06566-8576	DarkGreen	31.926272	11.109461	37.26
2	pallen@yahoo.com	24645 Valerie Unions Suite 582\nCobbborough, D...	Bisque	33.000915	11.330278	37.11
3	riverarebecca@gmail.com	1414 David Throughway\nPort Jason, OH 22070-1220	SaddleBrown	34.305557	13.717514	36.72
4	mstephens@davidson-herman.com	14023 Rodriguez Passage\nPort Jacobville, PR 3...	MediumAquaMarine	33.330673	12.795189	37.53
5	alvareznancy@lucas.biz	645 Martha Park Apt. 611\nJeffreychester, MN 6...	FloralWhite	33.871038	12.026925	34.47
6	katherine20@yahoo.com	68388 Reyes Lights Suite 692\nJosephbury, WV 9...	DarkSlateBlue	32.021596	11.366348	36.68
7	awatkins@yahoo.com	Unit 6538 Box 8980\nDPO AP 09026-4941	Aqua	32.739143	12.351959	37.37
8	vchurch@walter-martinez.com	860 Lee Key\nWest Debra, SD 97450-0495	Salmon	33.987773	13.386235	37.53
9	bonnie69@lin.biz	PSC 2734, Box 5255\nAPO AA 98456-7482	Brown	31.936549	11.814128	37.14

1.Summarize the data. How much data is present? What attributes/features are continuous valued? Which attributes are categorical?

In [3]:
```
1 ecommerce_df.info()
2 ecommerce_df.shape
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 500 entries, 0 to 499
Data columns (total 8 columns):
 #   Column                Non-Null Count  Dtype  
---  ------                --------------  -----  
 0   Email                 500 non-null    object 
 1   Address               500 non-null    object 
 2   Avatar                500 non-null    object 
 3   Avg. Session Length   500 non-null    float64
 4   Time on App           500 non-null    float64
 5   Time on Website       500 non-null    float64
 6   Length of Membership  500 non-null    float64
 7   Yearly Amount Spent   500 non-null    float64
dtypes: float64(5), object(3)
memory usage: 31.4+ KB
```

Out[3]: (500, 8)

Ans 1.There are 500 rows/data entries and 8 columns/attributes.There are 5 continuous valued attributes which are ['Avg. Session Length , Time on App ,Time on Website,Length of Membership,Yearly Amount Spent'] and 3 categorical attributes which are ['Email,Address,Avatar']. There are no null values for any of the attributes. Also,it is recommended to use a larger dataset whenever possible to train linear regression model, as this will give us a better understanding of the relationship between the independent and dependent variables.However, we can also choose to use a smaller subset of data for linear regression. This can be useful in various scenarios such as when we want to make a quick exploration of data or when we want to train a model with a smaller dataset to reduce the computational cost.

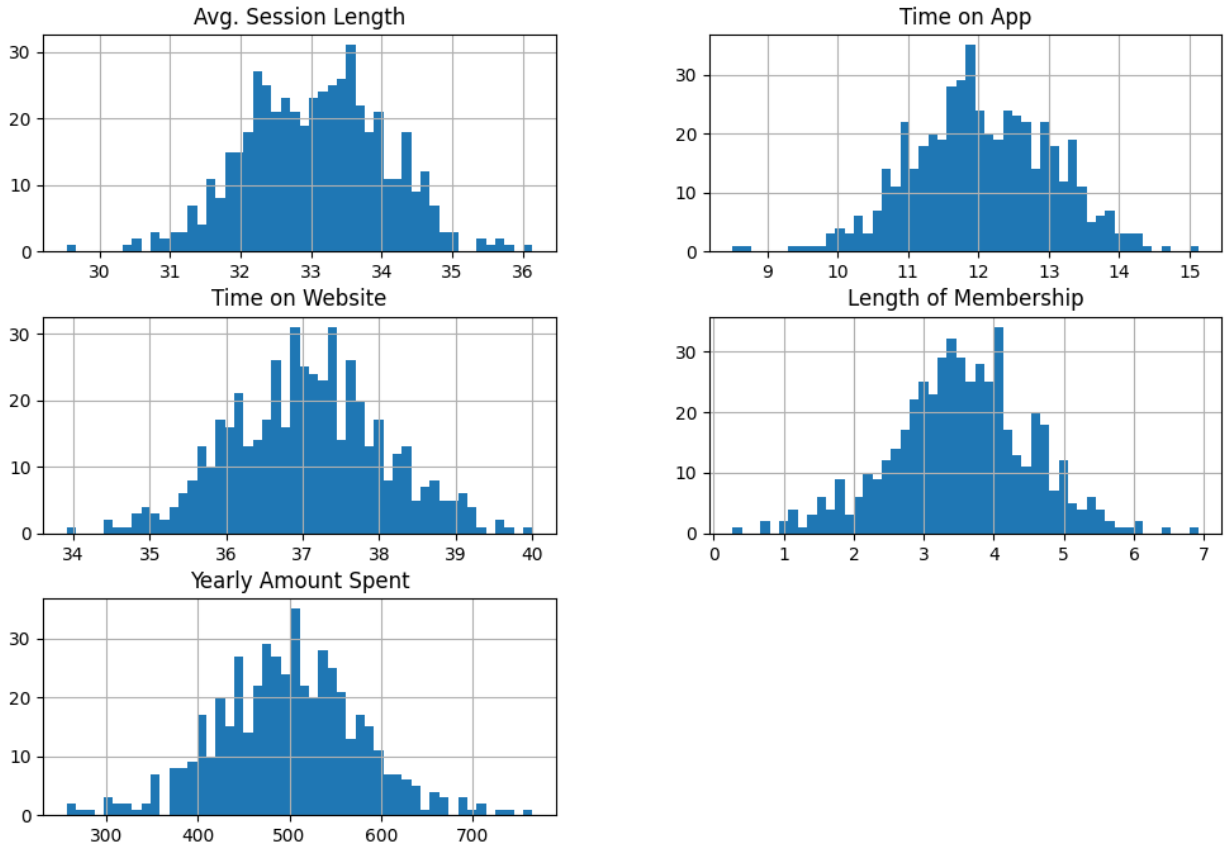
2.Display the statistical values for each of the attributes, along with visualizations (e.g. histogram) of the distributions for each attribute. Explain noticeable traits for key attributes. Are there any attributes that might require special treatment? If so, what special treatment might they require?

In [4]:
```
#statistical value of each attribute
ecommerce_df.describe()
```

Out[4]:

	Avg. Session Length	Time on App	Time on Website	Length of Membership	Yearly Amount Spent
count	500.000000	500.000000	500.000000	500.000000	500.000000
mean	33.053194	12.052488	37.060445	3.533462	499.314038
std	0.992563	0.994216	1.010489	0.999278	79.314782
min	29.532429	8.508152	33.913847	0.269901	256.670582
25%	32.341822	11.388153	36.349257	2.930450	445.038277
50%	33.082008	11.983231	37.069367	3.533975	498.887875
75%	33.711985	12.753850	37.716432	4.126502	549.313828
max	36.139662	15.126994	40.005182	6.922689	765.518462

In [5]:
```
# histogram for each attributes

ecommerce_df.hist(bins=50, figsize=(12, 8))
plt.show()
```



In [6]:
```
#calculating the skeweness
ecommerce_df.skew().sort_values(ascending=False)
```

```
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/2243137382.py:2: FutureWarning: Dropping of nuisance columns in DataFrame reductions (with 'numeric_only=None') is deprecated; in a future version this will raise TypeError.  Select only valid columns before calling the reduction.
  ecommerce_df.skew().sort_values(ascending=False)
```

Out[6]:
```
Yearly Amount Spent     0.034790
Time on Website         0.012142
Avg. Session Length    -0.032175
Time on App            -0.089121
Length of Membership   -0.106608
dtype: float64
```

Ans 2: We observe the skeweness value to be between -0.1 to 0.03 .Skewness is a measure of the asymmetry of a probability distribution, and a skewness value between -0.5 and 0.5 usually indicates a relatively symmetrical distribution. A symmetrical distribution has its mean, median, and mode all equal and its shape is roughly bell-shaped.Additionally, we observe bell shaped curve from the plot, this further supports the conclusion that the data has a symmetrical distribution. Also there are no null values for any of the attributes.No special treatment is needed for this dataset except for dropping the categorical values.

3. Analyze and discuss the relationships between the data attributes, and between the data attributes and label. This involves computing the Pearson Correlation Coefficient (PCC) and generating scatter plots.

In [7]:
```
#pearson coefficient of data attributes
ecommerce_df.drop(['Yearly Amount Spent'],axis=1).corr(method ='pearson
```

Out[7]:

	Avg. Session Length	Time on App	Time on Website	Length of Membership
Avg. Session Length	1.000000	-0.027826	-0.034987	0.060247
Time on App	-0.027826	1.000000	0.082388	0.029143
Time on Website	-0.034987	0.082388	1.000000	-0.047582
Length of Membership	0.060247	0.029143	-0.047582	1.000000

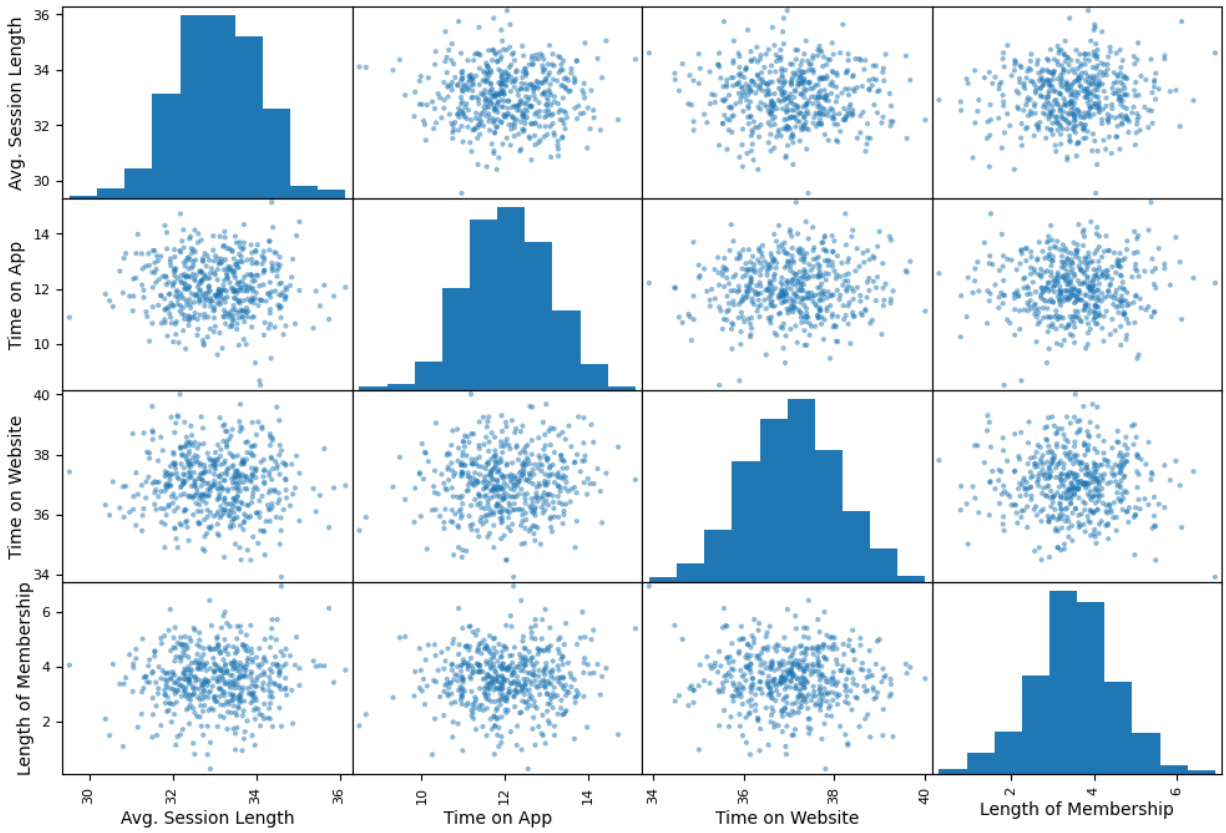
In [8]:
```
#pearson coefficient between data attributes and label
corr_matrix=ecommerce_df.corr(method ='pearson')
corr_matrix['Yearly Amount Spent'] #label
```

Out[8]:
```
Avg. Session Length     0.355088
Time on App             0.499328
Time on Website        -0.002641
Length of Membership    0.809084
Yearly Amount Spent     1.000000
Name: Yearly Amount Spent, dtype: float64
```

From the chart we conclude that'length of membership' has highest coefficient of 0.8.

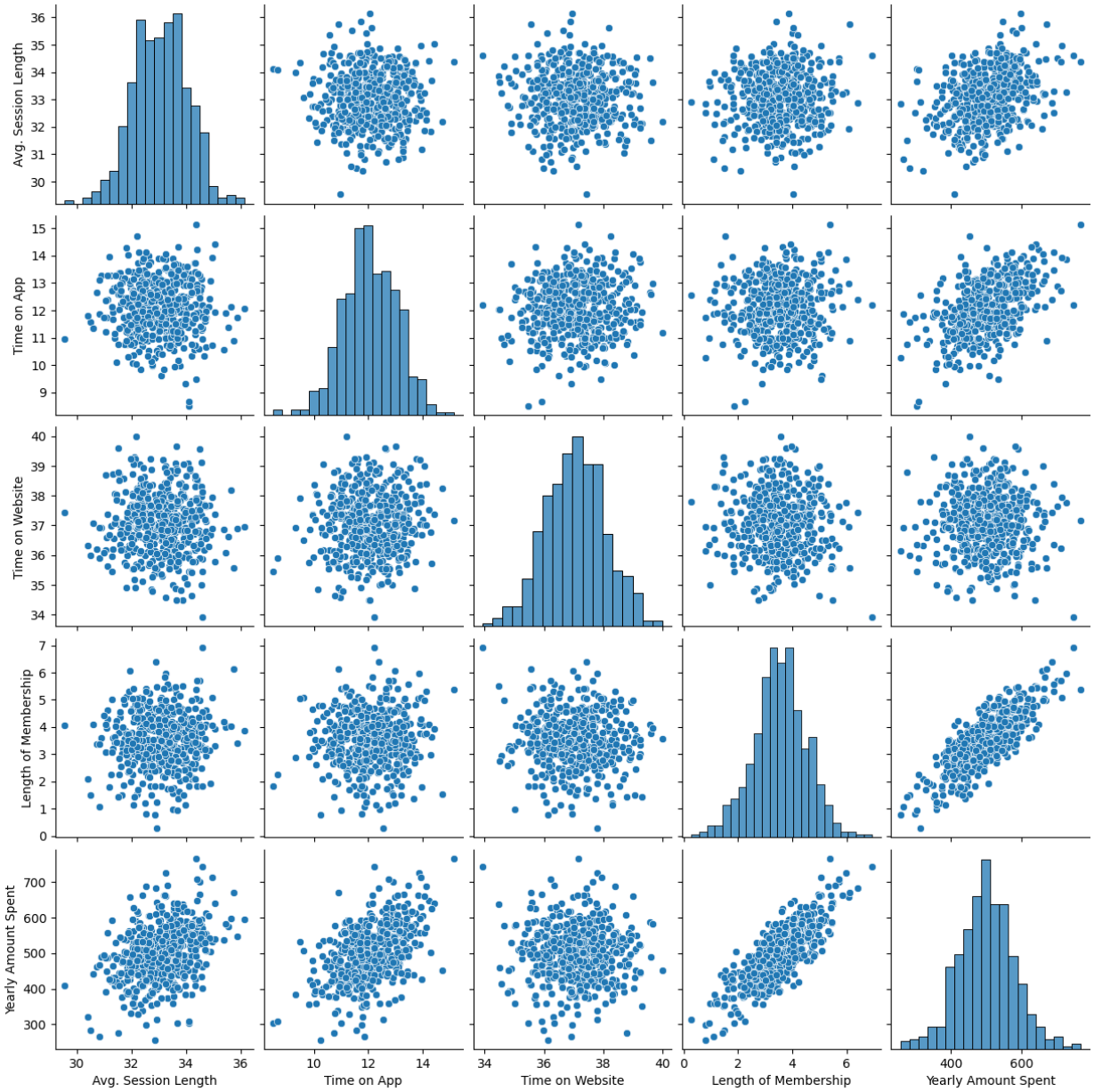
In [9]:
```
#scatter matrix plot between important data attributes

from pandas.plotting import scatter_matrix
attributes = ["Avg. Session Length", "Time on App", "Time on Website",
              "Length of Membership"]
scatter_matrix(ecommerce_df[attributes], figsize=(12, 8))

```

Out[9]:
```
array([[<AxesSubplot:xlabel='Avg. Session Length', ylabel='Avg. Session L
ength'>,
        <AxesSubplot:xlabel='Time on App', ylabel='Avg. Session Length'>,
        <AxesSubplot:xlabel='Time on Website', ylabel='Avg. Session Lengt
h'>,
        <AxesSubplot:xlabel='Length of Membership', ylabel='Avg. Session
Length'>],
       [<AxesSubplot:xlabel='Avg. Session Length', ylabel='Time on App'>,
        <AxesSubplot:xlabel='Time on App', ylabel='Time on App'>,
        <AxesSubplot:xlabel='Time on Website', ylabel='Time on App'>,
        <AxesSubplot:xlabel='Length of Membership', ylabel='Time on Ap
p'>],
       [<AxesSubplot:xlabel='Avg. Session Length', ylabel='Time on Websit
e'>,
        <AxesSubplot:xlabel='Time on App', ylabel='Time on Website'>,
        <AxesSubplot:xlabel='Time on Website', ylabel='Time on Website'>,
        <AxesSubplot:xlabel='Length of Membership', ylabel='Time on Websi
te'>],
       [<AxesSubplot:xlabel='Avg. Session Length', ylabel='Length of Memb
ership'>,
        <AxesSubplot:xlabel='Time on App', ylabel='Length of Membershi
p'>,
        <AxesSubplot:xlabel='Time on Website', ylabel='Length of Membersh
ip'>,
        <AxesSubplot:xlabel='Length of Membership', ylabel='Length of Mem
bership'>]],
      dtype=object)
```

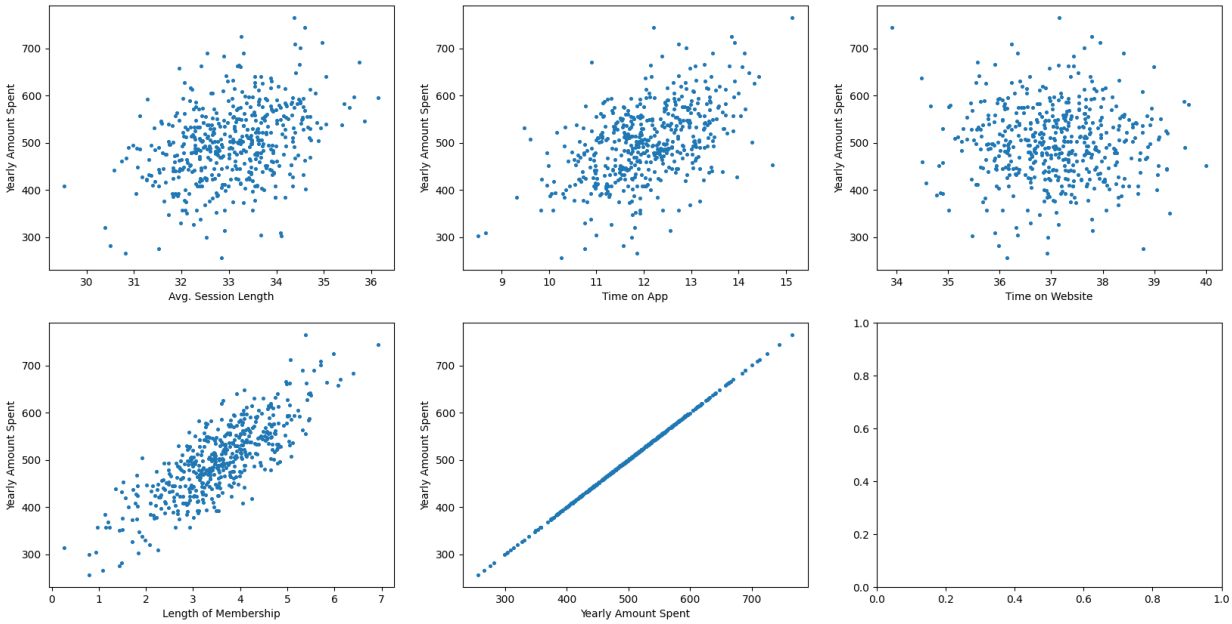

Avg. Session Length
Time on App
Time on Website
Length of Membership
30
32
34
36
10
12
14
34
36
38
40
2
4
6

In [10]:
```
#relation between various attributes using pairplot

import seaborn as sns
sns.pairplot(ecommerce_df)

```

Out[10]: <seaborn.axisgrid.PairGrid at 0x149274be0>



In [11]:
```
# Scatter plots of attributes versus output label

col = ['Avg. Session Length','Time on App','Time on Website','Length of

fig, ax = plt.subplots(2,3, figsize=(20,10))
j = 0
k = 0
for i in range(len(col )):
    if(i == (3)):
        j = 0
        k = 1
  # scatter plot
    ax[k,j].scatter(x=ecommerce_df[col [i]],y=ecommerce_df['Yearly Amou
    ax[k,j].set_xlabel(col [i])
    ax[k,j].set_ylabel('Yearly Amount Spent')
    j = j + 1
```



If the data points in a scatter plot are close to each other and form a linear pattern, it suggests that there is a strong linear relationship between the two variables. In this case, the data points for "length of membership" and "yearly amount spent" are close to each other and form a linear pattern in the scatter plot or pairplot this suggests that these two variables are more closely related to each other, i.e., there is a strong linear relationship between them.

4. Select 20% of the data for testing. Describe how you did that and verify that your test portion of the data is representative of the entire dataset.

In [12]:
```
# Converting 'Length of Membership' to a categorical type

ecommerce_df["Length of Membership_cat"] = pd.cut(ecommerce_df["Length
                                  bins=[0., 1.5, 3.0, 4.5, 6., np.inf],
                                  labels=[1, 2, 3, 4, 5])

#dropping categorical columns and label
x = ecommerce_df.drop(columns=['Yearly Amount Spent','Email','Address',
y = ecommerce_df['Yearly Amount Spent']

# Randomized sampling
x_train_1, x_test_1, y_train_1, y_test_1 = train_test_split(x, y, test_

# Stratified
y_strat_bin = ecommerce_df['Length of Membership_cat']
x_train, x_test, y_train, y_test = train_test_split(x, y, test_size=0.2

print(x_train_1.shape,y_train_1.shape, x_test_1.shape, y_test_1.shape)

# Distribution ratio of 'Length of Membership_cat' in the overall datas
dist_overall = ecommerce_df["Length of Membership_cat"].value_counts()

# Distribution ratio of 'Length of Membership_cat' in the randomized te
dist_randomized = x_test_1["Length of Membership_cat"].value_counts() /

# Distribution ratio of 'Length of Membership_cat' in the stratified te
dist_stratified = x_test["Length of Membership_cat"].value_counts() / l

compare = pd.DataFrame({"Overall":dist_overall,"Randomized":dist_random
compare
```

```
(400, 5) (400,) (100, 5) (100,)
```

Out[12]:

	Overall	Randomized	Stratified
1	0.032	0.04	0.03
2	0.242	0.27	0.24
3	0.554	0.48	0.55
4	0.164	0.19	0.17
5	0.008	0.02	0.01

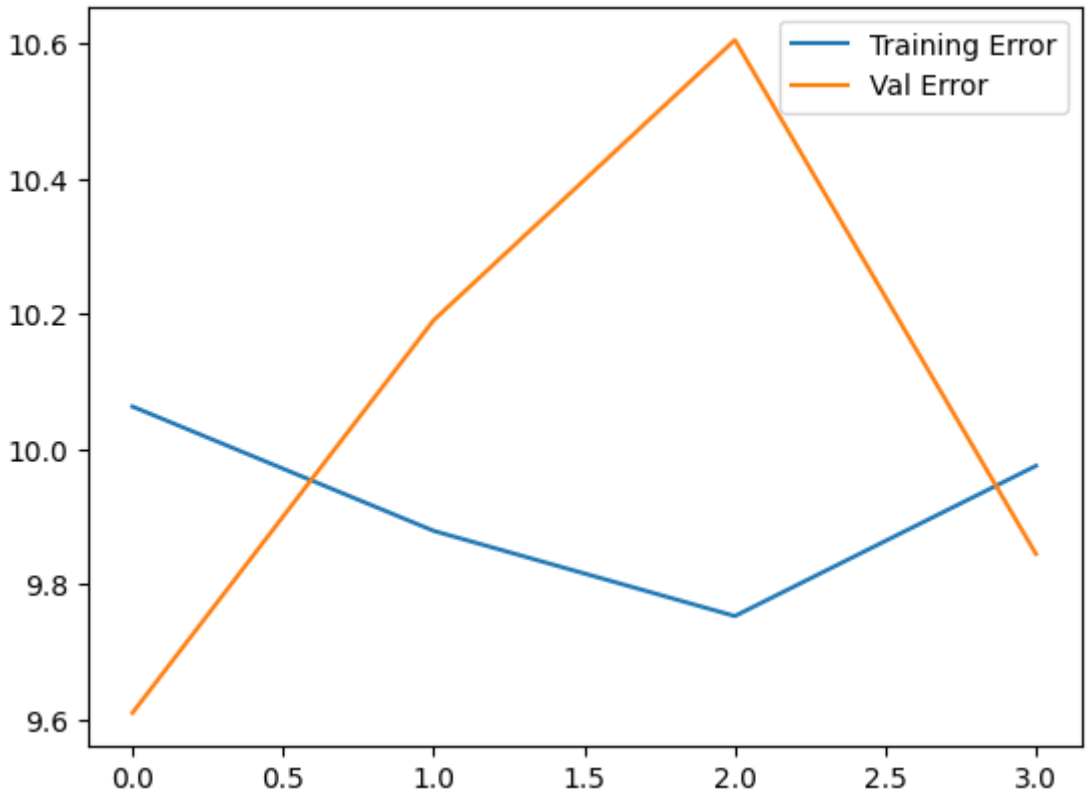
Ans 4:- We have created our test dataset using 2 methods: randomized and stratified. For randomized, we simply used the test_train_split function. For stratified, we first create a categorical attribute which can be used as stratas. We choose 'Length of Membership' attribute since it has the highest Pearson Correlation Coefficient with our target variable.. Then we compared the label distributions of the overall dataset, randomized test dataset and stratified test dataset. We see that stratified method produces better results than randomized methods. It produces a test set which is representative of the entire dataset.

In [13]:
```
# Converting training and test sets into numpy arrays
x_train, x_test, y_train, y_test = np.array(x_train), np.array(x_test),

#defining mean squared error
def mse(y_pred,y_true):
    return np.sum((y_true.reshape((y_true.shape[0],))-y_pred.reshape((y

#define metric dictionary
metric_dict= {}
```

5. Train a Linear Regression model using the training data with four-fold cross-validation using appropriate evaluation metric. Do this with closed form solution (using the Normal Equation or SVD) and with SGD. Perform Ridge, Lasso and Elastic Net regularization – try a few values of penalty term and describe its impact. Explore the impact of other hyperparameters, like batch size and learning rate (no need for grid search). Describe your findings. Display the training and validation loss as a function of training iterations.

In [14]:
```
# Linear regression model using a closed-form solution with four-fold c
train_plot = []
val_plot = []
kf = KFold(n_splits=4)
theta_closed=[]

for i, (train_index, val_index) in enumerate(kf.split(x_train,y_train)):
    print(f"Fold {i}:")
    x = x_train[train_index]
    y = y_train[train_index]
    x_val = x_train[val_index]
    y_val = y_train[val_index]

    X_b = np.c_[np.ones((x.shape[0], 1)), x]
    theta = np.linalg.inv(X_b.T.dot(X_b)).dot(X_b.T).dot(y)
    theta_closed.append(theta)

    X_train_b = np.c_[np.ones((x.shape[0], 1)), x]
    y_predict_train = X_train_b.dot(theta)
    print("Train error:",np.sqrt(mse(y_predict_train,y)))

    X_val_b = np.c_[np.ones((x_val.shape[0], 1)), x_val]
    y_predict_val = X_val_b.dot(theta)
    print("validation error:",np.sqrt(mse(y_predict_val,y_val)))
    train_plot.append(np.sqrt(mse(y_predict_train,y)))
    val_plot.append(np.sqrt(mse(y_predict_val,y_val)))
plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
plt.legend()

 #for prediction
theta_closed_best=np.mean(theta_closed,axis=0)    #calculating the mean
X_test_b = np.c_[np.ones((x_test.shape[0], 1)), x_test]
y_test_pred = X_test_b.dot(theta_closed_best)
metric_dict['Linear regression'] = np.sqrt(mse(y_test, y_test_pred))
```

```
Fold 0:
Train error: 10.062843403919272
validation error: 9.609864146877783
Fold 1:
Train error: 9.878993395744269
validation error: 10.190564788675019
Fold 2:
Train error: 9.752971016584787
validation error: 10.60495288369457
Fold 3:
Train error: 9.975176680690792
validation error: 9.8447974376302
```

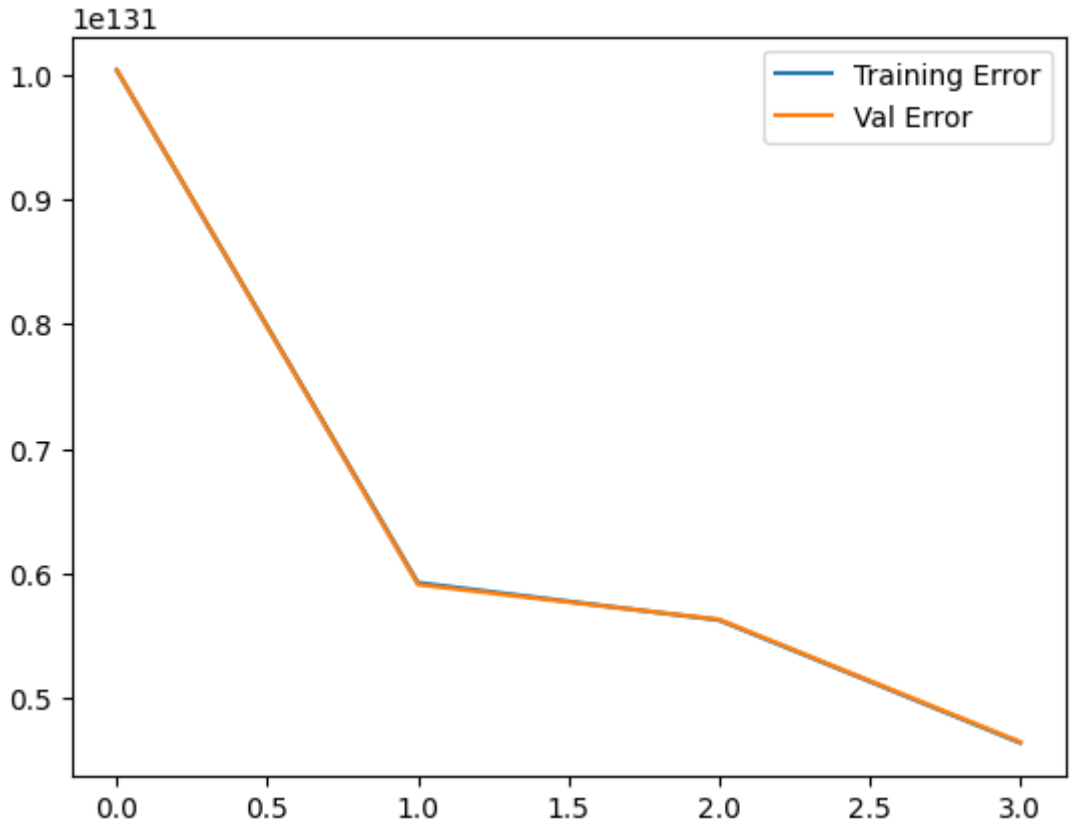


In [15]:

```
# Using Stochastic Gradient Descent with learning schedule parameters
train_plot = []
val_plot = []
k = 4
kf = KFold(n_splits=4)
for i, (train_index, val_index) in enumerate(kf.split(x_train,y_train)):

    print(f"Fold {i}:")
    x = x_train[train_index]
    y = y_train[train_index]
    x_val = x_train[val_index]
    y_val = y_train[val_index]
    X_b = np.c_[np.ones((x.shape[0], 1)), x]

    theta_path_sgd = []
    m = len(X_b)
    np.random.seed(42)

    n_epochs = 8
    t0, t1 = 5, 15         # learning schedule hyperparameters
    theta_sgd=[]
    def learning_schedule(t):
        return t0 / (t + t1)
    theta = np.random.randn(6,1)

    for epoch in range(n_epochs):
        for i in range (10):
            random_index = np.random.randint(m)
            xi = X_b[random_index:random_index+1]
            yi = y[random_index:random_index+1]
            gradients = 2 * xi.T.dot(xi.dot(theta) - yi)
            eta = learning_schedule(epoch * m + i)
            theta = theta - eta * gradients
            theta_path_sgd.append(theta)


    theta_sgd.append(theta)
    X_train_b = np.c_[np.ones((x.shape[0], 1)), x]
    y_predict_train = X_train_b.dot(theta)
    print("Train error:",np.sqrt(mse(y_predict_train,y)))

    X_val_b = np.c_[np.ones((x_val.shape[0], 1)), x_val]
    y_predict_val = X_val_b.dot(theta)
    print("val error:",np.sqrt(mean_squared_error(y_predict_val,y_val)))
    train_plot.append(np.sqrt(mse(y_predict_train,y)))
    val_plot.append(np.sqrt(mse(y_predict_val,y_val)))
plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
plt.legend()

theta_sgd_best=np.mean(theta_sgd,axis=0)
X_test_b = np.c_[np.ones((x_test.shape[0], 1)), x_test]
y_test_pred = X_test_b.dot(theta_sgd_best)
#metric_dict['sgd-learning schedule parameters'] = np.sqrt(mse(y_test,
```

```
55
```

```
Fold 0:
Train error: 1.0045414630051186e+131
val error: 1.0044454204319585e+131
Fold 1:
Train error: 5.920654212281748e+130
val error: 5.905568477111588e+130
Fold 2:
Train error: 5.619986916244871e+130
val error: 5.624709662432002e+130
Fold 3:
Train error: 4.632522816117348e+130
val error: 4.639381686620146e+130
```

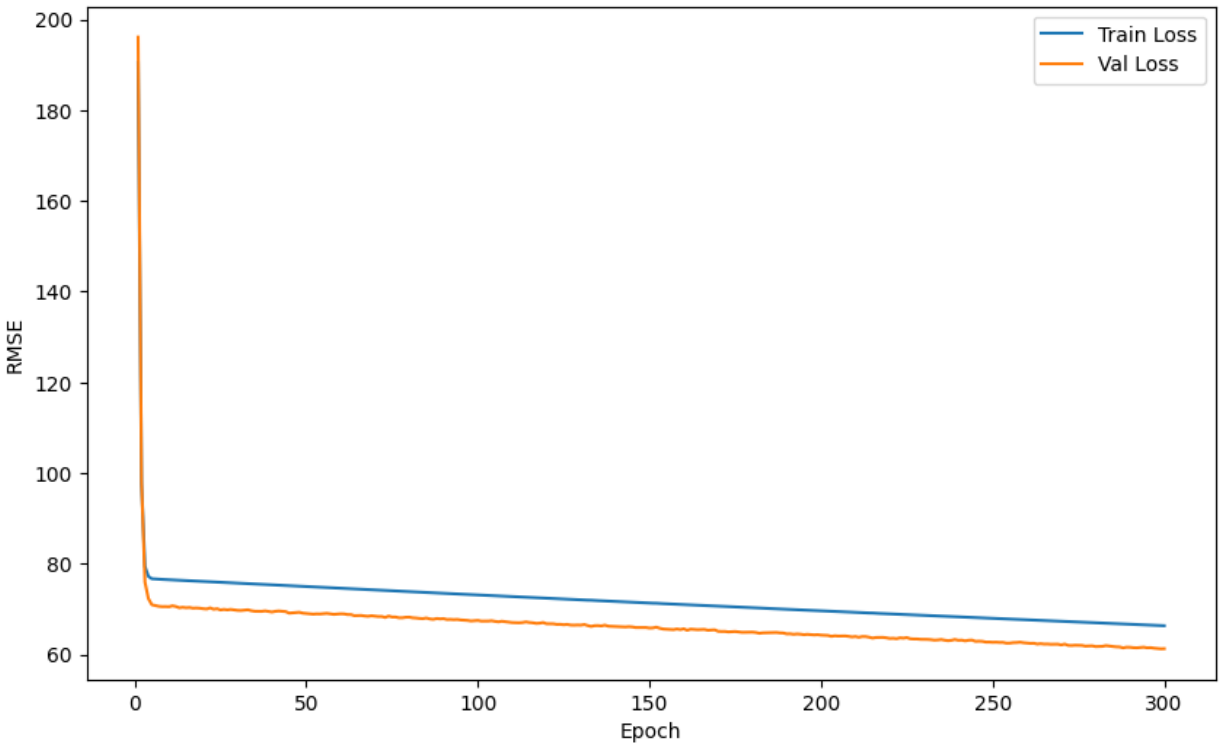


Fix - Added learning rates to SGD and plotted the loss vs epochs graphs

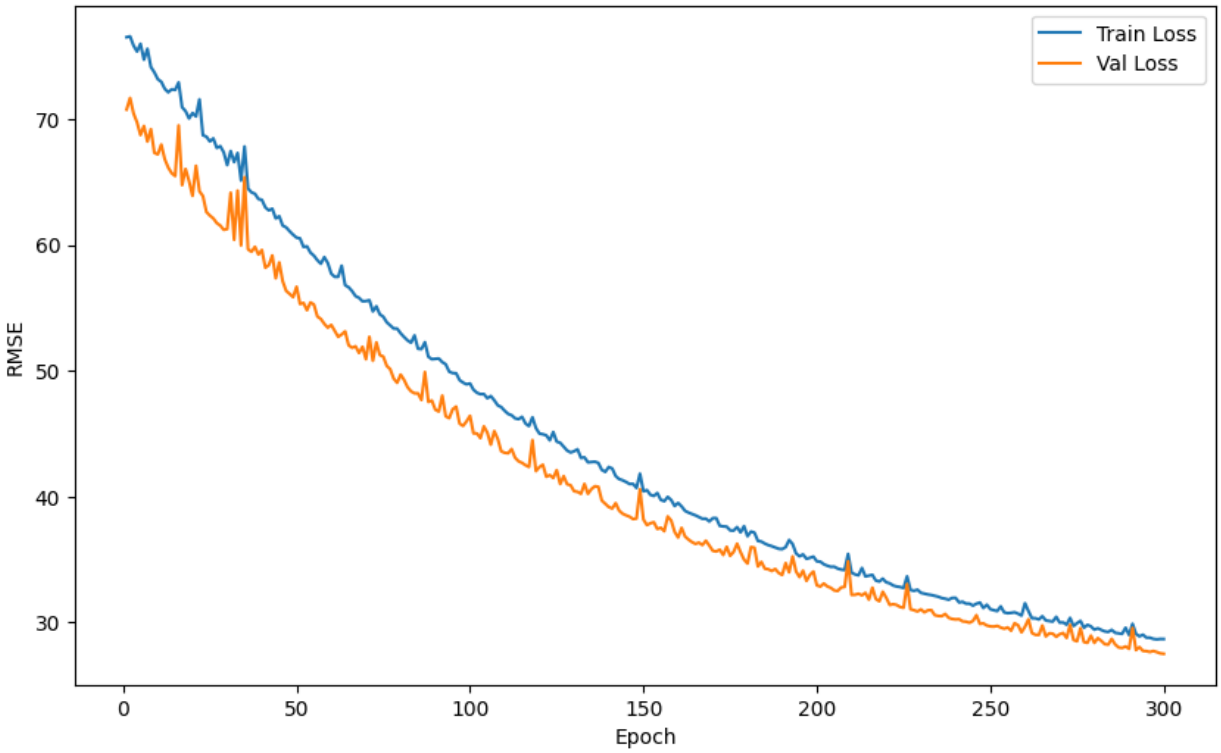
In [16]:
```
SGD with different learning rate

arning_rates = [0.000001, 0.00001, 0.0001, 0.001, 0.01]
epochs = 300

ain_losses = np.zeros((len(learning_rates), n_epochs))
l_losses = np.zeros((len(learning_rates), n_epochs))
st_rmse_scores = []

r i, learning_rate in enumerate(learning_rates):
  sgd = SGDRegressor(max_iter=1, warm_start=True, eta0=learning_rate, learni
  for epoch in range(n_epochs):
    sgd.partial_fit(x_train, y_train)
    y_pred_train = sgd.predict(x_train)
    y_pred_val = sgd.predict(x_val)
    train_losses[i, epoch] = np.sqrt(mse(y_pred_train, y_train))
    val_losses[i, epoch] = np.sqrt(mse(y_pred_val, y_val))

  # Plot train and val losses for each epoch and each learning rate separate
  fig, axs = plt.subplots(figsize=(10, 6))
  fig.suptitle(f"Learning Rate: {learning_rate}")
  axs.plot(range(1, n_epochs + 1), train_losses[i], label="Train Loss")
  axs.plot(range(1, n_epochs + 1), val_losses[i], label="Val Loss")
  axs.set_xlabel("Epoch")
  axs.set_ylabel("RMSE")
  axs.legend()

  # Calculate RMSE score on test set
  y_pred_test = sgd.predict(x_test)
  test_rmse_score = np.sqrt(mse(y_pred_test, y_test))
  metric_dict['SGD with learning rate ='+str(learning_rate)] = test_rmse_sc
  print(f"RMSE score between y_pred_val and y_test for learning_rate={learni
  test_rmse_scores.append(test_rmse_score)

t.show()
```

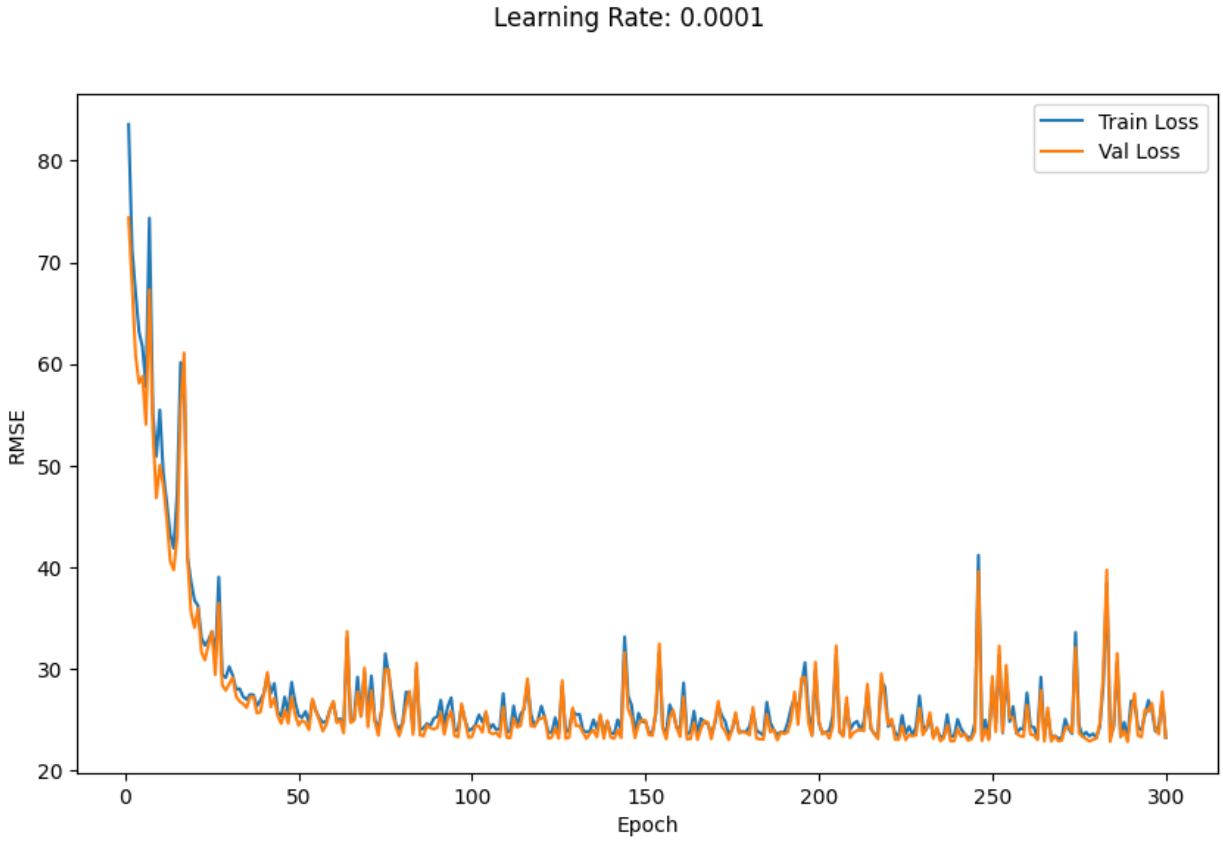
```
RMSE score between y_pred_val and y_test for learning_rate=1e-06: 60.3493
95135527104
RMSE score between y_pred_val and y_test for learning_rate=1e-05: 25.9960
01317630146
RMSE score between y_pred_val and y_test for learning_rate=0.0001: 23.349
901277079177
RMSE score between y_pred_val and y_test for learning_rate=0.001: 1488544
586411.5103
RMSE score between y_pred_val and y_test for learning_rate=0.01: 18562893
045068.81
```

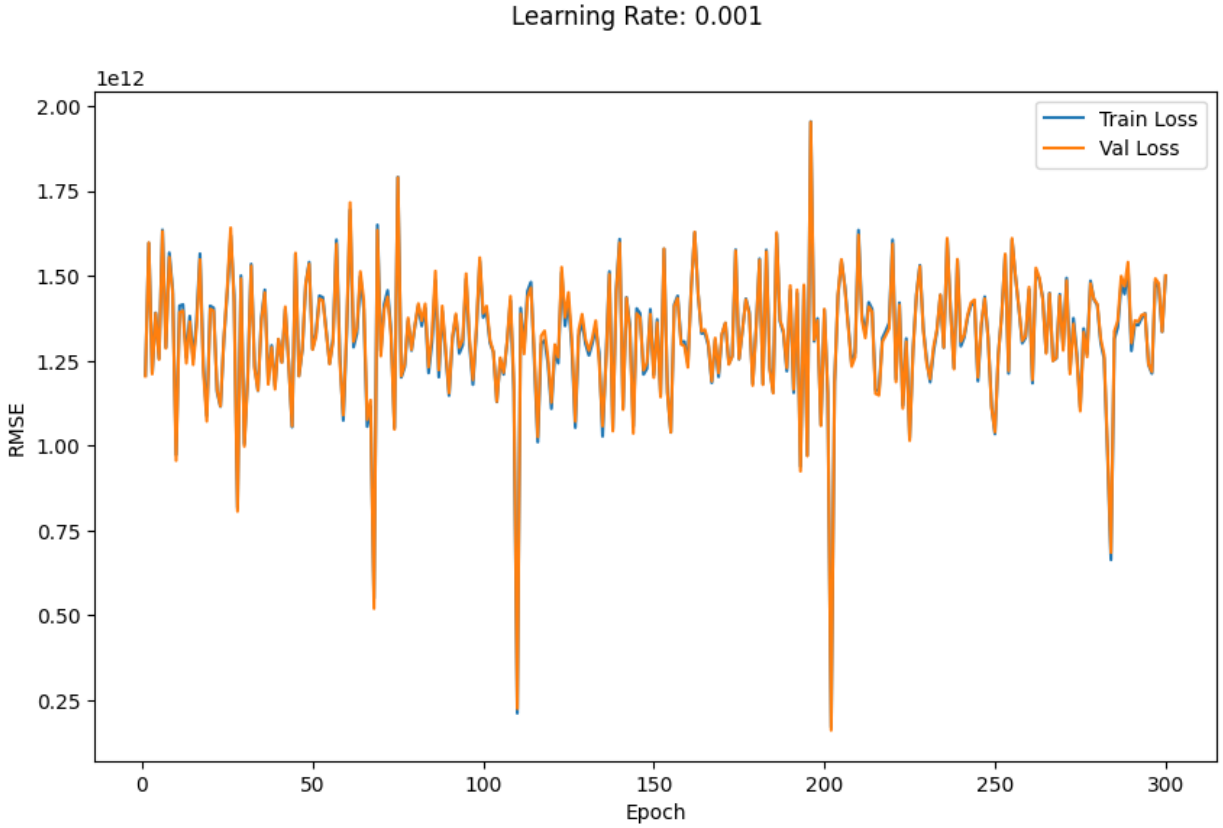
Learning Rate: 1e-06

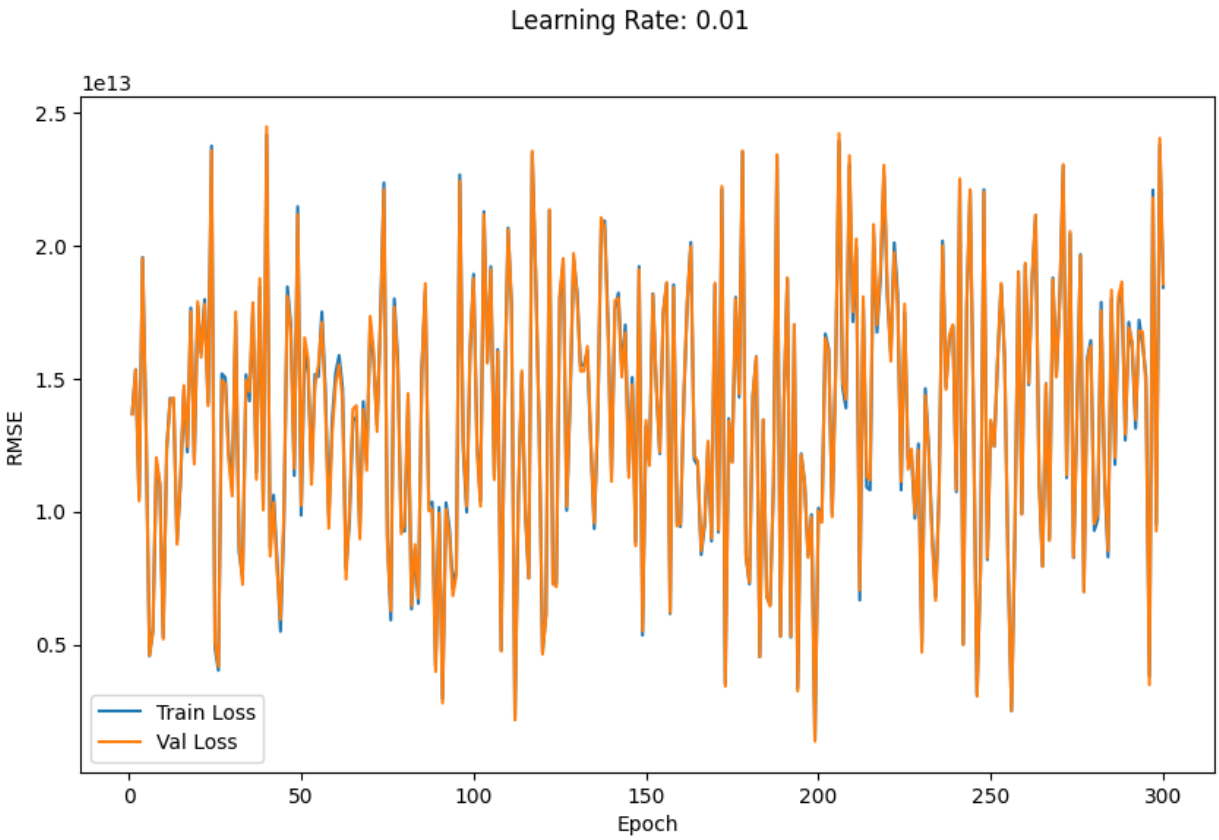


Learning Rate: 1e-05




Learning Rate: 0.0001
Train Loss
Val Loss
RMSE
Epoch
80
70
60
50
40
30
20
0
50
100
150
200
250
300

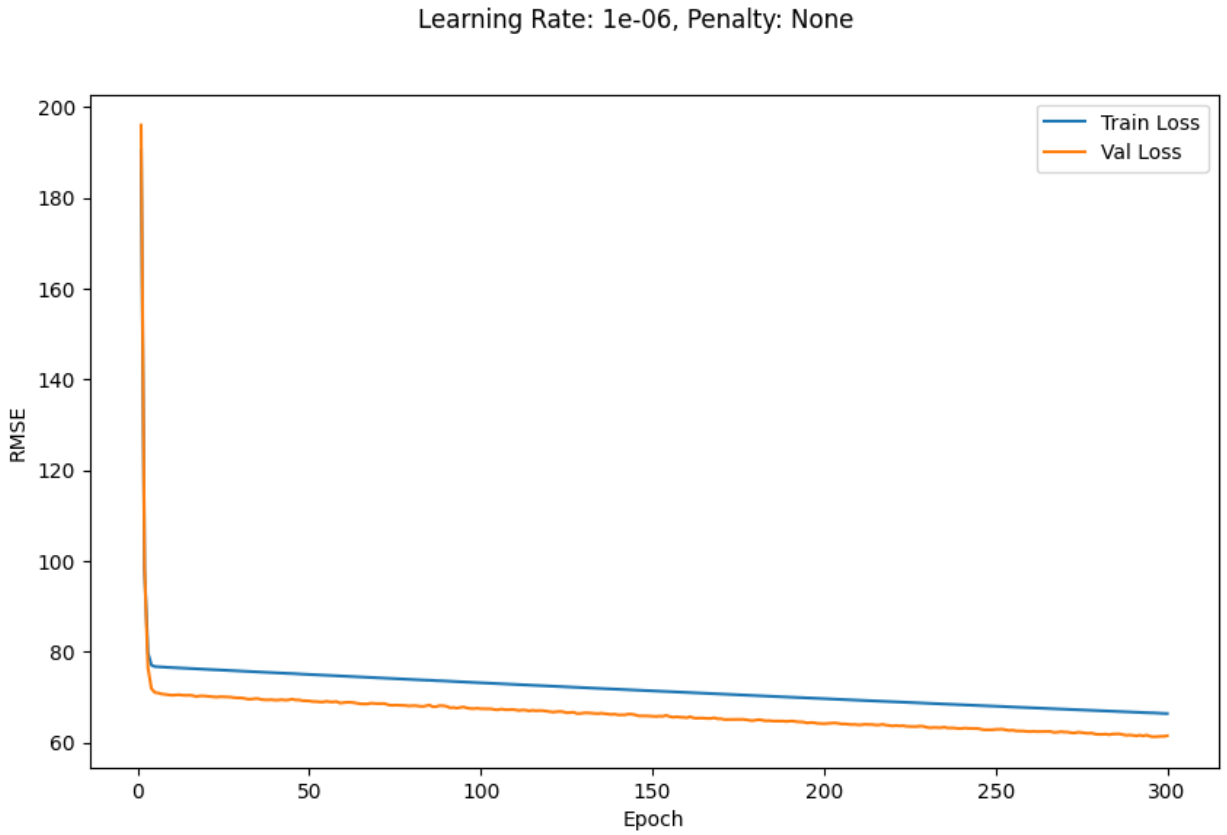

Learning Rate: 0.001
1e12
Train Loss
Val Loss
RMSE
Epoch
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0
50
100
150
200
250
300

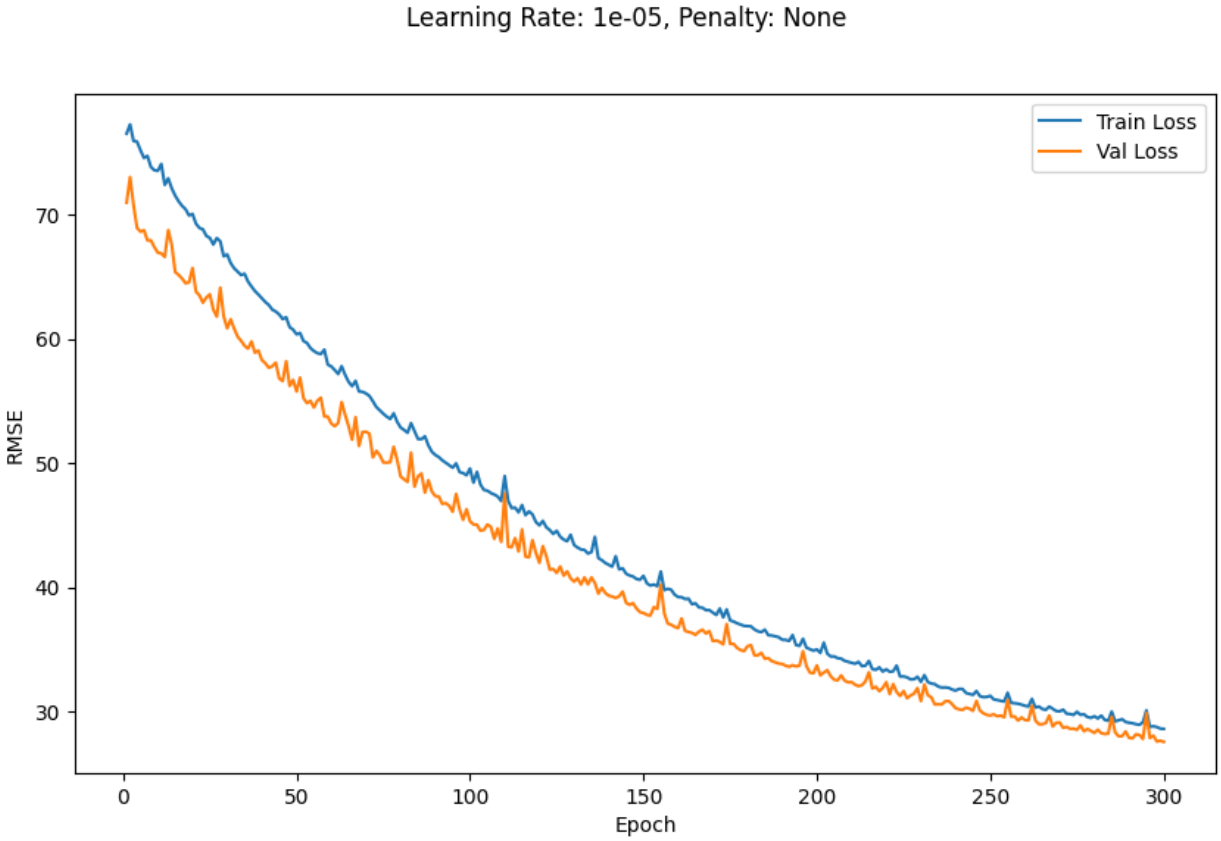

Learning Rate: 0.01
1e13
2.5
2.0
1.5
1.0
0.5
RMSE
Train Loss
Val Loss
0
50
100
150
200
250
300
Epoch

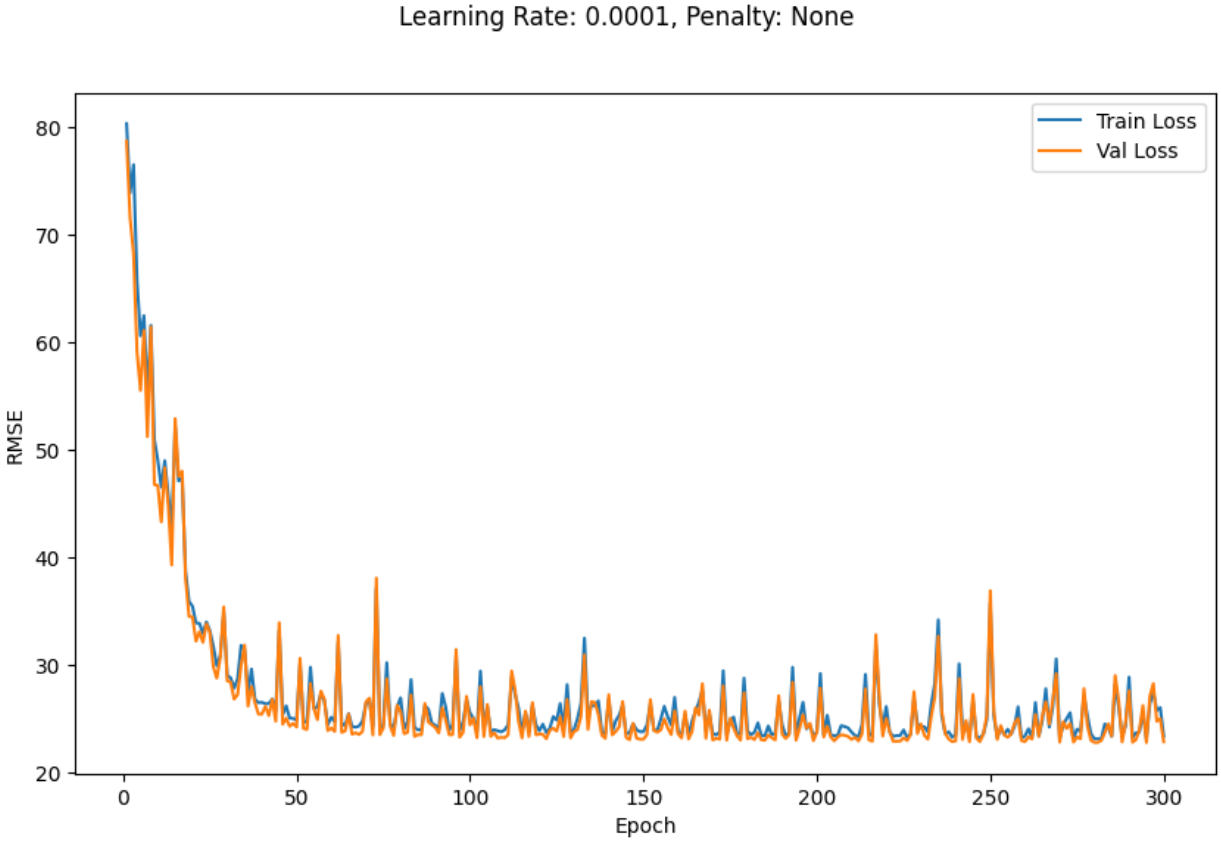
In [17]:

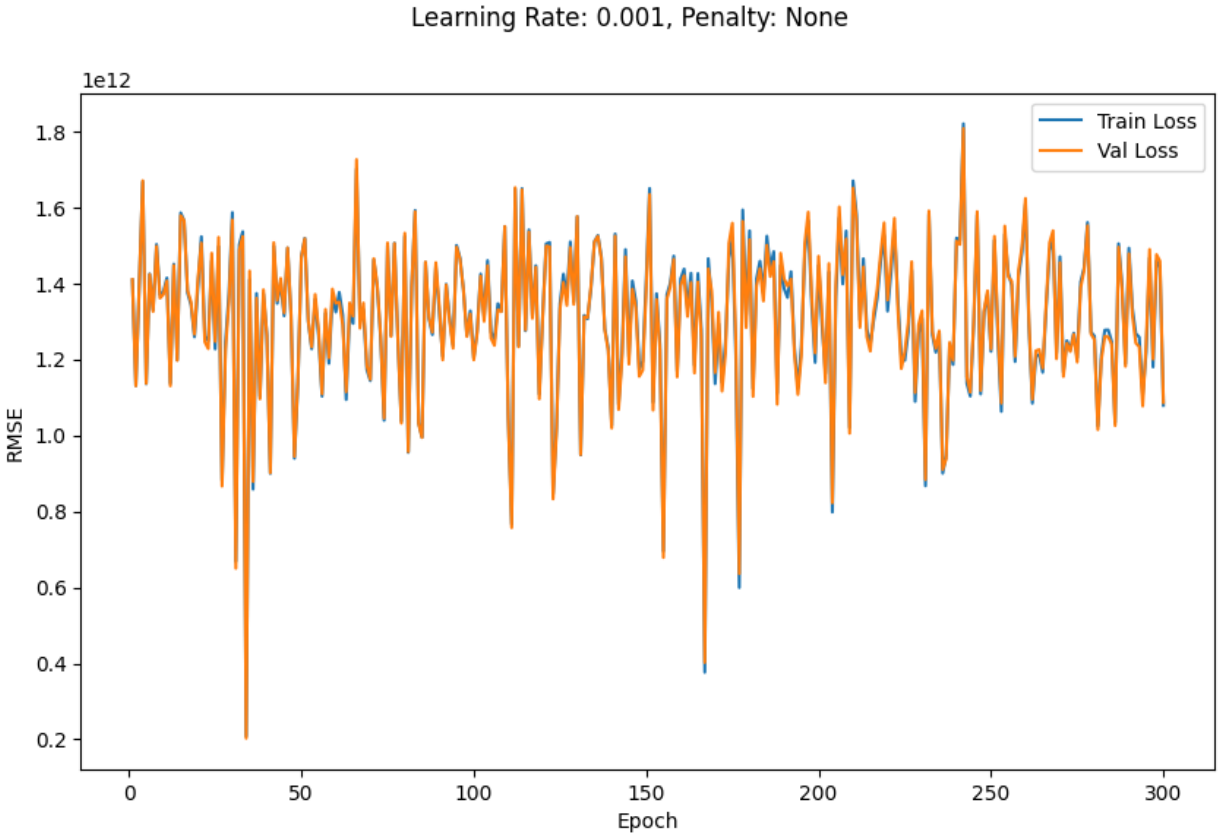
```
# SGD with different penalty terms

penalty_factors = [None, 'l1', 'l2']
learning_rates = [0.000001, 0.00001, 0.0001, 0.001, 0.01]
n_epochs = 300

for penalty in penalty_factors:
    for i, learning_rate in enumerate(learning_rates):
        sgd = SGDRegressor(max_iter=1, warm_start=True, eta0=learning_r
        train_losses = np.zeros(n_epochs)
        val_losses = np.zeros(n_epochs)
        for epoch in range(n_epochs):
            sgd.partial_fit(x_train, y_train)
            y_pred_train = sgd.predict(x_train)
            y_pred_val = sgd.predict(x_val)
            train_losses[epoch] = np.sqrt(mse(y_pred_train, y_train))
            val_losses[epoch] = np.sqrt(mse(y_pred_val, y_val))

        # Plot train and val losses for each epoch and each learning ra
        fig, axs = plt.subplots(figsize=(10, 6))
        fig.suptitle(f"Learning Rate: {learning_rate}, Penalty: {penalt
        axs.plot(range(1, n_epochs + 1), train_losses, label="Train Los
        axs.plot(range(1, n_epochs + 1), val_losses, label="Val Loss")
        axs.set_xlabel("Epoch")
        axs.set_ylabel("RMSE")
        axs.legend()

        # Calculate RMSE score on test set
        y_pred_test = sgd.predict(x_test)
        test_rmse_score = np.sqrt(mse(y_pred_test, y_test))
        metric_dict['SGD with learning rate ='+str(learning_rate)+' and
        print(f"RMSE score between y_pred_val and y_test for learning_r
        test_rmse_scores.append(test_rmse_score)

plt.show()
```

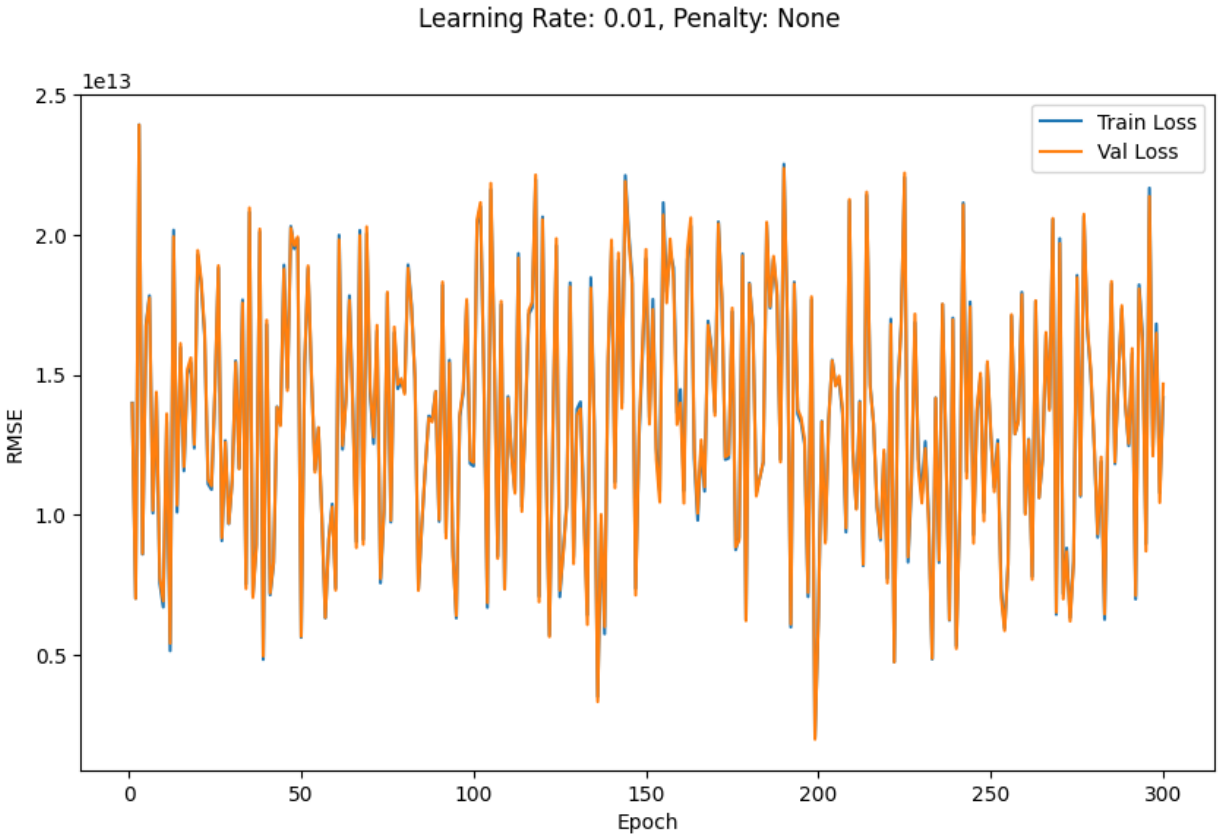
```
RMSE score between y_pred_val and y_test for learning_rate=1e-06 and pena
lty_factor=None: 60.451655627851046
RMSE score between y_pred_val and y_test for learning_rate=1e-05 and pena
lty_factor=None: 26.019897153035224
RMSE score between y_pred_val and y_test for learning_rate=0.0001 and pen
alty_factor=None: 23.068706461544643
RMSE score between y_pred_val and y_test for learning_rate=0.001 and pena
lty_factor=None: 1078176318229.8536
RMSE score between y_pred_val and y_test for learning_rate=0.01 and penal
ty_factor=None: 14256691494401.98
RMSE score between y_pred_val and y_test for learning_rate=1e-06 and pena
lty_factor=l1: 60.35799979471291
RMSE score between y_pred_val and y_test for learning_rate=1e-05 and pena
lty_factor=l1: 27.297829744206698
RMSE score between y_pred_val and y_test for learning_rate=0.0001 and pen
alty_factor=l1: 24.196112811617436
RMSE score between y_pred_val and y_test for learning_rate=0.001 and pena
lty_factor=l1: 1517469530288.826
RMSE score between y_pred_val and y_test for learning_rate=0.01 and penal
ty_factor=l1: 12320825983055.19
RMSE score between y_pred_val and y_test for learning_rate=1e-06 and pena
lty_factor=l2: 60.31959505021599
RMSE score between y_pred_val and y_test for learning_rate=1e-05 and pena
lty_factor=l2: 26.0436031325709
RMSE score between y_pred_val and y_test for learning_rate=0.0001 and pen
alty_factor=l2: 25.50362603647266
RMSE score between y_pred_val and y_test for learning_rate=0.001 and pena
lty_factor=l2: 1289996564600.162
RMSE score between y_pred_val and y_test for learning_rate=0.01 and penal
ty_factor=l2: 13336024816118.062
```

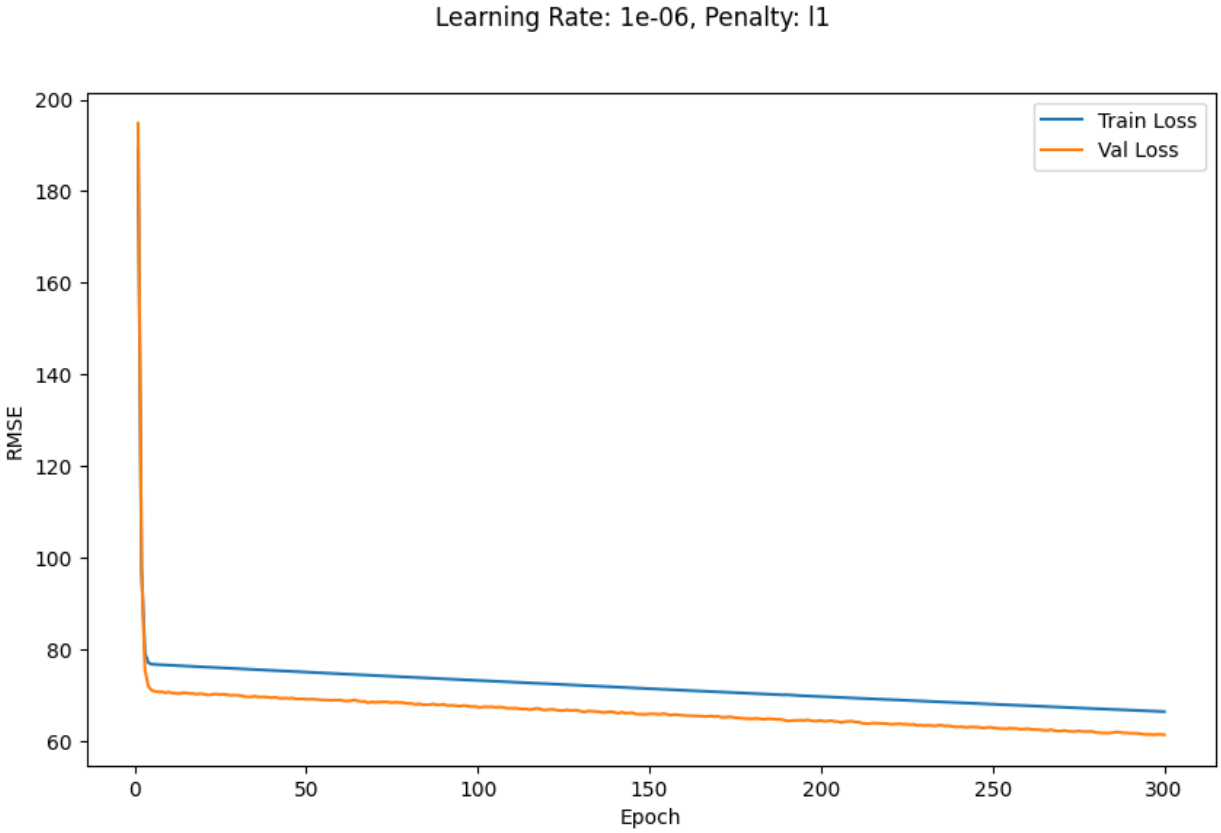


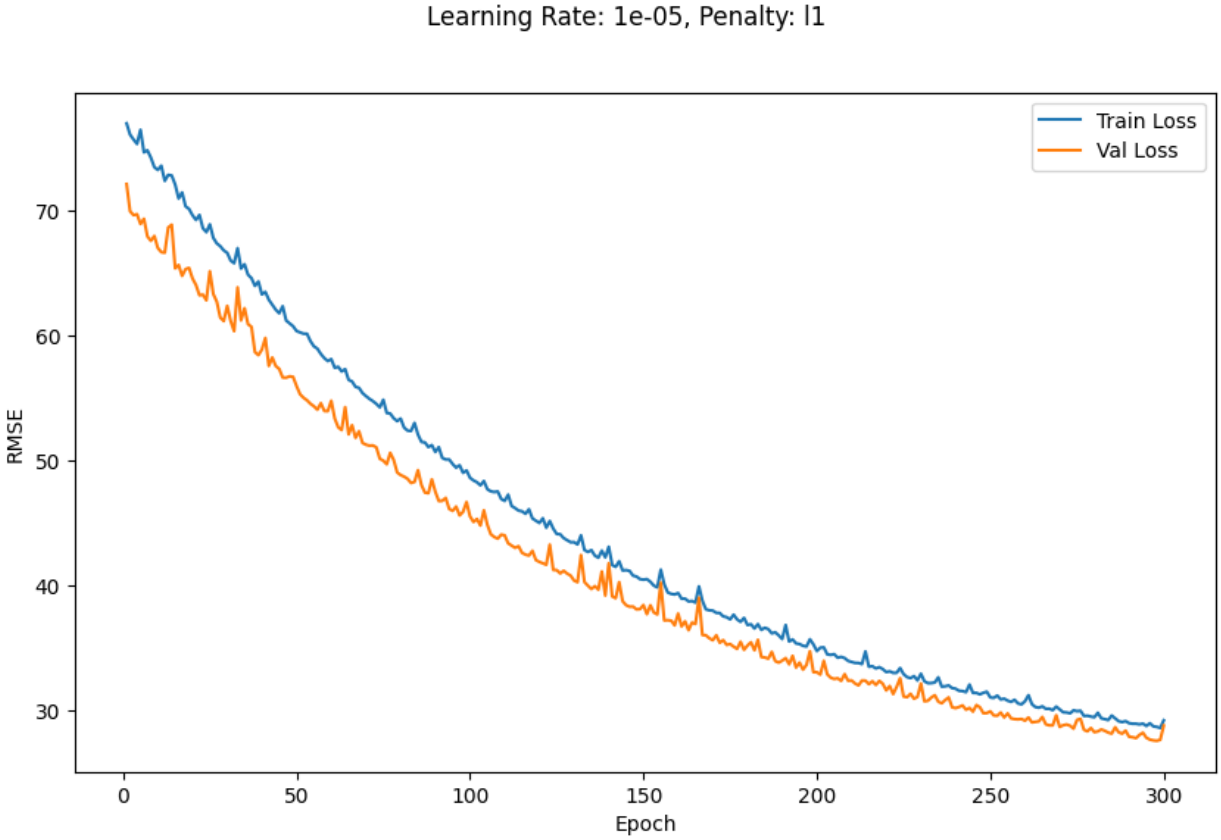

Learning Rate: 1e-05, Penalty: None
Train Loss
Val Loss
RMSE
Epoch
70
60
50
40
30
0
50
100
150
200
250
300

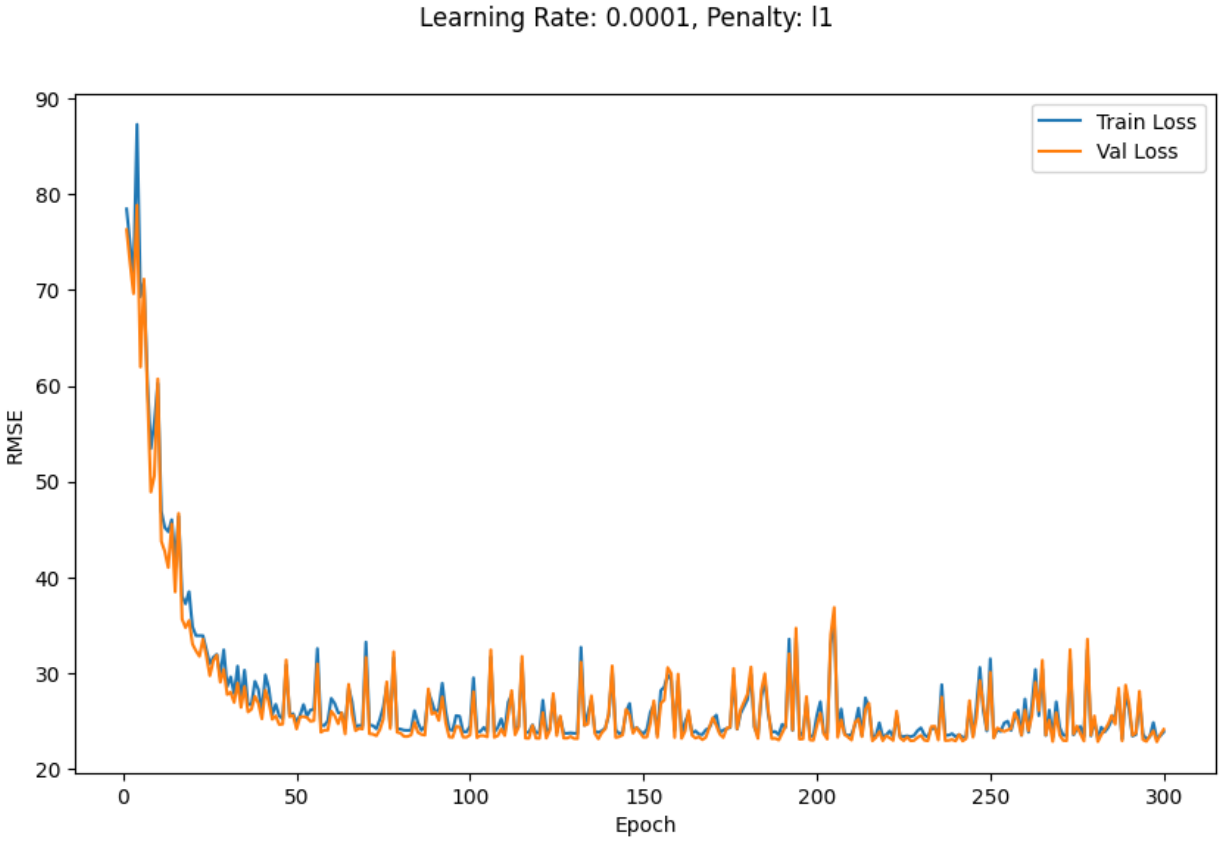

Learning Rate: 0.0001, Penalty: None
Train Loss
Val Loss
RMSE
Epoch
80
70
60
50
40
30
20
0
50
100
150
200
250
300

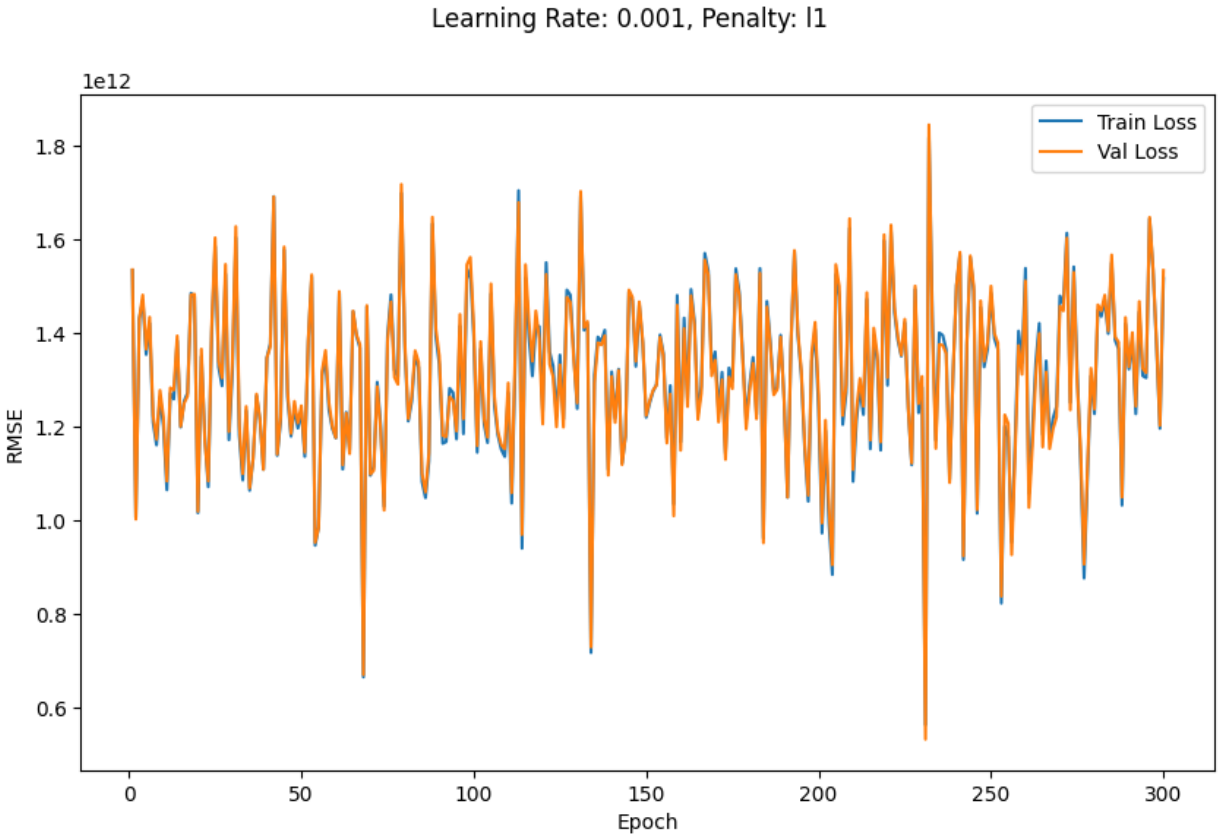

Learning Rate: 0.001, Penalty: None
1e12
Train Loss
Val Loss
RMSE
Epoch
0
50
100
150
200
250
300
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

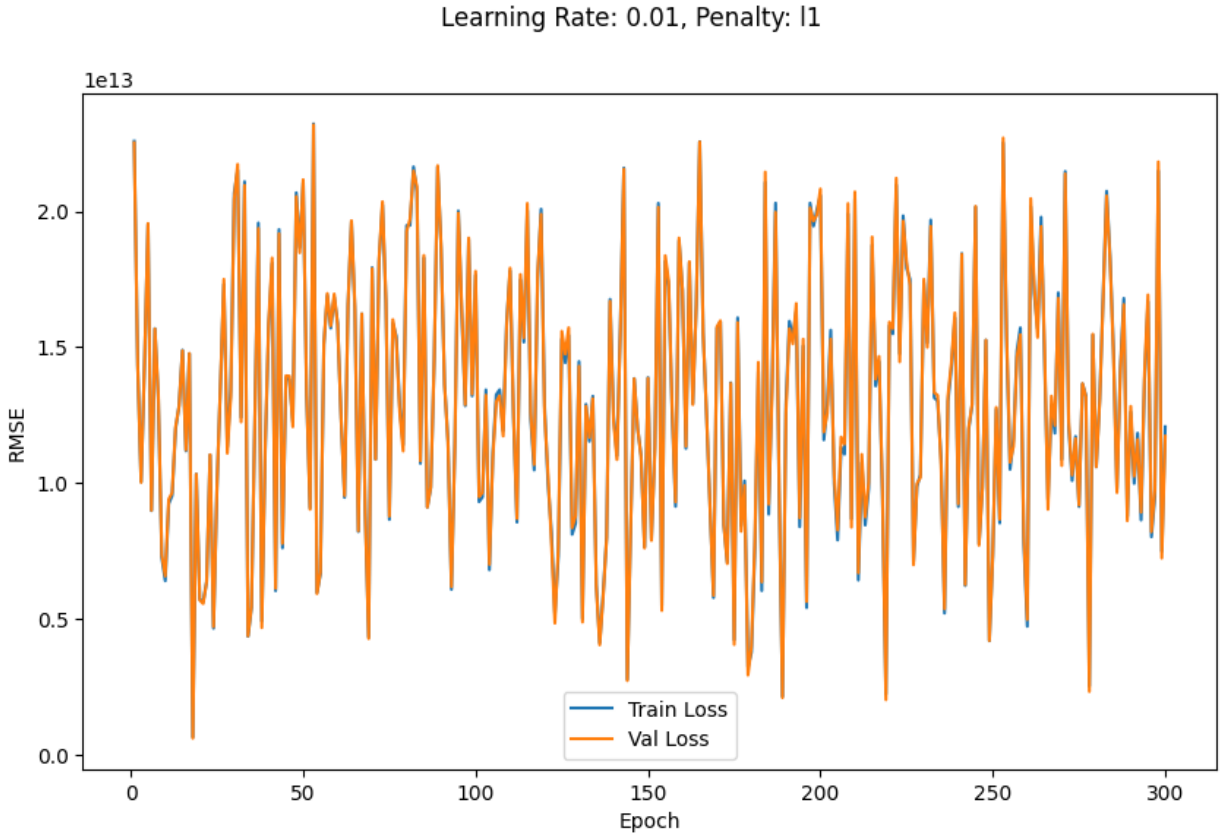

Learning Rate: 0.01, Penalty: None
1e13
Train Loss
Val Loss
RMSE
Epoch
0
50
100
150
200
250
300
0.5
1.0
1.5
2.0
2.5

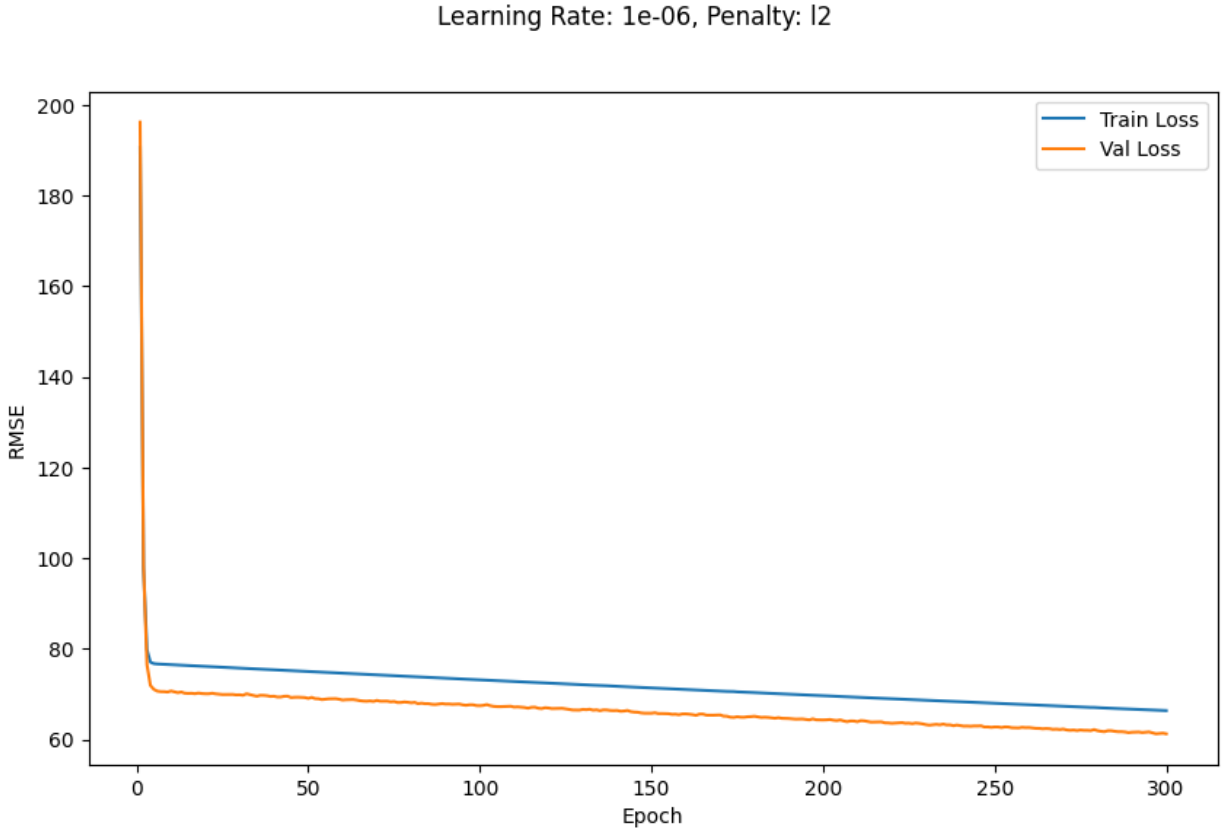

Learning Rate: 1e-06, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
200
180
160
140
120
100
80
60
0
50
100
150
200
250
300

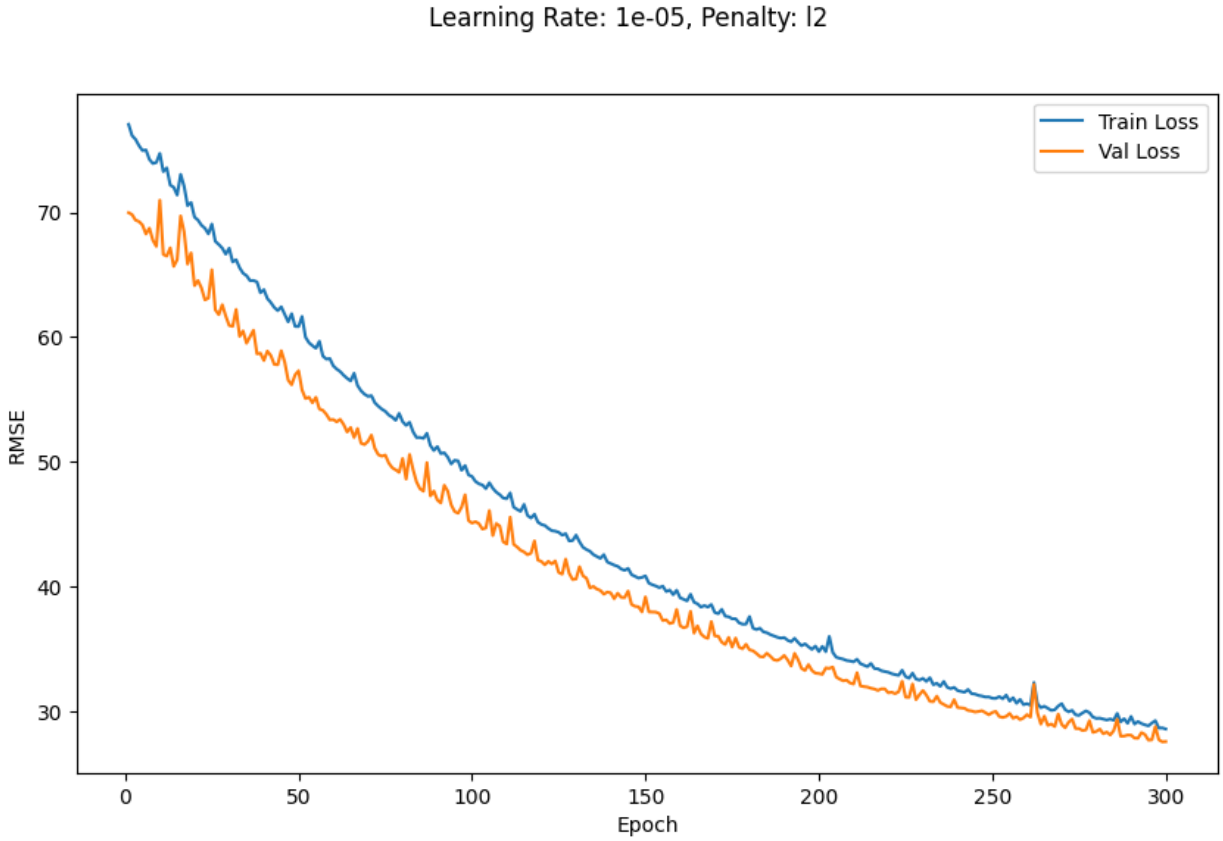

Learning Rate: 1e-05, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
70
60
50
40
30
0
50
100
150
200
250
300

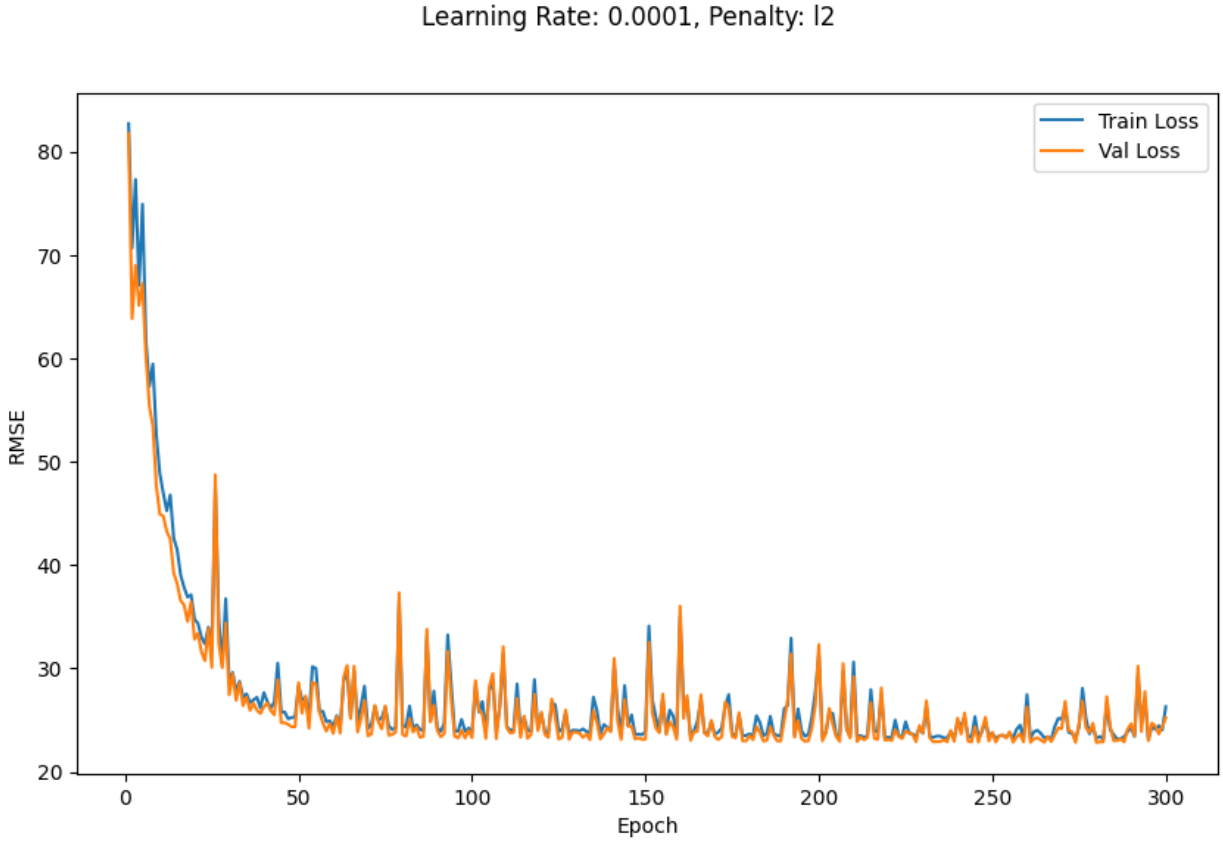

Learning Rate: 0.0001, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
90
80
70
60
50
40
30
20
0
50
100
150
200
250
300

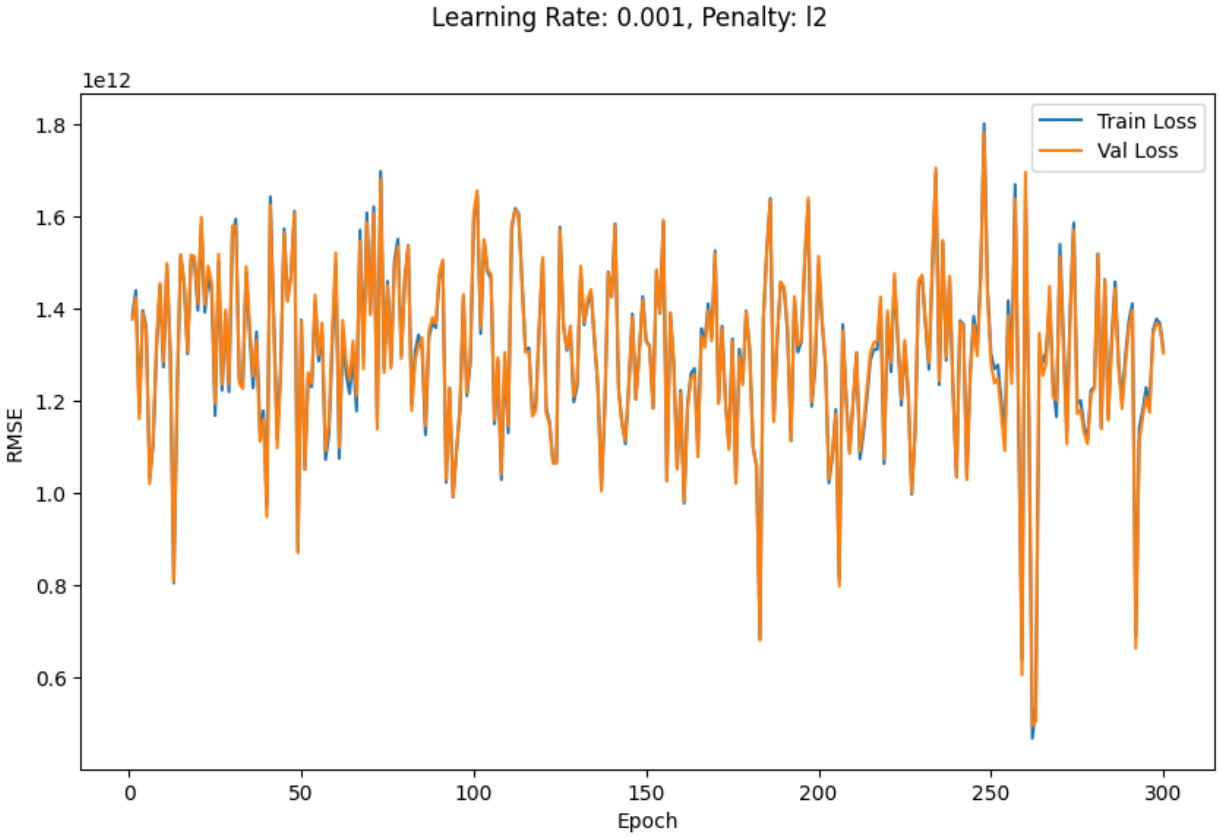

Learning Rate: 0.001, Penalty: l1
1e12
Train Loss
Val Loss
RMSE
Epoch
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
50
100
150
200
250
300

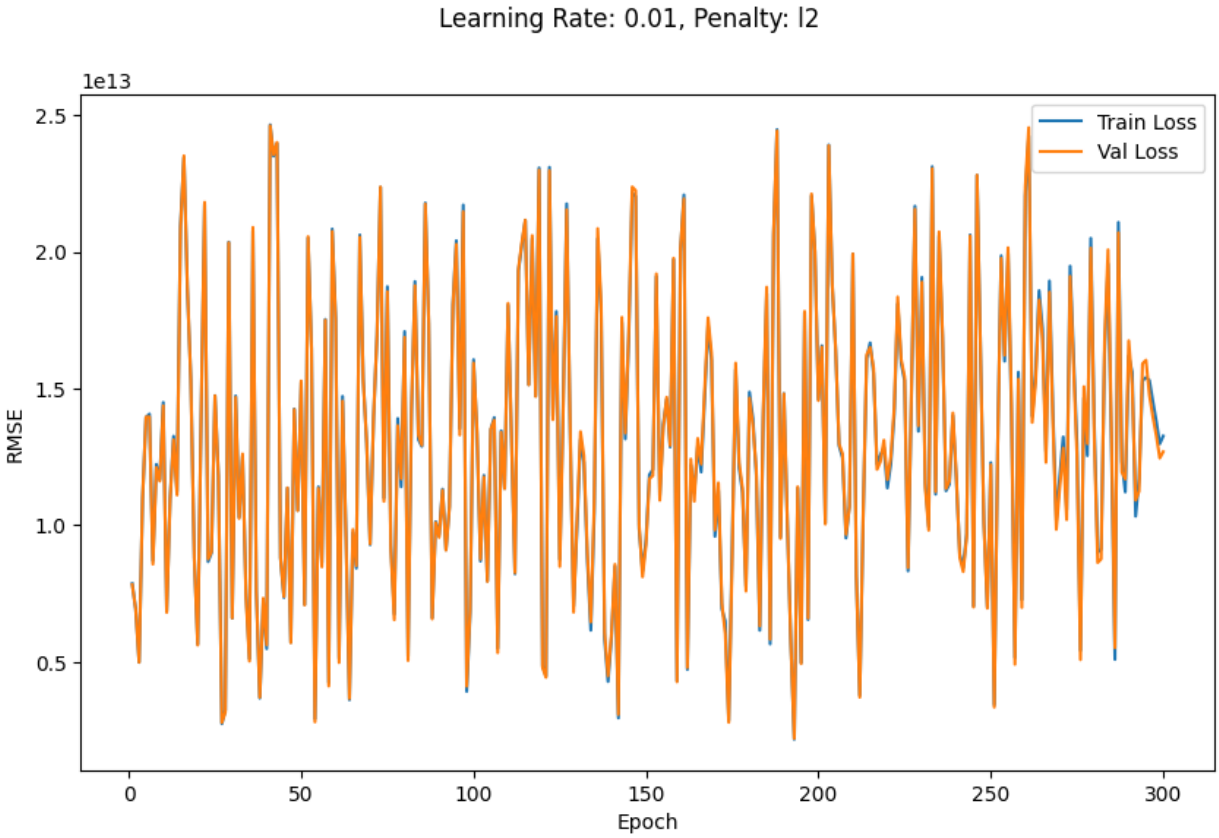

Learning Rate: 0.01, Penalty: l1
1e13
RMSE
Epoch
Train Loss
Val Loss


Learning Rate: 1e-06, Penalty: l2
RMSE
Epoch
Train Loss
Val Loss


Learning Rate: 1e-05, Penalty: l2
Train Loss
Val Loss
RMSE
Epoch
70
60
50
40
30
0
50
100
150
200
250
300


Learning Rate: 0.0001, Penalty: l2
Train Loss
Val Loss
RMSE
Epoch
80
70
60
50
40
30
20
0
50
100
150
200
250
300

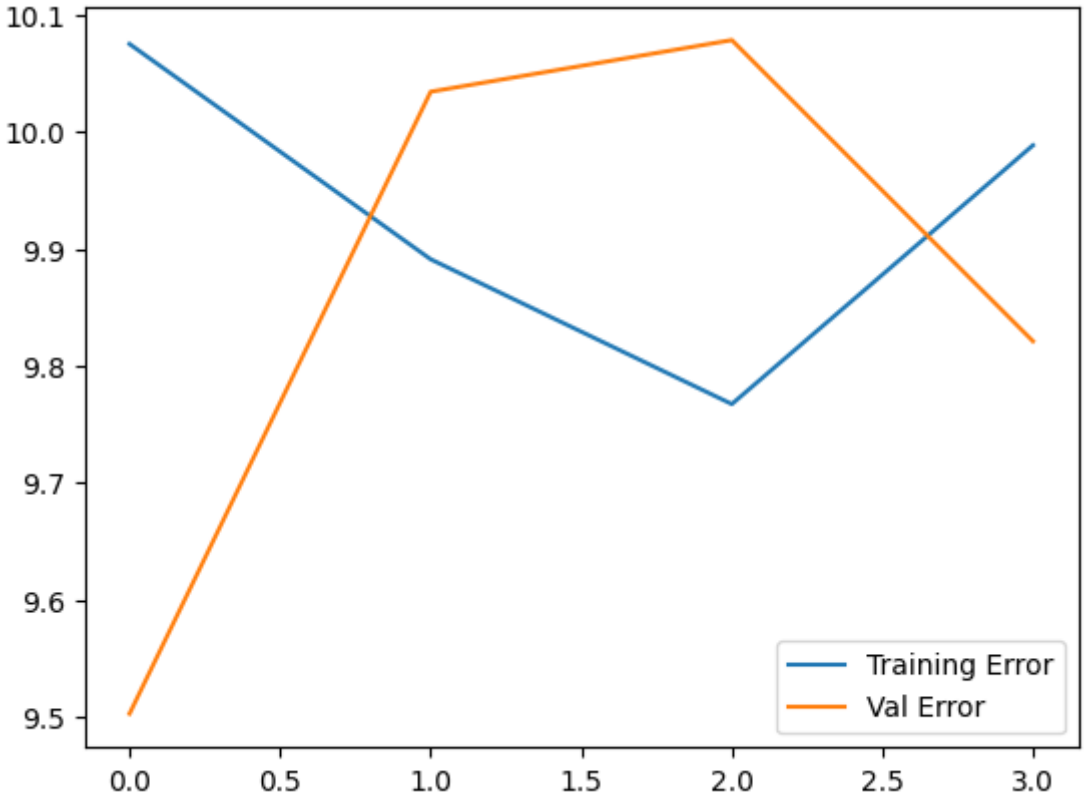

Learning Rate: 0.001, Penalty: l2
1e12
1.8
1.6
1.4
1.2
1.0
0.8
0.6
RMSE
0
50
100
150
200
250
300
Epoch
Train Loss
Val Loss


Learning Rate: 0.01, Penalty: l2
1e13
2.5
2.0
1.5
1.0
0.5
RMSE
0
50
100
150
200
250
300
Epoch
Train Loss
Val Loss

In [18]:
```
# Ridge
train_plot = []
val_plot = []
ridge = Ridge()
for cnt,var in enumerate(kf.split(x_train,y_train)):
    print(cnt)
    x = x_train[var[0],:]
    y = y_train[var[0]]
    x_val = x_train[var[1],:]
    y_val = y_train[var[1]]
    ridge.fit(x, y)
    y_pred_train = ridge.predict(x)
    ridge.fit(x_val, y_val)
    y_pred_val = ridge.predict(x_val)
    print("Train error:",np.sqrt(mse(y_pred_train,y)))
    print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
    train_plot.append(np.sqrt(mse(y_pred_train,y)))
    val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
plt.legend()
```

```
0
Train error: 10.075014442597382
Validation error: 9.502377464830563
1
Train error: 9.890932722228804
Validation error: 10.034209898388667
2
Train error: 9.767118608151108
Validation error: 10.078355357119019
3
Train error: 9.988418481539727
Validation error: 9.820884736133973
```

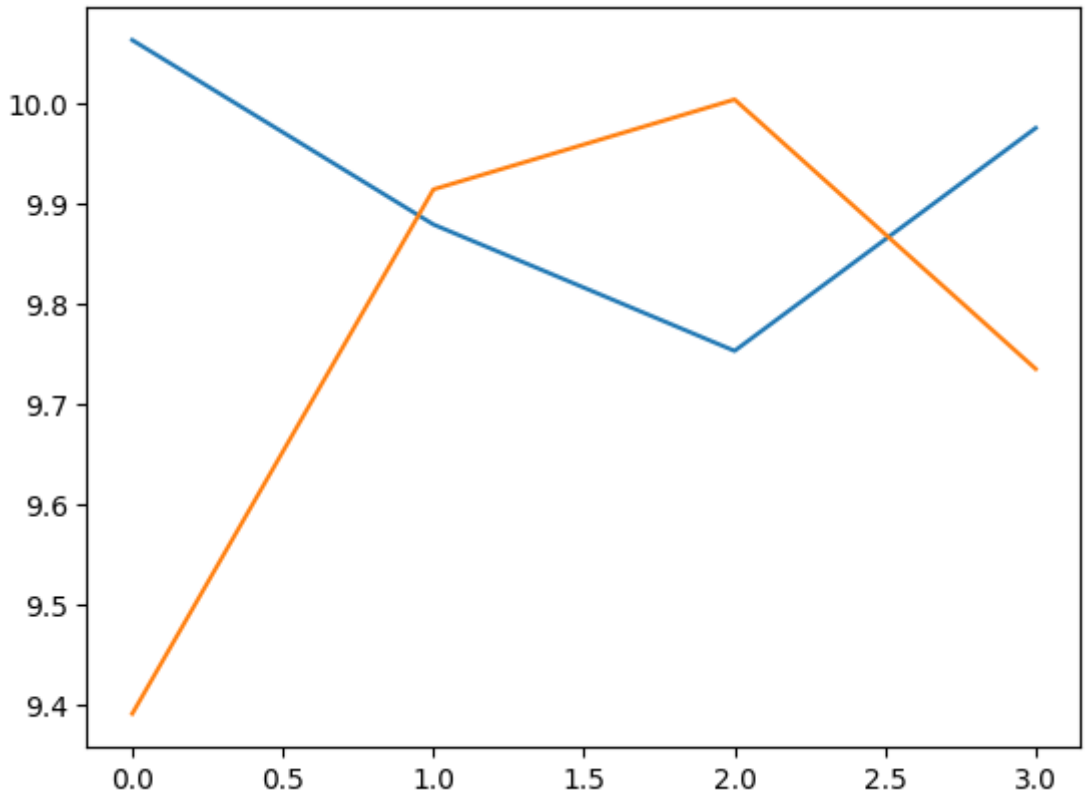
Out[18]:
```
<matplotlib.legend.Legend at 0x16a135f60>
```


10.1
10.0
9.9
9.8
9.7
9.6
9.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Training Error
Val Error

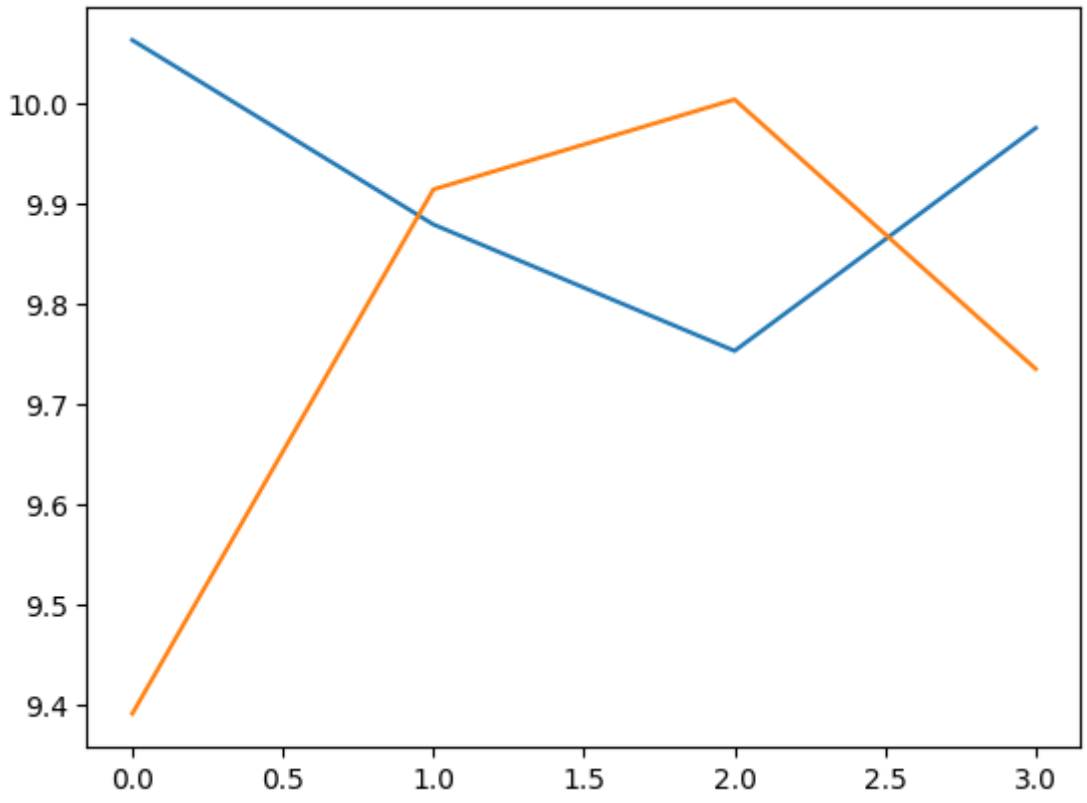
In [19]:

```
# Ridge with different learning rates

ridge_alpha = [0,0.001,1,10,100]

for r in ridge_alpha:
    print("Alpha value:", r)
    train_plot = []
    val_plot = []
    ridge_obj = Ridge(alpha=r)
    for cnt,var in enumerate(kf.split(x_train,y_train)):
        print(cnt)
        x = x_train[var[0],:]
        y = y_train[var[0]]
        x_val = x_train[var[1],:]
        y_val = y_train[var[1]]
        ridge_obj.fit(x, y)
        y_pred_train = ridge_obj.predict(x)
        ridge_obj.fit(x_val, y_val)
        y_pred_val = ridge_obj.predict(x_val)
        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
        y_pred_test = ridge_obj.predict(x_test)
        metric_dict['Ridge lr='+str(r)] = np.sqrt(mse(y_pred_test,y_tes
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

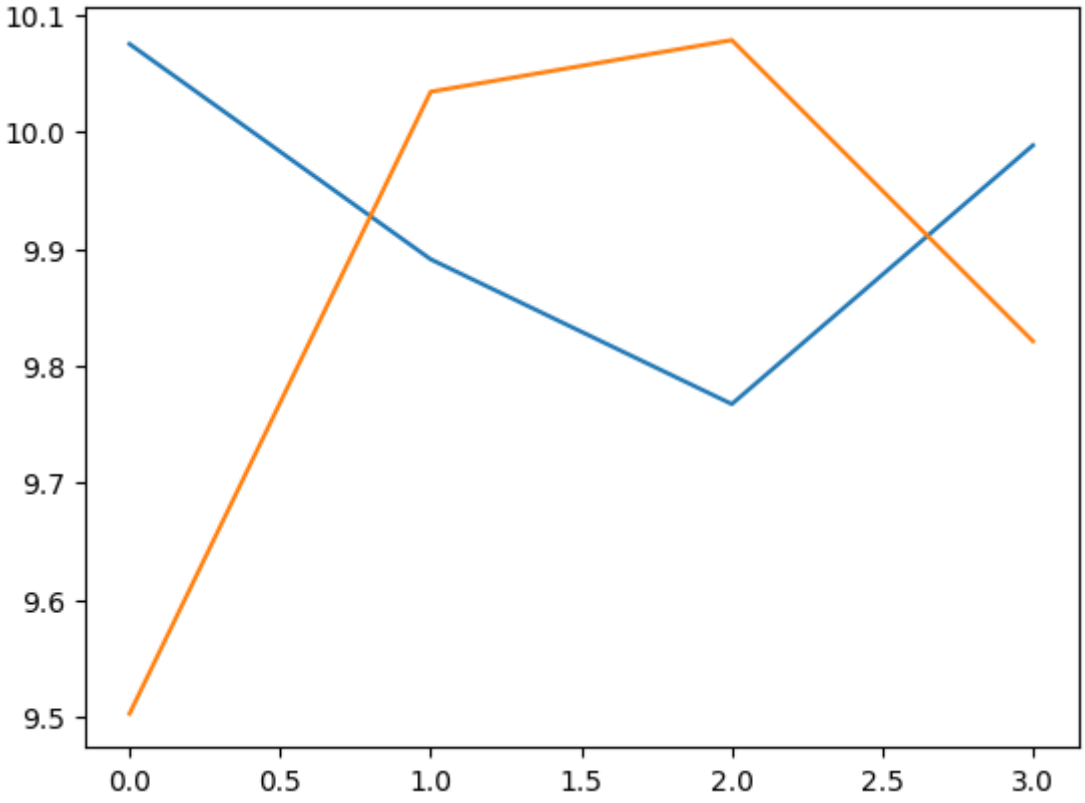
```
Alpha value: 0
0
Train error: 10.062843403919285
Validation error: 9.391231433490518
1
Train error: 9.878993395744255
Validation error: 9.913985002120688
2
Train error: 9.752971016584786
Validation error: 10.003613806207307
3
Train error: 9.975176680690797
Validation error: 9.734967169150917
```



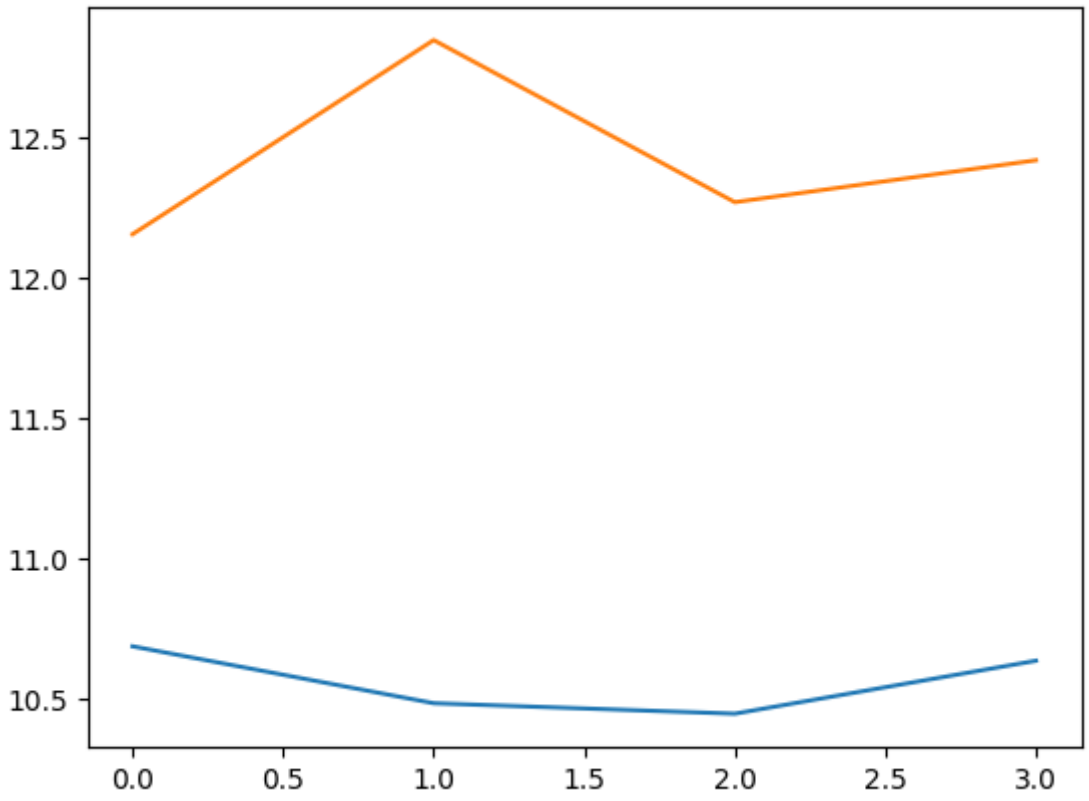

```
Alpha value: 0.001
0
Train error: 10.062843417351411
Validation error: 9.39123158642983
1
Train error: 9.878993408967082
Validation error: 9.913985164888548
2
Train error: 9.752971032315887
Validation error: 10.003613903644876
3
Train error: 9.975176695353953
Validation error: 9.734967283310652
```



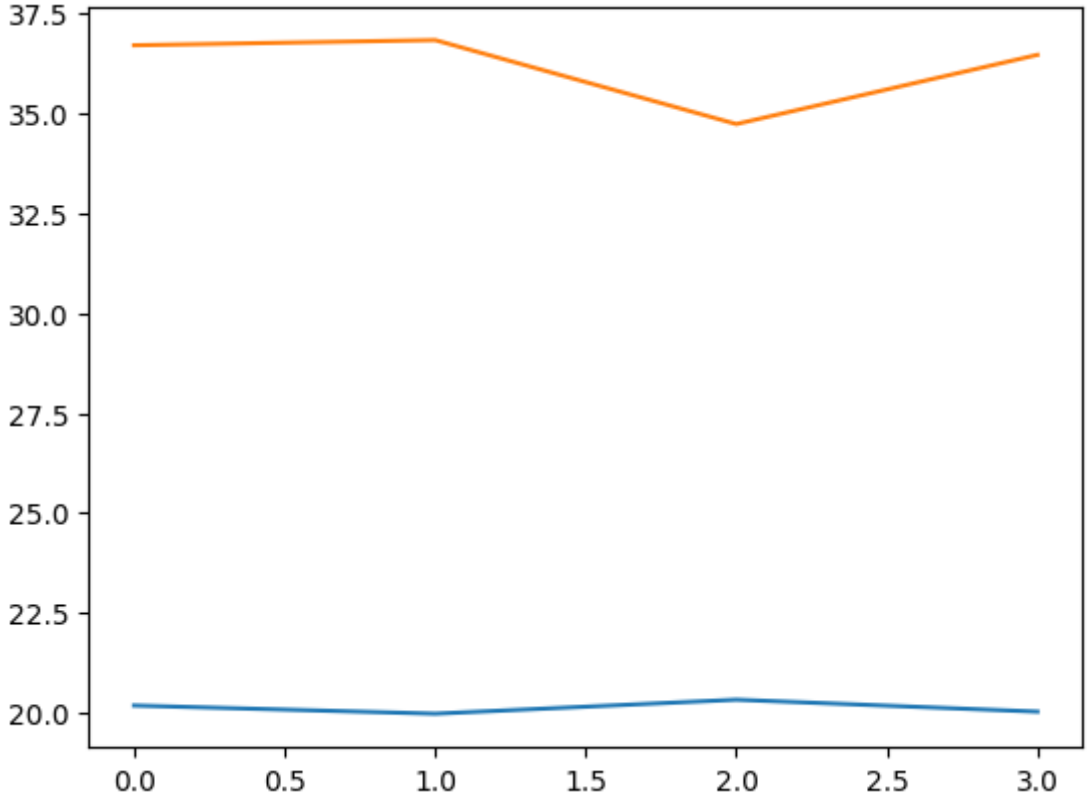

```
Alpha value: 1
0
Train error: 10.075014442597382
Validation error: 9.502377464830563
1
Train error: 9.890932722228804
Validation error: 10.034209898388667
2
Train error: 9.767118608151108
Validation error: 10.078355357119019
3
Train error: 9.988418481539727
Validation error: 9.820884736133973
```




```
Alpha value: 10
0
Train error: 10.686116855178412
Validation error: 12.155927576717094
1
Train error: 10.483130017367701
Validation error: 12.84845239403423
2
Train error: 10.445766419570973
Validation error: 12.2702473141902
3
Train error: 10.63468005910493
Validation error: 12.419920298358887
```



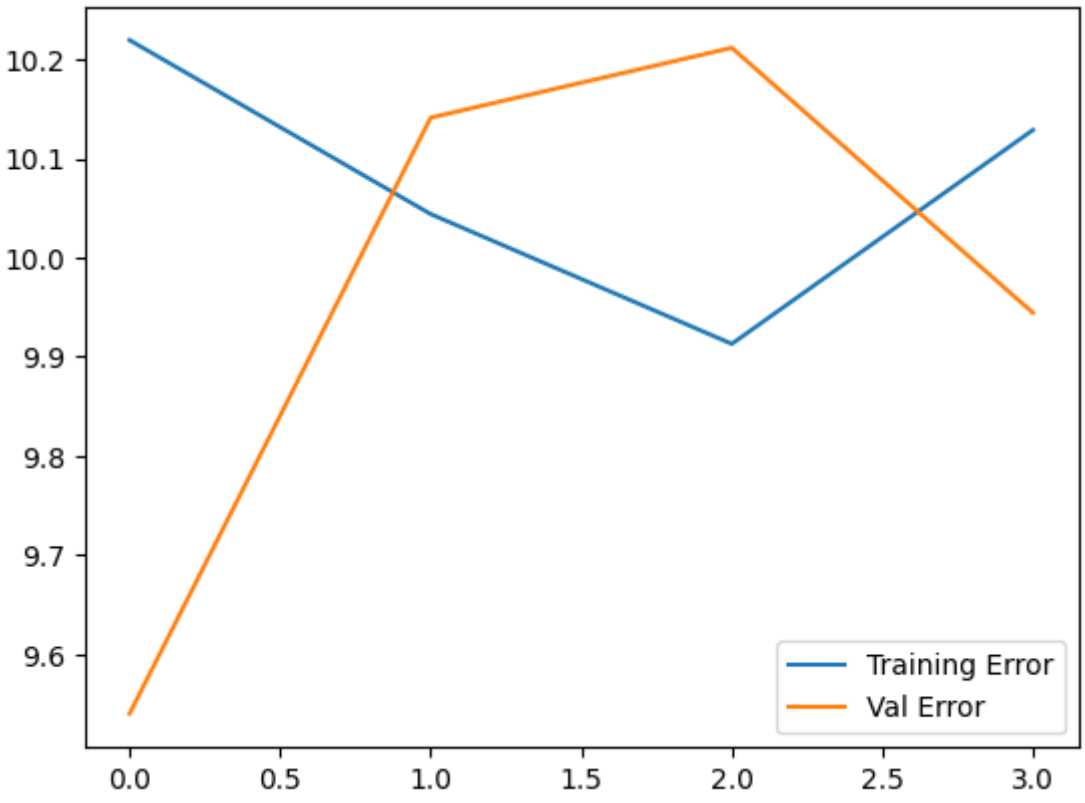

```
Alpha value: 100
0
Train error: 20.19030403381998
Validation error: 36.69165165120153
1
Train error: 19.98578189577161
Validation error: 36.821800985078454
2
Train error: 20.33571244388779
Validation error: 34.72340089890306
3
Train error: 20.036914858987636
Validation error: 36.453041902307795
```


37.5
35.0
32.5
30.0
27.5
25.0
22.5
20.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [20]:
```
# Lasso

train_plot = []
val_plot = []
lasso = Lasso()
for cnt,var in enumerate(kf.split(x_train,y_train)) :
    print(cnt)
    x = x_train[var[0],:]
    y = y_train[var[0]]
    x_val = x_train[var[1],:]
    y_val = y_train[var[1]]
    lasso.fit(x, y)
    y_pred_train = lasso.predict(x)
    lasso.fit(x_val, y_val)
    y_pred_val = lasso.predict(x_val)
    print("Train error:",np.sqrt(mse(y_pred_train,y)))
    print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
    train_plot.append(np.sqrt(mse(y_pred_train,y)))
    val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
plt.legend()
```

```
0
Train error: 10.219694278283034
Validation error: 9.539696669606949
1
Train error: 10.044070912840805
Validation error: 10.141392050235021
2
Train error: 9.913037805782604
Validation error: 10.211917568347353
3
Train error: 10.12899168807305
Validation error: 9.944373569630216
```

Out[20]: <matplotlib.legend.Legend at 0x2825b0dc0>


10.2
10.1
10.0
9.9
9.8
9.7
9.6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Training Error
Val Error

In [21]:

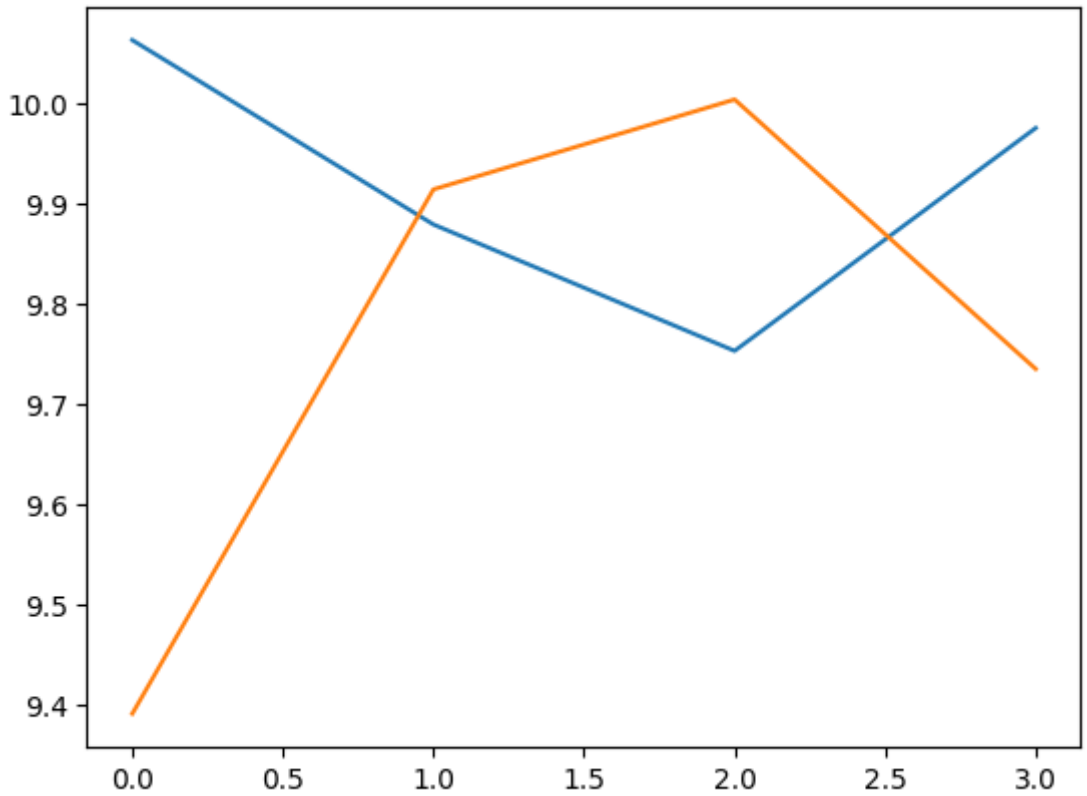
```
# Lasso with different learning rates

lasso_alpha = [0,0.001,1,10,100]

for r in lasso_alpha:
    print("Alpha value:", r)
    train_plot = []
    val_plot = []
    lasso_obj = Lasso(alpha=r)
    for cnt,var in enumerate(kf.split(x_train,y_train)):
        print(cnt)
        x = x_train[var[0],:]
        y = y_train[var[0]]
        x_val = x_train[var[1],:]
        y_val = y_train[var[1]]
        lasso_obj.fit(x, y)
        y_pred_train = lasso_obj.predict(x)
        lasso_obj.fit(x_val, y_val)
        y_pred_val = lasso_obj.predict(x_val)
        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
        y_pred_test = lasso_obj.predict(x_test)
        metric_dict['Lasso lr='+str(r)] = np.sqrt(mse(y_pred_test,y_tes
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()

    print('\n')
```

```
Alpha value: 0
0
Train error: 10.062843403919281
Validation error: 9.391231433490526
1
Train error: 9.878993395744269
Validation error: 9.913985002120674
2
Train error: 9.752971016584786
Validation error: 10.003613806207298
3
Train error: 9.975176680690788
Validation error: 9.73496716915092
```

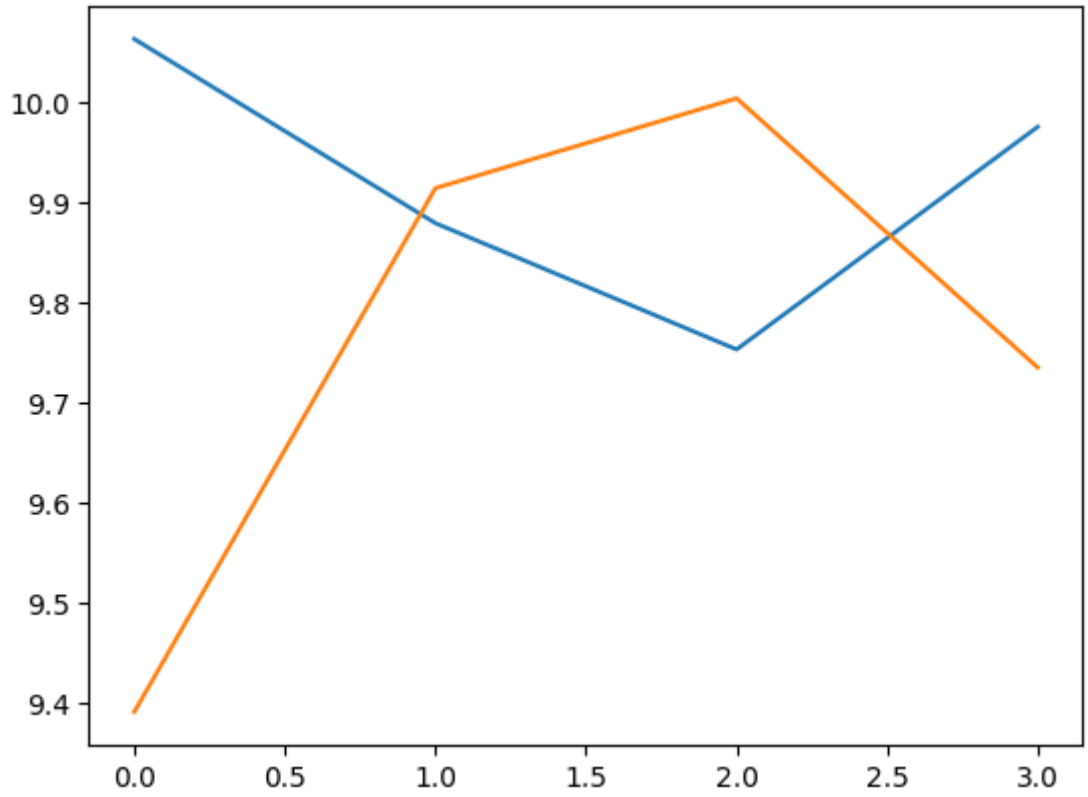
```
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1783271431.py:16: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
  lasso_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coordinate descent with no regularization may lead to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 1.519e+04, tolerance: 1.882e+02 Linear regression models with null weight for the l1 regularization term are more efficiently fitted using one of the solvers implemented in sklearn.linear_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1783271431.py:18: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
  lasso_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coordinate descent with no regularization may lead to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 4.410e+03, tolerance: 7.026e+01 Linear regression models with null weight for the l1 regularization term are more efficiently fitted using one of the solvers implemented in sklearn.linear_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1783271431.py:16: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
  lasso_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coordinate descent with no regularization may lead to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 1.464e+04, tolerance: 1.902e+02 Linear regression models with null weight for the l1 regularization term are more efficiently fitted using one of the solvers implemented in sklearn.linear_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1783271431.py:18: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
```

```
  lasso_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.914e+03, tolerance: 6.748e+01 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/17832714
31.py:16: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  lasso_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.427e+04, tolerance: 1.950e+02 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/17832714
31.py:18: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  lasso_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 5.004e+03, tolerance: 6.342e+01 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/17832714
31.py:16: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  lasso_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
```

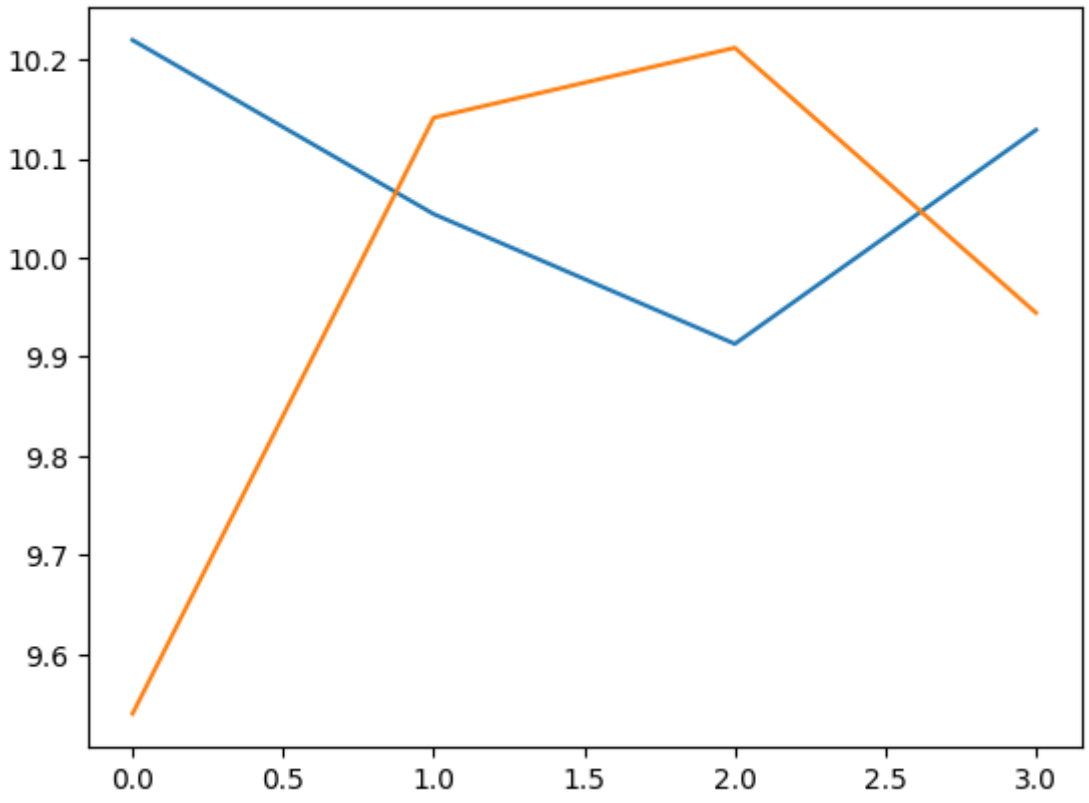
```
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.493e+04, tolerance: 2.020e+02 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/17832714
31.py:18: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  lasso_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.738e+03, tolerance: 5.558e+01 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
```



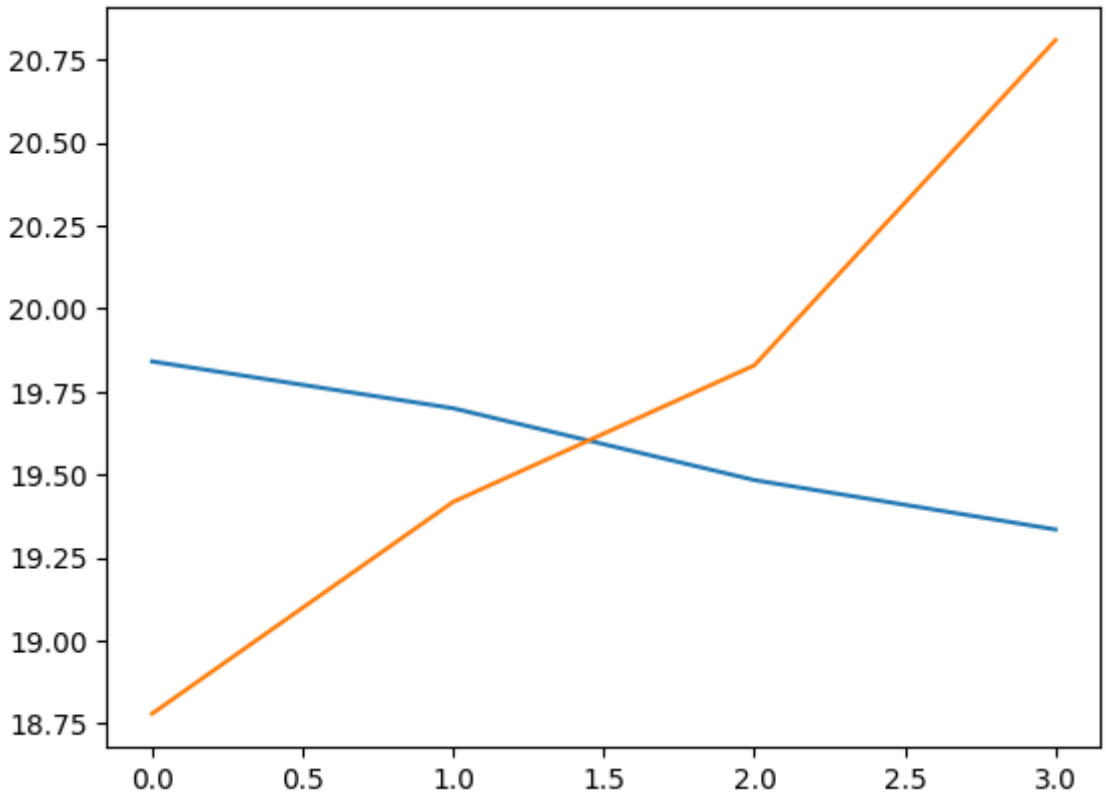
```
Alpha value: 0.001
0
Train error: 10.062845085474532
Validation error: 9.391233780784408
1
Train error: 9.878995150597623
Validation error: 9.913986977594016
2
Train error: 9.752973143016666
Validation error: 10.00361580113528
3
Train error: 9.975178672344041
Validation error: 9.734968694901601
```



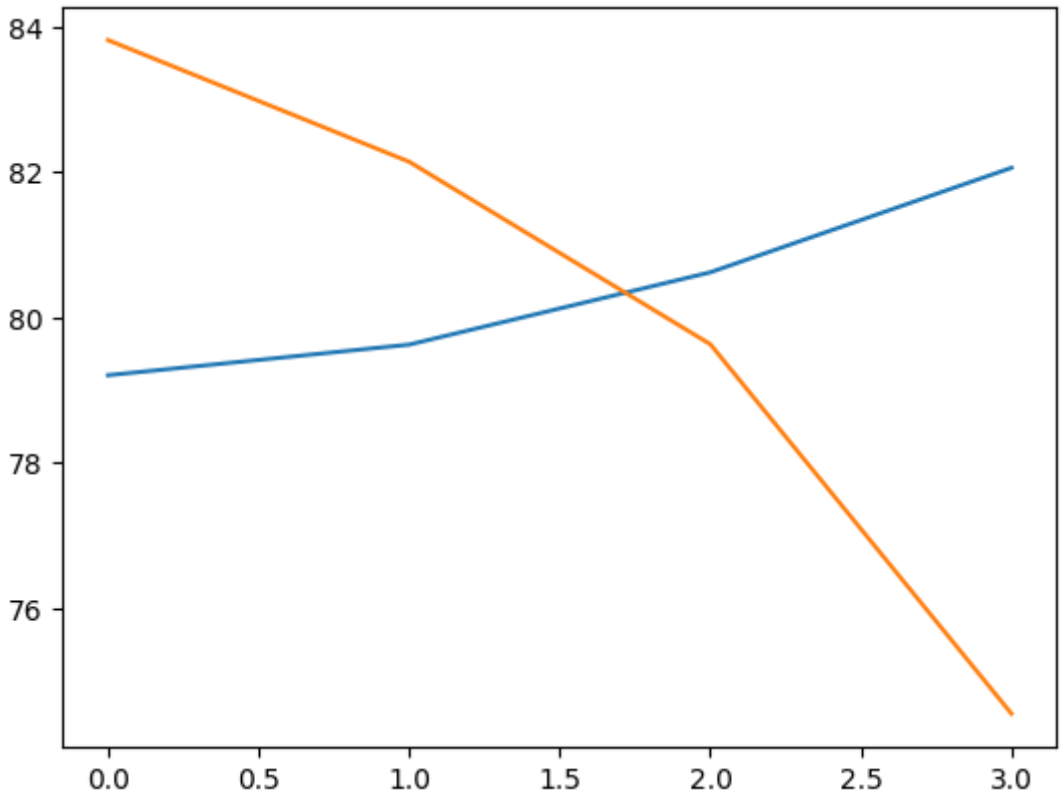

```
Alpha value: 1
0
Train error: 10.219694278283034
Validation error: 9.539696669606949
1
Train error: 10.044070912840805
Validation error: 10.141392050235021
2
Train error: 9.913037805782604
Validation error: 10.211917568347353
3
Train error: 10.12899168807305
Validation error: 9.944373569630216
```




```
Alpha value: 10
0
Train error: 19.840470827292478
Validation error: 18.77929413140439
1
Train error: 19.699262113428578
Validation error: 19.417353551247025
2
Train error: 19.483042378315115
Validation error: 19.828601022220337
3
Train error: 19.334194455294675
Validation error: 20.80837936724053
```



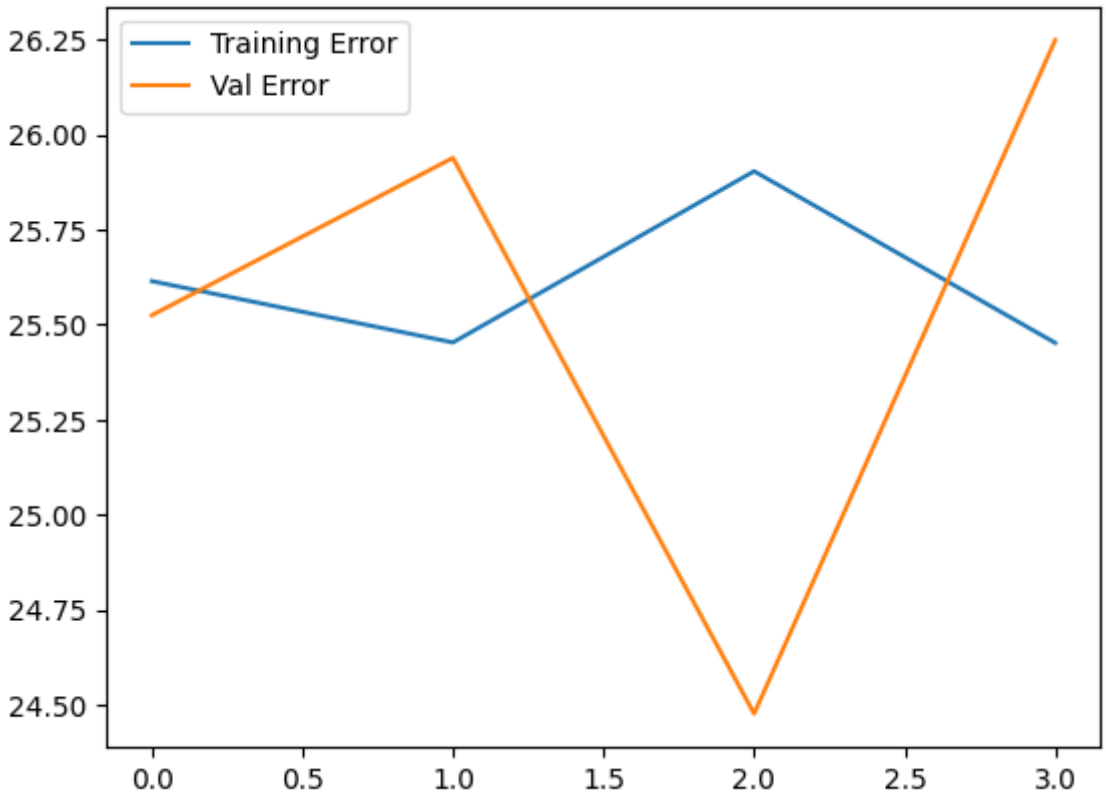

```
Alpha value: 100
0
Train error: 79.20838820557084
Validation error: 83.81848506960355
1
Train error: 79.62938661239804
Validation error: 82.14526360098415
2
Train error: 80.62425356391
Validation error: 79.63488104956944
3
Train error: 82.06165770411914
Validation error: 74.55281933102287
```


84
82
80
78
76
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [22]:
```
# Elastic Net

train_plot = []
val_plot = []
el_net = ElasticNet()
for cnt,var in enumerate(kf.split(x_train,y_train)) :
    print(cnt)
    x = x_train[var[0],:]
    y = y_train[var[0]]
    x_val = x_train[var[1],:]
    y_val = y_train[var[1]]
    el_net.fit(x, y)
    y_pred_train = el_net.predict(x)
    el_net.fit(x_val, y_val)
    y_pred_val = el_net.predict(x_val)
    print("Train error:",np.sqrt(mse(y_pred_train,y)))
    print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
    train_plot.append(np.sqrt(mse(y_pred_train,y)))
    val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
plt.legend()
```

```
0
Train error: 25.614383916124034
Validation error: 25.524551262807616
1
Train error: 25.452943656745983
Validation error: 25.938834604364793
2
Train error: 25.903733921650957
Validation error: 24.475707632798215
3
Train error: 25.451785149777752
Validation error: 26.24884171636151
```

Out[22]: <matplotlib.legend.Legend at 0x2848e60e0>


Training Error
Val Error
26.25
26.00
25.75
25.50
25.25
25.00
24.75
24.50
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [23]:

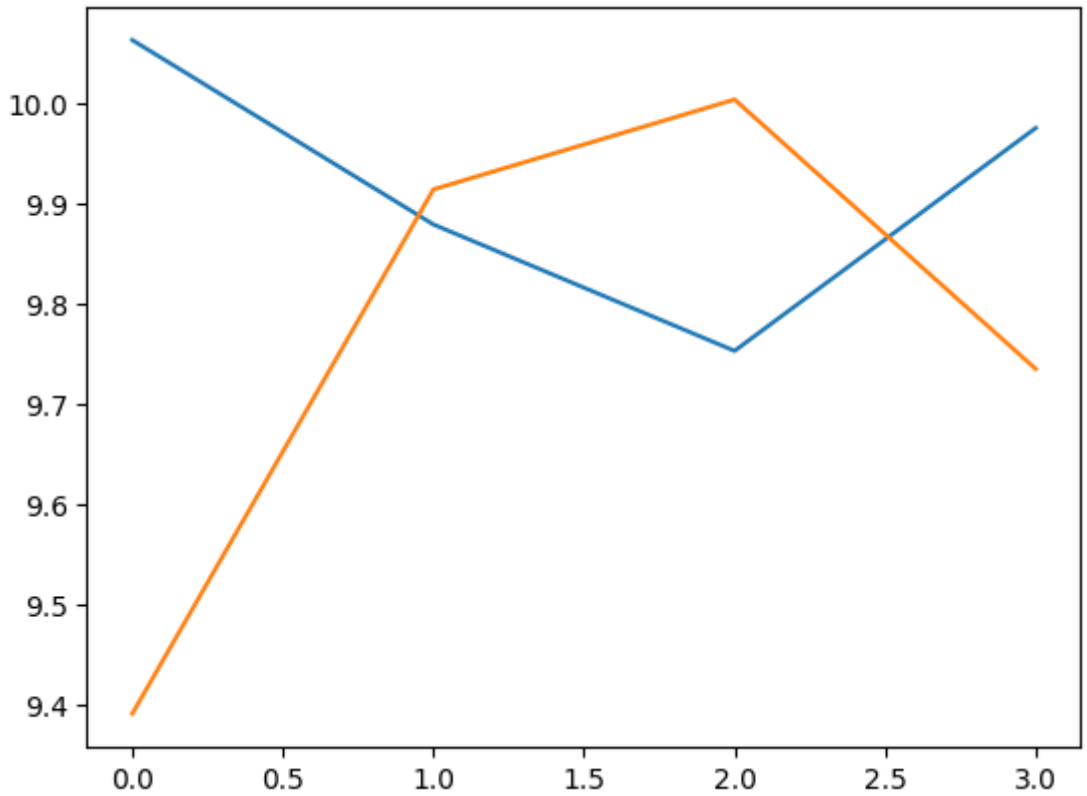
```
# ElasticNet with different learning rates

en_alpha = [0,0.001,1,10,100]

for r in en_alpha:
    print("Alpha value:", r)
    train_plot = []
    val_plot = []
    en_obj = ElasticNet(alpha=r)
    for cnt,var in enumerate(kf.split(x_train,y_train)):
        print(cnt)
        x = x_train[var[0],:]
        y = y_train[var[0]]
        x_val = x_train[var[1],:]
        y_val = y_train[var[1]]
        en_obj.fit(x, y)
        y_pred_train = en_obj.predict(x)
        en_obj.fit(x_val, y_val)
        y_pred_val = en_obj.predict(x_val)
        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
        y_pred_test = en_obj.predict(x_test)
        metric_dict['ElasticNet lr='+str(r)] = np.sqrt(mse(y_pred_test,
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

```
Alpha value: 0
0
Train error: 10.062843403919281
Validation error: 9.391231433490526
1
Train error: 9.878993395744269
Validation error: 9.913985002120674
2
Train error: 9.752971016584786
Validation error: 10.003613806207298
3
Train error: 9.975176680690788
Validation error: 9.73496716915092
```

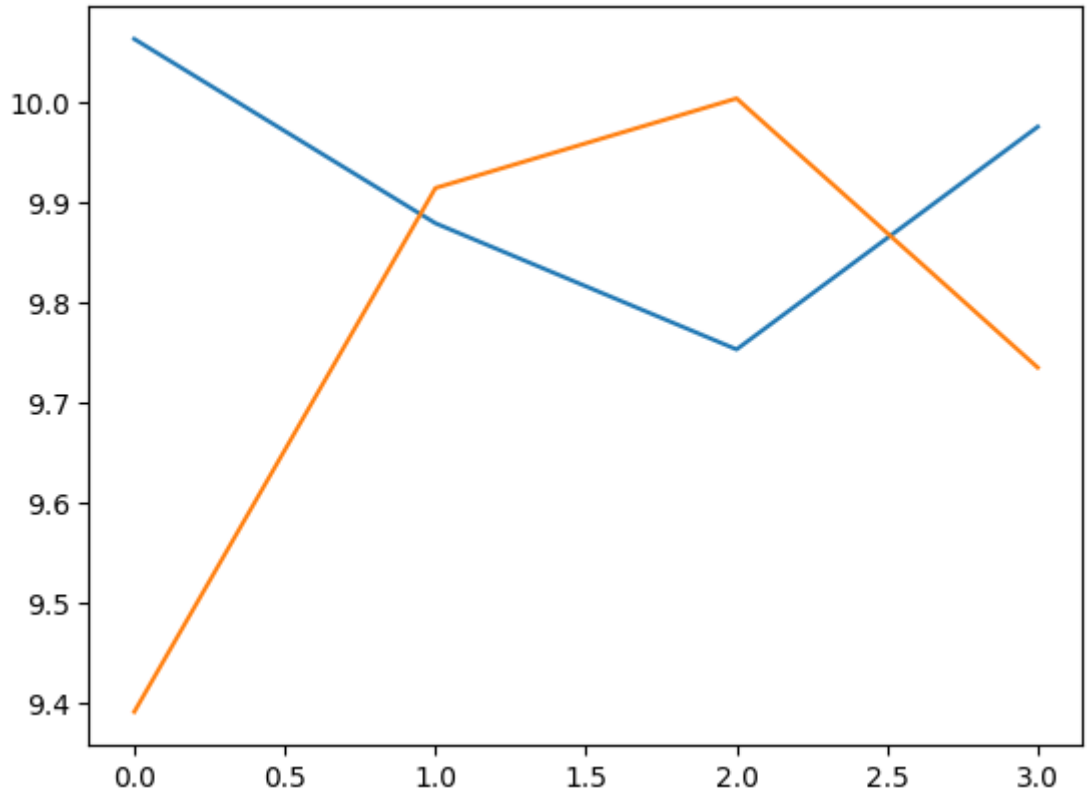
```
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1473329763.py:16: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
  en_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coordinate descent with no regularization may lead to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 1.519e+04, tolerance: 1.882e+02 Linear regression models with null weight for the l1 regularization term are more efficiently fitted using one of the solvers implemented in sklearn.linear_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1473329763.py:18: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
  en_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coordinate descent with no regularization may lead to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 4.410e+03, tolerance: 7.026e+01 Linear regression models with null weight for the l1 regularization term are more efficiently fitted using one of the solvers implemented in sklearn.linear_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1473329763.py:16: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
  en_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coordinate descent with no regularization may lead to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-packages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations, check the scale of the features or consider increasing regularisation. Duality gap: 1.464e+04, tolerance: 1.902e+02 Linear regression models with null weight for the l1 regularization term are more efficiently fitted using one of the solvers implemented in sklearn.linear_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/1473329763.py:18: UserWarning: With alpha=0, this algorithm does not converge well. You are advised to use the LinearRegression estimator
```

```
  en_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.914e+03, tolerance: 6.748e+01 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/14733297
63.py:16: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  en_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.427e+04, tolerance: 1.950e+02 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/14733297
63.py:18: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  en_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 5.004e+03, tolerance: 6.342e+01 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/14733297
63.py:16: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  en_obj.fit(x, y)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
```

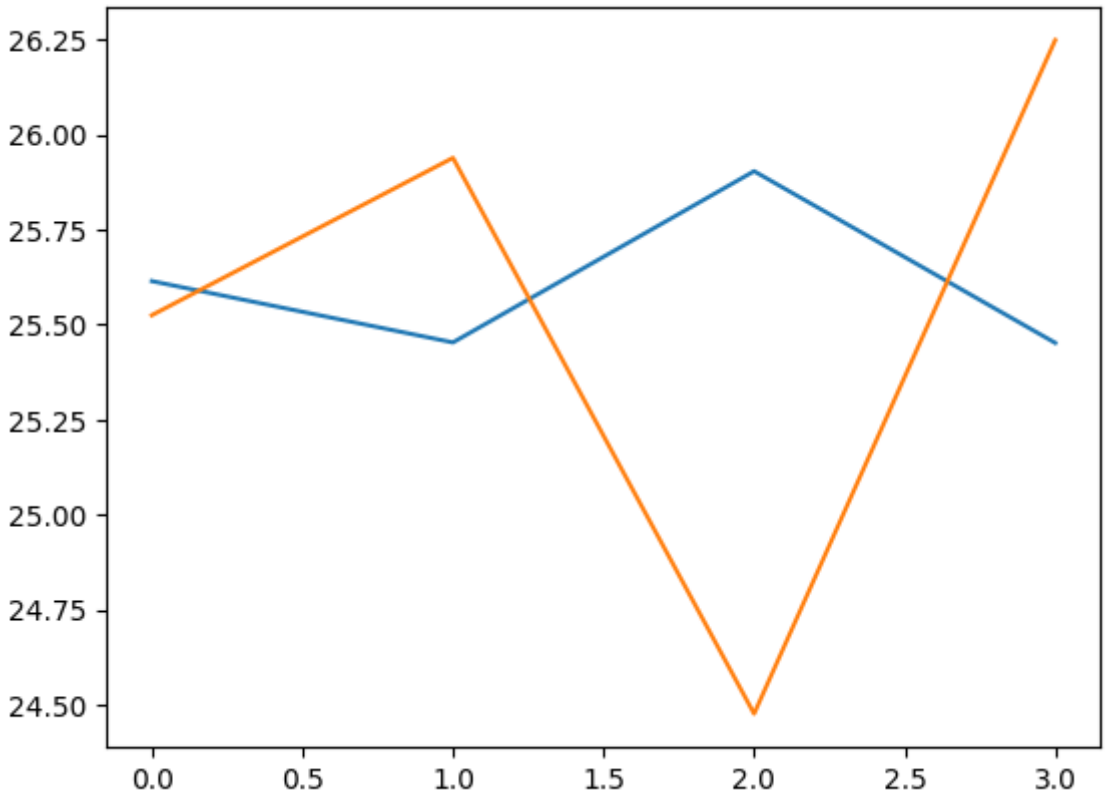
```
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.493e+04, tolerance: 2.020e+02 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
/var/folders/rp/kf6_svf16298_vgdvzcttf600000gn/T/ipykernel_66341/14733297
63.py:18: UserWarning: With alpha=0, this algorithm does not converge wel
l. You are advised to use the LinearRegression estimator
  en_obj.fit(x_val, y_val)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: UserWarning: Coor
dinate descent with no regularization may lead to unexpected results and
is discouraged.
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.738e+03, tolerance: 5.558e+01 Linear regression m
odels with null weight for the l1 regularization term are more efficientl
y fitted using one of the solvers implemented in sklearn.linear_model.Rid
ge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
```



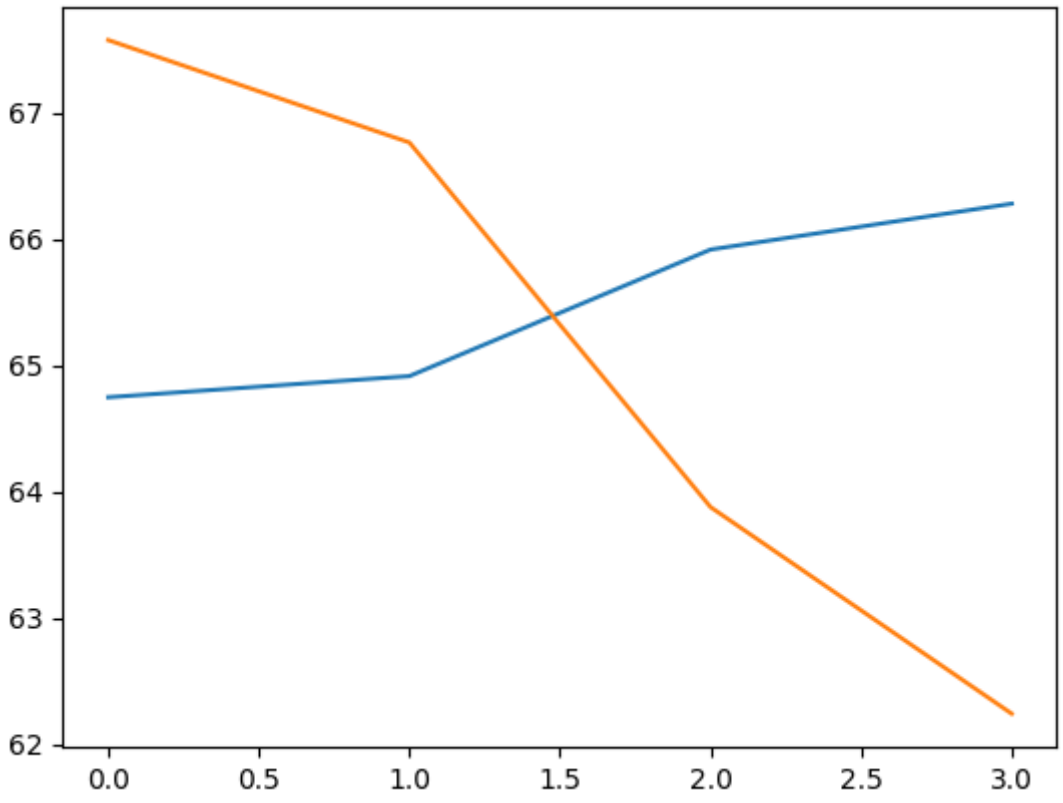
```
Alpha value: 0.001
0
Train error: 10.063121780435612
Validation error: 9.391636294643193
1
Train error: 9.879268200576313
Validation error: 9.914412625167968
2
Train error: 9.753345717058739
Validation error: 10.003836299689803
3
Train error: 9.97552638250847
Validation error: 9.735235100663695
```



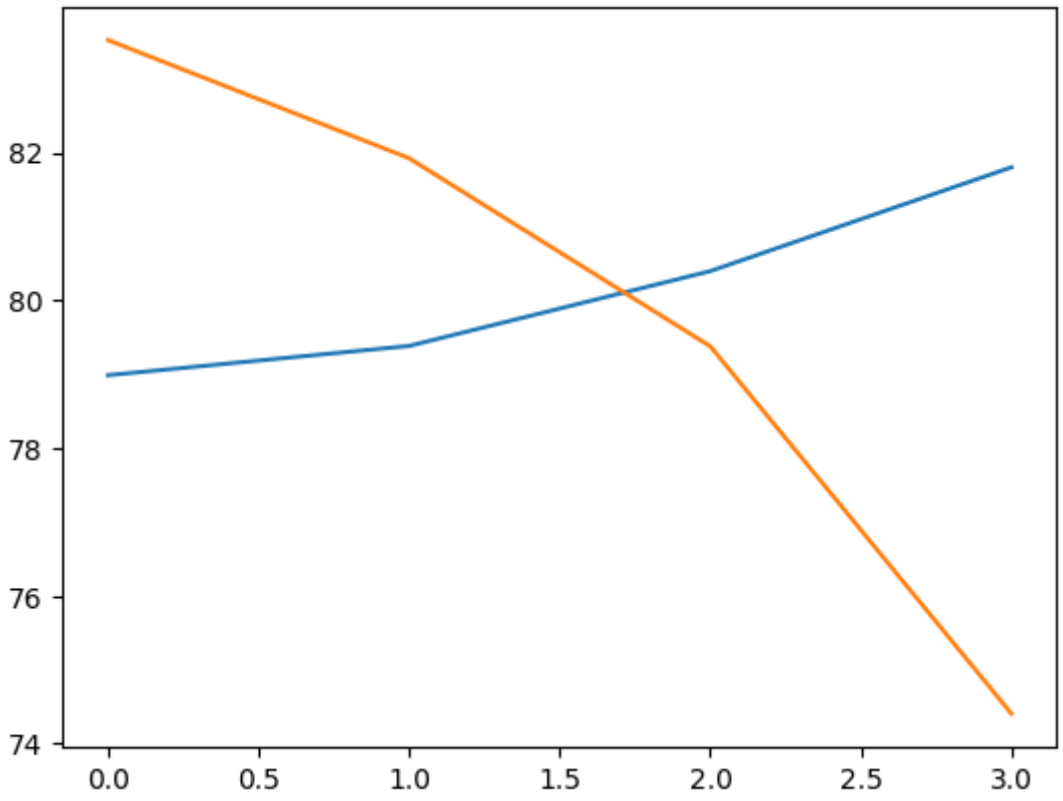

```
Alpha value: 1
0
Train error: 25.614383916124034
Validation error: 25.524551262807616
1
Train error: 25.452943656745983
Validation error: 25.938834604364793
2
Train error: 25.903733921650957
Validation error: 24.475707632798215
3
Train error: 25.451785149777752
Validation error: 26.24884171636151
```




```
Alpha value: 10
0
Train error: 64.74586471327686
Validation error: 67.57308550842096
1
Train error: 64.91314923908723
Validation error: 66.76257715971037
2
Train error: 65.91492793794157
Validation error: 63.87643550224843
3
Train error: 66.27821148278801
Validation error: 62.241126410258424
```




```
Alpha value: 100
0
Train error: 78.9860744198535
Validation error: 83.52142430500763
1
Train error: 79.38220360797499
Validation error: 81.92236353846627
2
Train error: 80.39468579243739
Validation error: 79.37696999204462
3
Train error: 81.80031900002213
Validation error: 74.405092802859
```



Answer:- The above results show the performance of different linear regression models with different hyperparameters on the dataset. From the results, we can see that the choice of hyperparameters such as learning rate and penalty factor has a significant impact on the performance of the models.

In the case of SGD, we can see that the performance of the model is influenced by the learning rate and penalty factor.

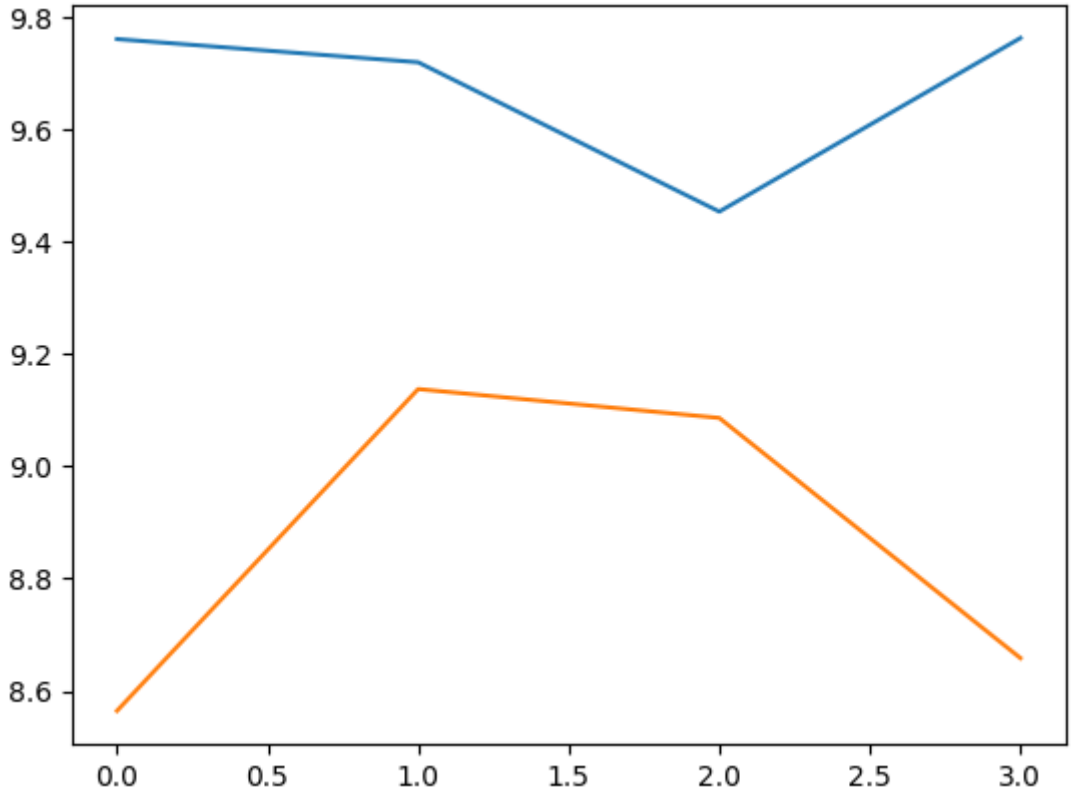
Based on the above results, we can conclude that ridge model is the best performing model among the linear regression models.

6.Repeat the previous step with polynomial regression. Using validation loss, explore if your model overfits/underfits the data.

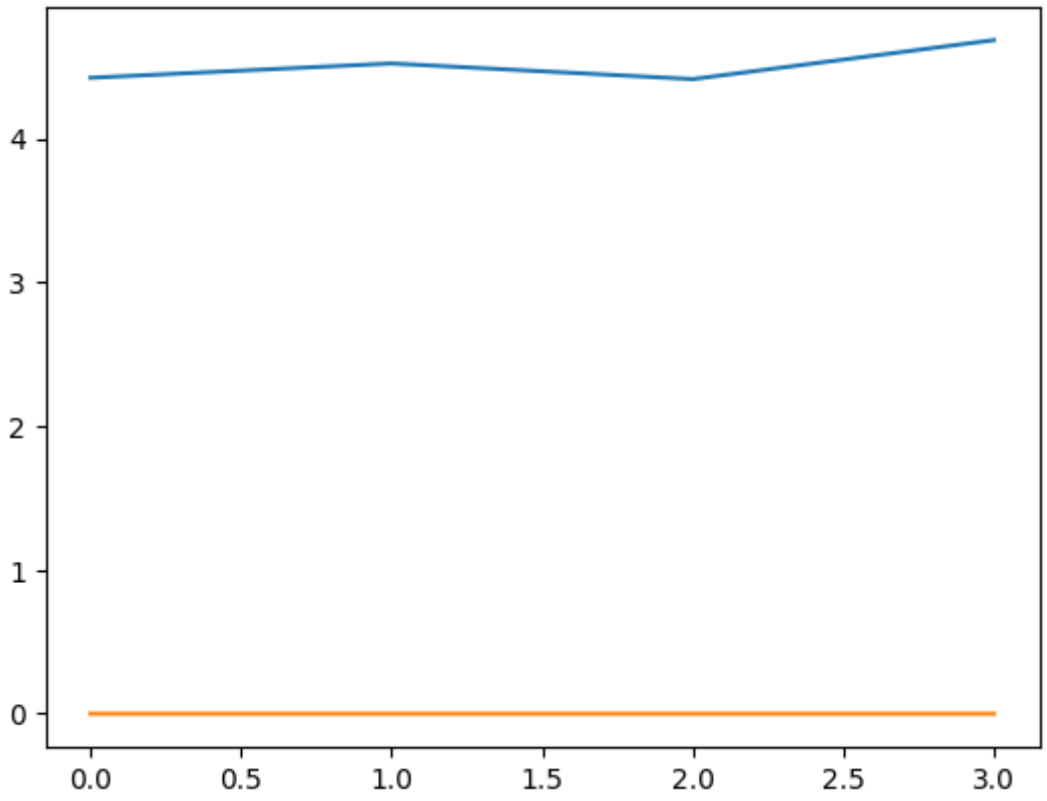
Fix - added ridge, lasso, etc for polynomial regressor.

In [24]:
```
# Polynomial regression using various degrees

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    lin_reg = LinearRegression()
    for cnt,var in enumerate(kf.split(x_train,y_train)):
        print(cnt)
        x = x_train[var[0],:]
        y = y_train[var[0]]
        x_val = x_train[var[1],:]
        y_val = y_train[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        lin_reg.fit(X_poly_1, y)
        y_pred_train=lin_reg.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        lin_reg.fit(X_poly_2, y_val)
        y_pred_val=lin_reg.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test)
        y_pred_test = lin_reg.predict(X_test_poly)
        metric_dict['Polynomial degree= '+str(r)] = np.sqrt(mse(y_pred_
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

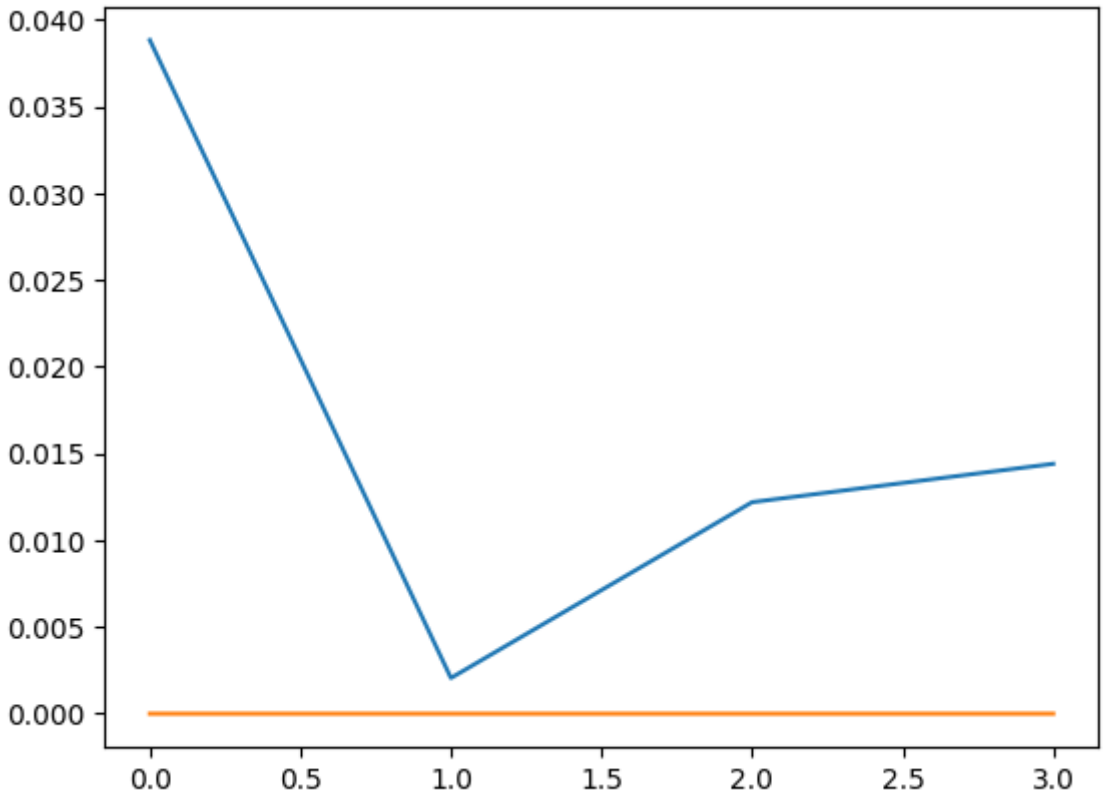
```
Degree: 2
0
Train error: 9.76013070976515
Validation error: 8.564470702777683
1
Train error: 9.718830185172754
Validation error: 9.13719085748416
2
Train error: 9.453076511101303
Validation error: 9.085760813520182
3
Train error: 9.762004677157083
Validation error: 8.65826662158363
```




```
Degree: 5
0
Train error: 4.42320117333612
Validation error: 1.9280863796003466e-07
1
Train error: 4.522924897798512
Validation error: 1.9835058900715033e-06
2
Train error: 4.414017676385926
Validation error: 4.7395288059359964e-08
3
Train error: 4.685520465432909
Validation error: 6.780744545007008e-08
```



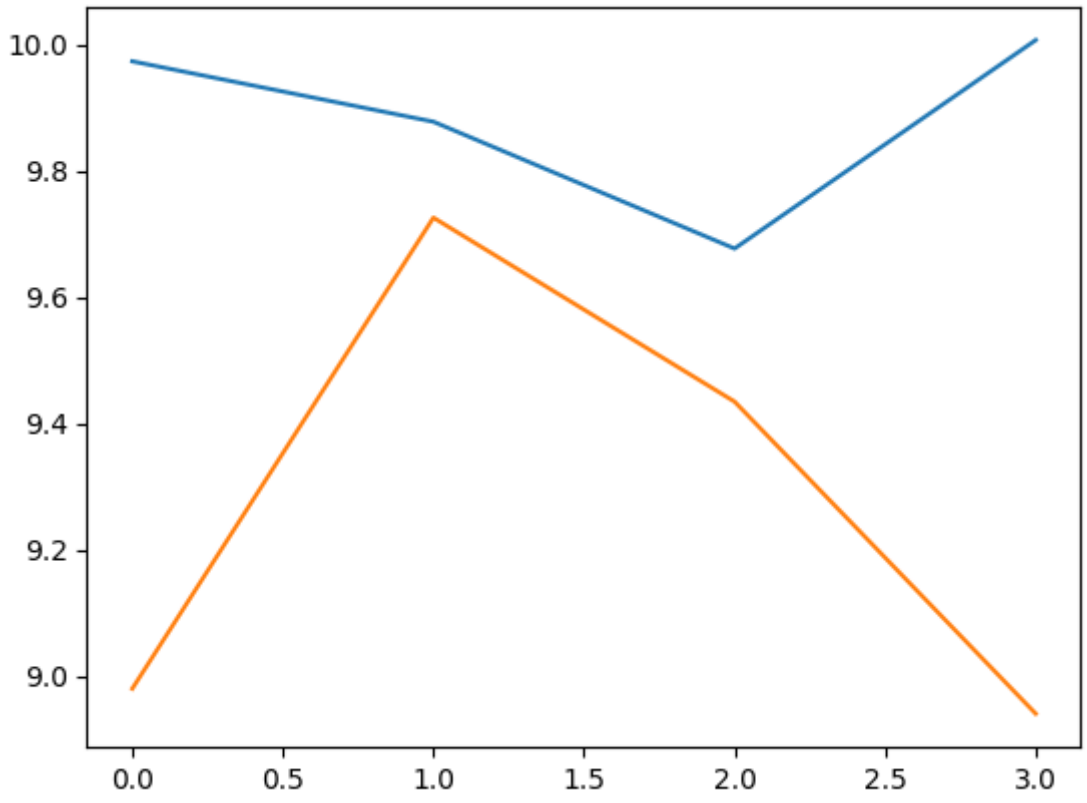

```
Degree: 10
0
Train error: 0.03882512790275171
Validation error: 1.2077238934433841e-08
1
Train error: 0.0020574117977613295
Validation error: 4.6613507019115987e-08
2
Train error: 0.012192259496120893
Validation error: 7.156039567916752e-08
3
Train error: 0.014402553397965928
Validation error: 5.15867155212226e-08
```


0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [25]:

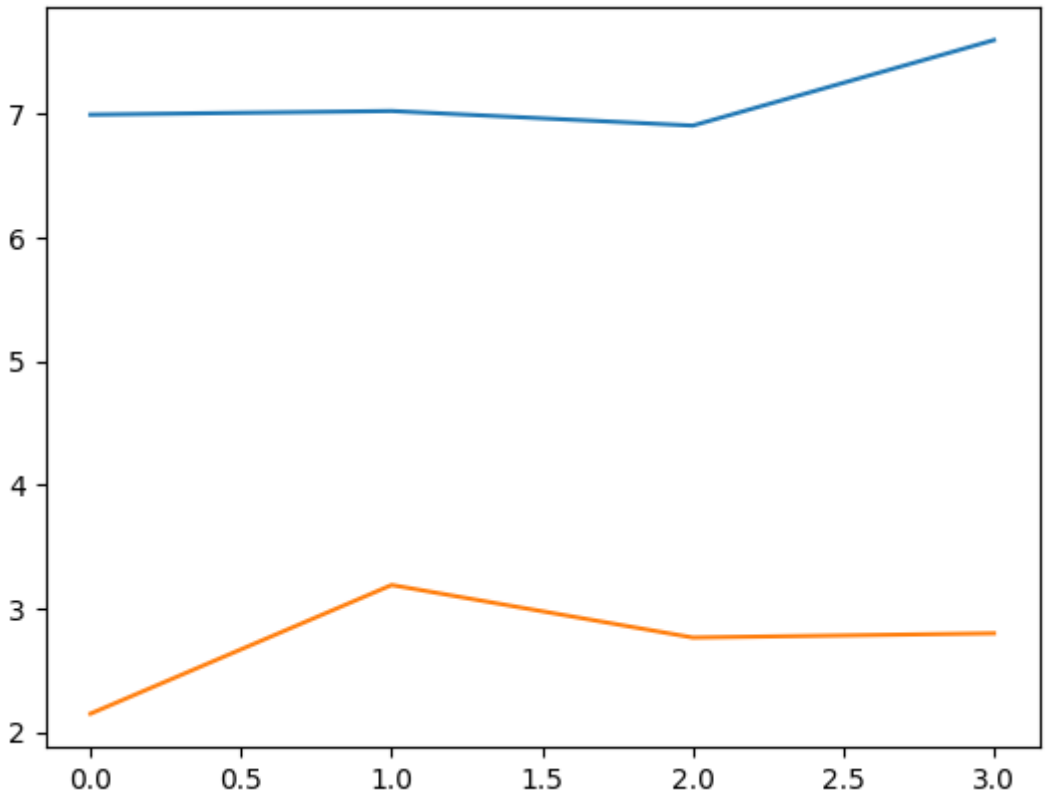
```
# Polynomial regression using ridge

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    ridge = Ridge()
    for cnt,var in enumerate(kf.split(x_train,y_train)):
        print(cnt)
        x = x_train[var[0],:]
        y = y_train[var[0]]
        x_val = x_train[var[1],:]
        y_val = y_train[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        ridge.fit(X_poly_1, y)
        y_pred_train=ridge.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        ridge.fit(X_poly_2, y_val)
        y_pred_val=ridge.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test)
        y_pred_test = ridge.predict(X_test_poly)
        metric_dict['Polynomial degree for ridge= '+str(r)] = np.sqrt(m
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

```
Degree: 2
0
Train error: 9.972991151121494
Validation error: 8.979952566115916
1
Train error: 9.877358330761425
Validation error: 9.725490296562116
2
Train error: 9.6765546513985
Validation error: 9.434203744416093
3
Train error: 10.006579648231806
Validation error: 8.940280963689755
```



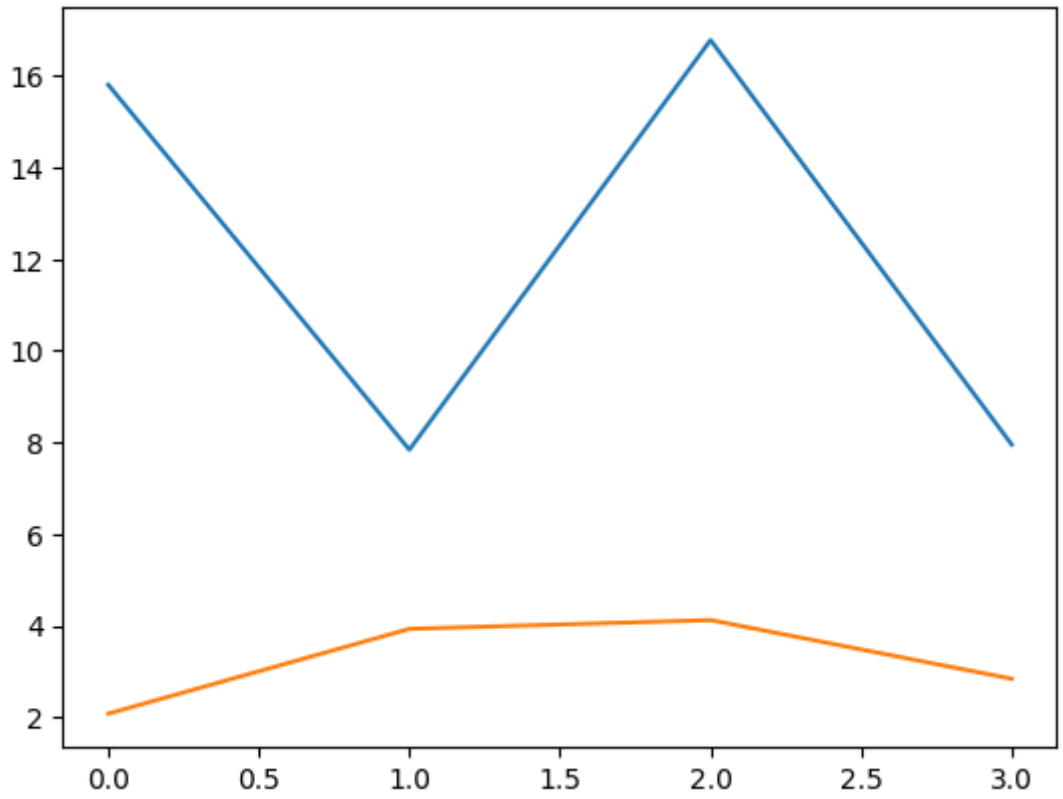

```
Degree: 5
0
Train error: 6.994982032474704
Validation error: 2.1473403441374415
1
Train error: 7.0244979882915635
Validation error: 3.1887220999344263
2
Train error: 6.90652736531292
Validation error: 2.763591889467526
3
```

```
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:212: LinAlgWarning: Ill-conditioned
matrix (rcond=3.07389e-18): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:249: LinAlgWarning: Ill-conditioned
matrix (rcond=7.16618e-18): result may not be accurate.
  dual_coef = linalg.solve(K, y, assume_a="pos", overwrite_a=False)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:212: LinAlgWarning: Ill-conditioned
matrix (rcond=3.22856e-18): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:249: LinAlgWarning: Ill-conditioned
matrix (rcond=7.44511e-18): result may not be accurate.
  dual_coef = linalg.solve(K, y, assume_a="pos", overwrite_a=False)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:212: LinAlgWarning: Ill-conditioned
matrix (rcond=3.01177e-18): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:249: LinAlgWarning: Ill-conditioned
matrix (rcond=7.7028e-18): result may not be accurate.
  dual_coef = linalg.solve(K, y, assume_a="pos", overwrite_a=False)
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:212: LinAlgWarning: Ill-conditioned
matrix (rcond=3.11504e-18): result may not be accurate.
  return linalg.solve(A, Xy, assume_a="pos", overwrite_a=True).T
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:249: LinAlgWarning: Ill-conditioned
matrix (rcond=7.40515e-18): result may not be accurate.
  dual_coef = linalg.solve(K, y, assume_a="pos", overwrite_a=False)

Train error: 7.598493218408641
Validation error: 2.7978097941925846
```




```
Degree: 10
0

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(

Train error: 15.811562761898207
Validation error: 2.0757433849606413
1

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(
```

```
Train error: 7.841224288667445
Validation error: 3.928138075586169
2

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(

Train error: 16.78711530058343
Validation error: 4.120097002849179
3
Train error: 7.953268031055733
Validation error: 2.8386054773469644

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_ridge.py:251: UserWarning: Singular matrix i
n solving dual problem. Using least-squares solution instead.
  warnings.warn(
```

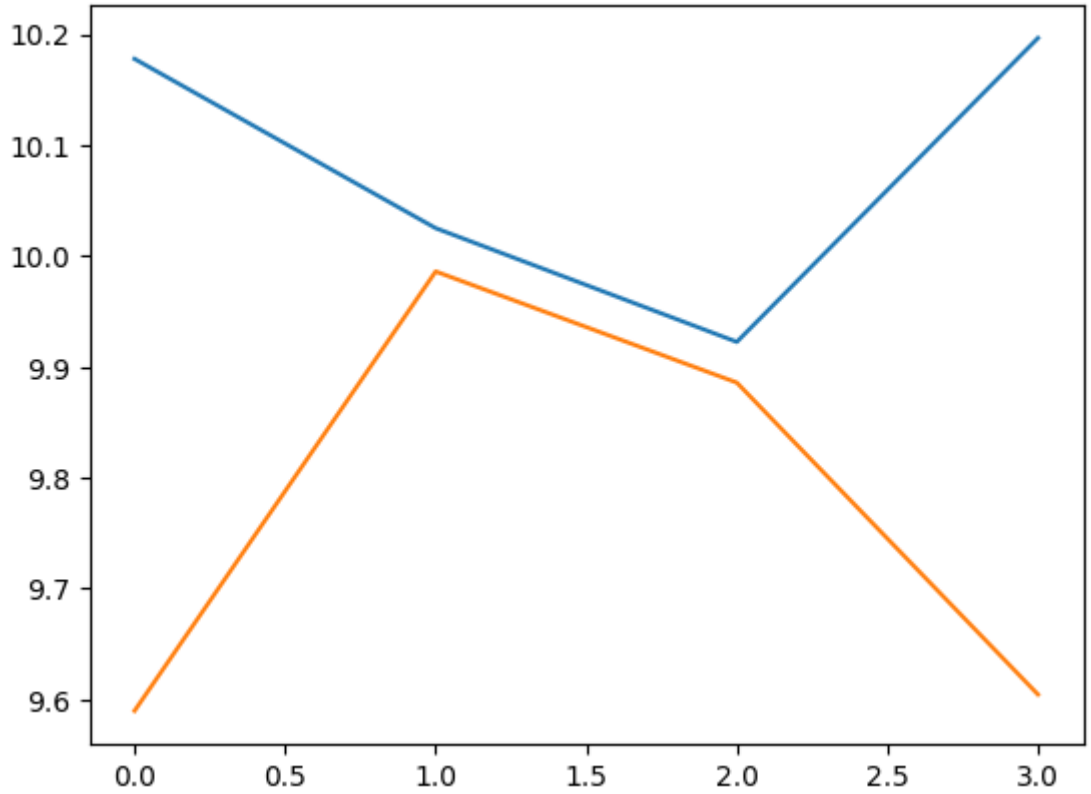


In [26]:

```
# Polynomial regression using lasso

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    lasso = Lasso()
    for cnt,var in enumerate(kf.split(x_train,y_train)):
        print(cnt)
        x = x_train[var[0],:]
        y = y_train[var[0]]
        x_val = x_train[var[1],:]
        y_val = y_train[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        lasso.fit(X_poly_1, y)
        y_pred_train=lasso.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        lasso.fit(X_poly_2, y_val)
        y_pred_val=lasso.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test)
        y_pred_test = lasso.predict(X_test_poly)
        metric_dict['Polynomial degree for Lasso = '+str(r)] = np.sqrt(
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

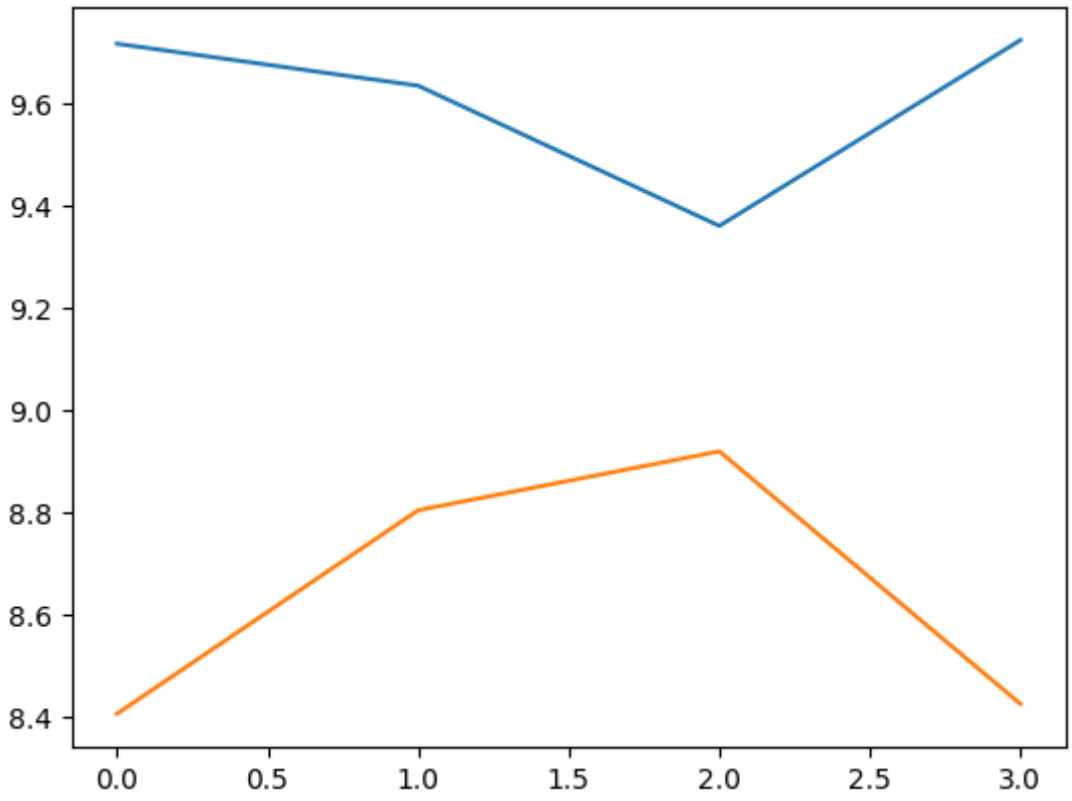
```
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.289e+02, tolerance: 1.882e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.569e+02, tolerance: 7.026e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 2.284e+02, tolerance: 1.902e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 7.345e+02, tolerance: 6.748e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.218e+02, tolerance: 1.950e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 2.277e+02, tolerance: 6.342e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.156e+03, tolerance: 2.020e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.731e+02, tolerance: 5.558e+01
  model = cd_fast.enet_coordinate_descent(
```

```
Degree: 2
0
Train error: 10.177702120533642
Validation error: 9.58984296614541
1
Train error: 10.025048679699102
Validation error: 9.986005036921377
2
Train error: 9.922503219779037
Validation error: 9.885775662560585
3
Train error: 10.196497020410424
Validation error: 9.604572280967375
```



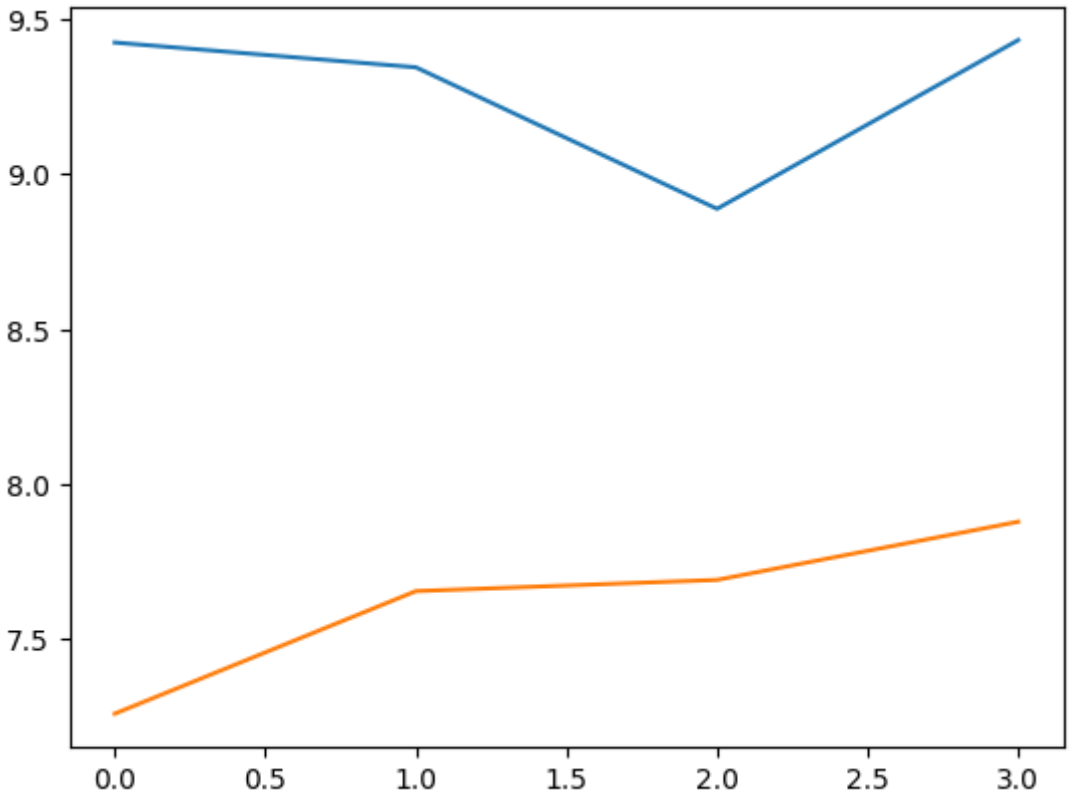

```
Degree: 5
0

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.541e+04, tolerance: 1.882e+02
  model = cd_fast.enet_coordinate_descent(

Train error: 9.71626768586068
Validation error: 8.407596671315675
1
```

```
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.013e+03, tolerance: 7.026e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.509e+04, tolerance: 1.902e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.366e+03, tolerance: 6.748e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.634266989542818
Validation error: 8.805036432572177
2

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.432e+04, tolerance: 1.950e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.341e+03, tolerance: 6.342e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.36050069017951
Validation error: 8.920099043903367
3

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.558e+04, tolerance: 2.020e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.827e+03, tolerance: 5.558e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.723420752105088
Validation error: 8.42674027067682
```



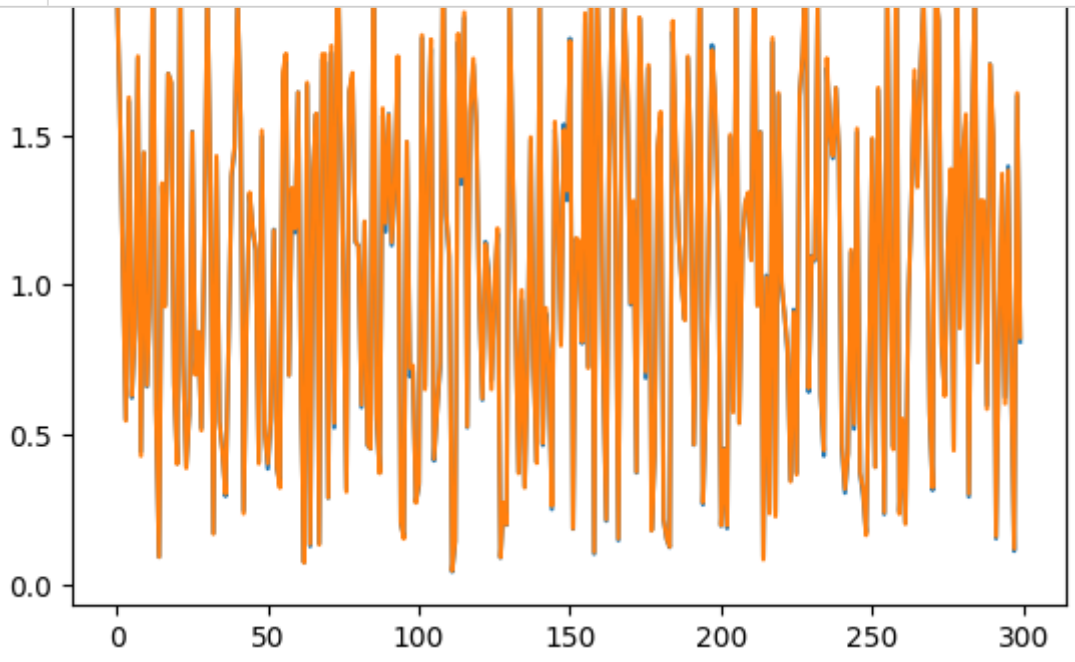

```
Degree: 10
0

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.425e+04, tolerance: 1.882e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.021e+03, tolerance: 7.026e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.424240456204537
Validation error: 7.26028476666213
1
```

```
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.395e+04, tolerance: 1.902e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.252e+03, tolerance: 6.748e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.34423087205704
Validation error: 7.655485117731198
2

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.267e+04, tolerance: 1.950e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.245e+03, tolerance: 6.342e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 8.888076168371121
Validation error: 7.691679344921441
3

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.431e+04, tolerance: 2.020e+02
  model = cd_fast.enet_coordinate_descent(

Train error: 9.432051169922236
Validation error: 7.8792058929730695

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.301e+03, tolerance: 5.558e+01
  model = cd_fast.enet_coordinate_descent(
```


9.5
9.0
8.5
8.0
7.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [27]:

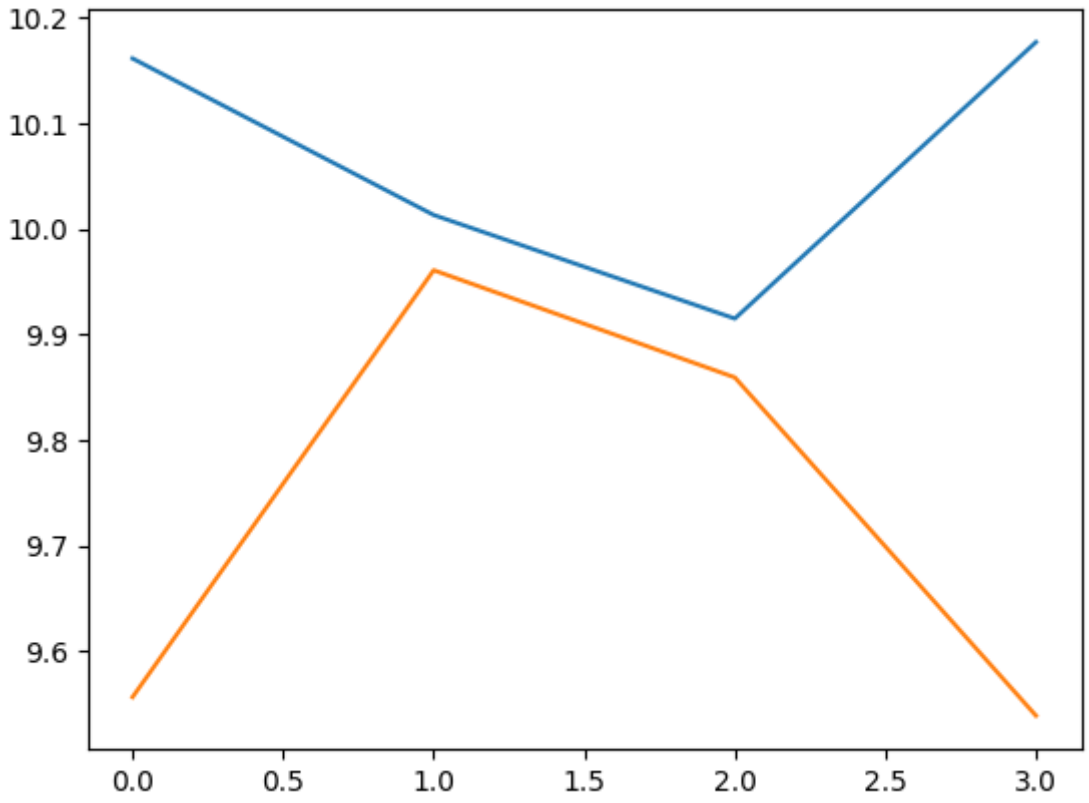
```
# Polynomial regression using SGD
sgd_pen = [None,'l1','l2']
learning_rates = [0.000001, 0.00001, 0.0001, 0.001, 0.01]
n_epochs = 300
pol_deg = [2,5]
for penalty in sgd_pen:
    for lr in learning_rates:
        for pol in pol_deg:
            print("Polynomial degree =:", pol)
            print("Penalty:", penalty)
            print("Learning Rate:", lr)
            train_loss = []
            val_loss = []
            sgd_obj = SGDRegressor(penalty=penalty, learning_rate='cons
            poly_features = PolynomialFeatures(degree=pol, include_bias
            X_train_poly = poly_features.fit_transform(x_train)
            X_val_poly = poly_features.transform(x_val)

            for epoch in range(n_epochs):
                sgd_obj.partial_fit(X_train_poly, y_train)
                y_pred_train = sgd_obj.predict(X_train_poly)
                y_pred_val = sgd_obj.predict(X_val_poly)
                train_loss.append(np.sqrt(mse(y_pred_train,y_train)))
                val_loss.append(np.sqrt(mse(y_pred_val,y_val)))

            plt.plot(range(n_epochs), train_loss, label = "Training Los
            plt.plot(range(n_epochs), val_loss, label = "Validation Los
            plt.legend()
            plt.show()

            # Calculate and print RMSE score between y_pred_val and y_t
            y_pred_test = sgd_obj.predict(poly_features.transform(x_tes
            rmse = np.sqrt(mse(y_pred_test, y_test))
            metric_dict['Polynomial degree for SGD= '+str(pol)+' Learni
            print("RMSE score on test data:", rmse)

```



In [28]:
```
# Polynomial regression using elastic net

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    el_net = ElasticNet()
    for cnt,var in enumerate(kf.split(x_train,y_train)):
        print(cnt)
        x = x_train[var[0],:]
        y = y_train[var[0]]
        x_val = x_train[var[1],:]
        y_val = y_train[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        el_net.fit(X_poly_1, y)
        y_pred_train=el_net.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        el_net.fit(X_poly_2, y_val)
        y_pred_val=el_net.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test)
        y_pred_test = el_net.predict(X_test_poly)
        metric_dict['Polynomial degree for elastic net= '+str(r)] = np.
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

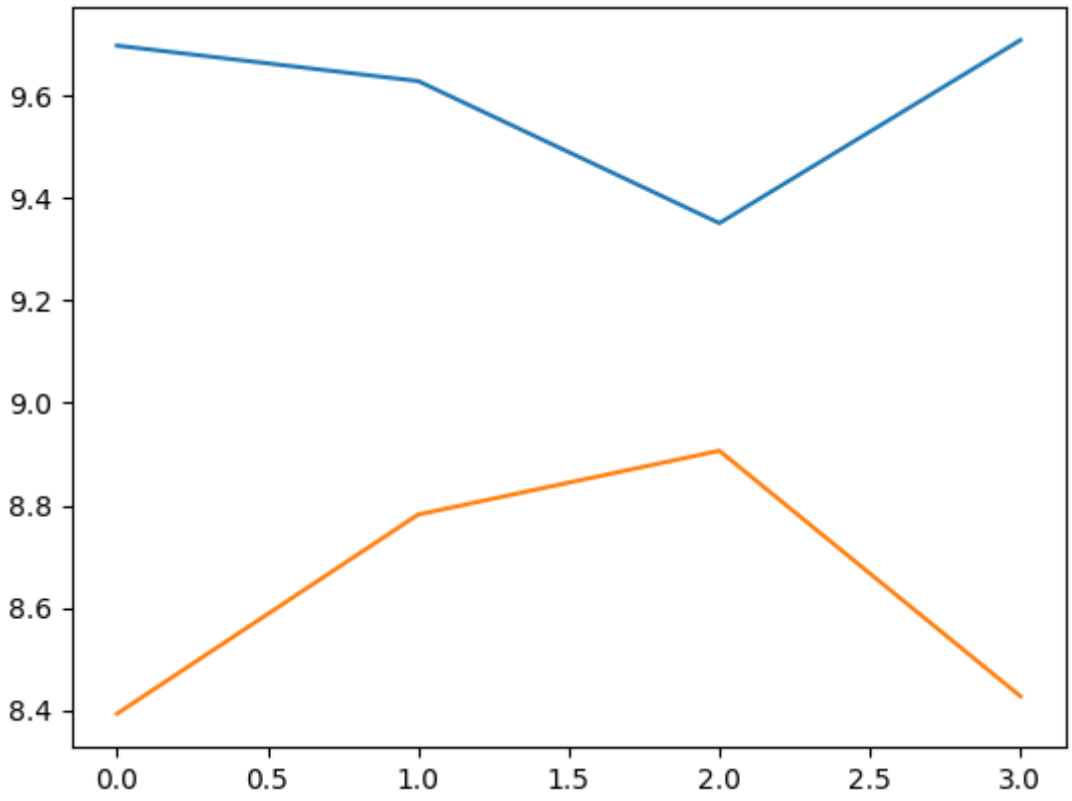
```
Degree: 2
0
Train error: 10.161712861049589
Validation error: 9.556382823561957
1
Train error: 10.013352613057389
Validation error: 9.960986562181604
2
Train error: 9.91516099217222
Validation error: 9.85937262845311
3
Train error: 10.17722328068957
Validation error: 9.538757835646235
```

```
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 7.034e+02, tolerance: 1.882e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 2.271e+02, tolerance: 7.026e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 2.062e+02, tolerance: 1.902e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 5.860e+02, tolerance: 6.748e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 6.267e+02, tolerance: 1.950e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.694e+02, tolerance: 6.342e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 5.338e+02, tolerance: 2.020e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.695e+02, tolerance: 5.558e+01
  model = cd_fast.enet_coordinate_descent(
```



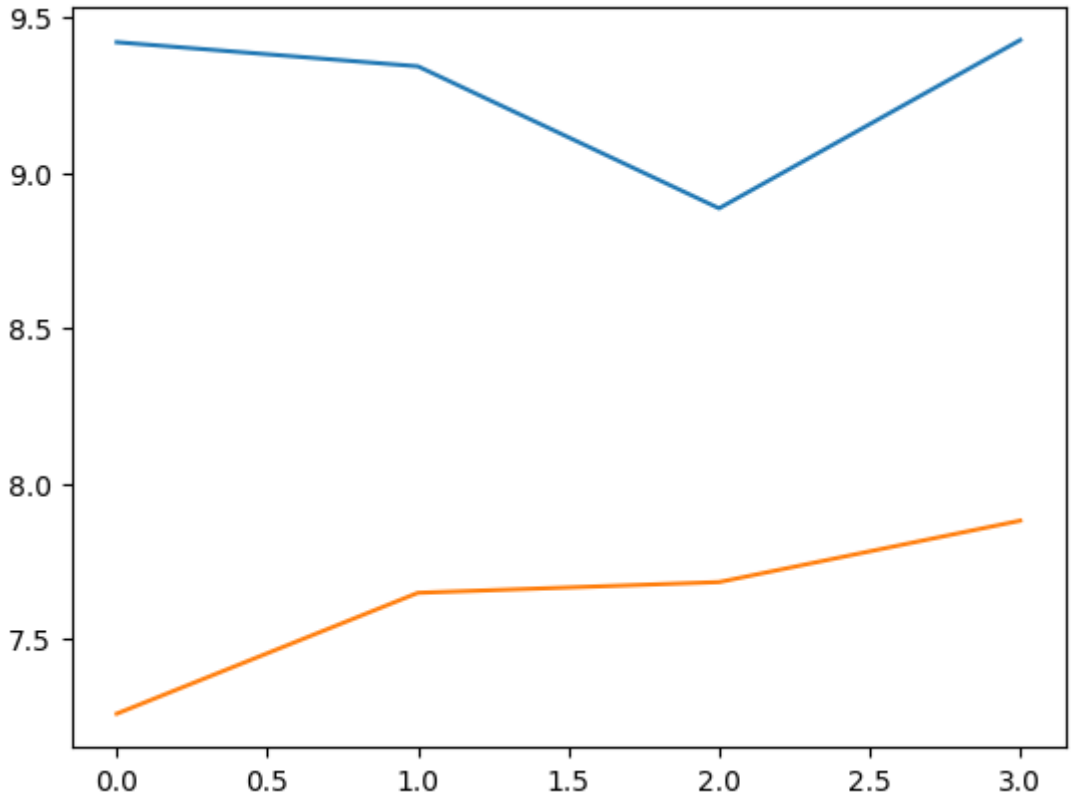

```
Degree: 5
0
Train error: 9.698549283296515
Validation error: 8.39251583865338
1

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.523e+04, tolerance: 1.882e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.966e+03, tolerance: 7.026e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.494e+04, tolerance: 1.902e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.320e+03, tolerance: 6.748e+01
  model = cd_fast.enet_coordinate_descent(
```

```
Train error: 9.629241967736391
Validation error: 8.781826920099554
2
Train error: 9.351512518846771
Validation error: 8.906621725124745
3

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.424e+04, tolerance: 1.950e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 4.294e+03, tolerance: 6.342e+01
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.547e+04, tolerance: 2.020e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.785e+03, tolerance: 5.558e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.70936241411298
Validation error: 8.42655168466262
```




```
Degree: 10
0

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.411e+04, tolerance: 1.882e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 2.988e+03, tolerance: 7.026e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.419182902996441
Validation error: 7.259838555097031
1
```

```
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.380e+04, tolerance: 1.902e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.206e+03, tolerance: 6.748e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 9.341588015927428
Validation error: 7.648505709521227
2

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.254e+04, tolerance: 1.950e+02
  model = cd_fast.enet_coordinate_descent(
/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.185e+03, tolerance: 6.342e+01
  model = cd_fast.enet_coordinate_descent(

Train error: 8.884955270681097
Validation error: 7.683070350343215
3

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 1.416e+04, tolerance: 2.020e+02
  model = cd_fast.enet_coordinate_descent(

Train error: 9.425949816477585
Validation error: 7.88059126710222

/Library/Frameworks/Python.framework/Versions/3.10/lib/python3.10/site-pa
ckages/sklearn/linear_model/_coordinate_descent.py:648: ConvergenceWarnin
g: Objective did not converge. You might want to increase the number of i
terations, check the scale of the features or consider increasing regular
isation. Duality gap: 3.262e+03, tolerance: 5.558e+01
  model = cd_fast.enet_coordinate_descent(
```



The polynomial model with degree 10 has a lower training error than validation error, it is likely that the model is overfitting to the training data. This occurs because the higher-degree polynomial model can fit the noise in the training data too closely, which may not be representative of the underlying pattern in the data. As a result, the model may not generalize well to new, unseen data. In such cases, it may be necessary to consider other models or regularization techniques to prevent overfitting.

7. Make predictions of the labels on the test data, using the trained model with chosen hyperparameters. Summarize performance using the appropriate evaluation metric. Discuss the results. Include thoughts about what further can be explored to increase performance

In [29]:
```
#Below is the sample code step for predication which is repeated for al

ridge = Ridge()
for cnt,var in enumerate(kf.split(x_train,y_train)):
    x = x_train[var[0],:]
    y = y_train[var[0]]
    x_val = x_train[var[1],:]
    y_val = y_train[var[1]]
    ridge.fit(x, y)
    y_pred_train = ridge.predict(x)
    ridge.fit(x_val, y_val)
    y_pred_val = ridge.predict(x_val)
```

In [30]:

```
# metric_dict has all the predicted values

metric_dict

```

Out[30]:

```
{'Linear regression': 9.921837612791698,
 'SGD with learning rate =1e-06': 60.349395135527104,
 'SGD with learning rate =1e-05': 25.996001317630146,
 'SGD with learning rate =0.0001': 23.349901277079177,
 'SGD with learning rate =0.001': 1488544586411.5103,
 'SGD with learning rate =0.01': 18562893045068.81,
 'SGD with learning rate =1e-06 and penalty_factor =None': 60.45165562785
1046,
 'SGD with learning rate =1e-05 and penalty_factor =None': 26.01989715303
5224,
 'SGD with learning rate =0.0001 and penalty_factor =None': 23.0687064615
44643,
 'SGD with learning rate =0.001 and penalty_factor =None': 1078176318229.
8536,
 'SGD with learning rate =0.01 and penalty_factor =None': 14256691494401.
98,
 'SGD with learning rate =1e-06 and penalty_factor =l1': 60.3579997947129
1,
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 'SGD with learning rate =0.0001 and penalty_factor =l1': 24.196112811617
436,
 'SGD with learning rate =0.001 and penalty_factor =l1': 1517469530288.82
6,
 'SGD with learning rate =0.01 and penalty_factor =l1': 12320825983055.1
9,
 'SGD with learning rate =1e-06 and penalty_factor =l2': 60.3195950502159
9,
 'SGD with learning rate =1e-05 and penalty_factor =l2': 26.043603132570
9,
 'SGD with learning rate =0.0001 and penalty_factor =l2': 25.503626036472
66,
 'SGD with learning rate =0.001 and penalty_factor =l2': 1289996564600.16
2,
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 'Ridge lr=0.001': 9.961299722733118,
 'Ridge lr=1': 9.99724056783426,
 'Ridge lr=10': 11.489062579737597,
 'Ridge lr=100': 34.91241198195777,
 'Lasso lr=0': 9.961322851563667,
 'Lasso lr=0.001': 9.960594883008598,
 'Lasso lr=1': 9.842479052252486,
 'Lasso lr=10': 19.984329995675473,
 'Lasso lr=100': 74.29880947106363,
 'ElasticNet lr=0': 9.961322851563667,
 'ElasticNet lr=0.001': 9.959996289392535,
 'ElasticNet lr=1': 24.402130733832436,
 'ElasticNet lr=10': 61.622807515558456,
 'ElasticNet lr=100': 74.14978992198517,
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 'Polynomial degree for ridge= 5': 176.7852714514506,
 'Polynomial degree for ridge= 10': 252.41481442567473,
```

```
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 'Polynomial degree for Lasso = 5': 13.028834283592813,
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 'Polynomial degree for SGD= 5 Learning_rate= 1e-06 and penalty_term= Non
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 'Polynomial degree for SGD= 2 Learning_rate= 1e-05 and penalty_term= Non
e': 36433836480819.01,
 'Polynomial degree for SGD= 5 Learning_rate= 1e-05 and penalty_term= Non
e': 8.204661425341553e+22,
 'Polynomial degree for SGD= 2 Learning_rate= 0.0001 and penalty_term= No
ne': 416463262033802.8,
 'Polynomial degree for SGD= 5 Learning_rate= 0.0001 and penalty_term= No
ne': 7.103048292982905e+23,
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e': 1.1984390776529858e+24,
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1': 25829311073320.42,
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 'Polynomial degree for SGD= 2 Learning_rate= 0.0001 and penalty_term= l
1': 379646381713654.25,
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1': 8.321639480681627e+23,
 'Polynomial degree for SGD= 2 Learning_rate= 0.001 and penalty_term= l
1': 1210002725579238.8,
 'Polynomial degree for SGD= 5 Learning_rate= 0.001 and penalty_term= l
1': 2.1304709944235073e+25,
 'Polynomial degree for SGD= 2 Learning_rate= 0.01 and penalty_term= l1':
3.746375941863839e+16,
 'Polynomial degree for SGD= 5 Learning_rate= 0.01 and penalty_term= l1':
1.261283345804604e+26,
 'Polynomial degree for SGD= 2 Learning_rate= 1e-06 and penalty_term= l
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 'Polynomial degree for SGD= 5 Learning_rate= 1e-06 and penalty_term= l
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 'Polynomial degree for SGD= 5 Learning_rate= 0.0001 and penalty_term= l
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 'Polynomial degree for SGD= 2 Learning_rate= 0.001 and penalty_term= l
2': 812753064715253.2,
```

```
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 'Polynomial degree for SGD= 2 Learning_rate= 0.01 and penalty_term= l2':
1.3505621812076556e+16,
 'Polynomial degree for SGD= 5 Learning_rate= 0.01 and penalty_term= l2':
6.376806259232264e+25,
 'Polynomial degree for elastic net= 2': 10.147262133951827,
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 'Polynomial degree for elastic net= 10': 19.639272544843354}
```

From the metric above we observe that Linear regression works the best on my test data, since it has the lowest RMSE score.

To improve the performance we can do the following: 1.Increase the amount and quality of the training data 2.Optimize the model hyperparameters, such as the learning rate or batch size 3.Regularize the model to prevent overfitting, such as by using dropout or L2 regularization 4.Perform feature engineering to extract more informative features from the data.

In [ ]:
```
1
```