Applied Machine Learning

Homework 1 Fix

part 5 - Added learning rates to SGD and plotted the loss vs epochs graphs

part 6 - added ridge, lasso, etc for polynomial regressor.

In [1]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
from sklearn.pipeline import Pipeline
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import OneHotEncoder
from sklearn.compose import ColumnTransformer
from sklearn.model_selection import train_test_split
from sklearn.model_selection import StratifiedShuffleSplit
from sklearn.model_selection import KFold
from sklearn.linear_model import Ridge
from sklearn.linear_model import Lasso
from sklearn.linear_model import ElasticNet
from sklearn.linear_model import SGDRegressor
from sklearn.preprocessing import PolynomialFeatures
```

Question 1

In [2]:
```
# Loading the dataset as a dataframe

happiness_df = pd.read_csv('happiness_data.csv')
happiness_df.head(20)
```

Out[2]:

	Country name	year	Life Ladder	Log GDP per capita	Social support	Healthy life expectancy at birth	Freedom to make life choices	Generosity	Perceptions of corruption	I
0	Afghanistan	2008	3.724	7.370	0.451	50.80	0.718	0.168	0.882	
1	Afghanistan	2009	4.402	7.540	0.552	51.20	0.679	0.190	0.850	
2	Afghanistan	2010	4.758	7.647	0.539	51.60	0.600	0.121	0.707	
3	Afghanistan	2011	3.832	7.620	0.521	51.92	0.496	0.162	0.731	
4	Afghanistan	2012	3.783	7.705	0.521	52.24	0.531	0.236	0.776	
5	Afghanistan	2013	3.572	7.725	0.484	52.56	0.578	0.061	0.823	
6	Afghanistan	2014	3.131	7.718	0.526	52.88	0.509	0.104	0.871	
7	Afghanistan	2015	3.983	7.702	0.529	53.20	0.389	0.080	0.881	
8	Afghanistan	2016	4.220	7.697	0.559	53.00	0.523	0.042	0.793	
9	Afghanistan	2017	2.662	7.697	0.491	52.80	0.427	-0.121	0.954	
10	Afghanistan	2018	2.694	7.692	0.508	52.60	0.374	-0.094	0.928	
11	Afghanistan	2019	2.375	7.697	0.420	52.40	0.394	-0.108	0.924	
12	Albania	2007	4.634	9.142	0.821	65.80	0.529	-0.009	0.875	
13	Albania	2009	5.485	9.262	0.833	66.20	0.525	-0.158	0.864	
14	Albania	2010	5.269	9.303	0.733	66.40	0.569	-0.172	0.726	
15	Albania	2011	5.867	9.331	0.759	66.68	0.487	-0.205	0.877	
16	Albania	2012	5.510	9.347	0.785	66.96	0.602	-0.169	0.848	
17	Albania	2013	4.551	9.359	0.759	67.24	0.632	-0.127	0.863	
18	Albania	2014	4.814	9.378	0.626	67.52	0.735	-0.025	0.883	
19	Albania	2015	4.607	9.403	0.639	67.80	0.704	-0.081	0.885	

Part 1

Summarize the data. How much data is present? What attributes/features are continuous valued? Which attributes are categorical?

In [3]:
```
# Extracting the number of rows and columns of the dataframe

print(len(happiness_df))
print(len(happiness_df.columns))
```

```
1949
11
```

In [4]:
```
happiness_df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1949 entries, 0 to 1948
Data columns (total 11 columns):
 #   Column                            Non-Null Count  Dtype  
---  ------                            --------------  -----  
 0   Country name                      1949 non-null   object 
 1   year                              1949 non-null   int64  
 2   Life Ladder                       1949 non-null   float64
 3   Log GDP per capita                1913 non-null   float64
 4   Social support                    1936 non-null   float64
 5   Healthy life expectancy at birth  1894 non-null   float64
 6   Freedom to make life choices      1917 non-null   float64
 7   Generosity                        1860 non-null   float64
 8   Perceptions of corruption         1839 non-null   float64
 9   Positive affect                   1927 non-null   float64
 10  Negative affect                   1933 non-null   float64
dtypes: float64(9), int64(1), object(1)
memory usage: 167.6+ KB
```

Answer:- The dataframe has a size of 1949 rows and 11 columns i.e. there are 1949 data entries and 11 attributes/features. There are 9 continuous valued attributes which are ['Life Ladder', 'Log GDP per capita', 'Social support', 'Healthy life expectancy at birth', 'Freedom to make life choices', 'Generosity', 'Perceptions of corruption', 'Positive affect', 'Negative affect'] and 2 categorical attributes which are ['Country name', 'year']. Continuous values are those which can assume an infinite number of values within a given range. Categorical values are those which can assume values only from a given set. Depending on the context and dataset, 'year' can be treated as categorical or continuous.

Part 2

Display the statistical values for each of the attributes, along with visualizations (e.g. histogram) of the distributions for each attribute. Explain noticeable traits for key attributes. Are there any attributes that might require special treatment? If so, what special treatment might they require?

In [5]:
```
1 happiness_df.describe()
```

Out[5]:

	year	Life Ladder	Log GDP per capita	Social support	Healthy life expectancy at birth	Freedom to make life choices	Generos
count	1949.000000	1949.000000	1913.000000	1936.000000	1894.000000	1917.000000	1860.0000
mean	2013.216008	5.466705	9.368453	0.812552	63.359374	0.742558	0.0001
std	4.166828	1.115711	1.154084	0.118482	7.510245	0.142093	0.1622
min	2005.000000	2.375000	6.635000	0.290000	32.300000	0.258000	-0.3350
25%	2010.000000	4.640000	8.464000	0.749750	58.685000	0.647000	-0.1130
50%	2013.000000	5.386000	9.460000	0.835500	65.200000	0.763000	-0.0255
75%	2017.000000	6.283000	10.353000	0.905000	68.590000	0.856000	0.0910
max	2020.000000	8.019000	11.648000	0.987000	77.100000	0.985000	0.6980

In [6]:
```
1 happiness_df["Country name"].value_counts()
```

Out[6]:
```
Zimbabwe        15
South Africa    15
Tanzania        15
Denmark         15
Tajikistan      15
                ..
Maldives         1
Suriname         1
Cuba             1
Oman             1
Guyana           1
Name: Country name, Length: 166, dtype: int64
```

In [7]:
```
# Calculating the mean of each attribute for each unique country

happiness_df[["Country name", "Life Ladder", "Log GDP per capita", "Social
              "Freedom to make life choices", "Generosity", "Perceptions o
               "Negative affect"]].groupby("Country name").mean()
```

Out[7]:

Country name	Life Ladder	Log GDP per capita	Social support	Healthy life expectancy at birth	Freedom to make life choices	Generosity	Perceptions of corruption	Pos a
Afghanistan	3.594667	7.650833	0.508417	52.266667	0.518167	0.070083	0.843333	0.54
Albania	5.019385	9.384385	0.716231	67.546154	0.662923	-0.082692	0.869385	0.65
Algeria	5.389875	9.328875	0.803571	65.290000	0.519167	-0.144333	0.691000	0.60
Angola	4.420250	8.990000	0.738250	53.550000	0.456250	-0.088250	0.866750	0.61
Argentina	6.310133	10.033800	0.904400	67.900000	0.768200	-0.159867	0.842067	0.83
...	...	...	...	...	...	...	...	
Venezuela	6.019867	9.480083	0.910133	65.958667	0.666400	-0.164182	0.795200	0.80
Vietnam	5.305857	8.655286	0.829231	66.928571	0.882455	0.002615	0.784700	0.64
Yemen	3.912250	8.102700	0.739833	54.866667	0.622417	-0.126200	0.824667	0.53
Zambia	4.551714	8.066857	0.737714	51.707143	0.756786	0.013357	0.828857	0.72
Zimbabwe	3.882600	7.850333	0.799400	50.233333	0.587933	-0.083000	0.844200	0.71

166 rows × 9 columns

In [8]:
```
# Calculating the median of each attribute for each unique country

happiness_df[["Country name", "Life Ladder", "Log GDP per capita", "Social
              "Freedom to make life choices", "Generosity", "Perceptions o
               "Negative affect"]].groupby("Country name").median()
```

Out[8]:

Country name	Life Ladder	Log GDP per capita	Social support	Healthy life expectancy at birth	Freedom to make life choices	Generosity	Perceptions of corruption	Positive affect
Afghanistan	3.7535	7.6970	0.5210	52.48	0.5160	0.0920	0.8605	0.5595
Albania	4.9950	9.3780	0.7100	67.52	0.7040	-0.0810	0.8770	0.6690
Algeria	5.3290	9.3360	0.8070	65.32	0.5565	-0.1695	0.6950	0.6040
Angola	4.1485	8.9985	0.7380	53.55	0.4330	-0.1200	0.8700	0.6185
Argentina	6.4240	10.0510	0.9020	67.84	0.7470	-0.1570	0.8510	0.8400
...	...	...	...	...	...	...	...	...
Venezuela	6.2580	9.5495	0.9110	65.82	0.6420	-0.1690	0.8130	0.8190
Vietnam	5.2960	8.6470	0.8320	66.90	0.8890	-0.0060	0.7885	0.6650
Yemen	4.0145	8.2250	0.7415	54.30	0.6410	-0.1490	0.8290	0.5250
Zambia	4.7770	8.1240	0.7525	52.69	0.7645	0.0055	0.8105	0.7145
Zimbabwe	3.7350	7.9500	0.7990	50.96	0.6330	-0.0880	0.8310	0.7150

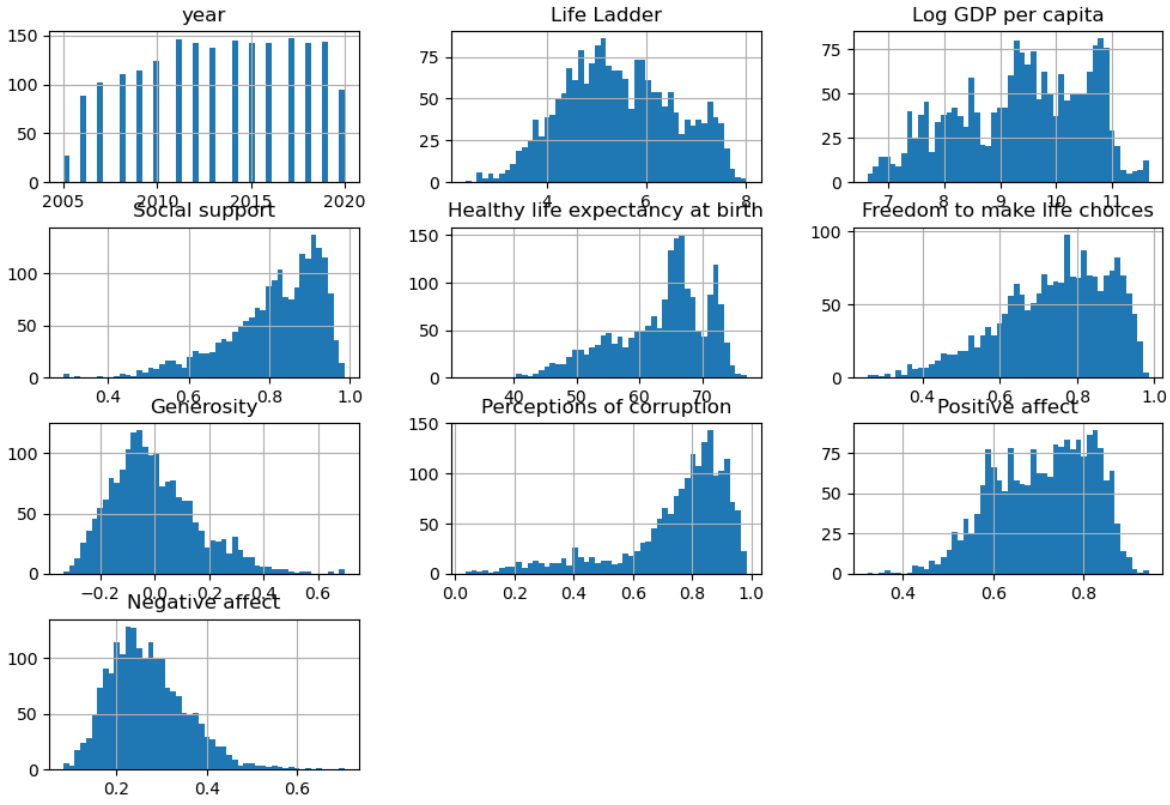
166 rows × 9 columns

In [9]:
```
happiness_df.skew()
```

```
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\4122175689.py:1: FutureWarn
ing: Dropping of nuisance columns in DataFrame reductions (with 'numeric_only
=None') is deprecated; in a future version this will raise TypeError.  Select
only valid columns before calling the reduction.
  happiness_df.skew()
```

Out[9]:
```
year                              -0.128238
Life Ladder                        0.068483
Log GDP per capita                -0.308453
Social support                    -1.110682
Healthy life expectancy at birth  -0.744426
Freedom to make life choices      -0.623019
Generosity                         0.807140
Perceptions of corruption         -1.496045
Positive affect                   -0.364197
Negative affect                    0.737166
dtype: float64
```

In [10]:
```
# Displaying the histogram for each attributes

happiness_df.hist(bins=50, figsize=(12, 8))
plt.show()
```



In [11]:
```
# Checking for null values

print(happiness_df["year"].isnull().values.any())
print(happiness_df["Life Ladder"].isnull().values.any())
print(happiness_df["Log GDP per capita"].isnull().values.any())
print(happiness_df["Social support"].isnull().values.any())
print(happiness_df["Healthy life expectancy at birth"].isnull().values.any
print(happiness_df["Freedom to make life choices"].isnull().values.any())
print(happiness_df["Generosity"].isnull().values.any())
print(happiness_df["Perceptions of corruption"].isnull().values.any())
print(happiness_df["Positive affect"].isnull().values.any())
print(happiness_df["Negative affect"].isnull().values.any())
```

```
False
False
True
True
True
True
True
True
True
True
```

Answer:- If we closely analyze the histograms and skewness values, these are the inferences:-

Log GDP per Capita: Its skewness value is -0.3, hence we can say the values are symmetrically distributed. The histogram confirms that.

Social support: The histogram is left skewed with a staggered bell shaped curve.

Healthy life expectancy at birth: The histogram is slightly left skewed since it has a skewness value of -0.7.

Freedom to make life choices: The histogram is slightly left skewed, but overall it can be considered symmetrically distributed.

Generosity: The histogram is right skewed with a staggered bell shaped curve.

Perceptions of corruption: The histogram is highly left skewed.

Positive affect: The values are symmetrically distributed.

Negative affect: The histogram is slightly right skewed.

All attributes except year and Life Ladder have null values in them. These attributes need special treatment and the null values need to be tackled. They can be either imputed or dropped. We will drop them.

Part 3

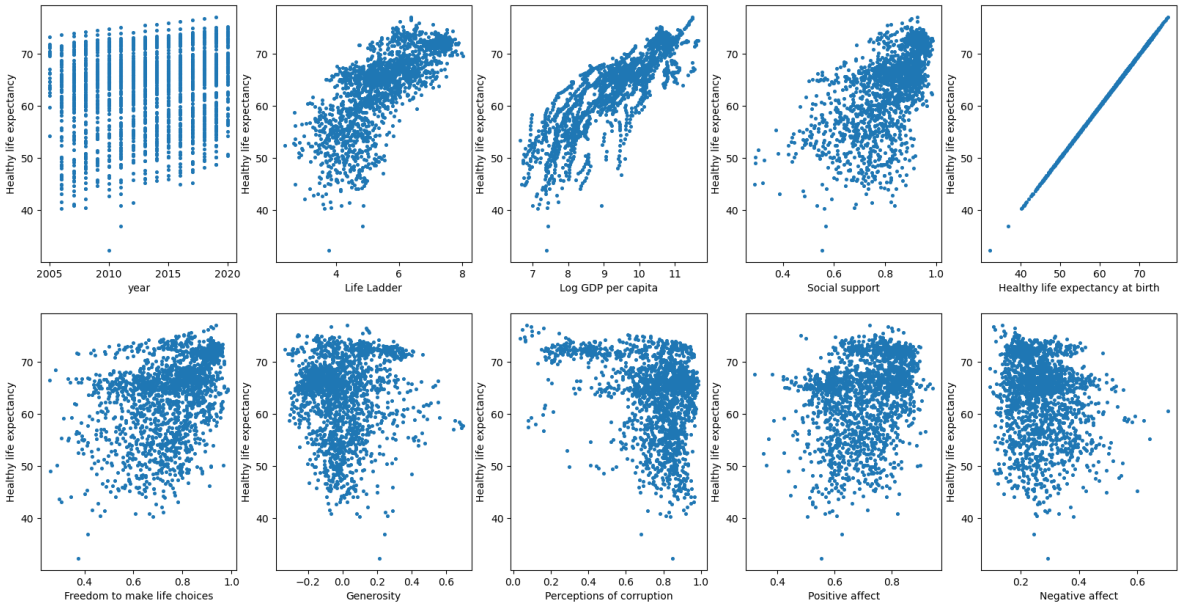
Analyze and discuss the relationships between the data attributes, and between the data attributes and label. This involves computing the Pearson Correlation Coefficient (PCC) and generating scatter plots.

In [12]:
```
# Finding the Pearson Correlation Coefficient (PCC) between the attributes

happiness_df.drop(columns=['Country name','year']).corr(method ='pearson')
```

Out[12]:

	Life Ladder	Log GDP per capita	Social support	Healthy life expectancy at birth	Freedom to make life choices	Generosity	Perceptions of corruption	P
Life Ladder	1.000000	0.790166	0.707806	0.744506	0.528063	0.190632	-0.427245	0.
Log GDP per capita	0.790166	1.000000	0.692602	0.848049	0.367932	-0.000915	-0.345511	0.
Social support	0.707806	0.692602	1.000000	0.616037	0.410402	0.067000	-0.219040	0.
Healthy life expectancy at birth	0.744506	0.848049	0.616037	1.000000	0.388681	0.020737	-0.322461	0.
Freedom to make life choices	0.528063	0.367932	0.410402	0.388681	1.000000	0.329300	-0.487883	0.
Generosity	0.190632	-0.000915	0.067000	0.020737	0.329300	1.000000	-0.290706	0.
Perceptions of corruption	-0.427245	-0.345511	-0.219040	-0.322461	-0.487883	-0.290706	1.000000	-0.
Positive affect	0.532273	0.302282	0.432152	0.318247	0.606114	0.358006	-0.296517	1.
Negative affect	-0.297488	-0.210781	-0.395865	-0.139477	-0.267661	-0.092542	0.264225	-0.

In [13]:
```
# Scatter plots of attributes versus output label

col = ['year','Life Ladder','Log GDP per capita','Social support','Healthy
 'Generosity','Perceptions of corruption','Positive affect','Negative affe

fig, ax = plt.subplots(2,int(len(col)/2 ), figsize=(20,10))
j = 0
k = 0
for i in range(len(col )):
    if(i == (len(col )/2)):
        j = 0
        k = 1
    # scatter plot
    ax[k,j].scatter(x=happiness_df[col [i]],y=happiness_df['Healthy life e
    ax[k,j].set_xlabel(col [i])
    ax[k,j].set_ylabel('Healthy life expectancy')
    j = j + 1
```



Answer:- From the correlation chart, we can say that attributes such as Life ladder, Log GDP per capita, Social support, Freedom to make life choices, Generosity and Positive affect have a positive correlation with the output label Healthy life expectancy at birth. Attributes like Perceptions of corruption, Negative affect have a negative correlation with the output label. Furthermore we can dissect Life ladder, Log GDP per capita, Social support have a high positive correlation and Freedom to make life choices, Generosity and Positive affect have low positive correlation.

Part 4

Select 20% of the data for testing. Describe how you did that and verify that your test portion of the data is representative of the entire dataset.

In [14]:
```
# Removing unwanted columns and NULL values

happiness_df_trimmed = happiness_df.drop(columns=['Country name','year',
happiness_df_trimmed = happiness_df_trimmed.dropna()
happiness_df_trimmed.head()
```

Out[14]:

	Log GDP per capita	Social support	Healthy life expectancy at birth	Freedom to make life choices	Generosity	Perceptions of corruption	Positive affect	Negative affect
0	7.370	0.451	50.80	0.718	0.168	0.882	0.518	0.258
1	7.540	0.552	51.20	0.679	0.190	0.850	0.584	0.237
2	7.647	0.539	51.60	0.600	0.121	0.707	0.618	0.275
3	7.620	0.521	51.92	0.496	0.162	0.731	0.611	0.267
4	7.705	0.521	52.24	0.531	0.236	0.776	0.710	0.268

In [15]:
```
# Converting 'Log GDP per capita' to a categorical type attribute 'GDP_Cat

happiness_df_trimmed['GDP_Cat'] = pd.cut(happiness_df_trimmed['Log GDP per
```

In [16]:
```
# Splitting the data intro training and testing using randomized method ar

x = happiness_df_trimmed.drop('Healthy life expectancy at birth', axis=1)
y = happiness_df_trimmed['Healthy life expectancy at birth']

# We are using the newly created 'GDP_Cat' as our stratified sampling bins
y_strat_bin = happiness_df_trimmed['GDP_Cat']

# Randomized
x_train_1, x_test_1, y_train_1, y_test_1 = train_test_split(x, y, test_si

# Stratified
x_train_2, x_test_2, y_train_2, y_test_2 = train_test_split(x, y, test_si

print(x_train_1.shape,y_train_1.shape, x_test_1.shape, y_test_1.shape)
```

```
(1366, 8) (1366,) (342, 8) (342,)
```

In [17]:
```
# Distribution ratio of 'GDP_Cat' in the overall dataset
dist_overall = happiness_df_trimmed["GDP_Cat"].value_counts() / len(happi

# Distribution ratio of 'GDP_Cat' in the randomized test dataset
dist_randomized = x_test_1["GDP_Cat"].value_counts() / len(x_test_1)

# Distribution ratio of 'GDP_Cat' in the stratified test dataset
dist_stratified = x_test_2["GDP_Cat"].value_counts() / len(x_test_2)

compare = pd.DataFrame({"Overall":dist_overall,"Randomized":dist_randomize
compare
```

Out[17]:

	Overall	Randomized	Stratified
1	0.170960	0.143275	0.172515
2	0.497658	0.505848	0.497076
3	0.331382	0.350877	0.330409

Answer:- Before train test split, we dropped the unwanted columns and also tackled the NULL values in the dataset. Then we created our test dataset using 2 methods: randomized and stratified. For randomized, we simply used the test_train_split function. For stratified, we first create a categorical attribute which can be used as stratas. We choose 'Log GDP per capita' attribute since it has the highest Pearson Correlation Coefficient with our target variable. We divide the 'Log GDP per capita' column into 3 labels 1,2,3 based on ranges. Then we compare the label distributions of the overall dataset, randomized test dataset and stratified test dataset. We see that stratified method produces better results than randomized methods. It produces a test set which is representative of the entire dataset.

Part 5 FIX!

Added learning rates to SGD and plotted the loss vs epochs graphs

Train a Linear Regression model using the training data with four-fold cross-validation using appropriate evaluation metric. Do this with closed form solution (using the Normal Equation or SVD) and with SGD. Perform Ridge, Lasso and Elastic Net regularization – try a few values of penalty term and describe its impact. Explore the impact of other hyperparameters, like batch size and learning rate (no need for grid search). Describe your findings. Display the training and validation loss as a function of training iterations.

In [18]:
```
# Converting training and test sets into numpy arrays

x_train_2, x_test_2, y_train_2, y_test_2 = np.array(x_train_2), np.array(
```

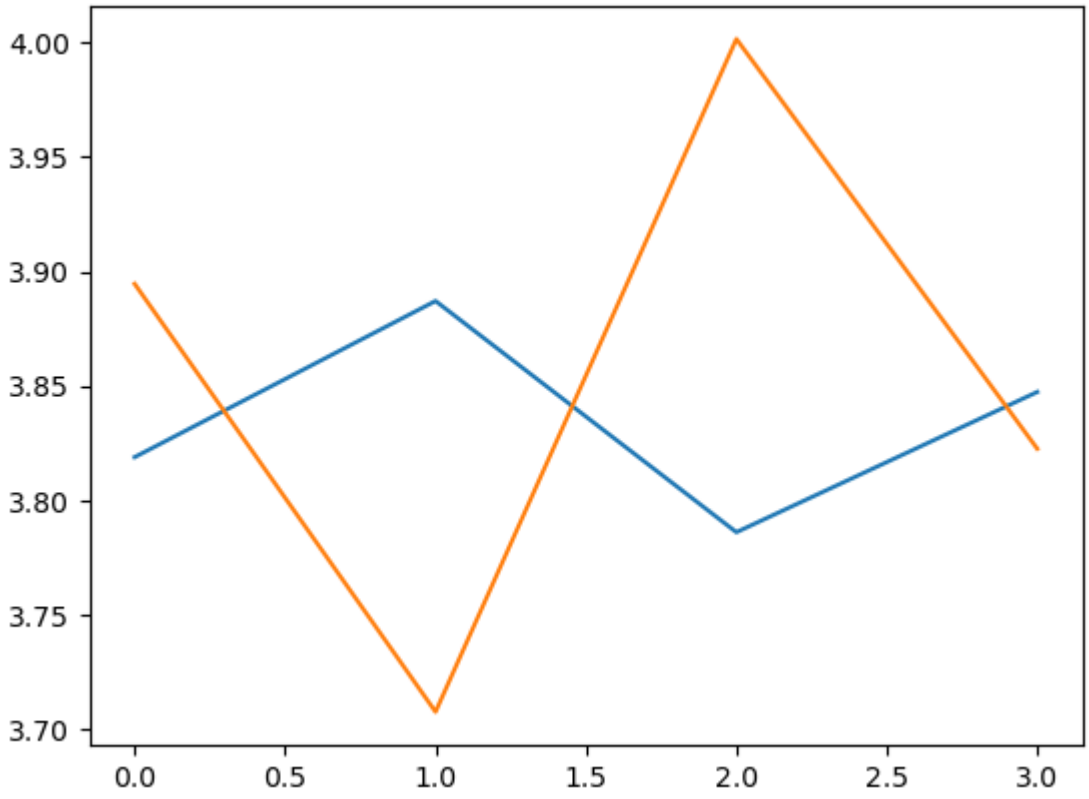
In [19]:

```
# Linear regression model using a closed-form solution (Normal equation) w
# We are using RMSE evaluation metric.

def normal(x,y):
    phi = np.hstack([x, np.ones((x.shape[0],1))])
    phi_t_phi = np.matmul(phi.T, phi)
    reg = 1*np.eye(phi_t_phi.shape[0])
    inv_term = np.linalg.inv(reg+phi_t_phi)
    w = np.matmul(inv_term,np.matmul(phi.T,y))
    return w

def mse(y_pred,y_true):
    return np.sum((y_true.reshape((y_true.shape[0],))-y_pred.reshape((y_t
```

In [20]:

```
kf = KFold(n_splits = 4)

train_plot = []
val_plot = []

for cnt,var in enumerate(kf.split(x_train_2,y_train_2)) :
    print(cnt)
    x = x_train_2[var[0],:]
    y = y_train_2[var[0]]
    x_val = x_train_2[var[1],:]
    y_val = y_train_2[var[1]]
    w = normal(x,y)
    phi = np.hstack([x, np.ones((x.shape[0],1))])
    y_pred_train = np.matmul(phi,w.reshape(9,1))
    phi_val = np.hstack([x_val, np.ones((x_val.shape[0],1))])
    y_pred_val = np.matmul(phi_val,w.reshape(9,1),)
    print("Train error:",np.sqrt(mse(y_pred_train,y)))
    print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
    train_plot.append(np.sqrt(mse(y_pred_train,y)))
    val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
test_stack = np.hstack([x_test_2, np.ones((x_test_2.shape[0],1))])
y_pred_test = np.matmul(test_stack,w.reshape(9,1),)
metric_dict = {}
metric_dict['Linear regression'] = np.sqrt(mse(y_pred_test,y_test_2))
```

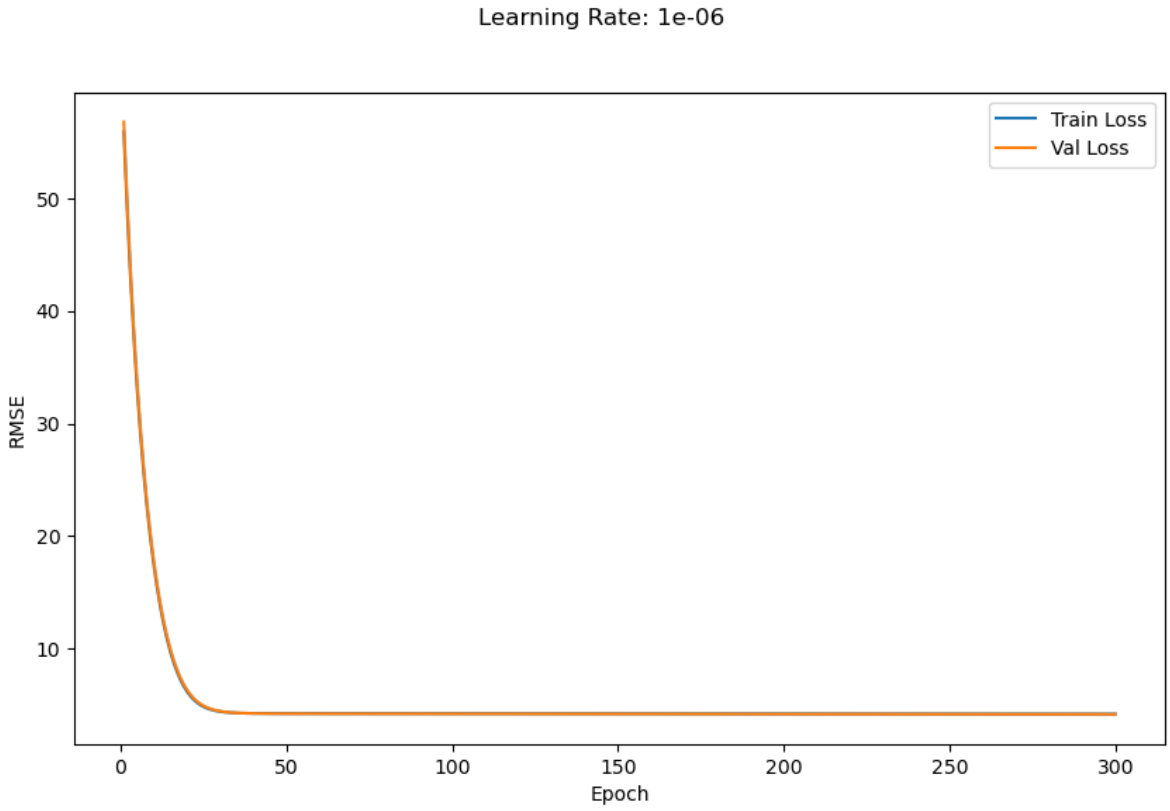
```
0
Train error: 3.818776080593649
Validation error: 3.8945433333753683
1
Train error: 3.8870500147623175
Validation error: 3.707428285591268
2
Train error: 3.7859531773863266
Validation error: 4.001585070020308
3
Train error: 3.8472813016461296
Validation error: 3.8224446443526774
```

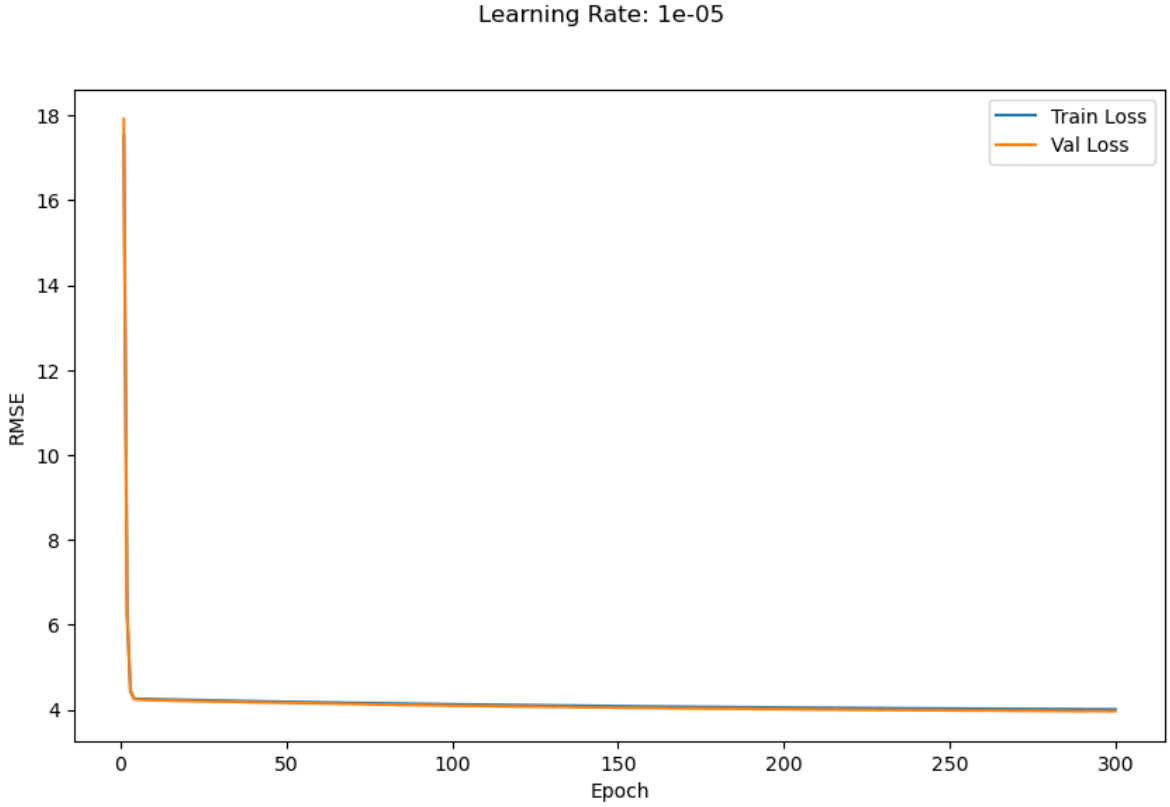

4.00
3.95
3.90
3.85
3.80
3.75
3.70
0.0
0.5
1.0
1.5
2.0
2.5
3.0

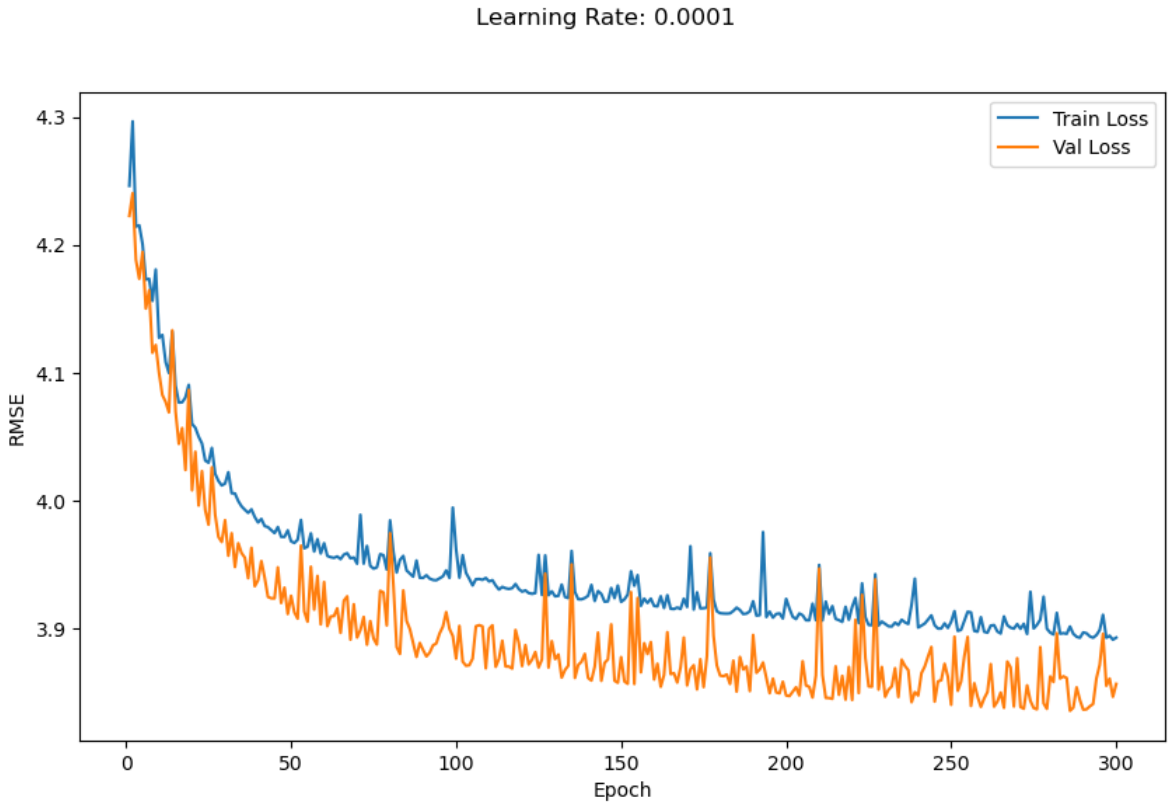
In [21]:

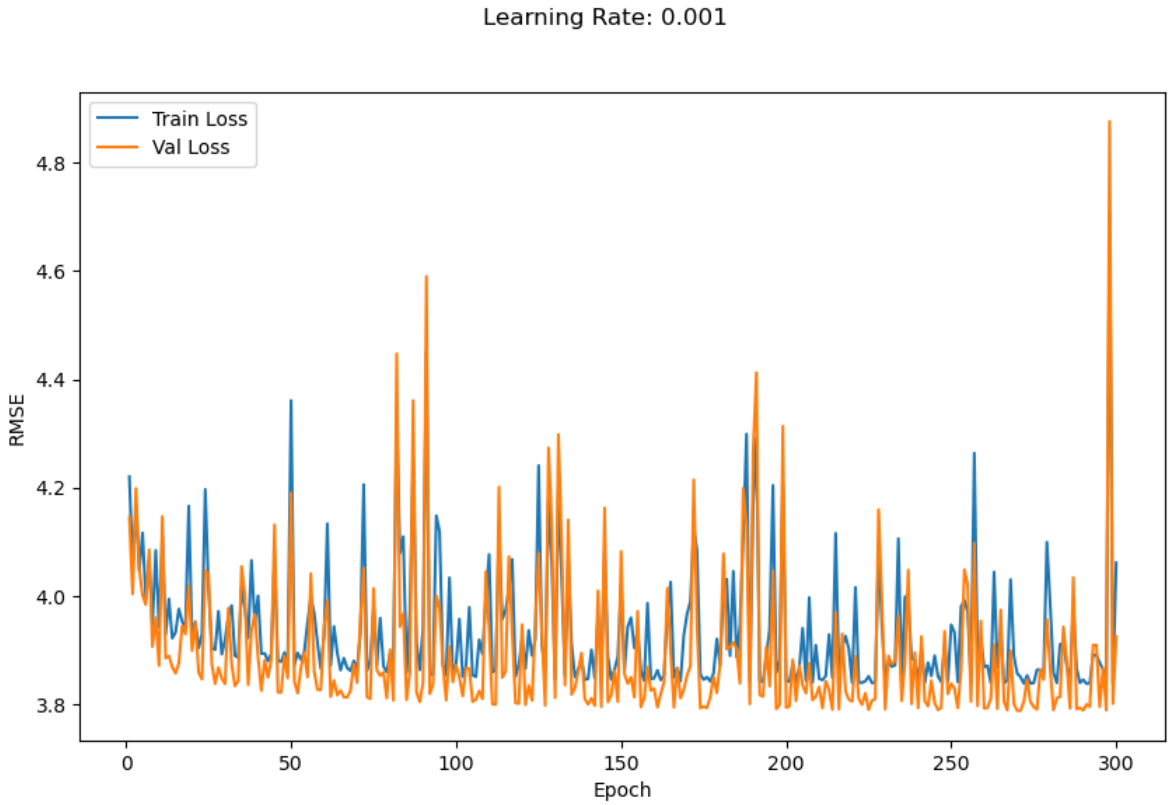
```
# SGD with different learning rate

learning_rates = [0.000001, 0.00001, 0.0001, 0.001, 0.01]
n_epochs = 300

train_losses = np.zeros((len(learning_rates), n_epochs))
val_losses = np.zeros((len(learning_rates), n_epochs))
test_rmse_scores = []

for i, learning_rate in enumerate(learning_rates):
    sgd = SGDRegressor(max_iter=1, warm_start=True, eta0=learning_rate, le
    for epoch in range(n_epochs):
        sgd.partial_fit(x_train_2, y_train_2)
        y_pred_train = sgd.predict(x_train_2)
        y_pred_val = sgd.predict(x_val)
        train_losses[i, epoch] = np.sqrt(mse(y_pred_train, y_train_2))
        val_losses[i, epoch] = np.sqrt(mse(y_pred_val, y_val))

    # Plot train and val losses for each epoch and each learning rate sepa
    fig, axs = plt.subplots(figsize=(10, 6))
    fig.suptitle(f"Learning Rate: {learning_rate}")
    axs.plot(range(1, n_epochs + 1), train_losses[i], label="Train Loss")
    axs.plot(range(1, n_epochs + 1), val_losses[i], label="Val Loss")
    axs.set_xlabel("Epoch")
    axs.set_ylabel("RMSE")
    axs.legend()

    # Calculate RMSE score on test set
    y_pred_test = sgd.predict(x_test_2)
    test_rmse_score = np.sqrt(mse(y_pred_test, y_test_2))
    metric_dict['SGD with learning rate ='+str(learning_rate)] = test_rmse
    print(f"RMSE score between y_pred_val and y_test for learning_rate={le
    test_rmse_scores.append(test_rmse_score)

plt.show()
```

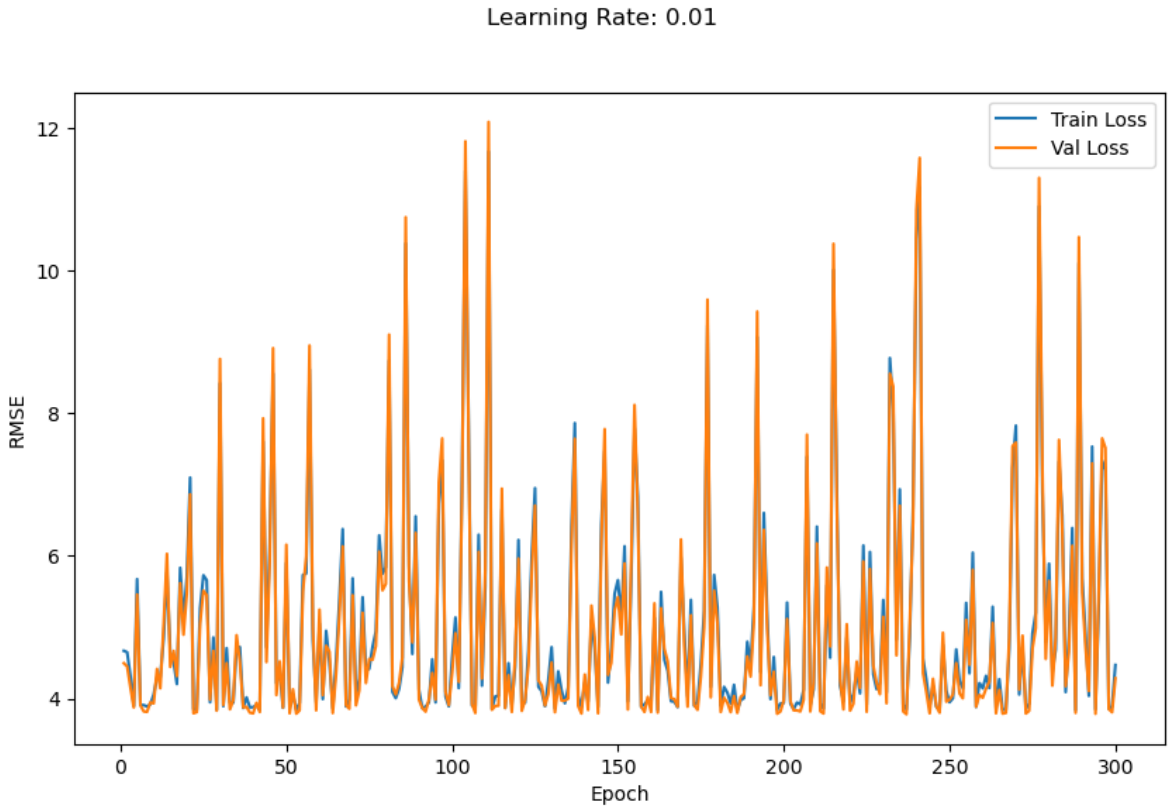
```
RMSE score between y_pred_val and y_test for learning_rate=1e-06: 3.969667490
79844
RMSE score between y_pred_val and y_test for learning_rate=1e-05: 3.767096303
4078917
RMSE score between y_pred_val and y_test for learning_rate=0.0001: 3.68929718
2286318
RMSE score between y_pred_val and y_test for learning_rate=0.001: 3.932298364
6063134
RMSE score between y_pred_val and y_test for learning_rate=0.01: 4.3745030436
89119
```


Learning Rate: 1e-06
Train Loss
Val Loss
RMSE
Epoch
50
40
30
20
10
0
50
100
150
200
250
300


Learning Rate: 1e-05
Train Loss
Val Loss
RMSE
Epoch
18
16
14
12
10
8
6
4
0
50
100
150
200
250
300


Learning Rate: 0.0001
Train Loss
Val Loss
RMSE
Epoch
4.3
4.2
4.1
4.0
3.9
0
50
100
150
200
250
300

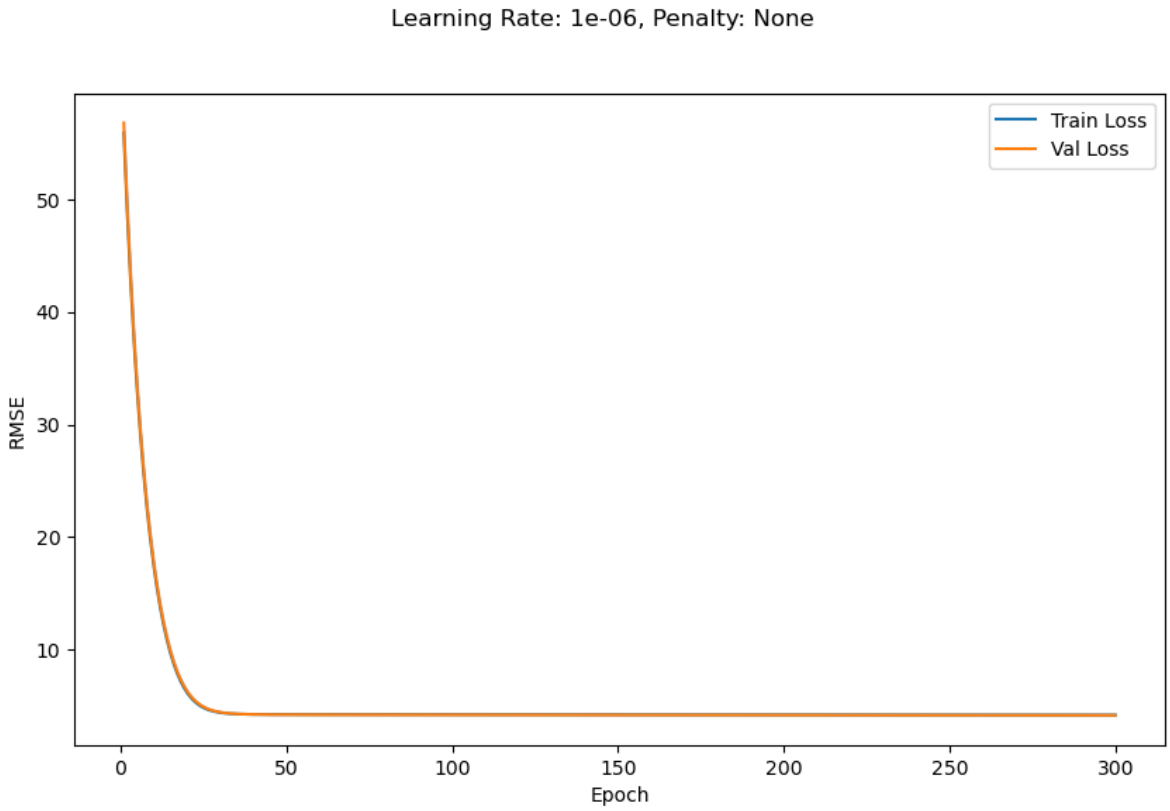

Learning Rate: 0.001
Train Loss
Val Loss
RMSE
Epoch
4.8
4.6
4.4
4.2
4.0
3.8
0
50
100
150
200
250
300

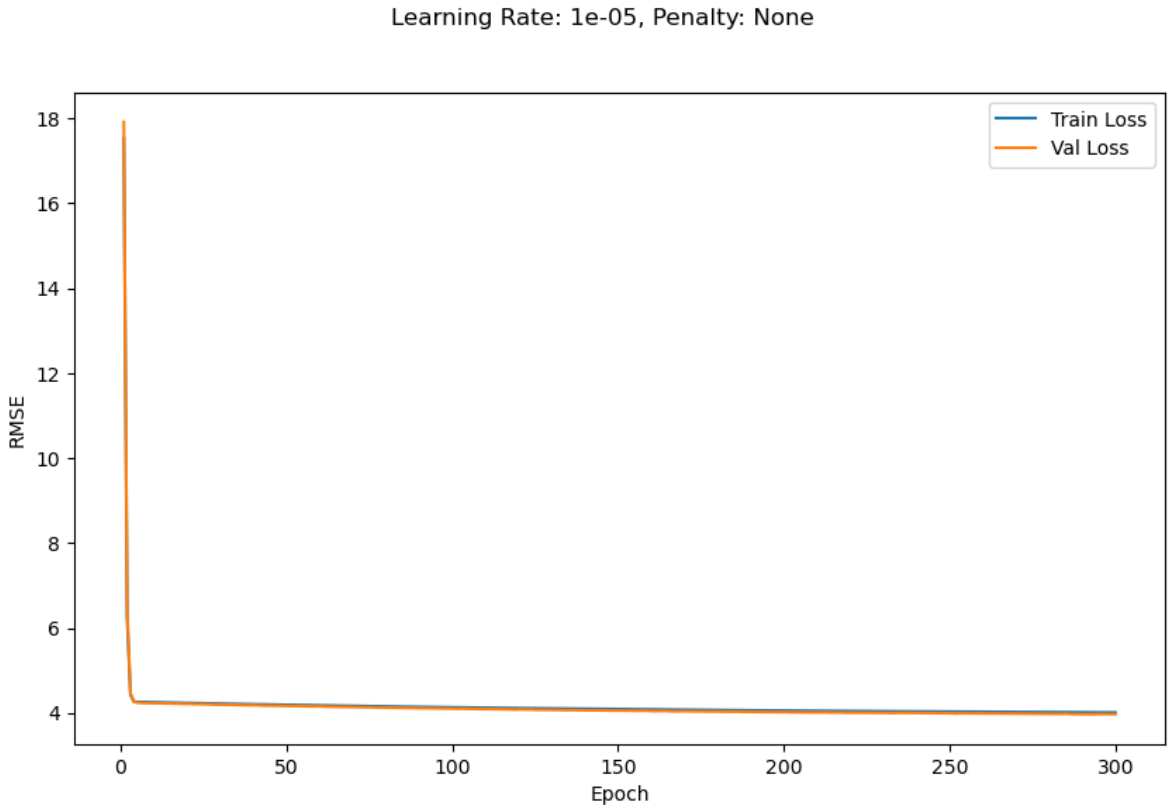

Learning Rate: 0.01
Train Loss
Val Loss
RMSE
Epoch
12
10
8
6
4
0
50
100
150
200
250
300

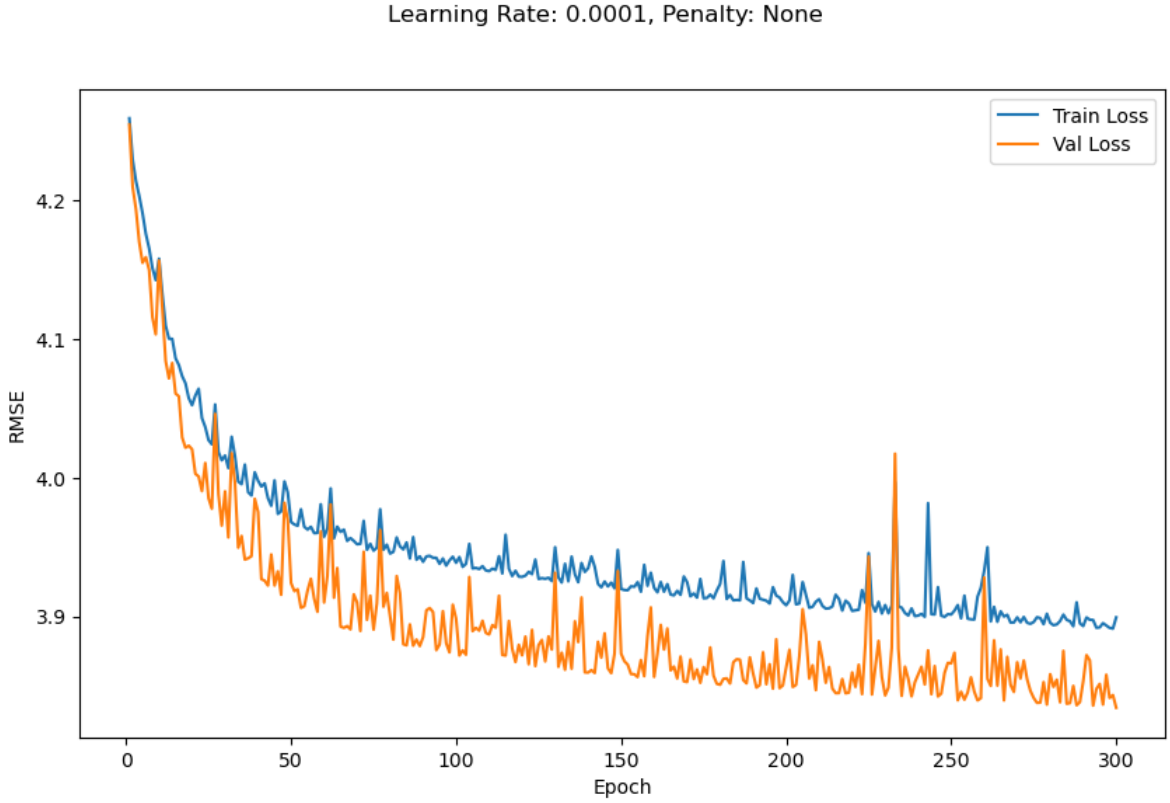
In [22]:

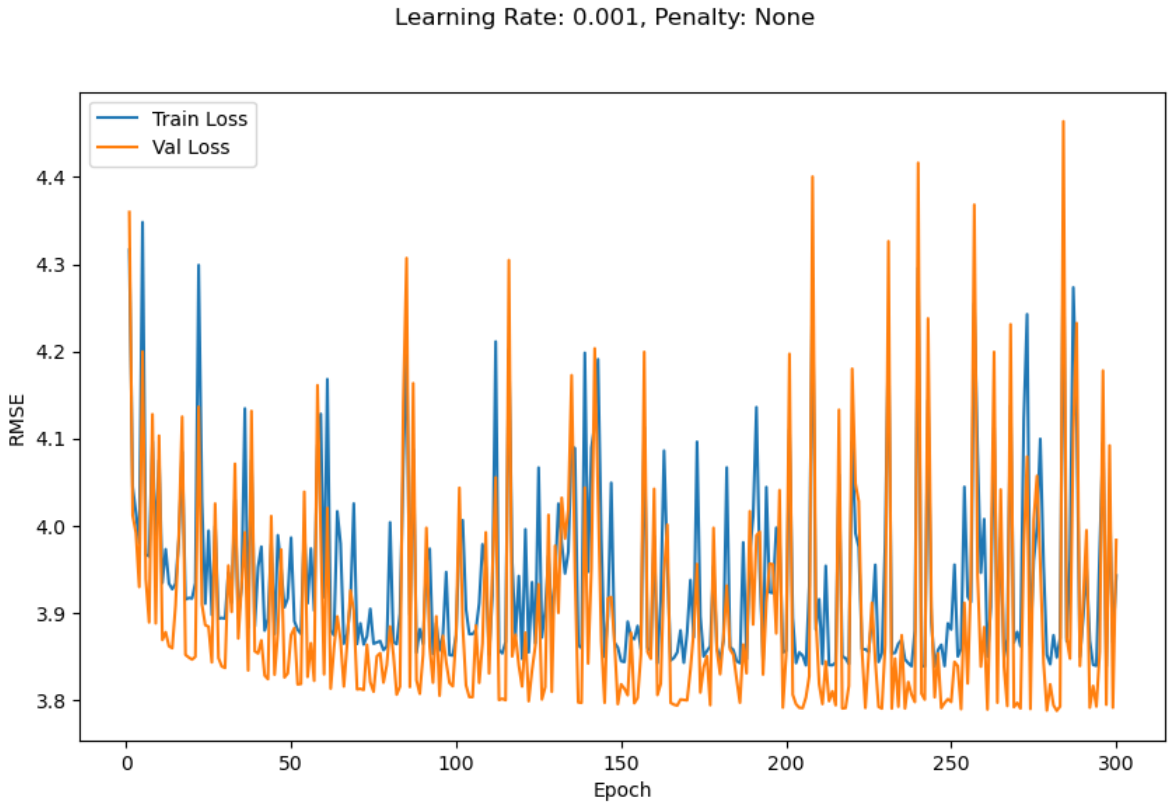
```
# SGD with different penalty terms

penalty_factors = [None, 'l1', 'l2']
learning_rates = [0.000001, 0.00001, 0.0001, 0.001, 0.01]
n_epochs = 300

for penalty in penalty_factors:
    for i, learning_rate in enumerate(learning_rates):
        sgd = SGDRegressor(max_iter=1, warm_start=True, eta0=learning_rate
        train_losses = np.zeros(n_epochs)
        val_losses = np.zeros(n_epochs)
        for epoch in range(n_epochs):
            sgd.partial_fit(x_train_2, y_train_2)
            y_pred_train = sgd.predict(x_train_2)
            y_pred_val = sgd.predict(x_val)
            train_losses[epoch] = np.sqrt(mse(y_pred_train, y_train_2))
            val_losses[epoch] = np.sqrt(mse(y_pred_val, y_val))

        # Plot train and val losses for each epoch and each learning rate
        fig, axs = plt.subplots(figsize=(10, 6))
        fig.suptitle(f"Learning Rate: {learning_rate}, Penalty: {penalty}"
        axs.plot(range(1, n_epochs + 1), train_losses, label="Train Loss"
        axs.plot(range(1, n_epochs + 1), val_losses, label="Val Loss")
        axs.set_xlabel("Epoch")
        axs.set_ylabel("RMSE")
        axs.legend()

        # Calculate RMSE score on test set
        y_pred_test = sgd.predict(x_test_2)
        test_rmse_score = np.sqrt(mse(y_pred_test, y_test_2))
        metric_dict['SGD with learning rate ='+str(learning_rate)+' and pe
        print(f"RMSE score between y_pred_val and y_test for learning_rate
        test_rmse_scores.append(test_rmse_score)

plt.show()
```

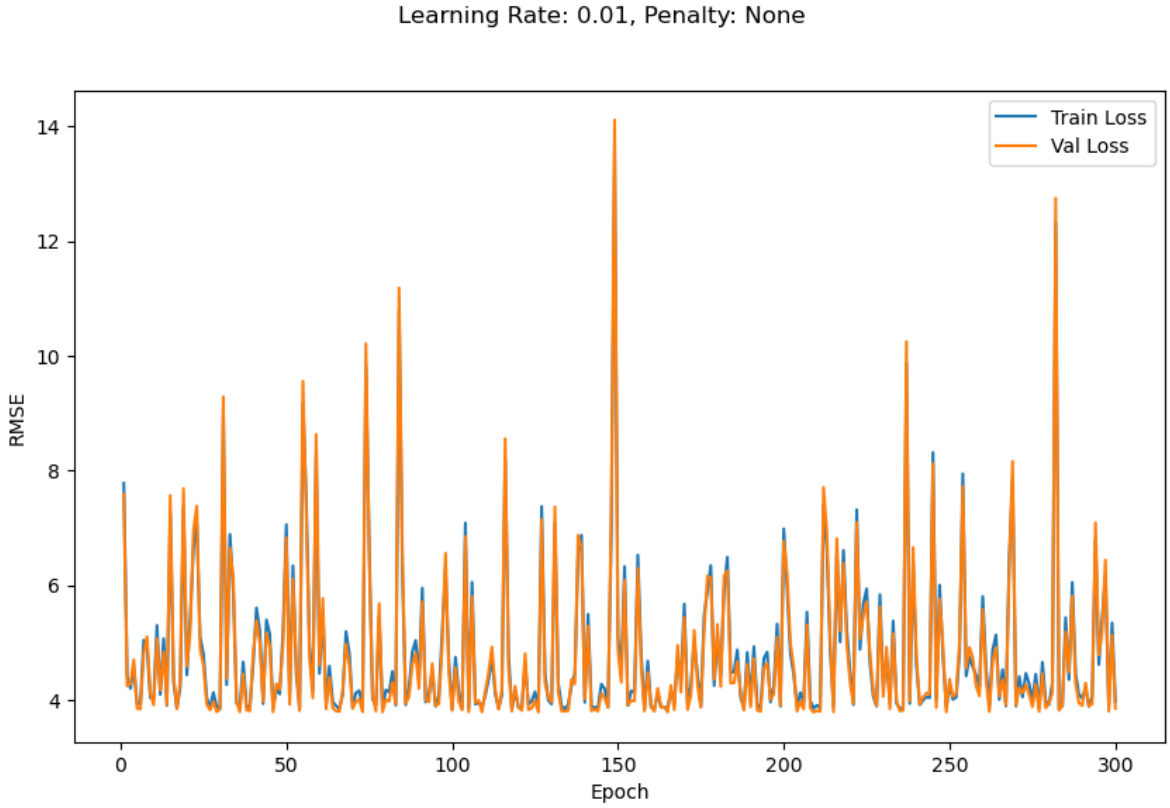
```
RMSE score between y_pred_val and y_test for learning_rate=1e-06 and penalty_
factor=None: 3.9696651820128945
RMSE score between y_pred_val and y_test for learning_rate=1e-05 and penalty_
factor=None: 3.767312827919267
RMSE score between y_pred_val and y_test for learning_rate=0.0001 and penalty
_factor=None: 3.699389148877477
RMSE score between y_pred_val and y_test for learning_rate=0.001 and penalty_
factor=None: 3.7830674990488733
RMSE score between y_pred_val and y_test for learning_rate=0.01 and penalty_f
actor=None: 3.836960346031612
RMSE score between y_pred_val and y_test for learning_rate=1e-06 and penalty_
factor=l1: 3.9696603920934166
RMSE score between y_pred_val and y_test for learning_rate=1e-05 and penalty_
factor=l1: 3.768378646001259
RMSE score between y_pred_val and y_test for learning_rate=0.0001 and penalty
_factor=l1: 3.7035761649960635
RMSE score between y_pred_val and y_test for learning_rate=0.001 and penalty_
factor=l1: 3.7274527732688942
RMSE score between y_pred_val and y_test for learning_rate=0.01 and penalty_f
actor=l1: 3.727742795406936
RMSE score between y_pred_val and y_test for learning_rate=1e-06 and penalty_
factor=l2: 3.96968115196651
RMSE score between y_pred_val and y_test for learning_rate=1e-05 and penalty_
factor=l2: 3.768136952869328
RMSE score between y_pred_val and y_test for learning_rate=0.0001 and penalty
_factor=l2: 3.689837070793864
RMSE score between y_pred_val and y_test for learning_rate=0.001 and penalty_
factor=l2: 3.797161489890803
RMSE score between y_pred_val and y_test for learning_rate=0.01 and penalty_f
actor=l2: 3.809220893603164
```

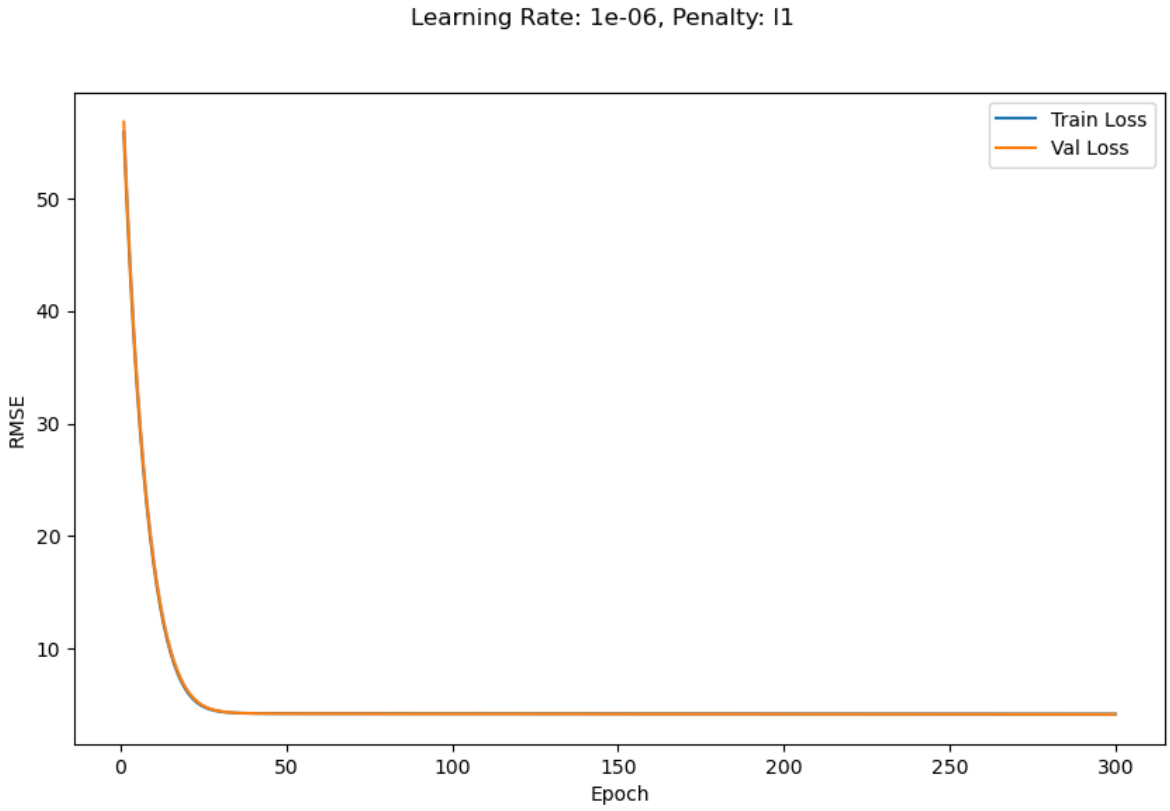


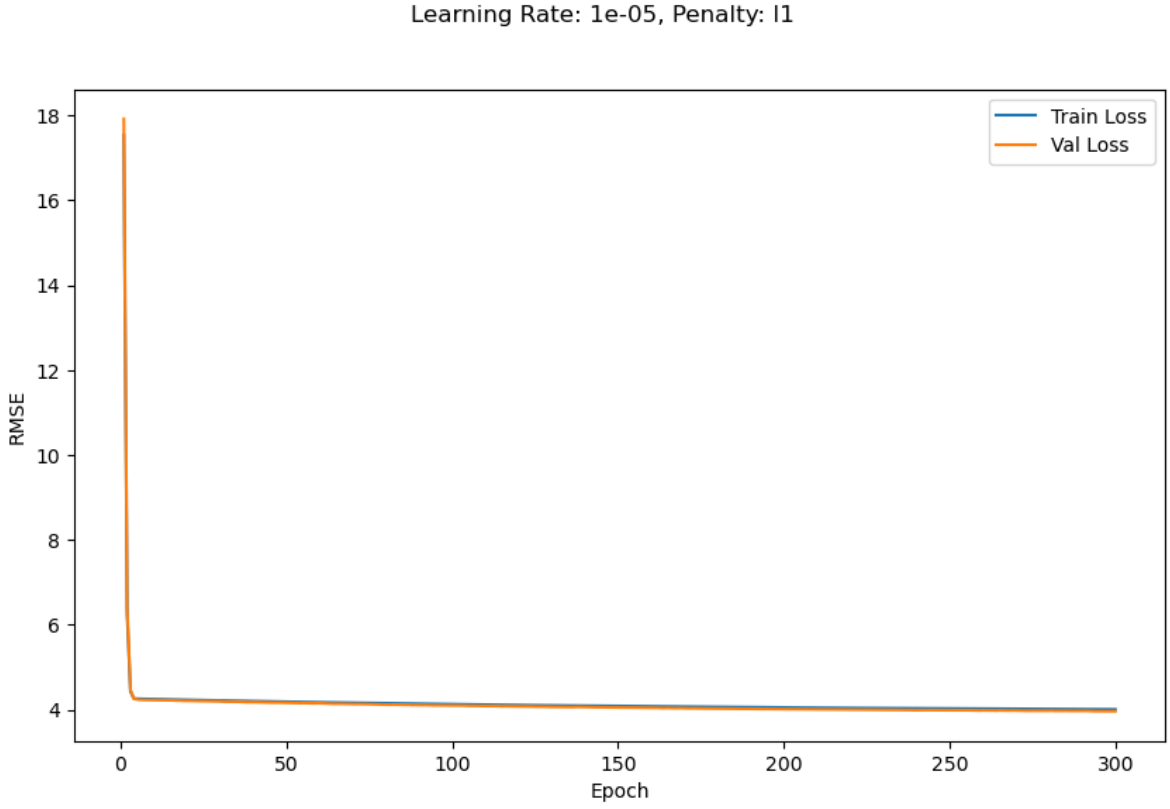

Learning Rate: 1e-05, Penalty: None
Train Loss
Val Loss
RMSE
Epoch
18
16
14
12
10
8
6
4
0
50
100
150
200
250
300

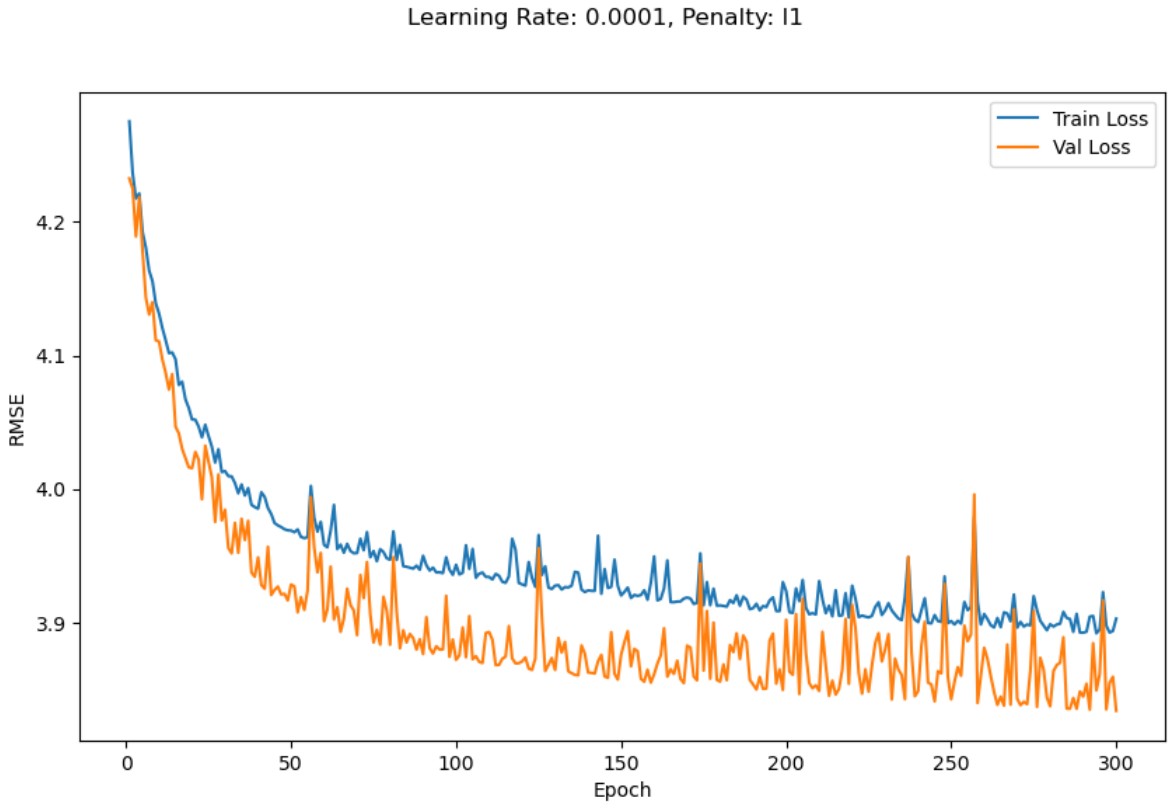

Learning Rate: 0.0001, Penalty: None
Train Loss
Val Loss
RMSE
Epoch
4.2
4.1
4.0
3.9
0
50
100
150
200
250
300

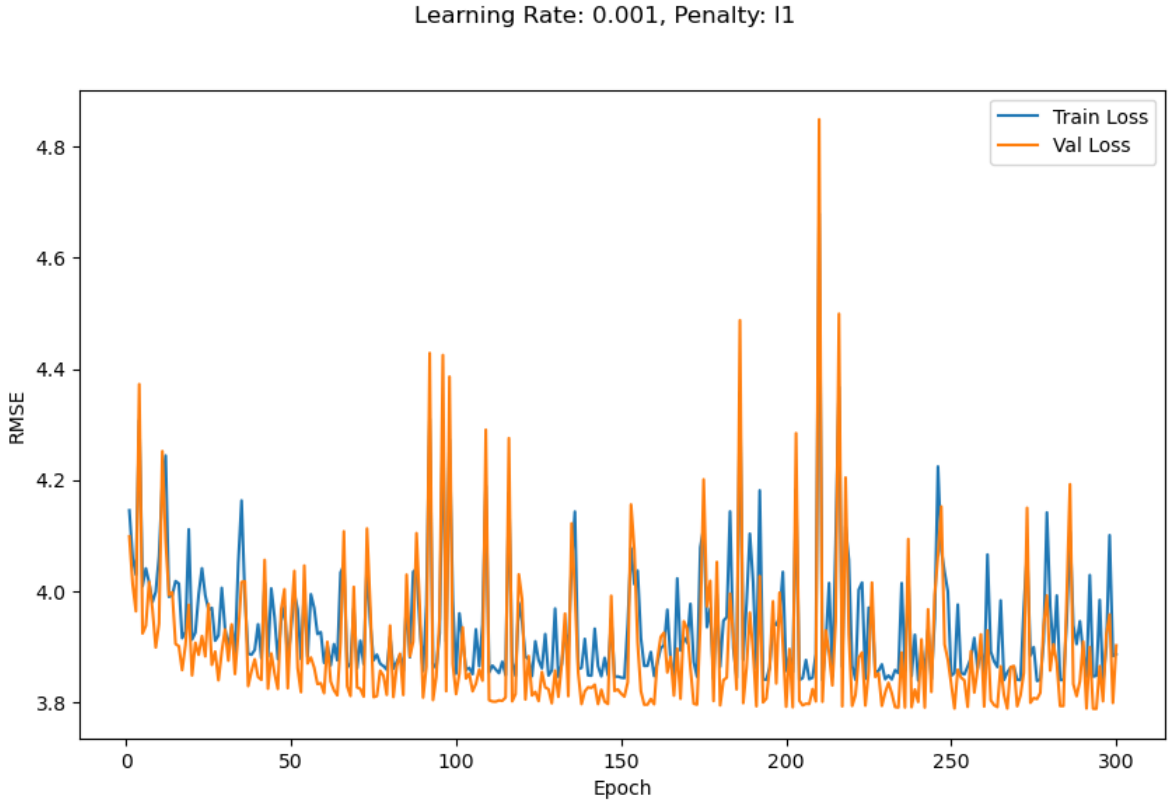

Learning Rate: 0.001, Penalty: None
Train Loss
Val Loss
RMSE
Epoch
4.4
4.3
4.2
4.1
4.0
3.9
3.8
0
50
100
150
200
250
300

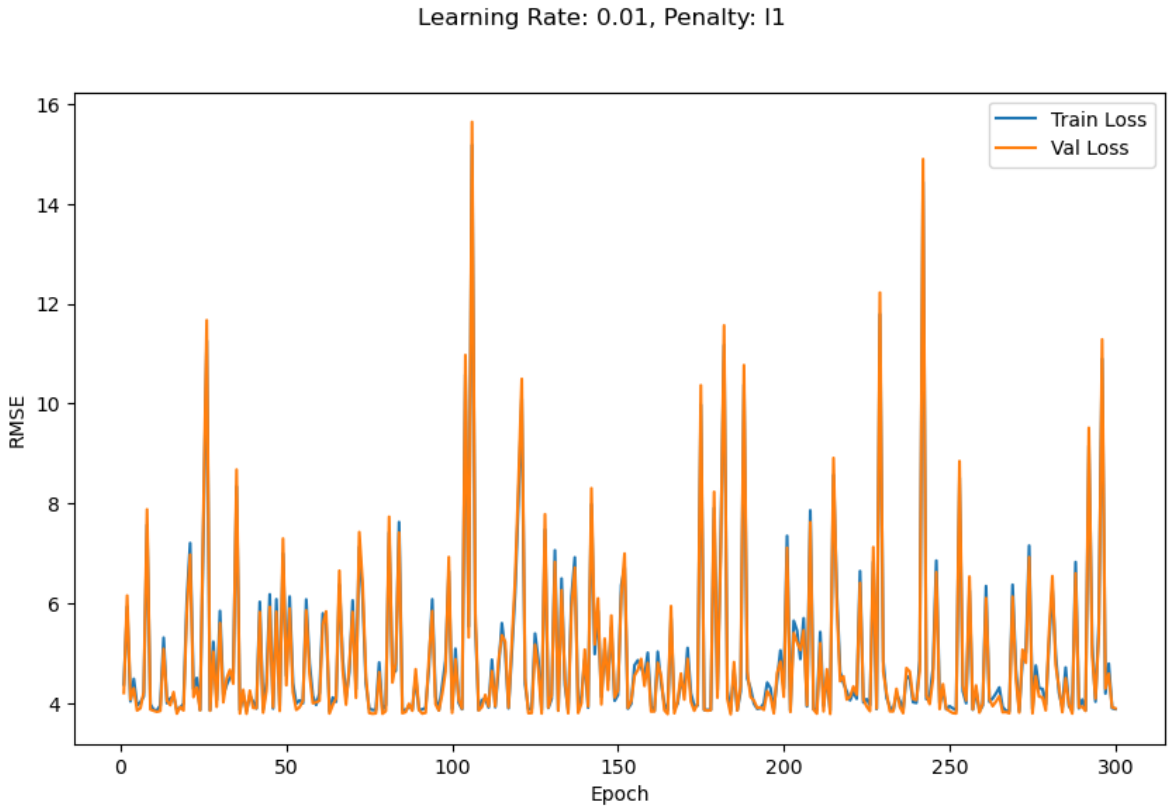

Learning Rate: 0.01, Penalty: None
Train Loss
Val Loss
RMSE
Epoch
14
12
10
8
6
4
0
50
100
150
200
250
300

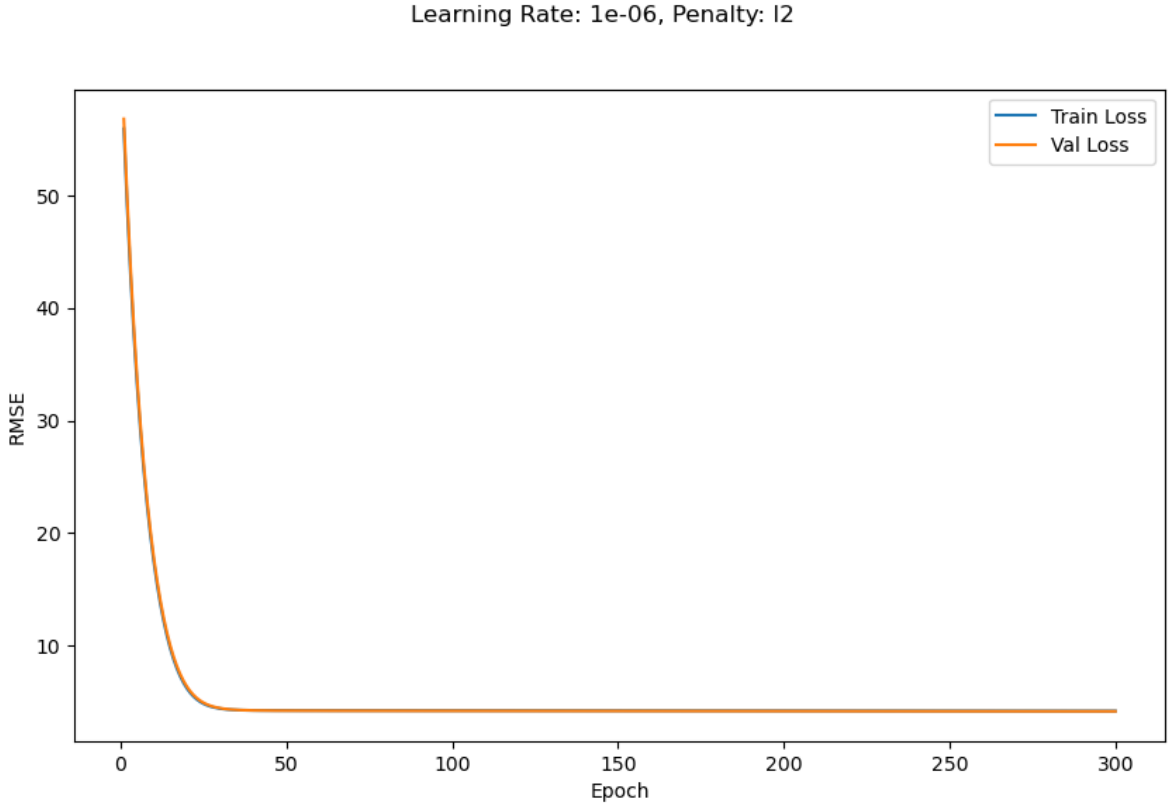

Learning Rate: 1e-06, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
50
40
30
20
10
0
50
100
150
200
250
300

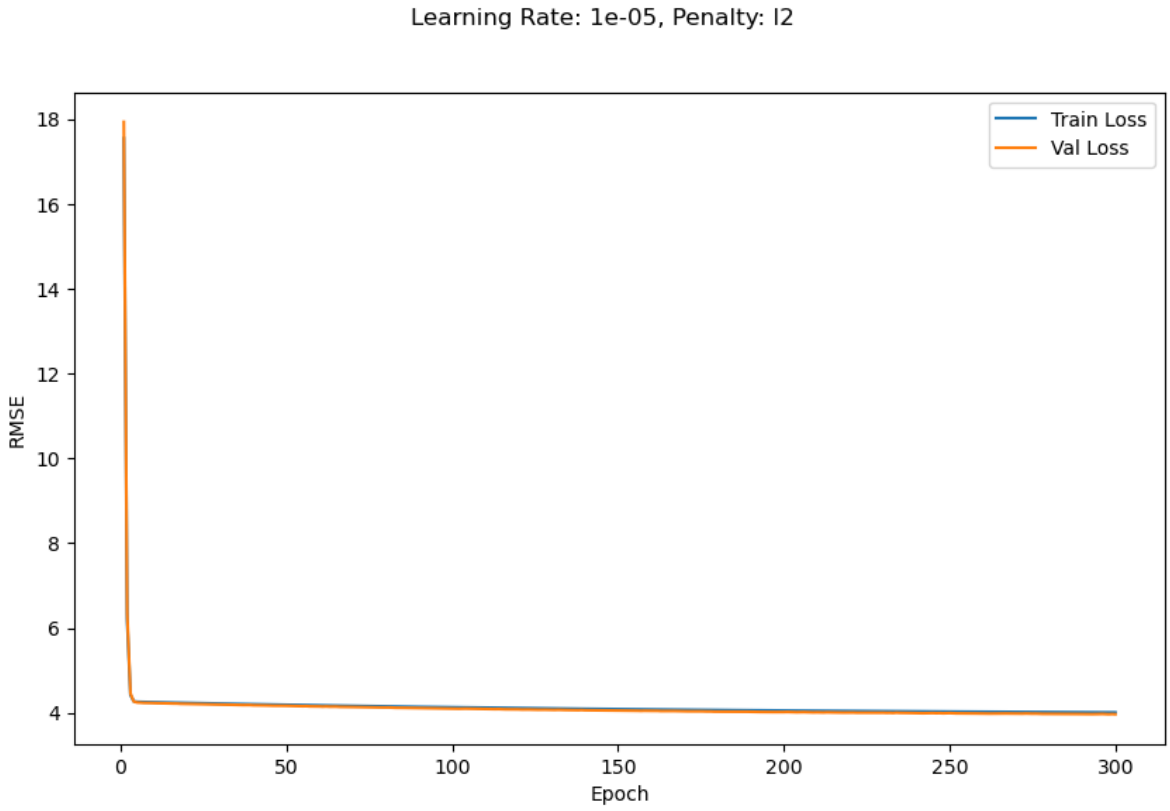

Learning Rate: 1e-05, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
18
16
14
12
10
8
6
4
0
50
100
150
200
250
300

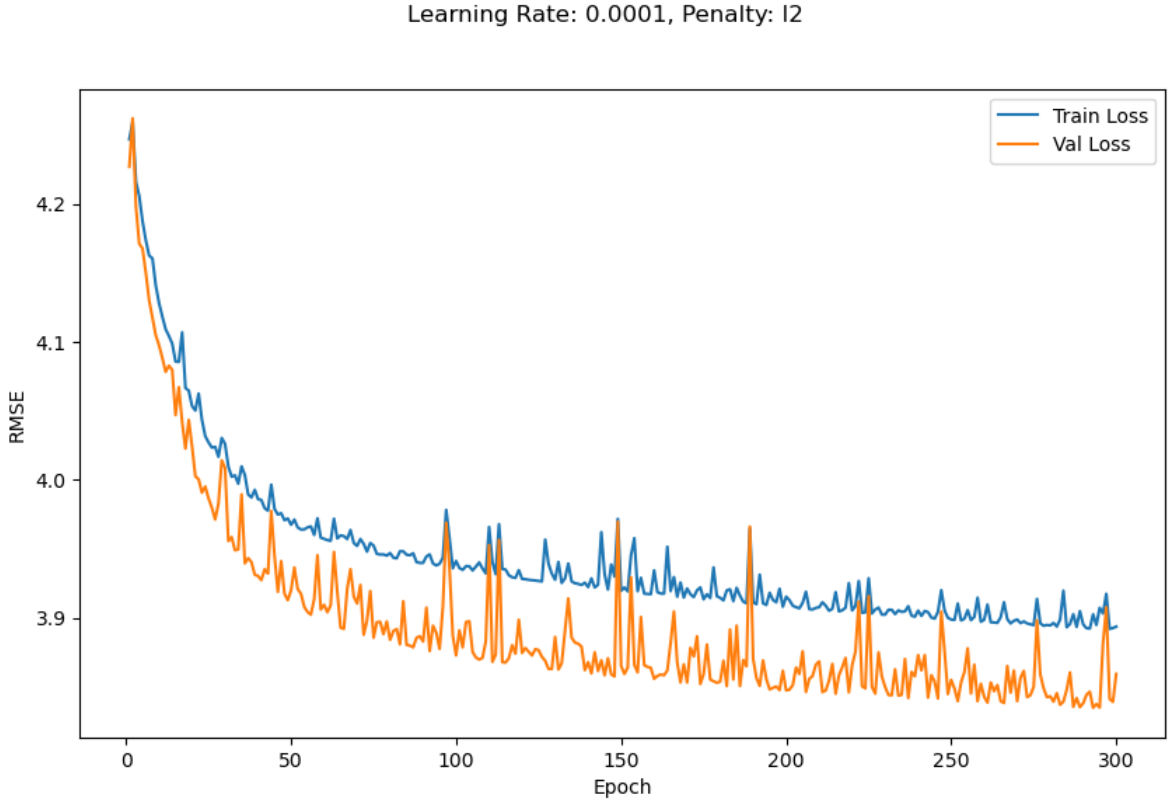

Learning Rate: 0.0001, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
4.2
4.1
4.0
3.9
0
50
100
150
200
250
300

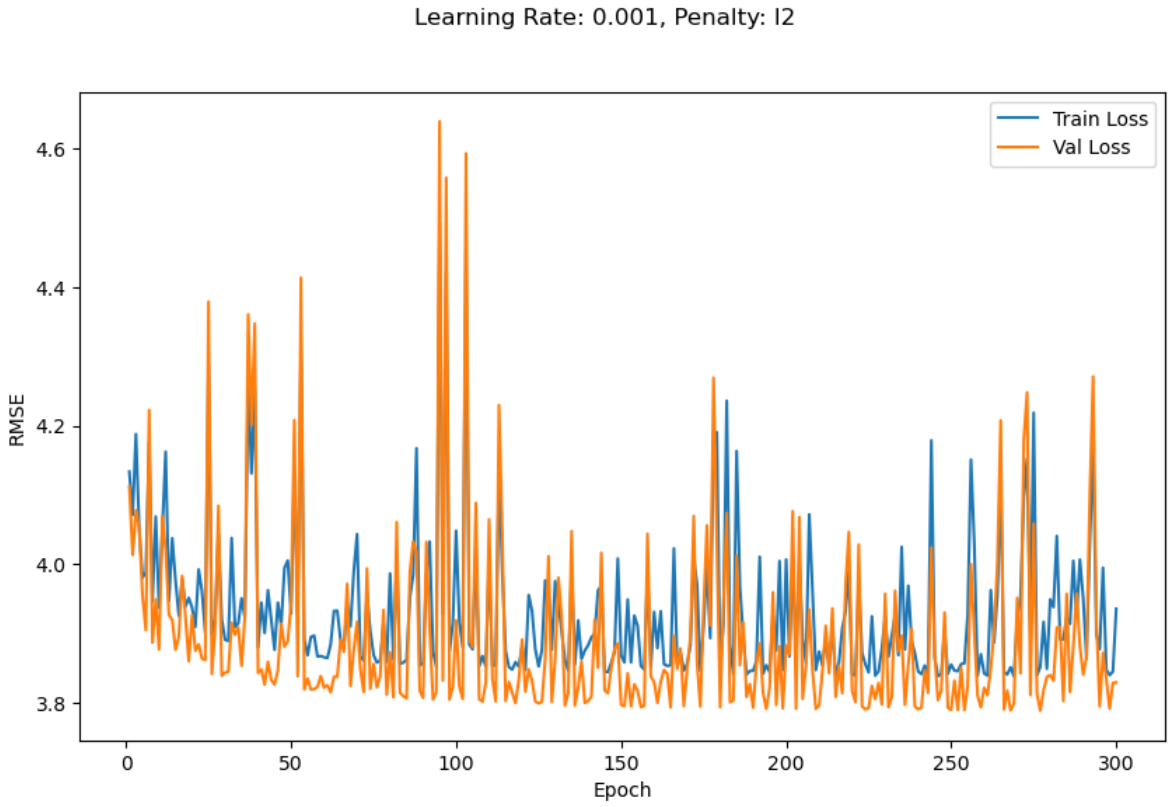

Learning Rate: 0.001, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
4.8
4.6
4.4
4.2
4.0
3.8
0
50
100
150
200
250
300

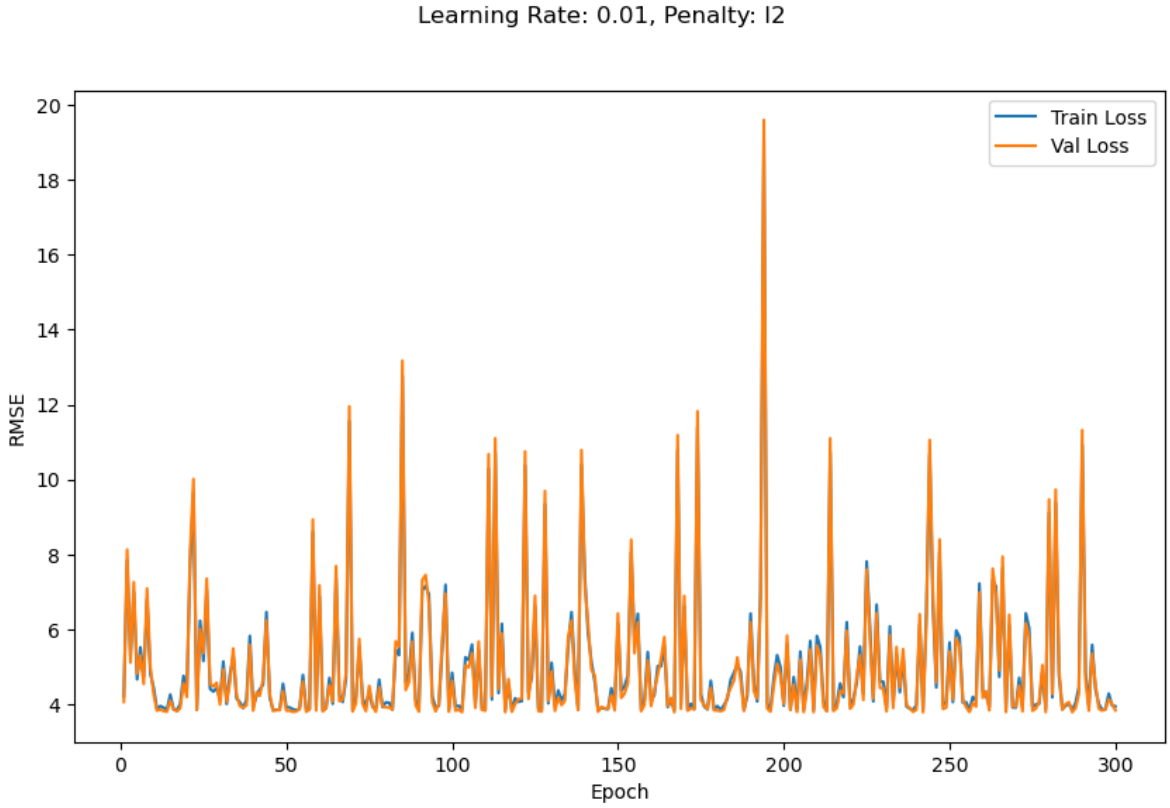

Learning Rate: 0.01, Penalty: l1
Train Loss
Val Loss
RMSE
Epoch
16
14
12
10
8
6
4
0
50
100
150
200
250
300


Learning Rate: 1e-06, Penalty: l2
Train Loss
Val Loss
RMSE
Epoch
50
40
30
20
10
0
50
100
150
200
250
300


Learning Rate: 1e-05, Penalty: l2
Train Loss
Val Loss
RMSE
Epoch
18
16
14
12
10
8
6
4
0
50
100
150
200
250
300


Learning Rate: 0.0001, Penalty: l2
Train Loss
Val Loss
RMSE
Epoch
4.2
4.1
4.0
3.9
0
50
100
150
200
250
300

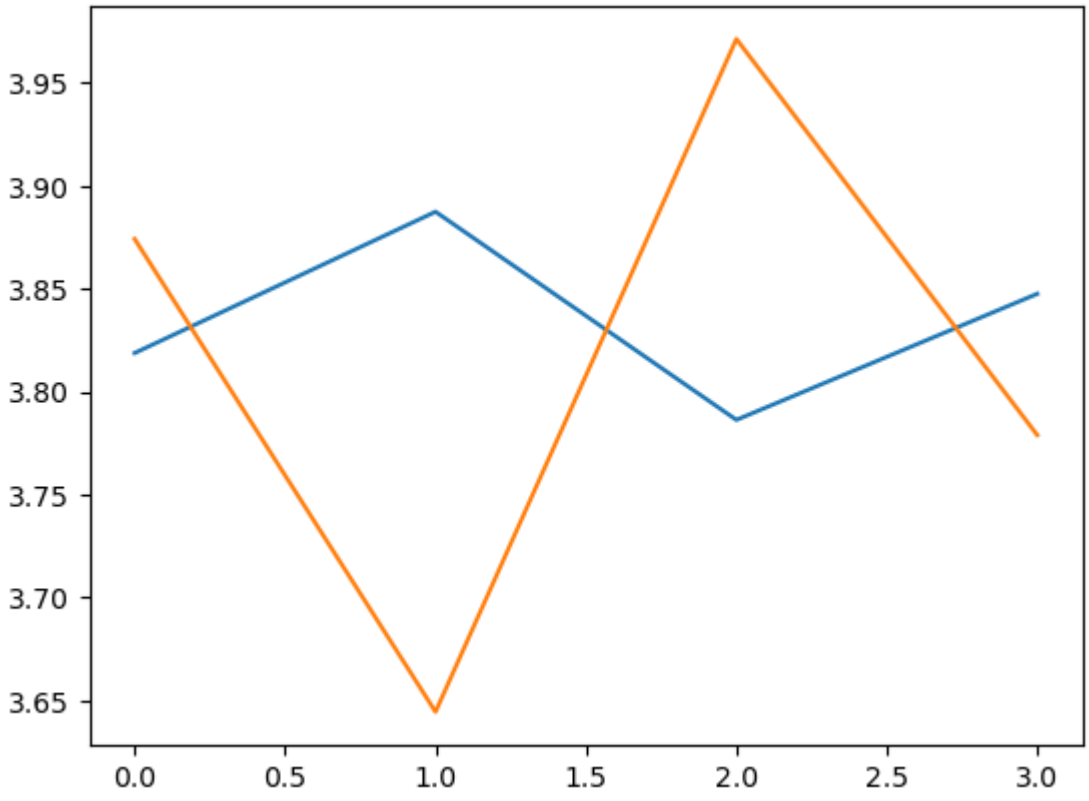

Learning Rate: 0.001, Penalty: l2
Train Loss
Val Loss
RMSE
Epoch
4.6
4.4
4.2
4.0
3.8
0
50
100
150
200
250
300


Learning Rate: 0.01, Penalty: l2
Train Loss
Val Loss
RMSE
Epoch
20
18
16
14
12
10
8
6
4
0
50
100
150
200
250
300

In [23]:
```
# Ridge

train_plot = []
val_plot = []
ridge = Ridge()
for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
    print(cnt)
    x = x_train_2[var[0],:]
    y = y_train_2[var[0]]
    x_val = x_train_2[var[1],:]
    y_val = y_train_2[var[1]]
    ridge.fit(x, y)
    y_pred_train = ridge.predict(x)
    ridge.fit(x_val, y_val)
    y_pred_val = ridge.predict(x_val)
    print("Train error:",np.sqrt(mse(y_pred_train,y)))
    print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
    train_plot.append(np.sqrt(mse(y_pred_train,y)))
    val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
```

```
0
Train error: 3.8186868297055447
Validation error: 3.874078616824582
1
Train error: 3.88731131963597
Validation error: 3.6444479315091987
2
Train error: 3.7862233026159204
Validation error: 3.9711948361115503
3
Train error: 3.8474595920132773
Validation error: 3.778825605436278
```

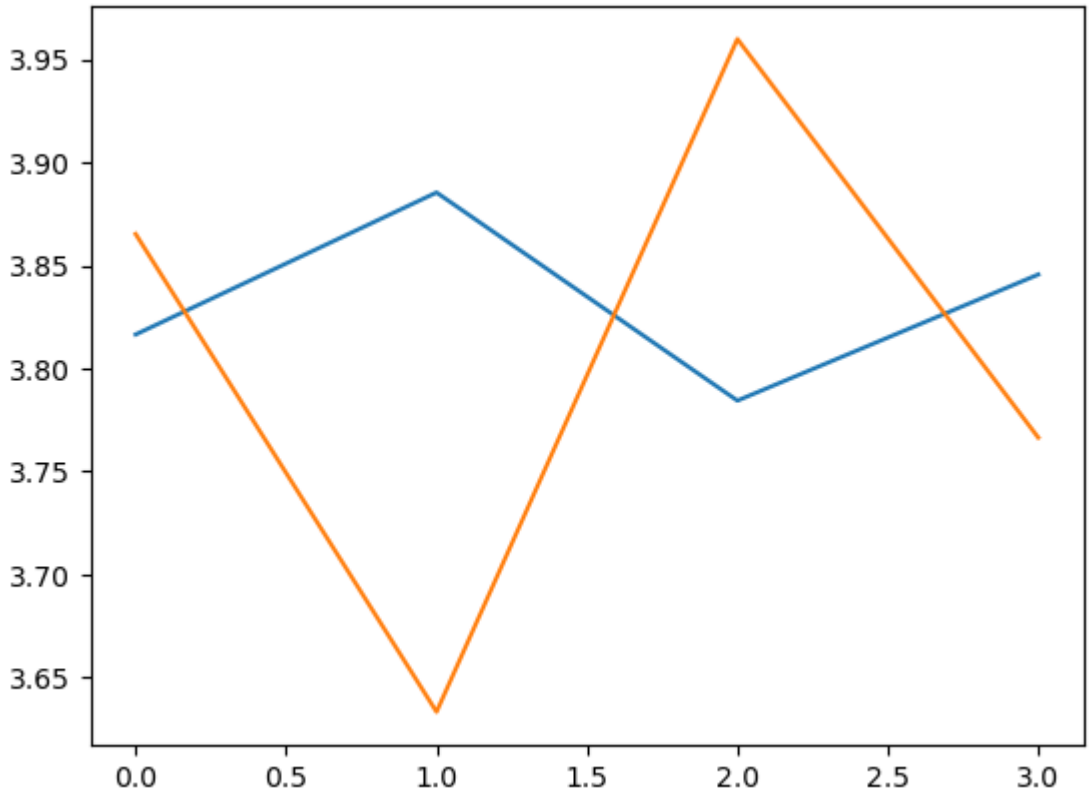
Out[23]: [<matplotlib.lines.Line2D at 0x1aafb022c70>]


3.95
3.90
3.85
3.80
3.75
3.70
3.65
0.0
0.5
1.0
1.5
2.0
2.5
3.0

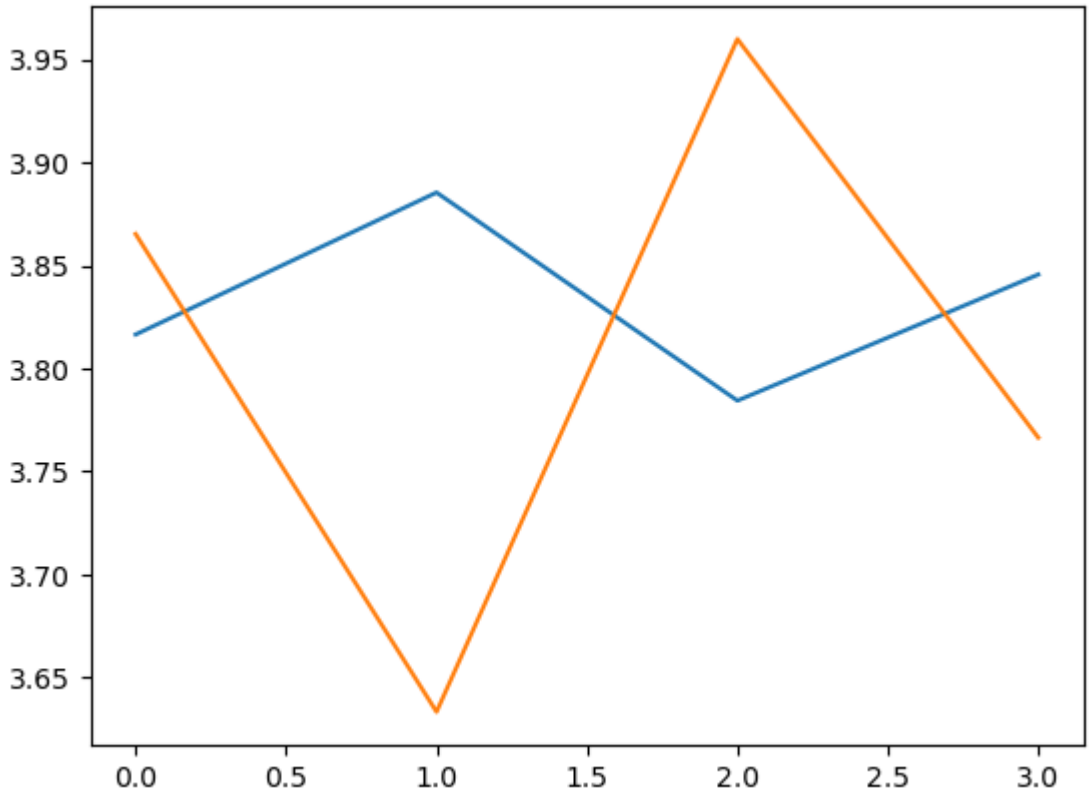
In [24]:

```
# Ridge with different learning rates

ridge_alpha = [0,0.001,1,10,100]

for r in ridge_alpha:
    print("Alpha value:", r)
    train_plot = []
    val_plot = []
    ridge_obj = Ridge(alpha=r)
    for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
        print(cnt)
        x = x_train_2[var[0],:]
        y = y_train_2[var[0]]
        x_val = x_train_2[var[1],:]
        y_val = y_train_2[var[1]]
        ridge_obj.fit(x, y)
        y_pred_train = ridge_obj.predict(x)
        ridge_obj.fit(x_val, y_val)
        y_pred_val = ridge_obj.predict(x_val)
        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
        y_pred_test = ridge_obj.predict(x_test_2)
        metric_dict['Ridge lr='+str(r)] = np.sqrt(mse(y_pred_test,y_test_
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

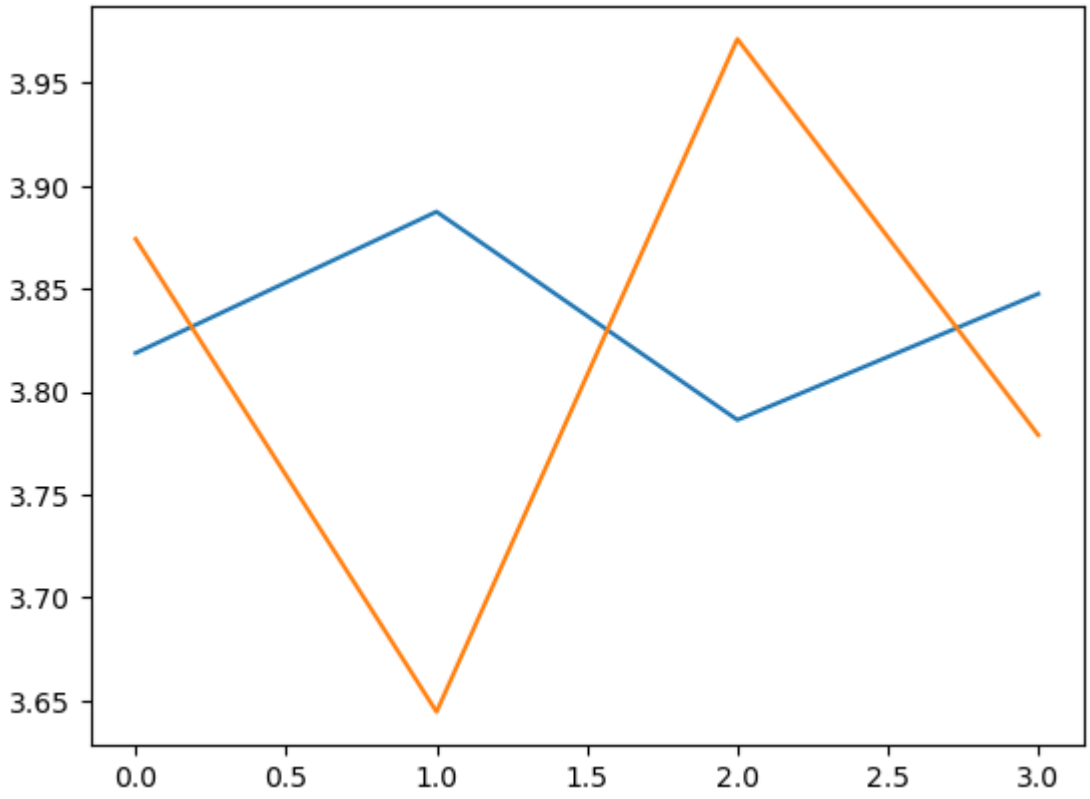
```
Alpha value: 0
0
Train error: 3.8164612972453944
Validation error: 3.865214725903877
1
Train error: 3.885364521098337
Validation error: 3.6333954981013745
2
Train error: 3.784381403215582
Validation error: 3.9598258620368276
3
Train error: 3.8455747398564712
Validation error: 3.766418213916159
```



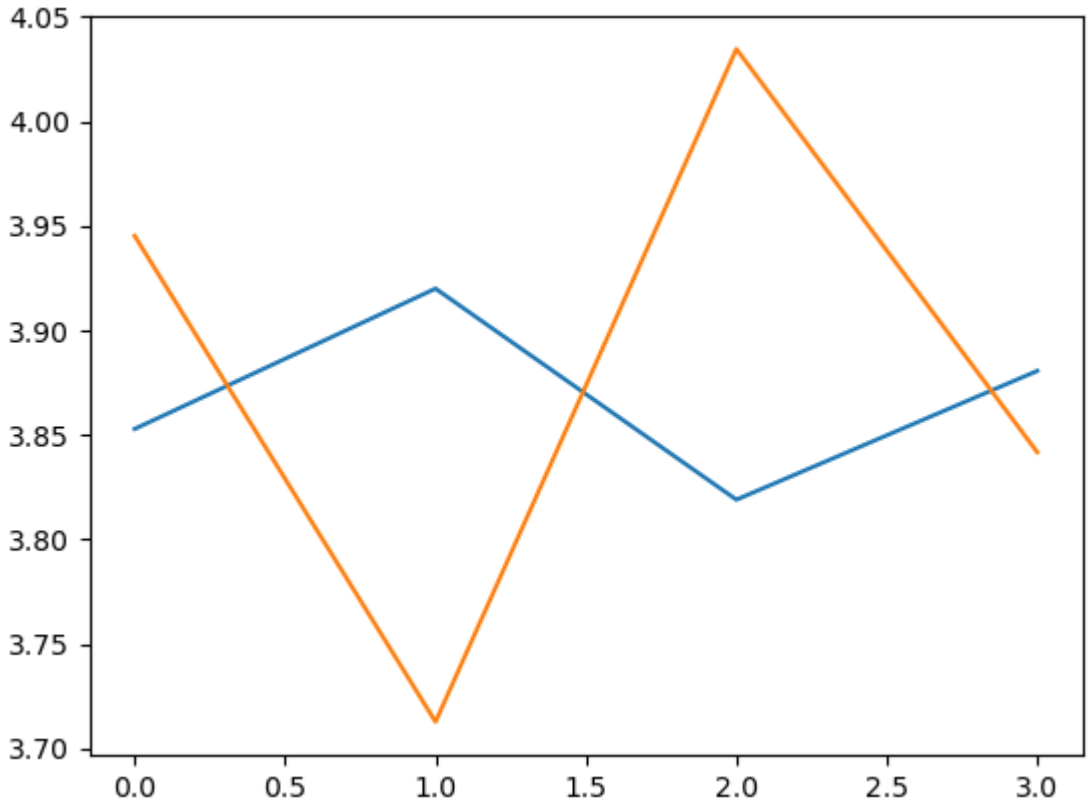

```
Alpha value: 0.001
0
Train error: 3.816461300573554
Validation error: 3.8652147455542765
1
Train error: 3.8853645239437653
Validation error: 3.6333955266443887
2
Train error: 3.7843814058771197
Validation error: 3.9598258914814157
3
Train error: 3.845574742575016
Validation error: 3.7664182493842664
```



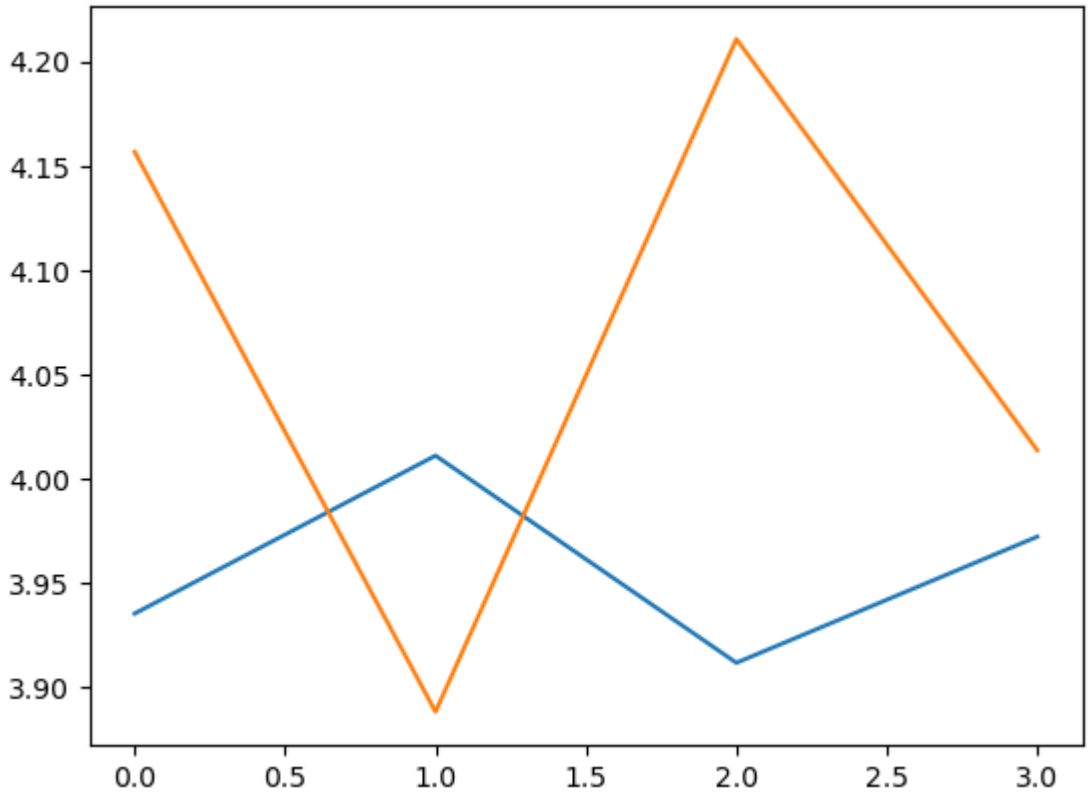

```
Alpha value: 1
0
Train error: 3.8186868297055447
Validation error: 3.874078616824582
1
Train error: 3.88731131963597
Validation error: 3.6444479315091987
2
Train error: 3.7862233026159204
Validation error: 3.9711948361115503
3
Train error: 3.8474595920132773
Validation error: 3.778825605436278
```




```
Alpha value: 10
0
Train error: 3.852839502605182
Validation error: 3.9452247522360193
1
Train error: 3.9199970199332586
Validation error: 3.7127295207848348
2
Train error: 3.8189913036504723
Validation error: 4.03460095127003
3
Train error: 3.880620352043506
Validation error: 3.8416972195632932
```



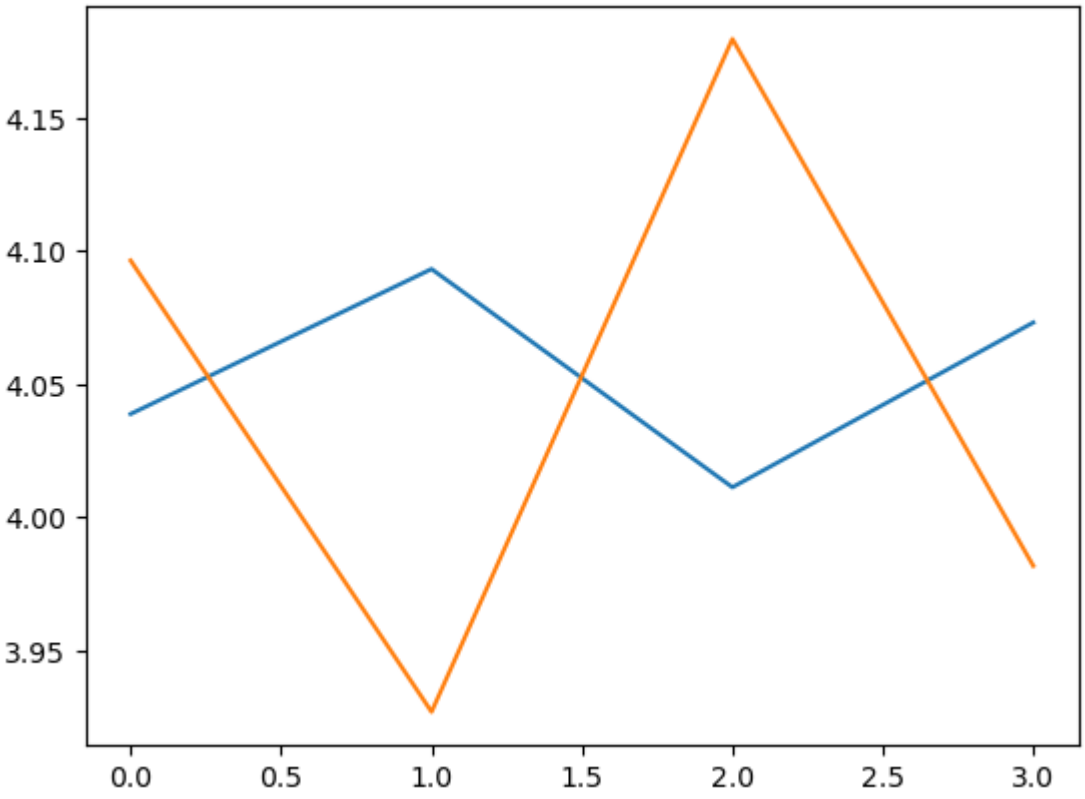

```
Alpha value: 100
0
Train error: 3.935456740365958
Validation error: 4.156639463680036
1
Train error: 4.011135347831783
Validation error: 3.888408358736641
2
Train error: 3.9118792669831643
Validation error: 4.210704490794341
3
Train error: 3.9723089888724608
Validation error: 4.013699803476162
```


4.20
4.15
4.10
4.05
4.00
3.95
3.90
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [25]:
```
# Lasso

train_plot = []
val_plot = []
lasso = Lasso()
for cnt,var in enumerate(kf.split(x_train_2,y_train_2)) :
    print(cnt)
    x = x_train_2[var[0],:]
    y = y_train_2[var[0]]
    x_val = x_train_2[var[1],:]
    y_val = y_train_2[var[1]]
    lasso.fit(x, y)
    y_pred_train = lasso.predict(x)
    lasso.fit(x_val, y_val)
    y_pred_val = lasso.predict(x_val)
    print("Train error:",np.sqrt(mse(y_pred_train,y)))
    print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
    train_plot.append(np.sqrt(mse(y_pred_train,y)))
    val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
```

```
0
Train error: 4.038797374934554
Validation error: 4.096396701079226
1
Train error: 4.093208809782641
Validation error: 3.926992058461979
2
Train error: 4.0112880311792205
Validation error: 4.1795496645221055
3
Train error: 4.073134414611913
Validation error: 3.98176966428072
```

Out[25]: [<matplotlib.lines.Line2D at 0x1aaf91117c0>]


4.15
4.10
4.05
4.00
3.95
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [26]:

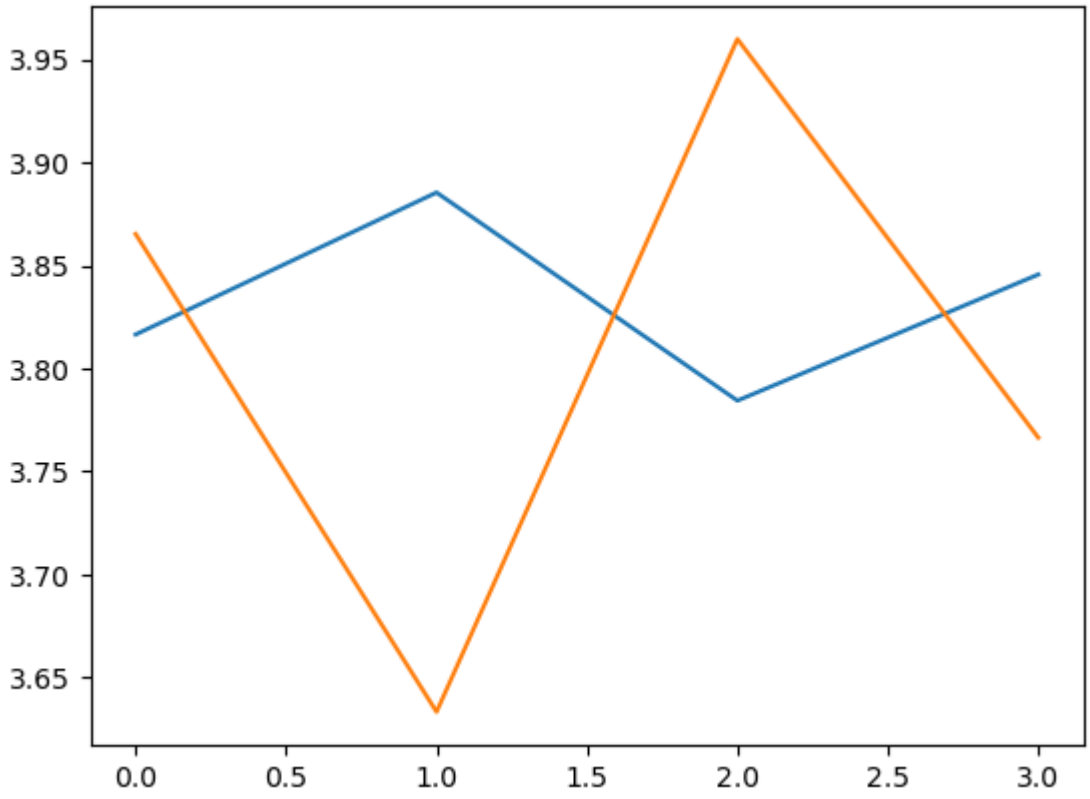
```
# Lasso with different learning rates

lasso_alpha = [0,0.001,1,10,100]

for r in lasso_alpha:
    print("Alpha value:", r)
    train_plot = []
    val_plot = []
    lasso_obj = Lasso(alpha=r)
    for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
        print(cnt)
        x = x_train_2[var[0],:]
        y = y_train_2[var[0]]
        x_val = x_train_2[var[1],:]
        y_val = y_train_2[var[1]]
        lasso_obj.fit(x, y)
        y_pred_train = lasso_obj.predict(x)
        lasso_obj.fit(x_val, y_val)
        y_pred_val = lasso_obj.predict(x_val)
        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
        y_pred_test = lasso_obj.predict(x_test_2)
        metric_dict['Lasso lr='+str(r)] = np.sqrt(mse(y_pred_test,y_test_
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

```
Alpha value: 0
0
Train error: 3.8164612972453944
Validation error: 3.865214725903877
1
Train error: 3.885364521098337
Validation error: 3.6333954981013736
2
Train error: 3.7843814032155816
Validation error: 3.959825862036827
3
Train error: 3.8455747398564717
Validation error: 3.7664182139161584
```

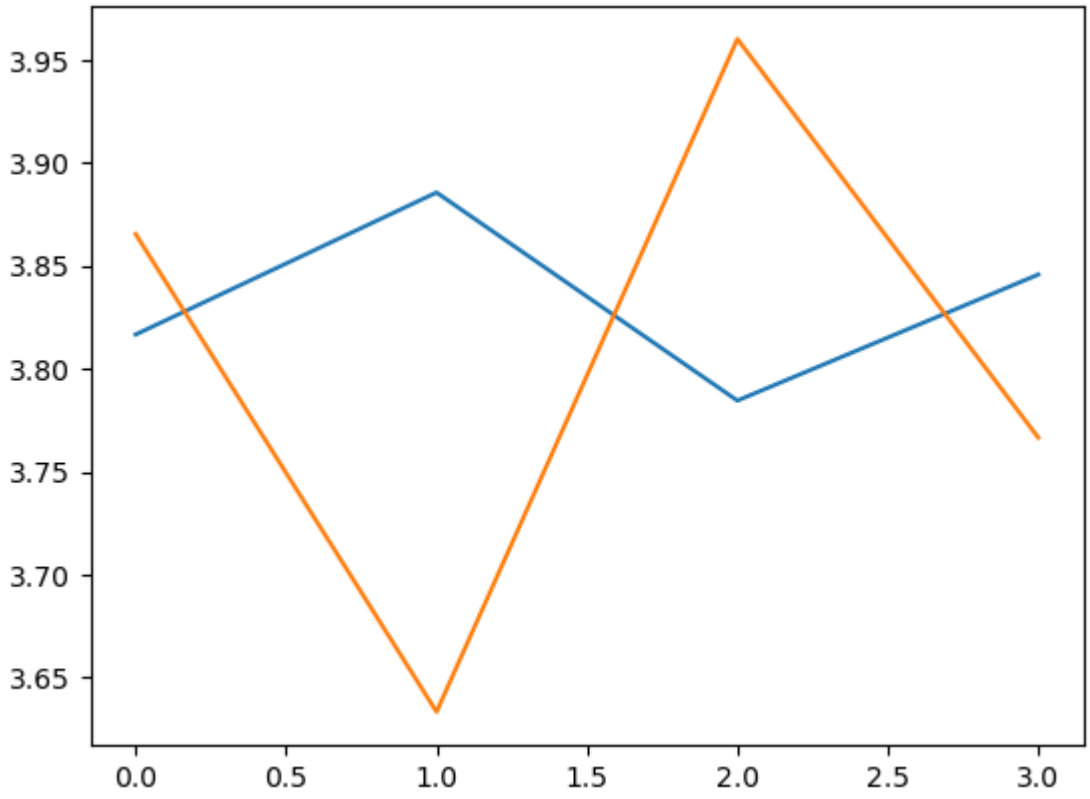
```
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:16: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
se the LinearRegression estimator
  lasso_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.457e+03, tolerance: 5.971e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:18: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
se the LinearRegression estimator
  lasso_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.555e+03, tolerance: 2.052e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:16: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
se the LinearRegression estimator
  lasso_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.729e+03, tolerance: 6.179e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:18: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
se the LinearRegression estimator
  lasso_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
```

```
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.257e+03, tolerance: 1.870e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:16: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
se the LinearRegression estimator
  lasso_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.340e+03, tolerance: 5.880e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:18: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
se the LinearRegression estimator
  lasso_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.673e+03, tolerance: 2.163e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:16: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
se the LinearRegression estimator
  lasso_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.579e+03, tolerance: 6.105e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\3184651632.py:18: UserWarni
ng: With alpha=0, this algorithm does not converge well. You are advised to u
```

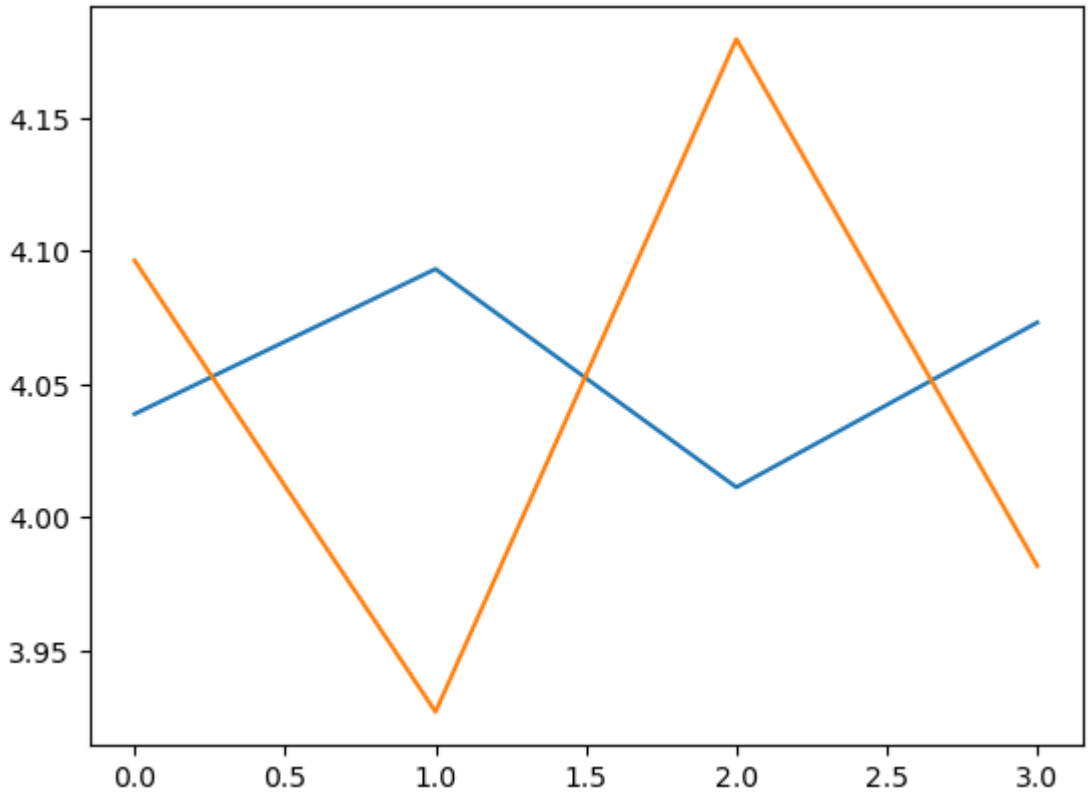
```
se the LinearRegression estimator
  lasso_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.419e+03, tolerance: 1.902e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
```



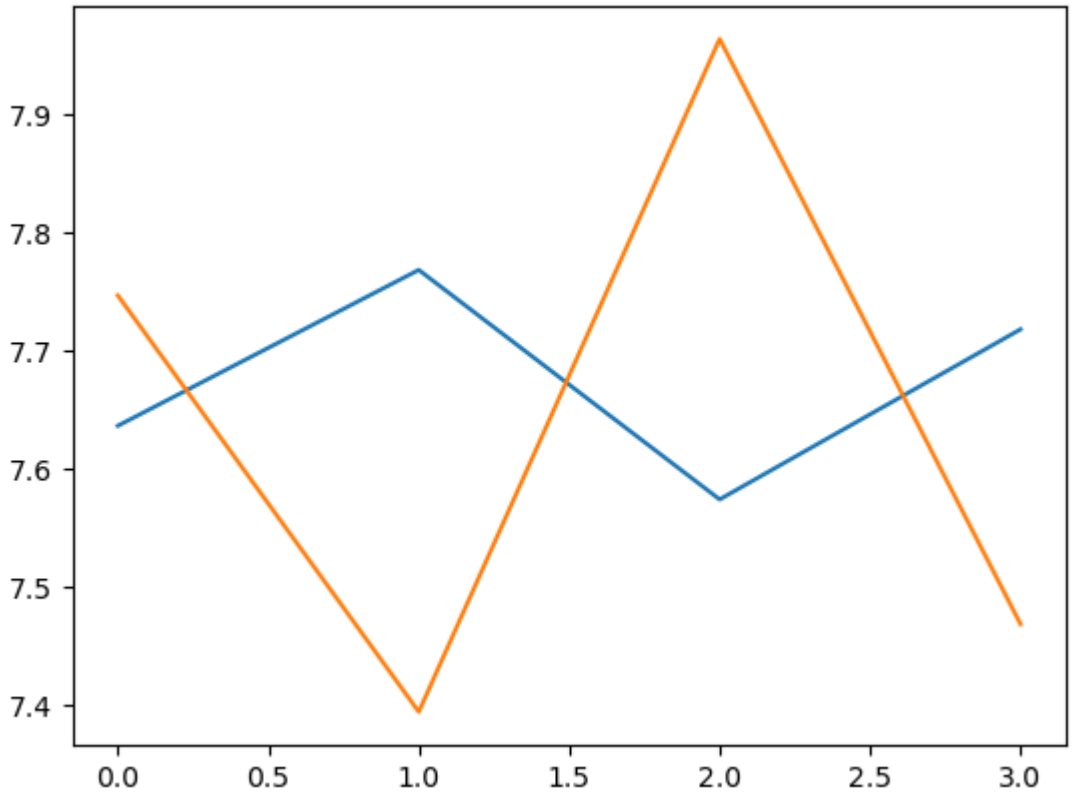

```
Alpha value: 0.001
0
Train error: 3.8165428504187138
Validation error: 3.8652885815082554
1
Train error: 3.8854637607861346
Validation error: 3.6334863391034093
2
Train error: 3.784485964434128
Validation error: 3.959932777446898
3
Train error: 3.845656624340425
Validation error: 3.7665211802799496
```



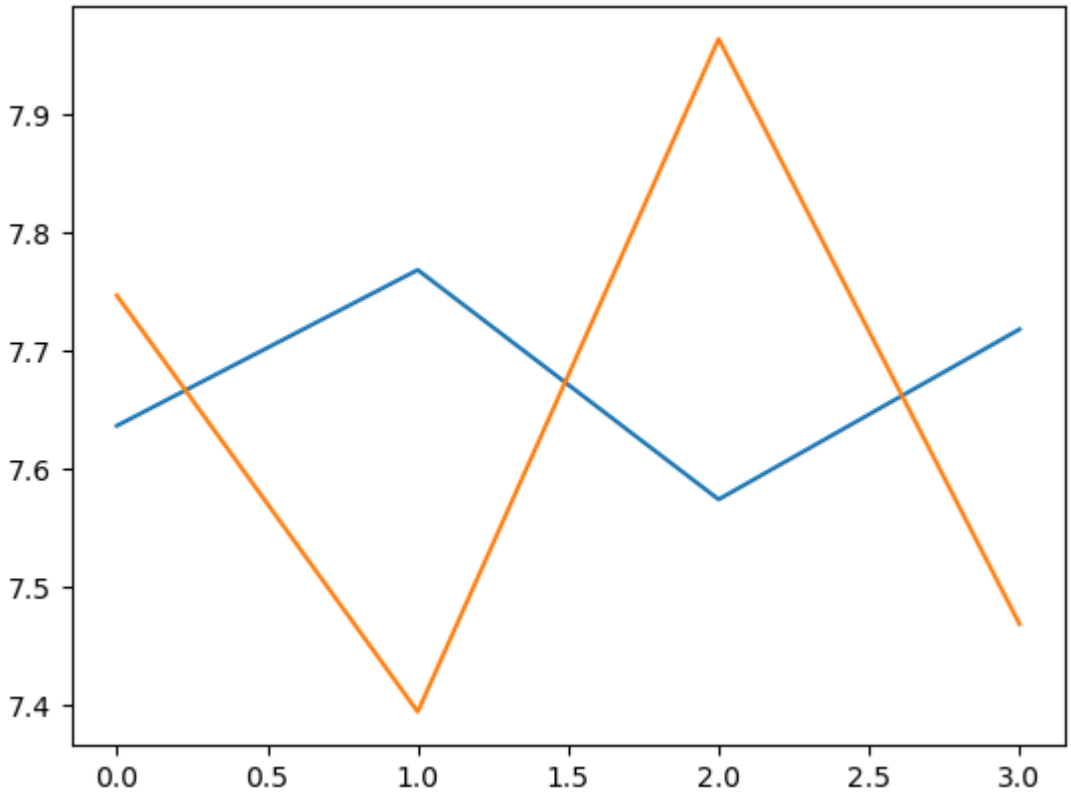

```
Alpha value: 1
0
Train error: 4.038797374934554
Validation error: 4.096396701079226
1
Train error: 4.093208809782641
Validation error: 3.926992058461979
2
Train error: 4.0112880311792205
Validation error: 4.1795496645221055
3
Train error: 4.073134414611913
Validation error: 3.98176966428072
```




```
Alpha value: 10
0
Train error: 7.63616284494717
Validation error: 7.746478390300121
1
Train error: 7.768110993028431
Validation error: 7.393978527563255
2
Train error: 7.573816670038543
Validation error: 7.963619835739025
3
Train error: 7.717837191561589
Validation error: 7.468266103185494
```



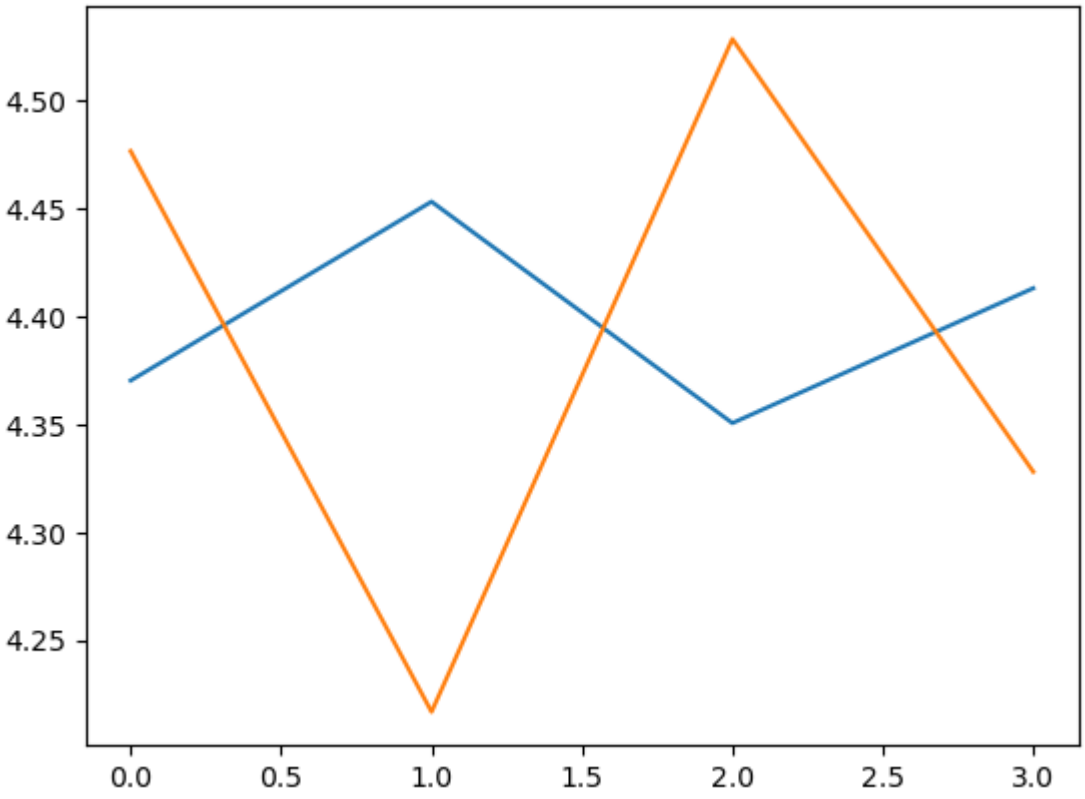

```
Alpha value: 100
0
Train error: 7.63616284494717
Validation error: 7.746478390300121
1
Train error: 7.768110993028431
Validation error: 7.393978527563255
2
Train error: 7.573816670038543
Validation error: 7.963619835739025
3
Train error: 7.717837191561589
Validation error: 7.468266103185494
```


7.9
7.8
7.7
7.6
7.5
7.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [27]:
```
# Elastic Net

train_plot = []
val_plot = []
el_net = ElasticNet()
for cnt,var in enumerate(kf.split(x_train_2,y_train_2)) :
    print(cnt)
    x = x_train_2[var[0],:]
    y = y_train_2[var[0]]
    x_val = x_train_2[var[1],:]
    y_val = y_train_2[var[1]]
    el_net.fit(x, y)
    y_pred_train = el_net.predict(x)
    el_net.fit(x_val, y_val)
    y_pred_val = el_net.predict(x_val)
    print("Train error:",np.sqrt(mse(y_pred_train,y)))
    print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
    train_plot.append(np.sqrt(mse(y_pred_train,y)))
    val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

plt.plot([0,1,2,3], train_plot, label = "Training Error")
plt.plot([0,1,2,3], val_plot, label = "Val Error")
```

```
0
Train error: 4.3702365712793885
Validation error: 4.4765321353181315
1
Train error: 4.453116189790851
Validation error: 4.216748083891603
2
Train error: 4.350523676313172
Validation error: 4.528462199812707
3
Train error: 4.413012157576303
Validation error: 4.327948718273263
```

Out[27]:
```
[<matplotlib.lines.Line2D at 0x1aafca34550>]
```


4.50
4.45
4.40
4.35
4.30
4.25
0.0
0.5
1.0
1.5
2.0
2.5
3.0

In [28]:

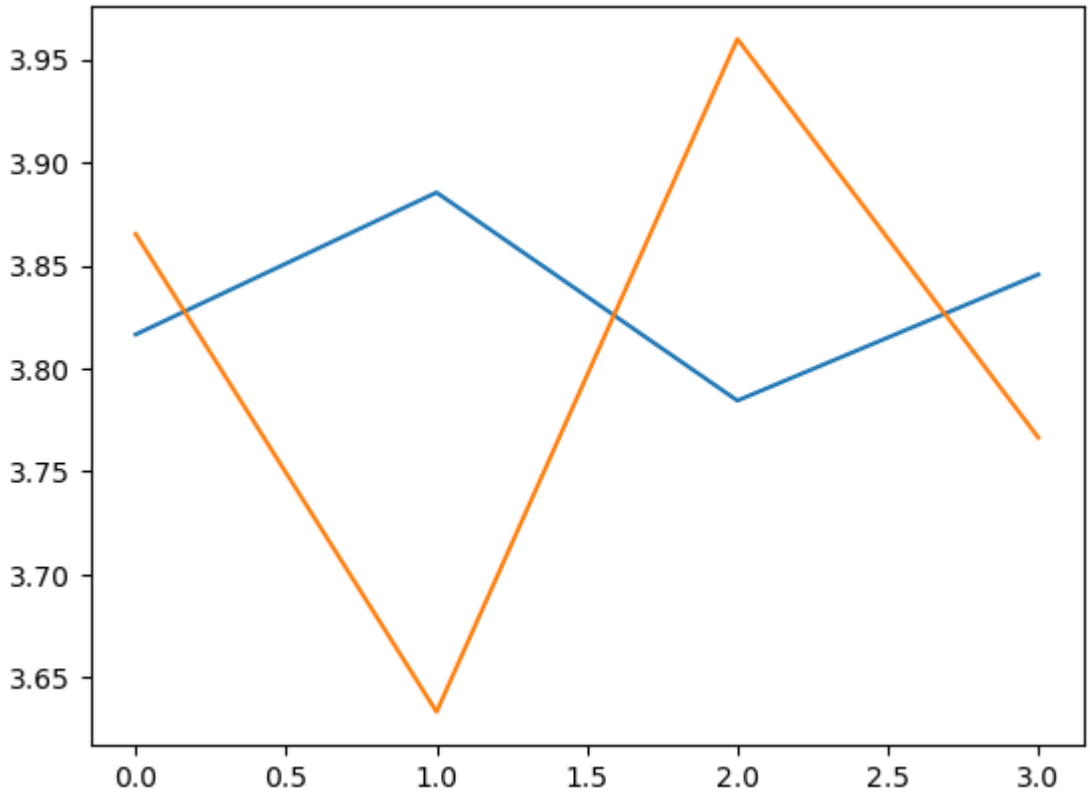
```
# ElasticNet with different learning rates

en_alpha = [0,0.001,1,10,100]

for r in en_alpha:
    print("Alpha value:", r)
    train_plot = []
    val_plot = []
    en_obj = ElasticNet(alpha=r)
    for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
        print(cnt)
        x = x_train_2[var[0],:]
        y = y_train_2[var[0]]
        x_val = x_train_2[var[1],:]
        y_val = y_train_2[var[1]]
        en_obj.fit(x, y)
        y_pred_train = en_obj.predict(x)
        en_obj.fit(x_val, y_val)
        y_pred_val = en_obj.predict(x_val)
        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))
        y_pred_test = en_obj.predict(x_test_2)
        metric_dict['ElasticNet lr='+str(r)] = np.sqrt(mse(y_pred_test,y_t
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

```
Alpha value: 0
0
Train error: 3.8164612972453944
Validation error: 3.865214725903877
1
Train error: 3.885364521098337
Validation error: 3.6333954981013736
2
Train error: 3.7843814032155816
Validation error: 3.959825862036827
3
Train error: 3.8455747398564717
Validation error: 3.7664182139161584
```

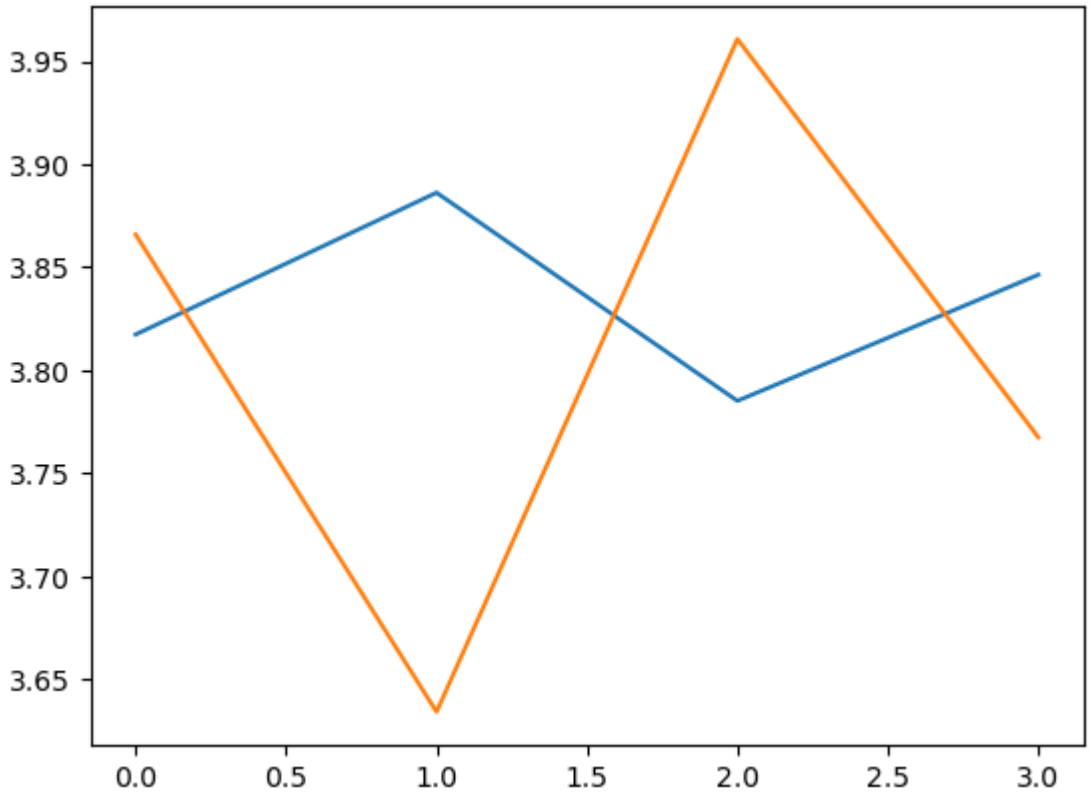
```
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:16: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
he LinearRegression estimator
  en_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.457e+03, tolerance: 5.971e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:18: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
he LinearRegression estimator
  en_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.555e+03, tolerance: 2.052e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:16: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
he LinearRegression estimator
  en_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.729e+03, tolerance: 6.179e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:18: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
he LinearRegression estimator
  en_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
```

```
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.257e+03, tolerance: 1.870e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:16: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
he LinearRegression estimator
  en_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.340e+03, tolerance: 5.880e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:18: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
he LinearRegression estimator
  en_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.673e+03, tolerance: 2.163e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:16: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
he LinearRegression estimator
  en_obj.fit(x, y)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.579e+03, tolerance: 6.105e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
C:\Users\jeets\AppData\Local\Temp\ipykernel_29384\9477209.py:18: UserWarning:
With alpha=0, this algorithm does not converge well. You are advised to use t
```

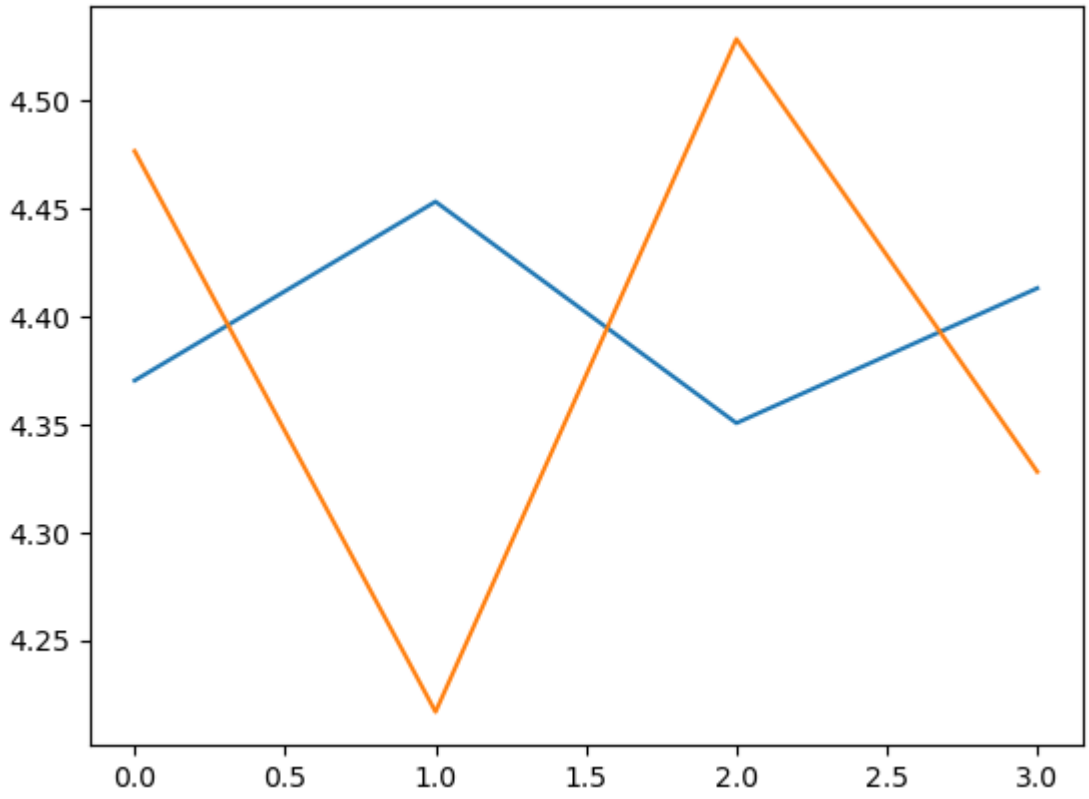
```
he LinearRegression estimator
  en_obj.fit(x_val, y_val)
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: UserWarning: Coordinate descent with no regularization may lea
d to unexpected results and is discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.419e+03, tolerance: 1.902e+00
Linear regression models with null weight for the l1 regularization term are
more efficiently fitted using one of the solvers implemented in sklearn.linea
r_model.Ridge/RidgeCV instead.
  model = cd_fast.enet_coordinate_descent(
```



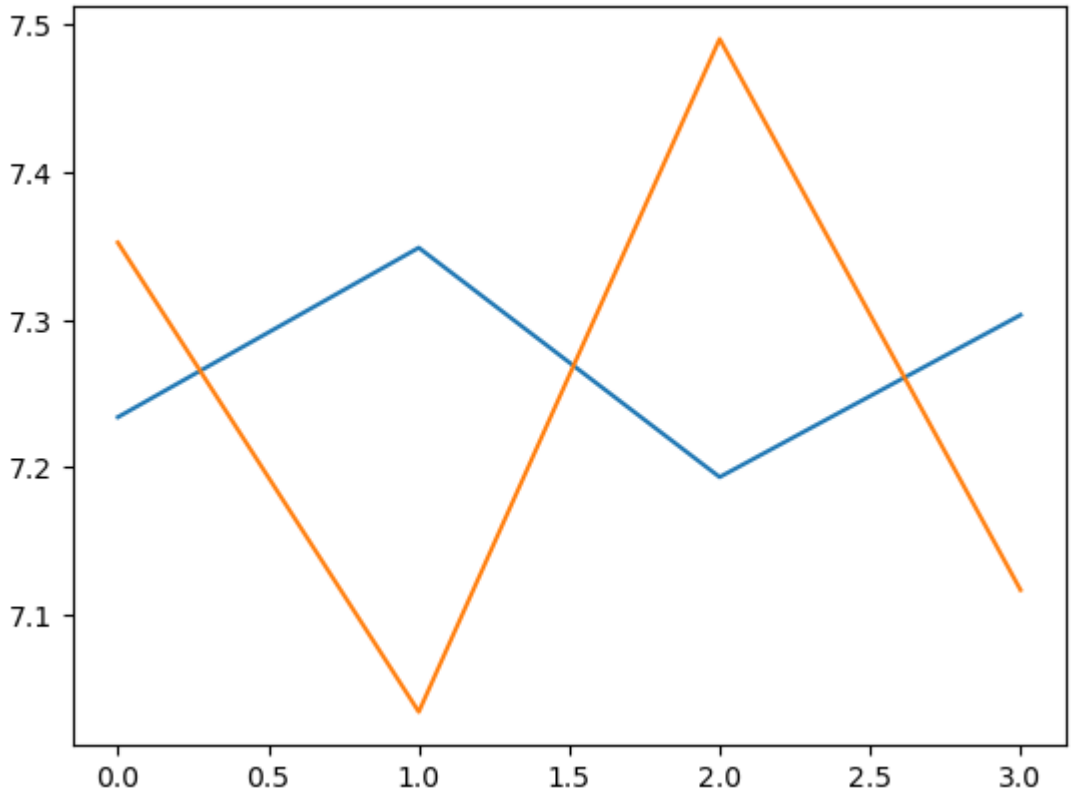

```
Alpha value: 0.001
0
Train error: 3.8173723278108436
Validation error: 3.865869484582345
1
Train error: 3.8861806530765493
Validation error: 3.6342771518819412
2
Train error: 3.785155090783941
Validation error: 3.96073348210214
3
Train error: 3.846346434889784
Validation error: 3.767491162795419
```



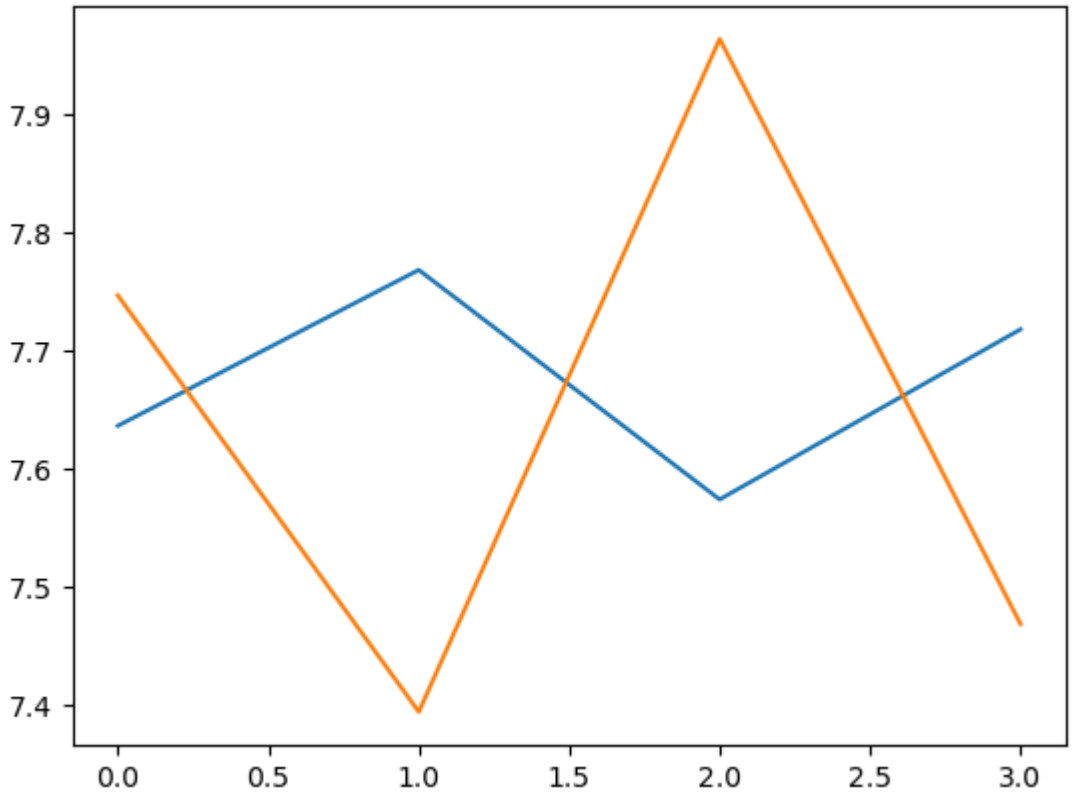

```
Alpha value: 1
0
Train error: 4.3702365712793885
Validation error: 4.4765321353181315
1
Train error: 4.453116189790851
Validation error: 4.216748083891603
2
Train error: 4.350523676313172
Validation error: 4.528462199812707
3
Train error: 4.413012157576303
Validation error: 4.327948718273263
```




```
Alpha value: 10
0
Train error: 7.234094340632255
Validation error: 7.35235457830052
1
Train error: 7.348853346128042
Validation error: 7.0345224480322
2
Train error: 7.193514583679743
Validation error: 7.4902096613872375
3
Train error: 7.303280342238653
Validation error: 7.117028796466902
```




```
Alpha value: 100
0
Train error: 7.63616284494717
Validation error: 7.746478390300121
1
Train error: 7.768110993028431
Validation error: 7.393978527563255
2
Train error: 7.573816670038543
Validation error: 7.963619835739025
3
Train error: 7.717837191561589
Validation error: 7.468266103185494
```



Answer:- The above results show the performance of different linear regression models with different hyperparameters on the dataset. From the results, we can see that the choice of hyperparameters such as learning rate and penalty factor has a significant impact on the performance of the models.

For example, we can see that the Ridge regression model with a learning rate of 1 performs better than the same model with a learning rate of 0, 0.001, or 10. Similarly, we can see that the Lasso and ElasticNet models perform better with a learning rate of 0.001 than 0 or 1.

In the case of SGD, we can see that the performance of the model is influenced by the learning rate and penalty factor. Among the SGD models, we can see that the SGD model with a learning rate of 0.0001 and no penalty performs the best. If we want to trde-off accuracy for best-fit model, we can select the model with lowest learning rate.

Based on the above results, we can conclude that the SGD model with a learning rate of 0.0001 and no penalty factor is the best performing model among the linear regression models.

Part 6 FIX!

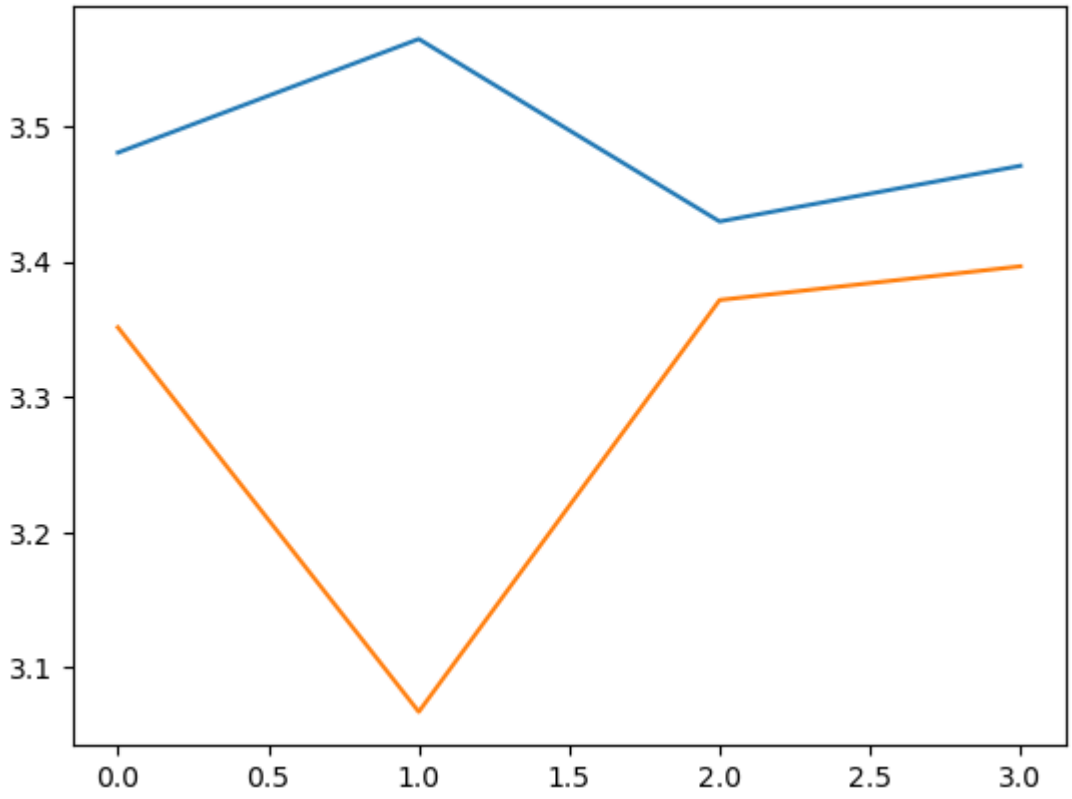
added ridge, lasso, etc for polynomial regressor.

Repeat the previous step with polynomial regression. Using validation loss, explore if your model overfits/underfits the data.

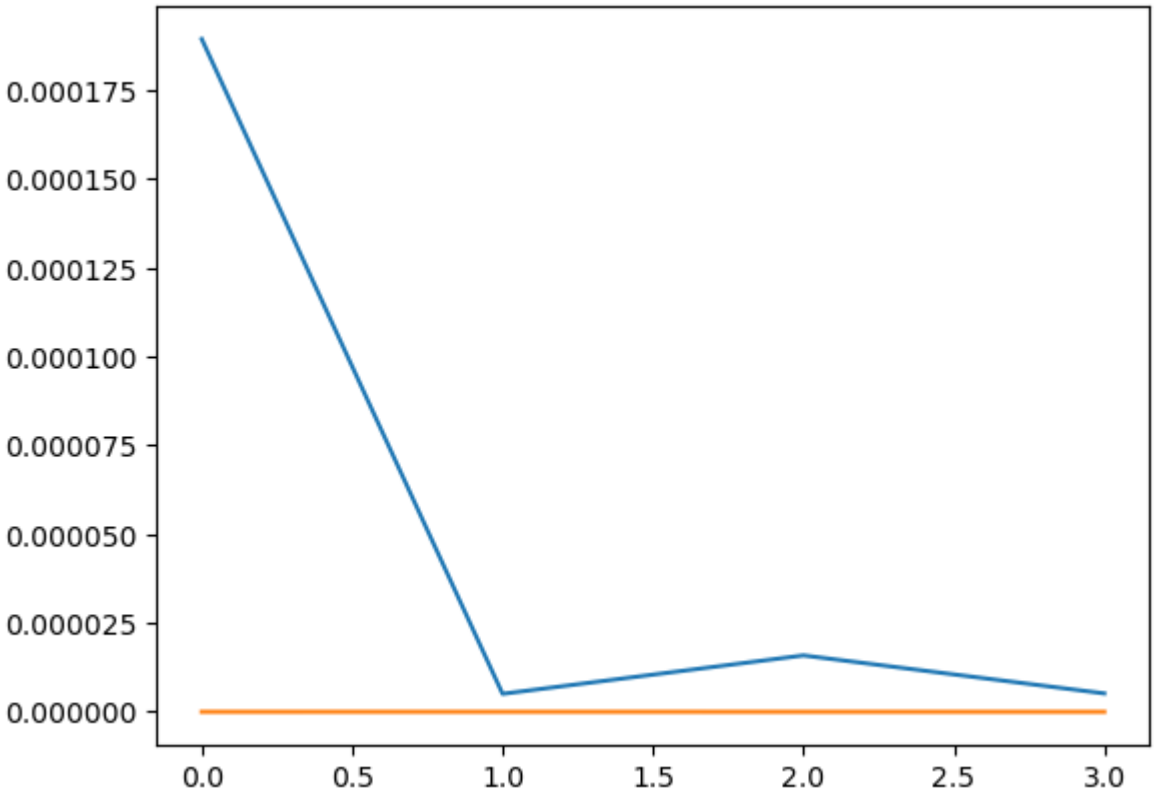
In [29]:

```
# Polynomial regression using various degrees

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    lin_reg = LinearRegression()
    for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
        print(cnt)
        x = x_train_2[var[0],:]
        y = y_train_2[var[0]]
        x_val = x_train_2[var[1],:]
        y_val = y_train_2[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        lin_reg.fit(X_poly_1, y)
        y_pred_train=lin_reg.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        lin_reg.fit(X_poly_2, y_val)
        y_pred_val=lin_reg.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test_2)
        y_pred_test = lin_reg.predict(X_test_poly)
        metric_dict['Polynomial degree= '+str(r)] = np.sqrt(mse(y_pred_te
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

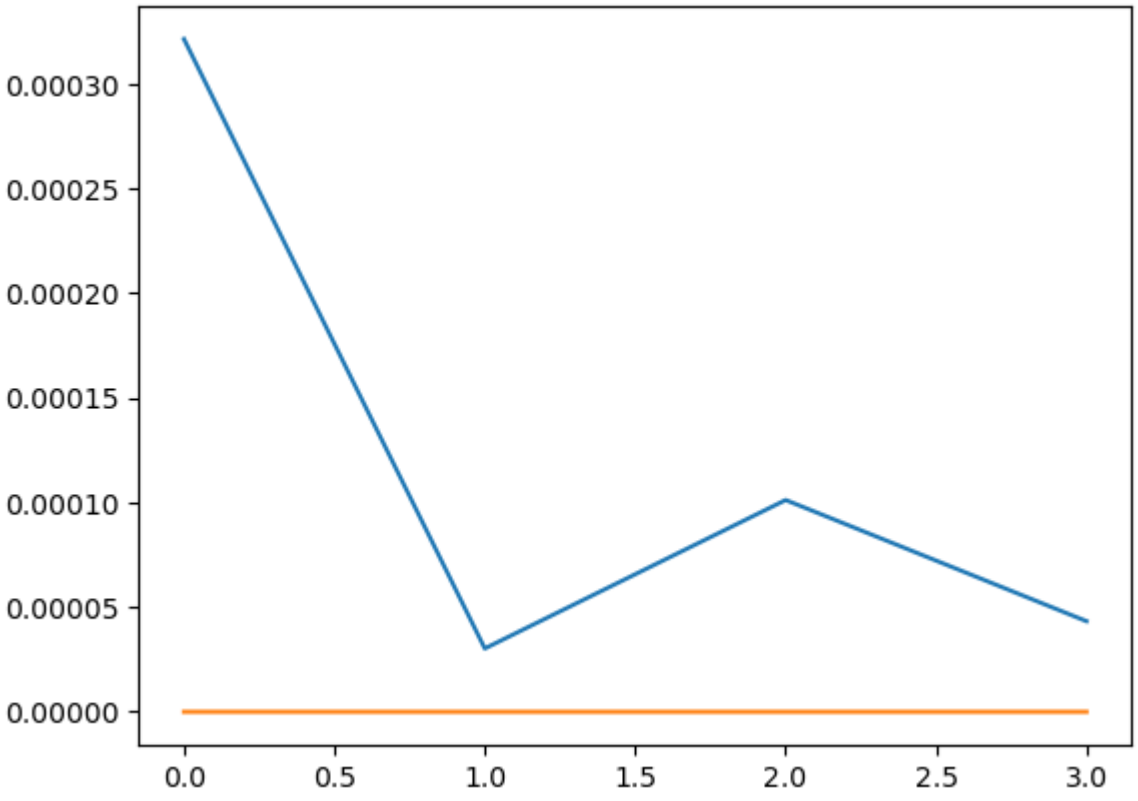
```
Degree: 2
0
Train error: 3.480439443923598
Validation error: 3.351374064617879
1
Train error: 3.5643830729882917
Validation error: 3.067284862232806
2
Train error: 3.4295680768999164
Validation error: 3.371557380615077
3
Train error: 3.470715748512232
Validation error: 3.3964585668919063
```




```
Degree: 5
0
Train error: 0.00018939667881141212
Validation error: 2.451125219907198e-09
1
Train error: 5.062712854654552e-06
Validation error: 7.071578381275772e-09
2
Train error: 1.5878902551751587e-05
Validation error: 1.5786709859717238e-09
3
Train error: 5.1821998857735465e-06
Validation error: 1.7168214995055538e-09
```



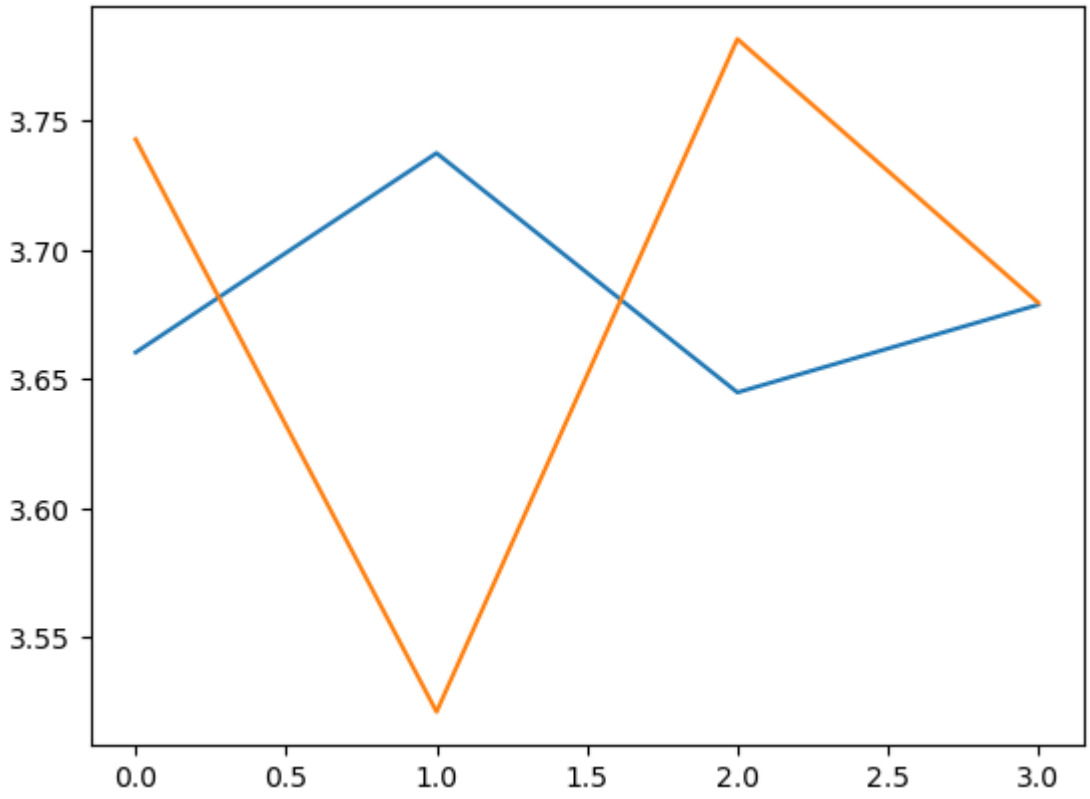

```
Degree: 10
0
Train error: 0.0003214144263393508
Validation error: 4.681995426008565e-09
1
Train error: 3.024337400238046e-05
Validation error: 2.6741039169683448e-09
2
Train error: 0.00010118785198348438
Validation error: 1.8587230792280205e-09
3
Train error: 4.328954718870466e-05
Validation error: 1.8945759387736444e-09
```


0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
0.00000
0.0
0.5
1.0
1.5
2.0
2.5
3.0

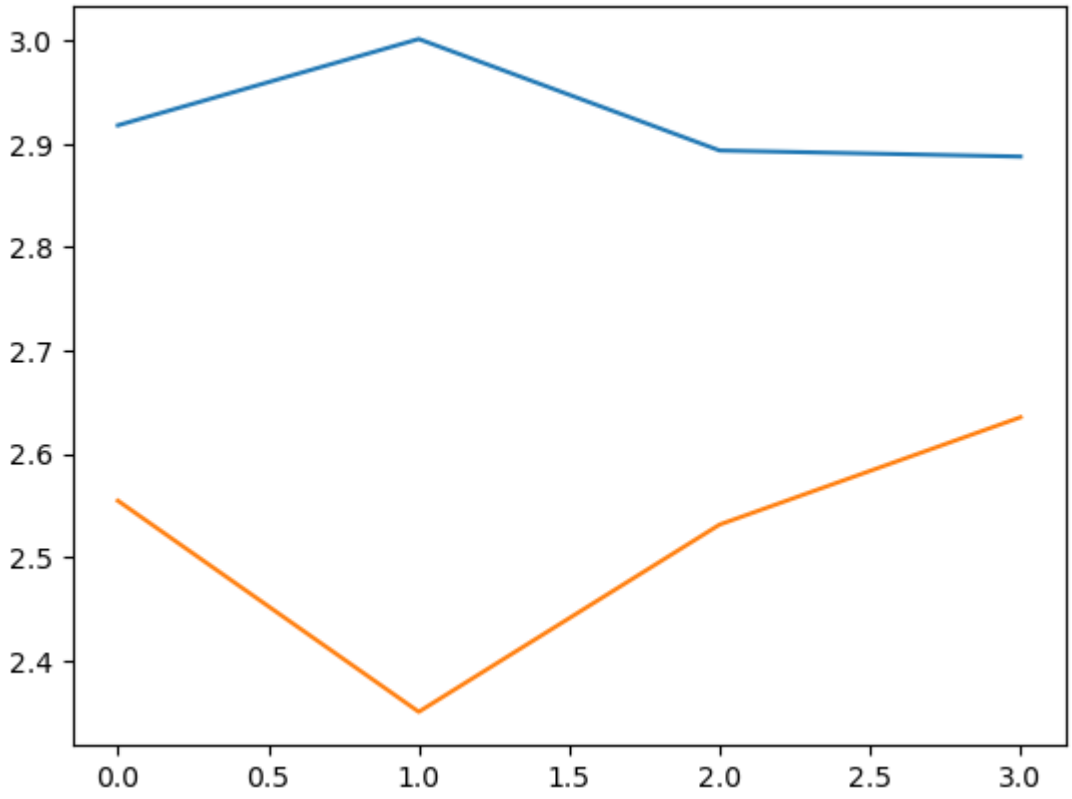
In [30]:

```
# Polynomial regression using ridge

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    ridge = Ridge()
    for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
        print(cnt)
        x = x_train_2[var[0],:]
        y = y_train_2[var[0]]
        x_val = x_train_2[var[1],:]
        y_val = y_train_2[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        ridge.fit(X_poly_1, y)
        y_pred_train=ridge.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        ridge.fit(X_poly_2, y_val)
        y_pred_val=ridge.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test_2)
        y_pred_test = ridge.predict(X_test_poly)
        metric_dict['Polynomial degree for ridge= '+str(r)] = np.sqrt(mse
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

```
Degree: 2
0
Train error: 3.6602909085079114
Validation error: 3.742760400310355
1
Train error: 3.737412793841303
Validation error: 3.5212350260206375
2
Train error: 3.6448151048627975
Validation error: 3.781549575381845
3
Train error: 3.6787414895501316
Validation error: 3.679502835184422
```

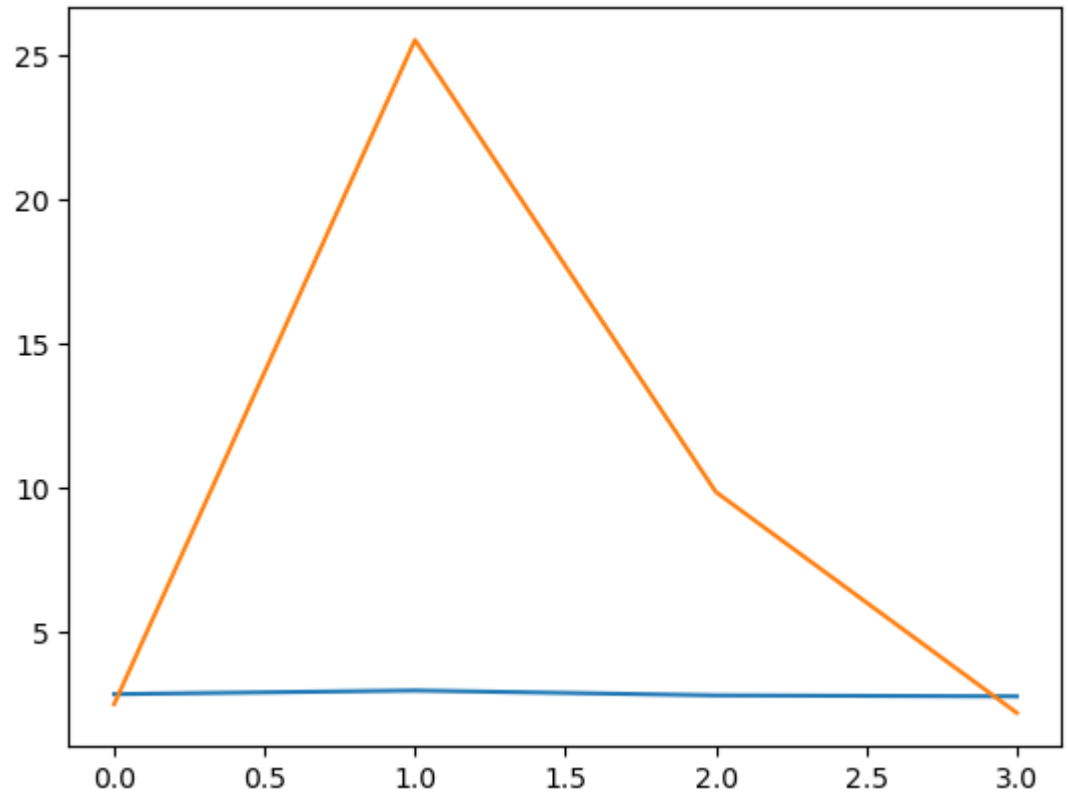



```
Degree: 5
0
Train error: 2.9179844038692258
Validation error: 2.554432896876155
1
Train error: 3.0015732246547286
Validation error: 2.349993268841313
2
Train error: 2.8936080554117076
Validation error: 2.5313295440635097
3
Train error: 2.887727509173558
Validation error: 2.6352916811729252
```




```
Degree: 10
0

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(

Train error: 2.8335826086736424
Validation error: 2.4871295827107733
1

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(

Train error: 2.9603305917893676
Validation error: 25.545416103975324
2
```

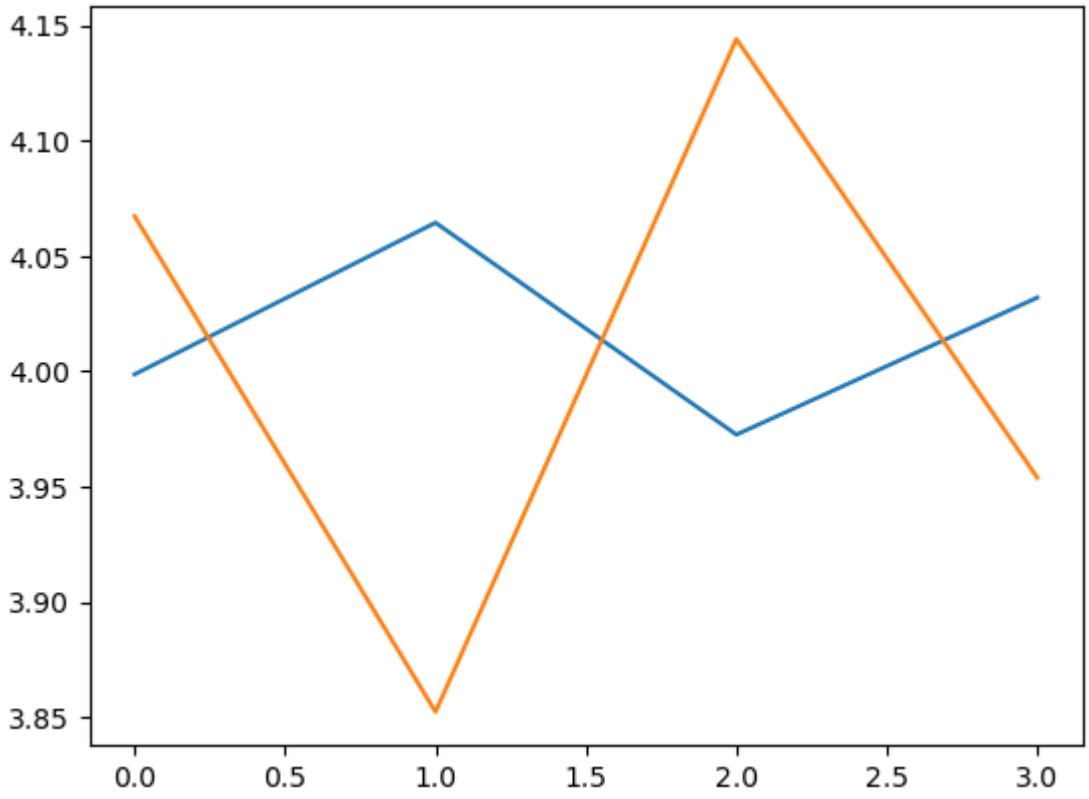
```
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(

Train error: 2.7915379552828097
Validation error: 9.852811014560062
3

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:19
6: UserWarning: Singular matrix in solving dual problem. Using least-squares
solution instead.
  warnings.warn(

Train error: 2.7579822730904318
Validation error: 2.185656583260377
```



In [31]:

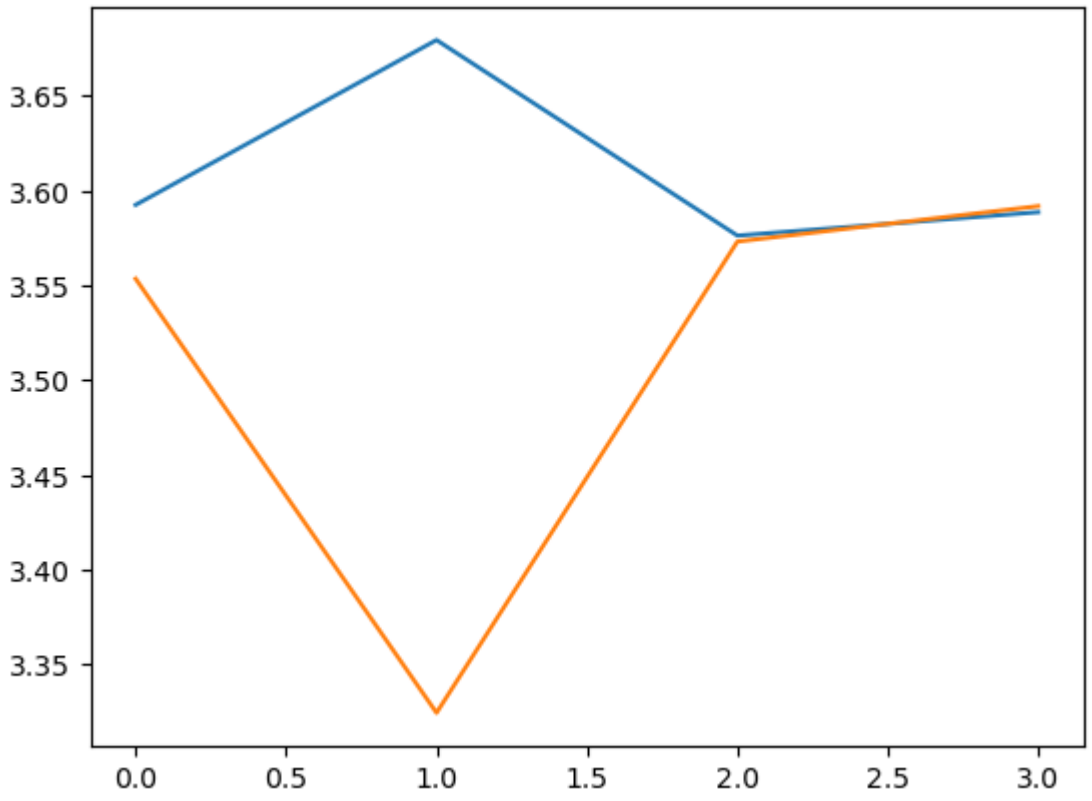
```
# Polynomial regression using lasso

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    lasso = Lasso()
    for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
        print(cnt)
        x = x_train_2[var[0],:]
        y = y_train_2[var[0]]
        x_val = x_train_2[var[1],:]
        y_val = y_train_2[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        lasso.fit(X_poly_1, y)
        y_pred_train=lasso.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        lasso.fit(X_poly_2, y_val)
        y_pred_val=lasso.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test_2)
        y_pred_test = lasso.predict(X_test_poly)
        metric_dict['Polynomial degree for Lasso = '+str(r)] = np.sqrt(mse
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

```
Degree: 2
0
Train error: 3.99869187802109
Validation error: 4.067223344251254
1
Train error: 4.064428555541588
Validation error: 3.8523874684629775
2
Train error: 3.9725429633156844
Validation error: 4.144051958874811
3
Train error: 4.031961214029916
Validation error: 3.9538730532276265
```




```
Degree: 5
0

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 6.806e+03, tolerance: 5.971e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.097e+03, tolerance: 2.052e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.592533436869899
Validation error: 3.553567654493042
1

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 7.053e+03, tolerance: 6.179e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.949e+03, tolerance: 1.870e+00
  model = cd_fast.enet_coordinate_descent(
```

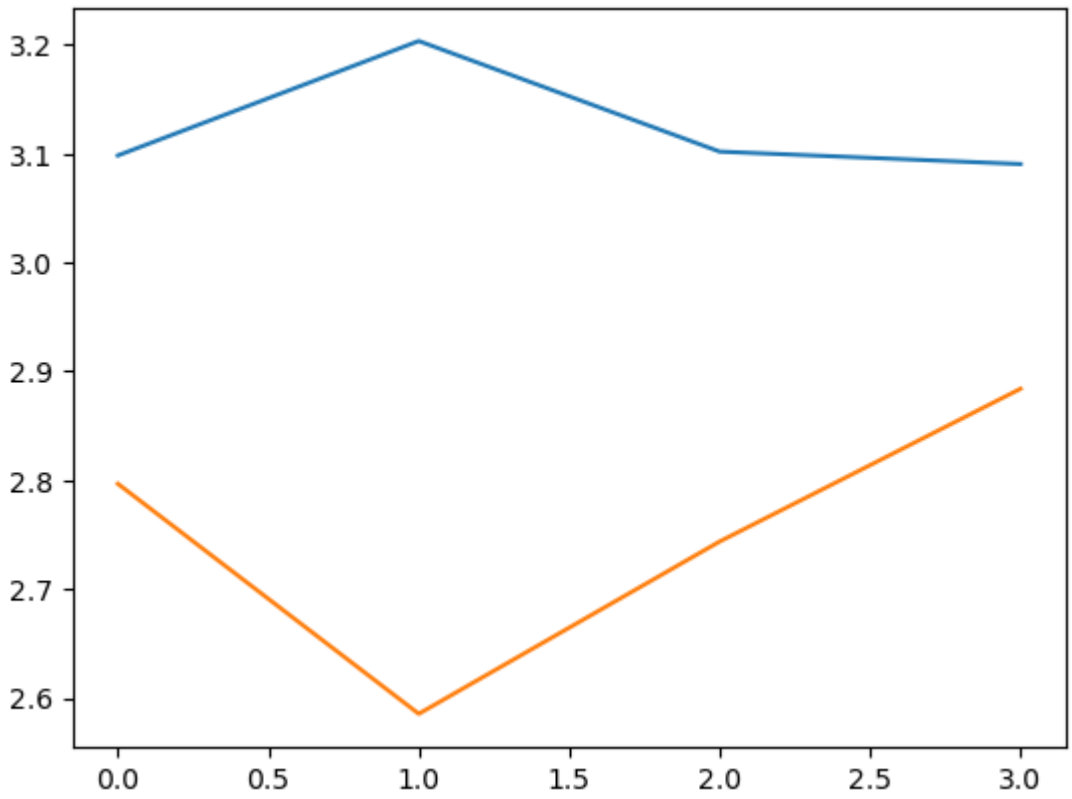
```
Train error: 3.6794849903060785
Validation error: 3.3245845983410924
2

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 6.706e+03, tolerance: 5.880e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.298e+03, tolerance: 2.163e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.5763128036350005
Validation error: 3.5731862311005425
3

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 6.756e+03, tolerance: 6.105e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.588692530944484
Validation error: 3.5918641573989802

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.274e+03, tolerance: 1.902e+00
  model = cd_fast.enet_coordinate_descent(
```



```
Degree: 10
0

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.282e+03, tolerance: 5.971e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.499e+03, tolerance: 2.052e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.098252087785686
Validation error: 2.7967890257848333
1

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.611e+03, tolerance: 6.179e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.287e+03, tolerance: 1.870e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.203328764270571
Validation error: 2.5856117987979634
2

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.273e+03, tolerance: 5.880e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.480e+03, tolerance: 2.163e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.1018034940110537
Validation error: 2.743790973073377
3
```

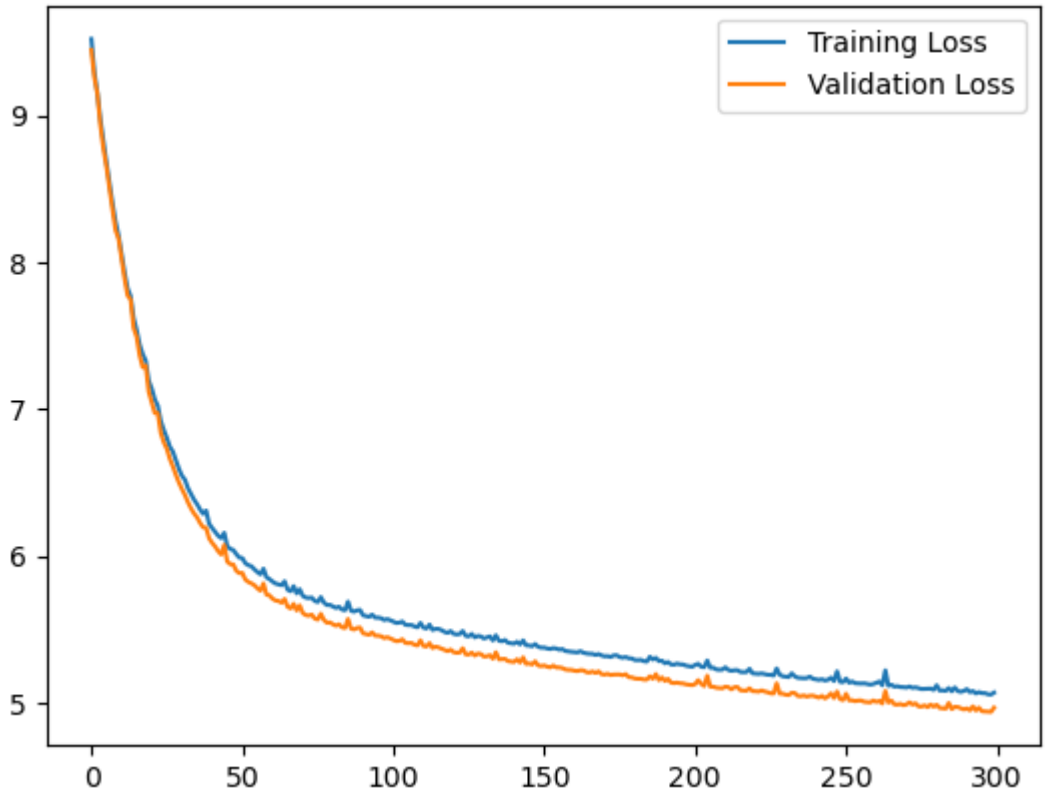
```
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.233e+03, tolerance: 6.105e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.616e+03, tolerance: 1.902e+00
  model = cd_fast.enet_coordinate_descent(
```

```
Train error: 3.0903722088814733
Validation error: 2.8840436612412015
```

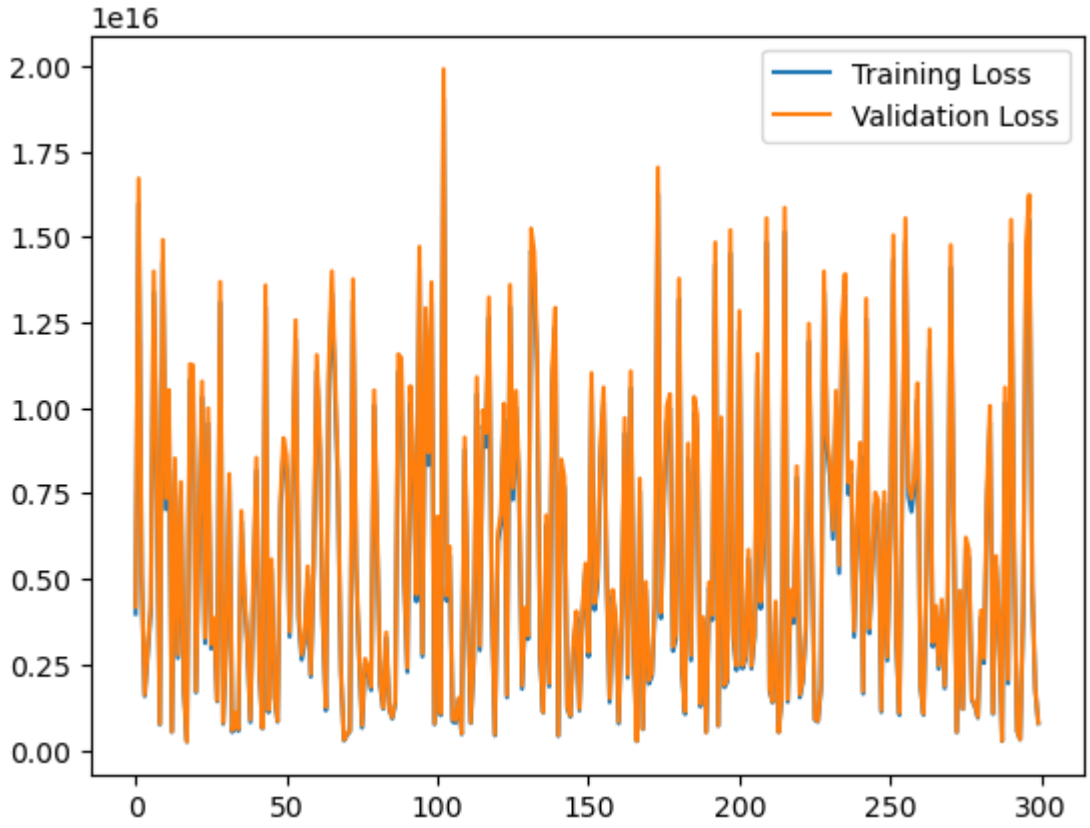


In [32]:
```
Polynomial regression using SGD
gd_pen = [None,'l1','l2']
earning_rates = [0.000001, 0.00001, 0.0001, 0.001, 0.01]
_epochs = 300
ol_deg = [2,5]
or penalty in sgd_pen:
  for lr in learning_rates:
    for pol in pol_deg:
        print("Polynomial degree =:", pol)
        print("Penalty:", penalty)
        print("Learning Rate:", lr)
        train_loss = []
        val_loss = []
        sgd_obj = SGDRegressor(penalty=penalty, learning_rate='constant', et
        poly_features = PolynomialFeatures(degree=pol, include_bias=False)
        X_train_poly = poly_features.fit_transform(x_train_2)
        X_val_poly = poly_features.transform(x_val)

        for epoch in range(n_epochs):
            sgd_obj.partial_fit(X_train_poly, y_train_2)
            y_pred_train = sgd_obj.predict(X_train_poly)
            y_pred_val = sgd_obj.predict(X_val_poly)
            train_loss.append(np.sqrt(mse(y_pred_train,y_train_2)))
            val_loss.append(np.sqrt(mse(y_pred_val,y_val)))

        plt.plot(range(n_epochs), train_loss, label = "Training Loss")
        plt.plot(range(n_epochs), val_loss, label = "Validation Loss")
        plt.legend()
        plt.show()

        # Calculate and print RMSE score between y_pred_val and y_test
        y_pred_test = sgd_obj.predict(poly_features.transform(x_test_2))
        rmse = np.sqrt(mse(y_pred_test, y_test_2))
        metric_dict['Polynomial degree for SGD= '+str(pol)+' Learning_rate=
        print("RMSE score on test data:", rmse)

```

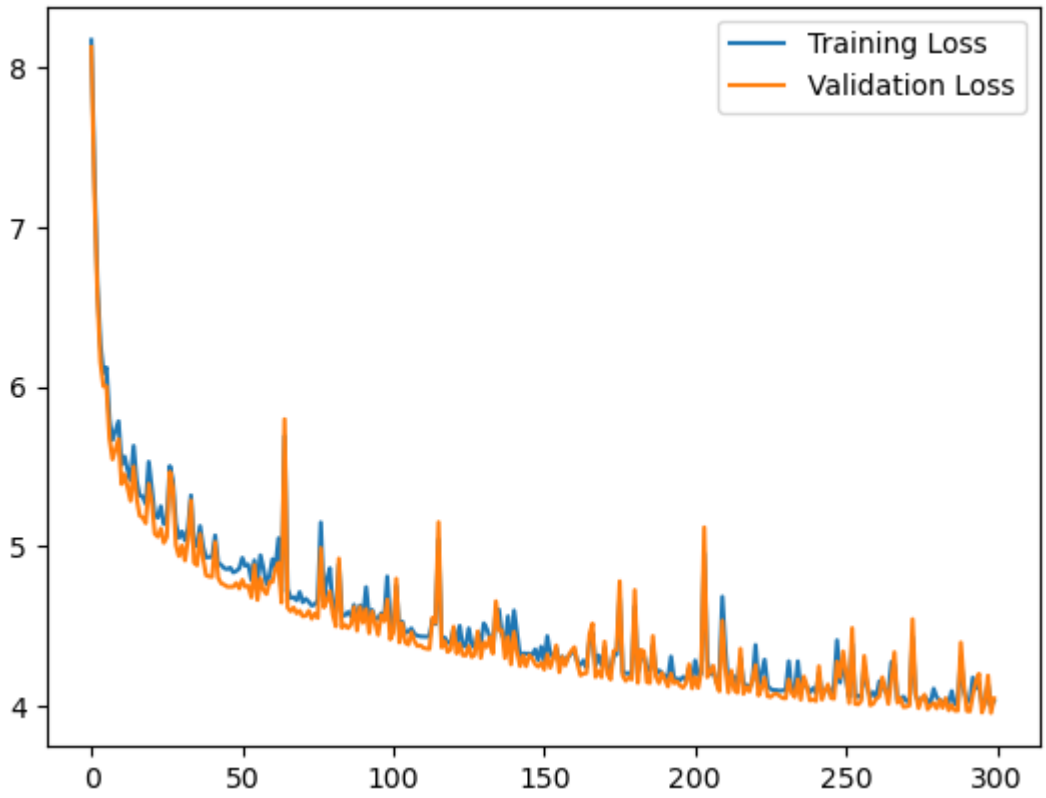
```
Polynomial degree =: 2
Penalty: None
Learning Rate: 1e-06
```



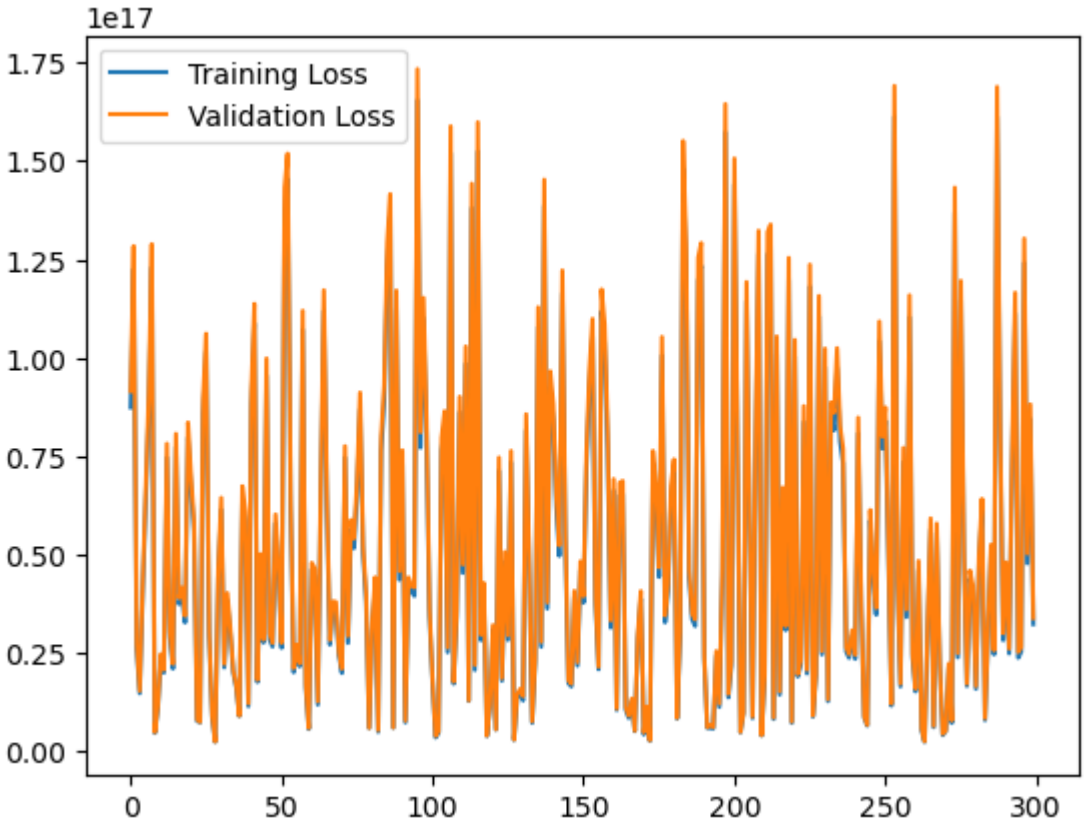

```
RMSE score on test data: 5.002836638073316
Polynomial degree =: 5
Penalty: None
Learning Rate: 1e-06
```



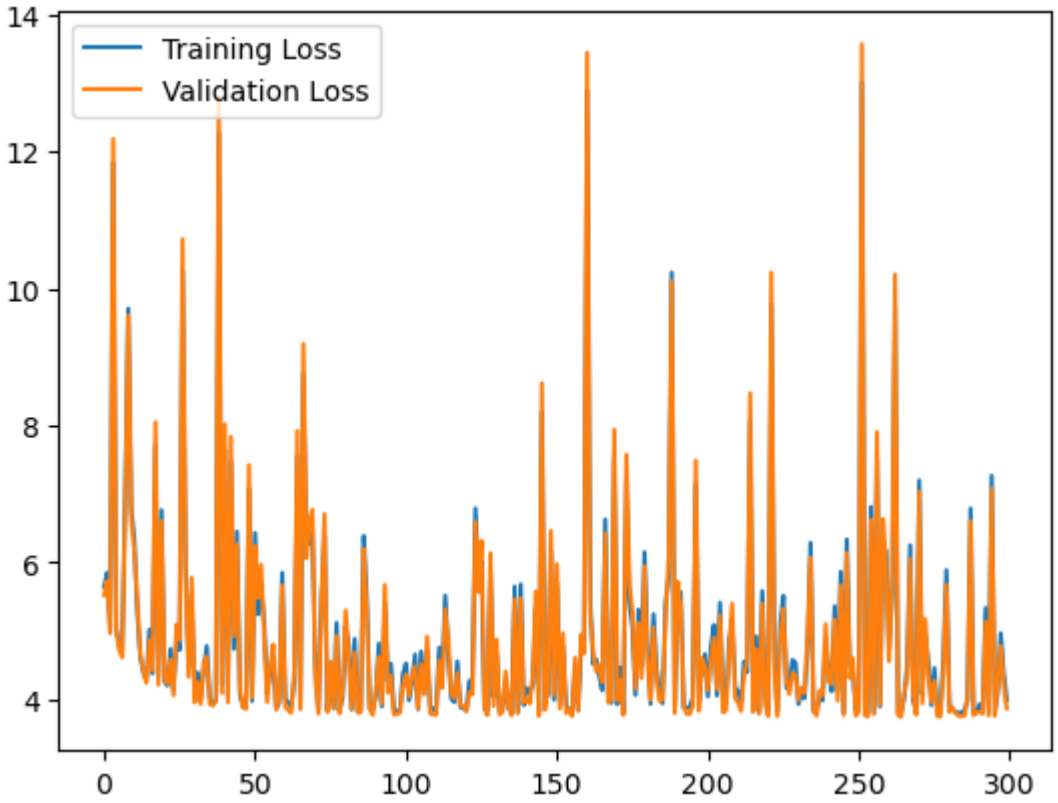
```
RMSE score on test data: 770328126591323.4
Polynomial degree =: 2
Penalty: None
Learning Rate: 1e-05
```



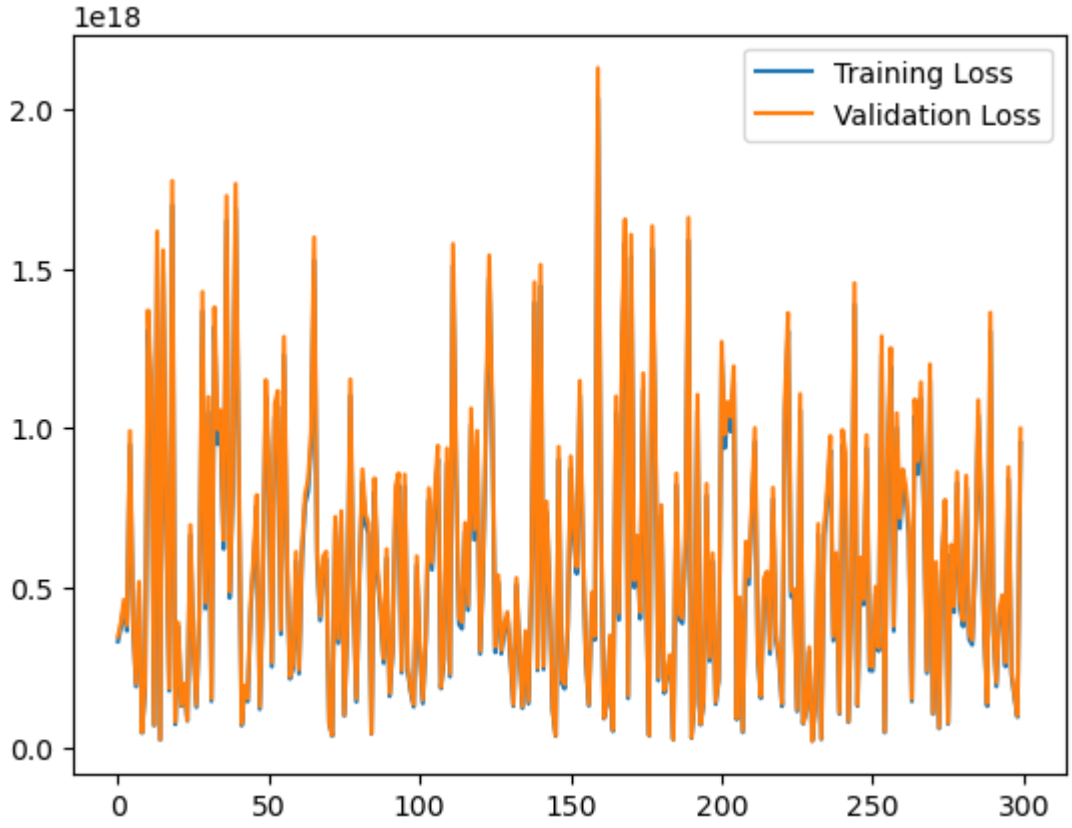

```
RMSE score on test data: 3.9412323193397887
Polynomial degree =: 5
Penalty: None
Learning Rate: 1e-05
```



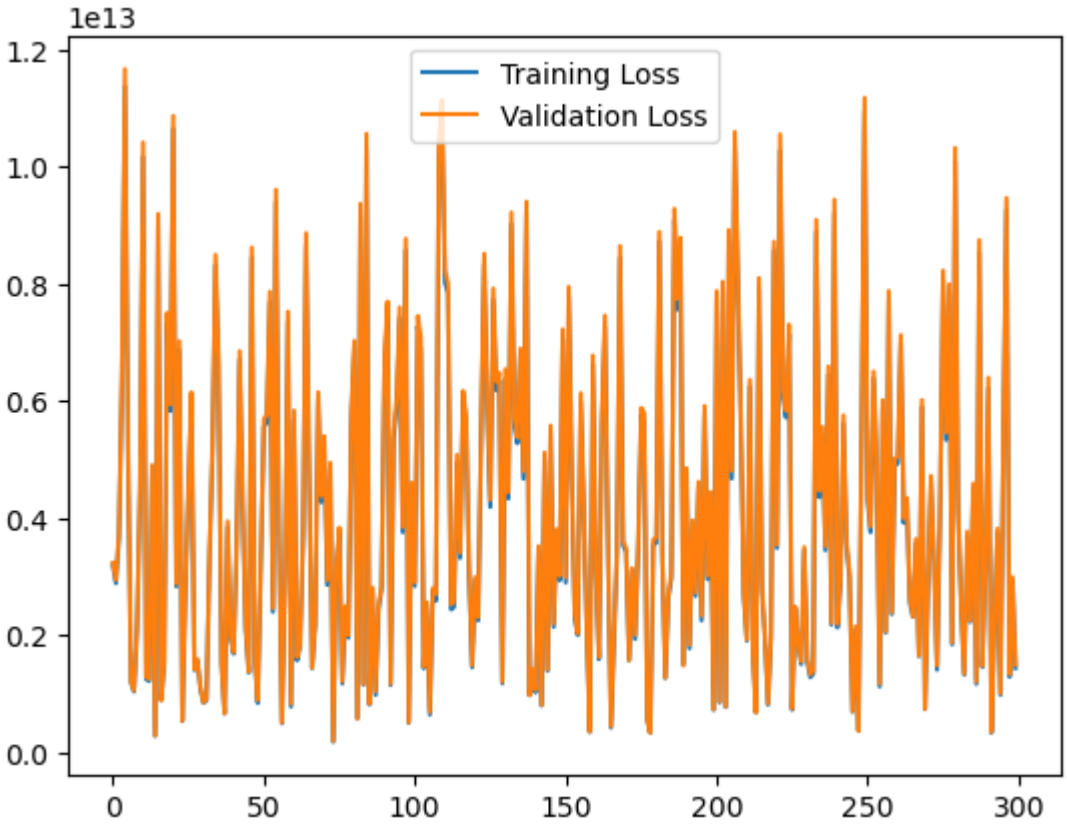

```
RMSE score on test data: 3.1996231306365056e+16
Polynomial degree =: 2
Penalty: None
Learning Rate: 0.0001
```



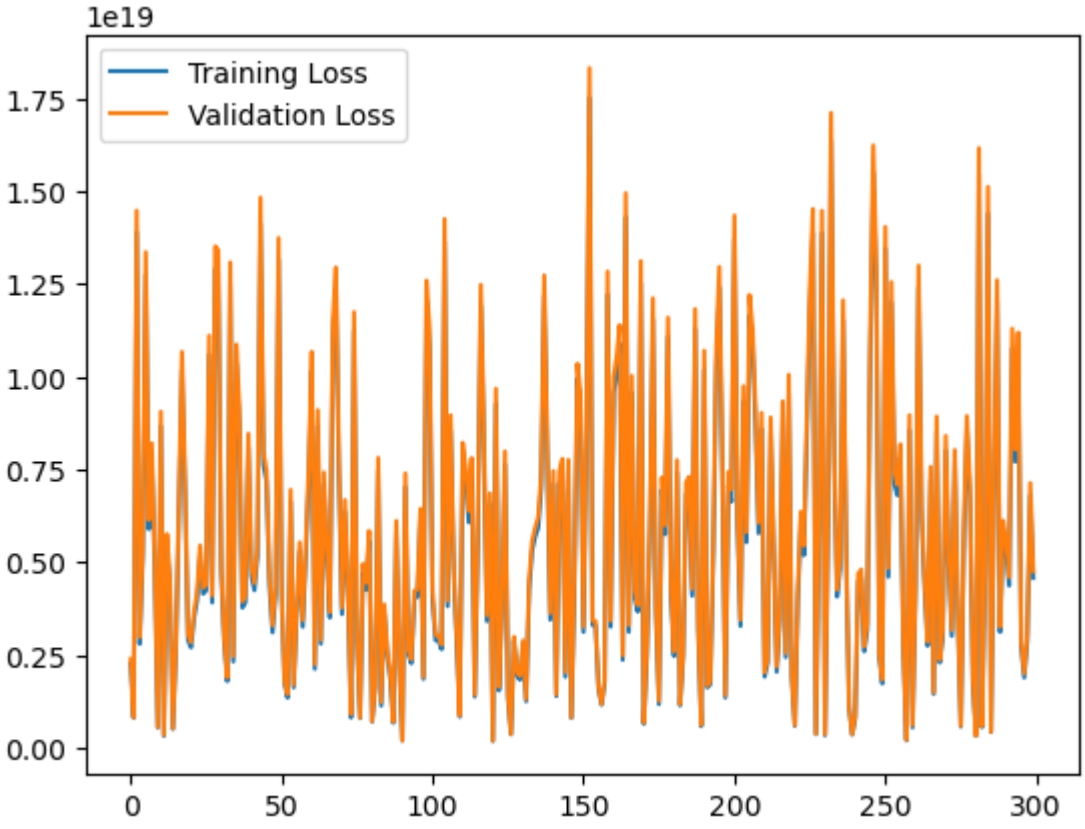
```
RMSE score on test data: 3.8693893503500916
Polynomial degree =: 5
Penalty: None
Learning Rate: 0.0001
```



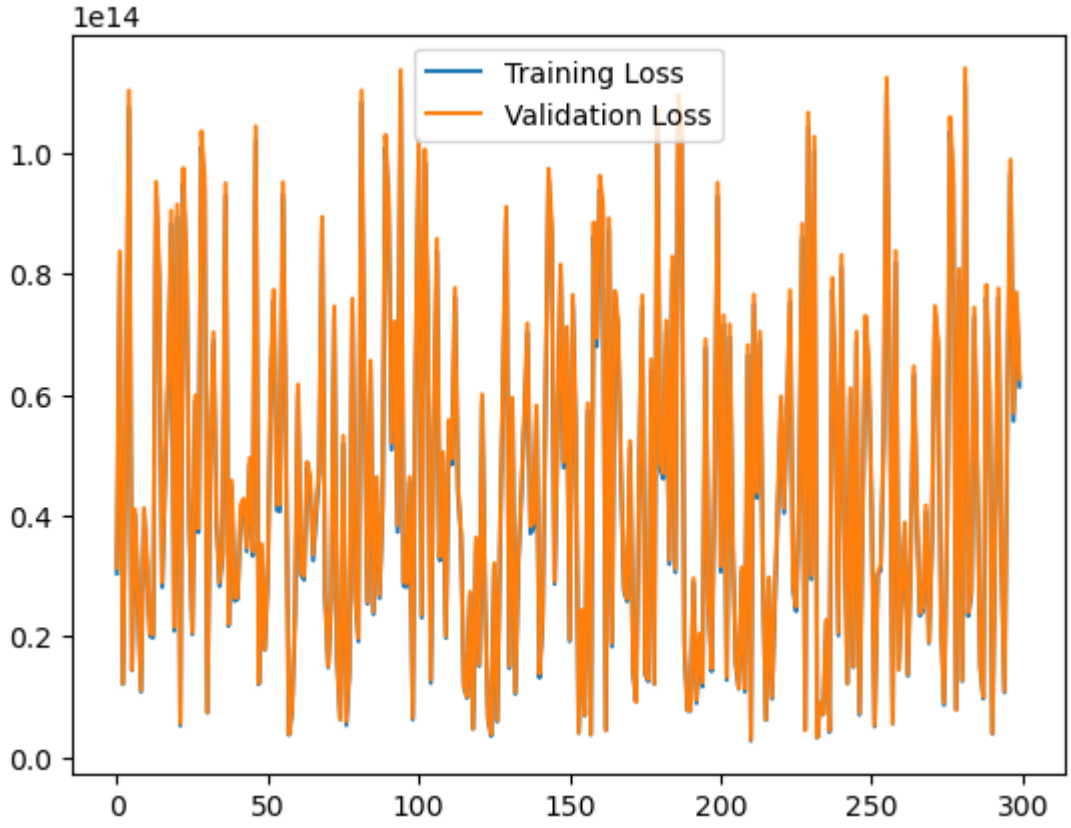

```
RMSE score on test data: 9.397934997545254e+17
Polynomial degree =: 2
Penalty: None
Learning Rate: 0.001
```



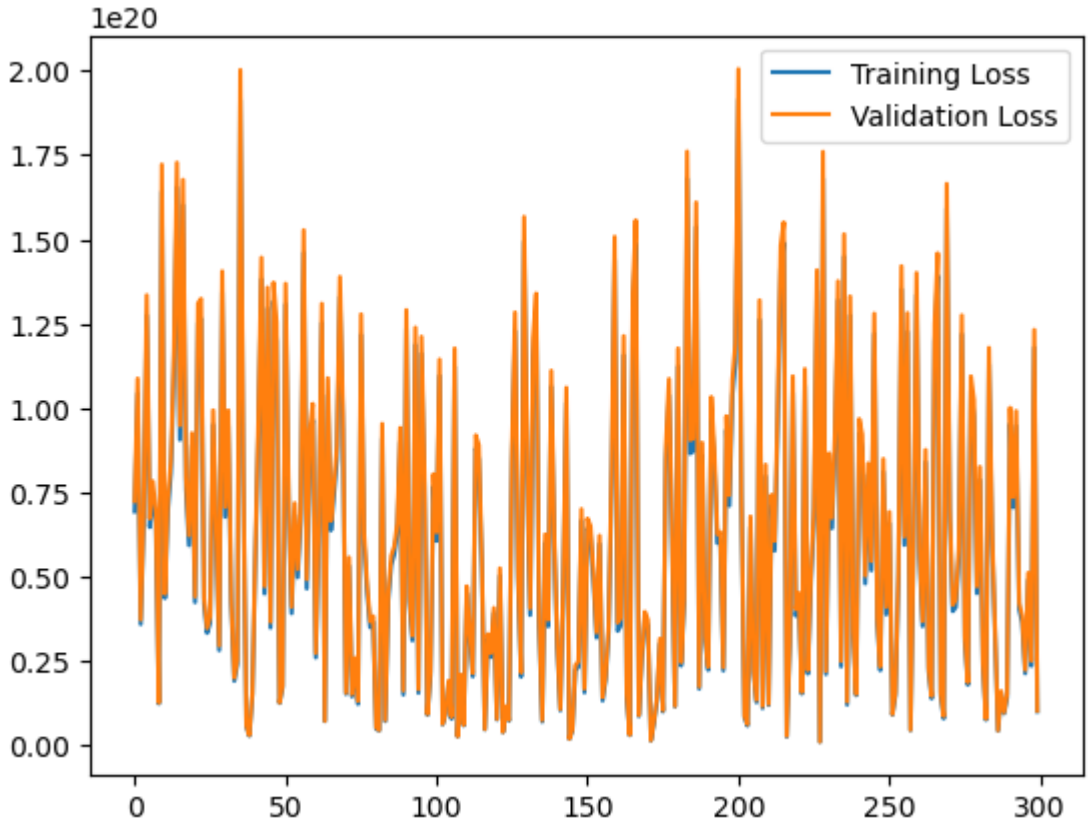

```
RMSE score on test data: 1411997102619.439
Polynomial degree =: 5
Penalty: None
Learning Rate: 0.001
```



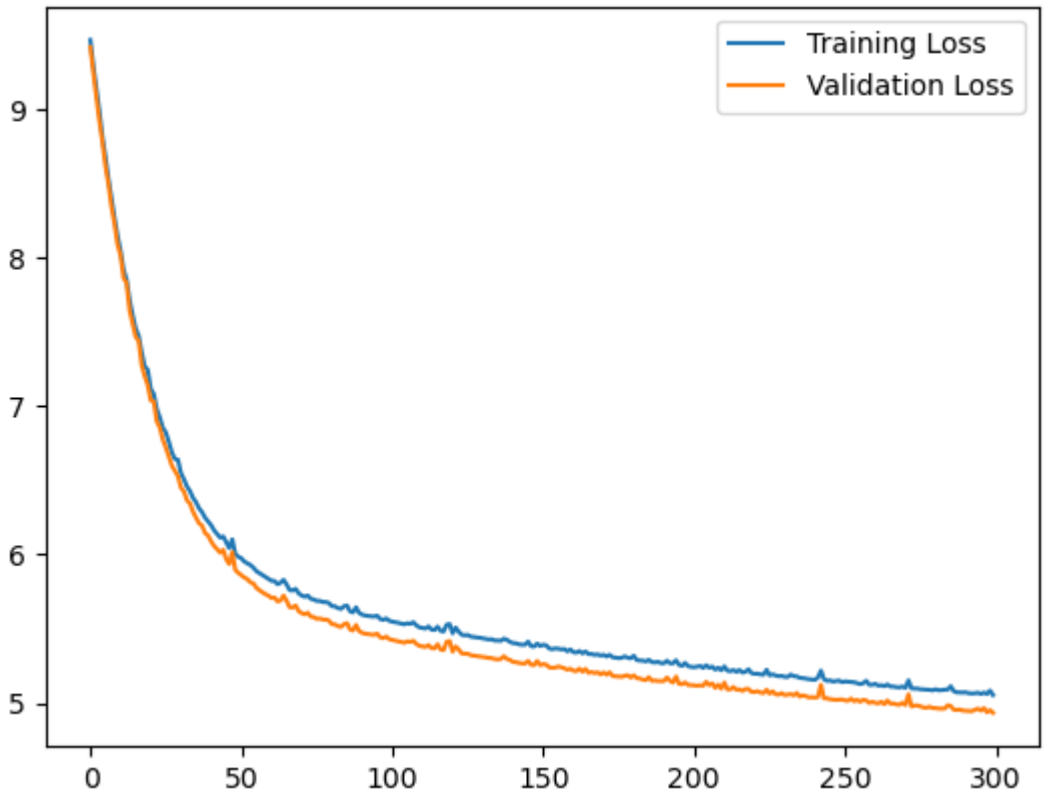
```
RMSE score on test data: 4.524759715010909e+18
Polynomial degree =: 2
Penalty: None
Learning Rate: 0.01
```



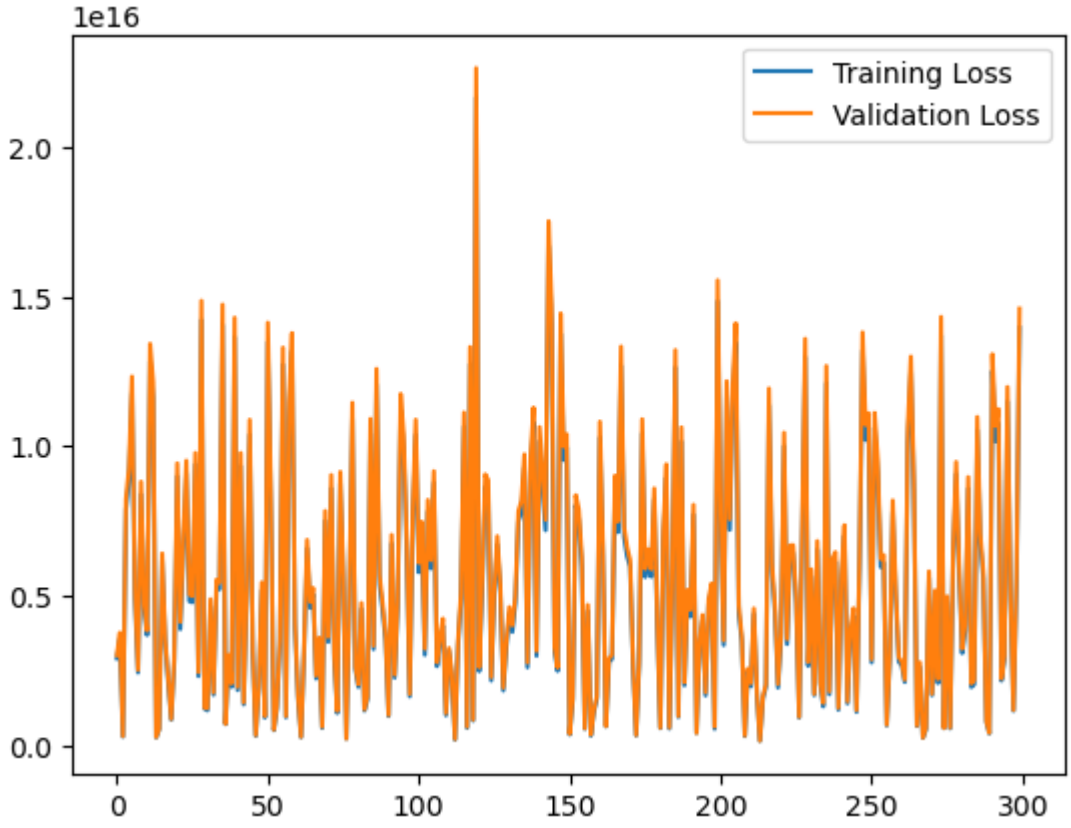

```
RMSE score on test data: 60805219622965.67
Polynomial degree =: 5
Penalty: None
Learning Rate: 0.01
```



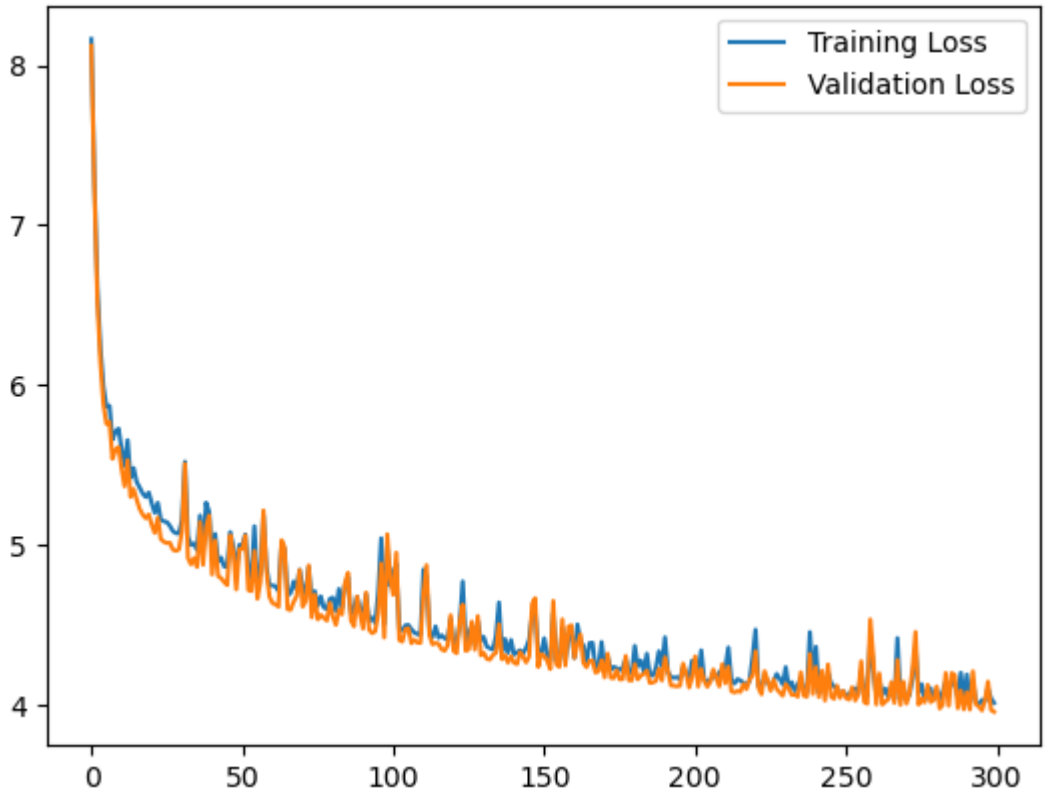

```
RMSE score on test data: 1.0193444668765428e+19
Polynomial degree =: 2
Penalty: l1
Learning Rate: 1e-06
```



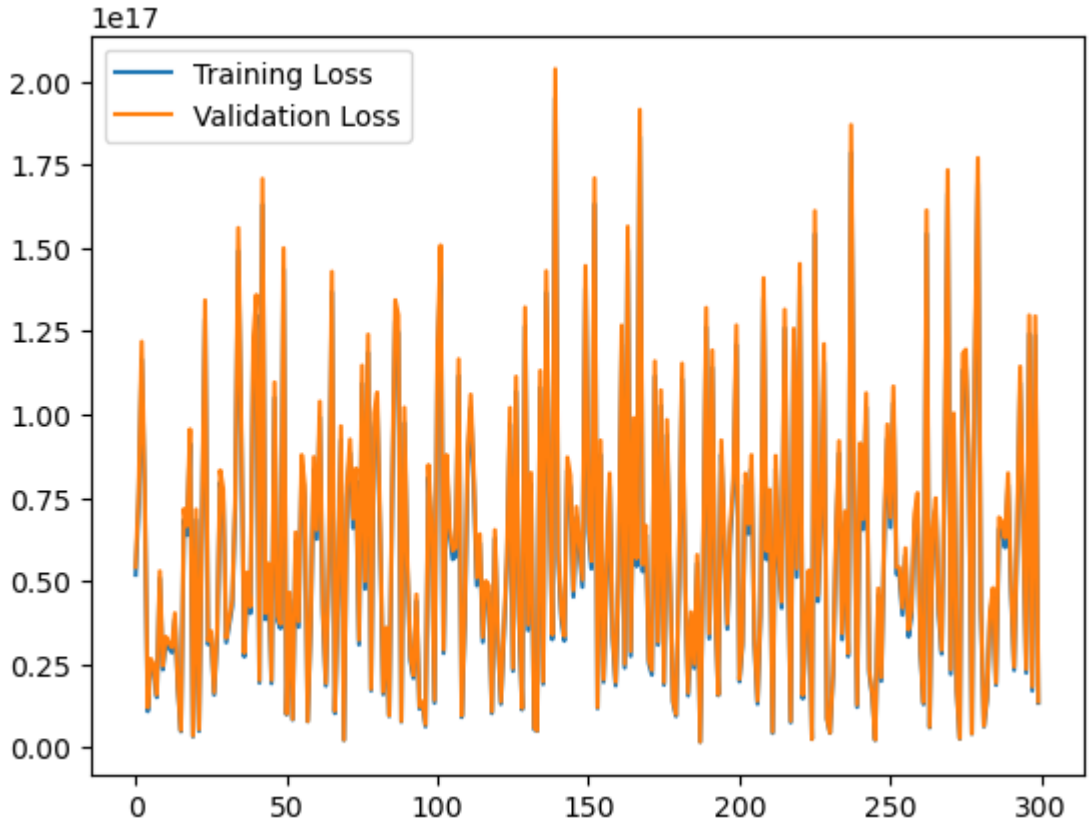
```
RMSE score on test data: 4.966885500381611
Polynomial degree =: 5
Penalty: l1
Learning Rate: 1e-06
```




```
RMSE score on test data: 1.3761579679256478e+16
Polynomial degree =: 2
Penalty: l1
Learning Rate: 1e-05
```

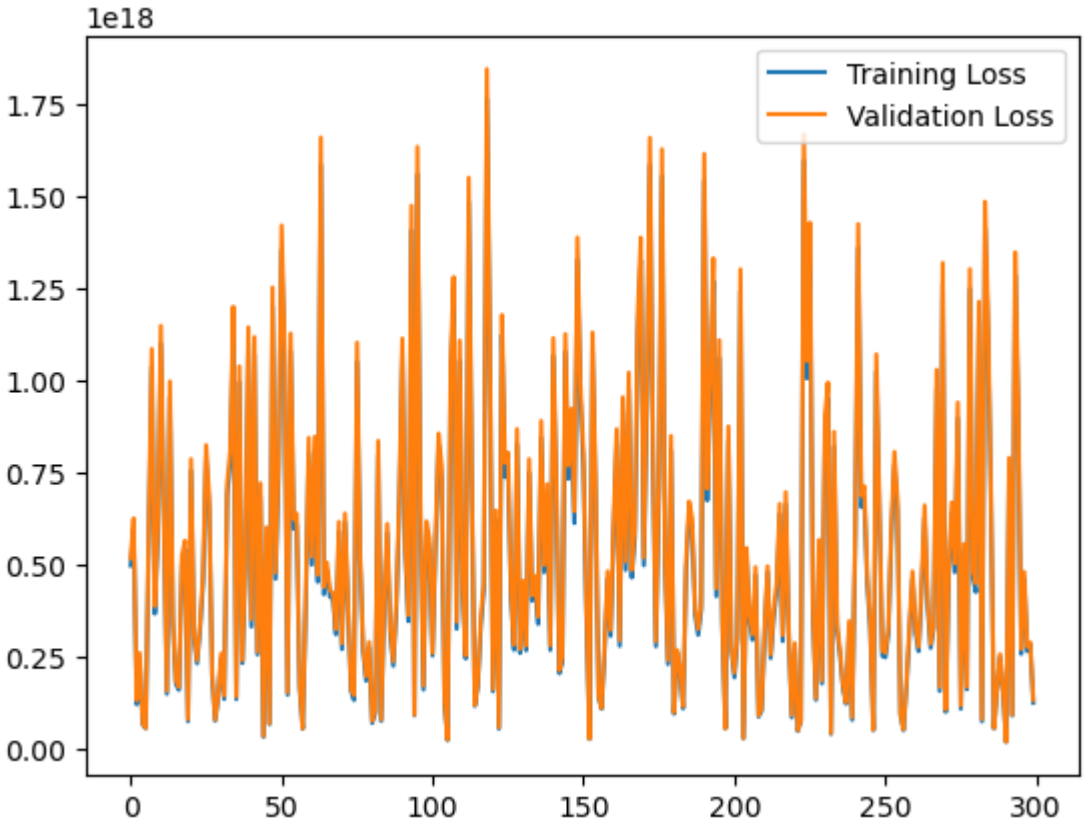



```
RMSE score on test data: 3.8999474542505728
Polynomial degree =: 5
Penalty: l1
Learning Rate: 1e-05
```

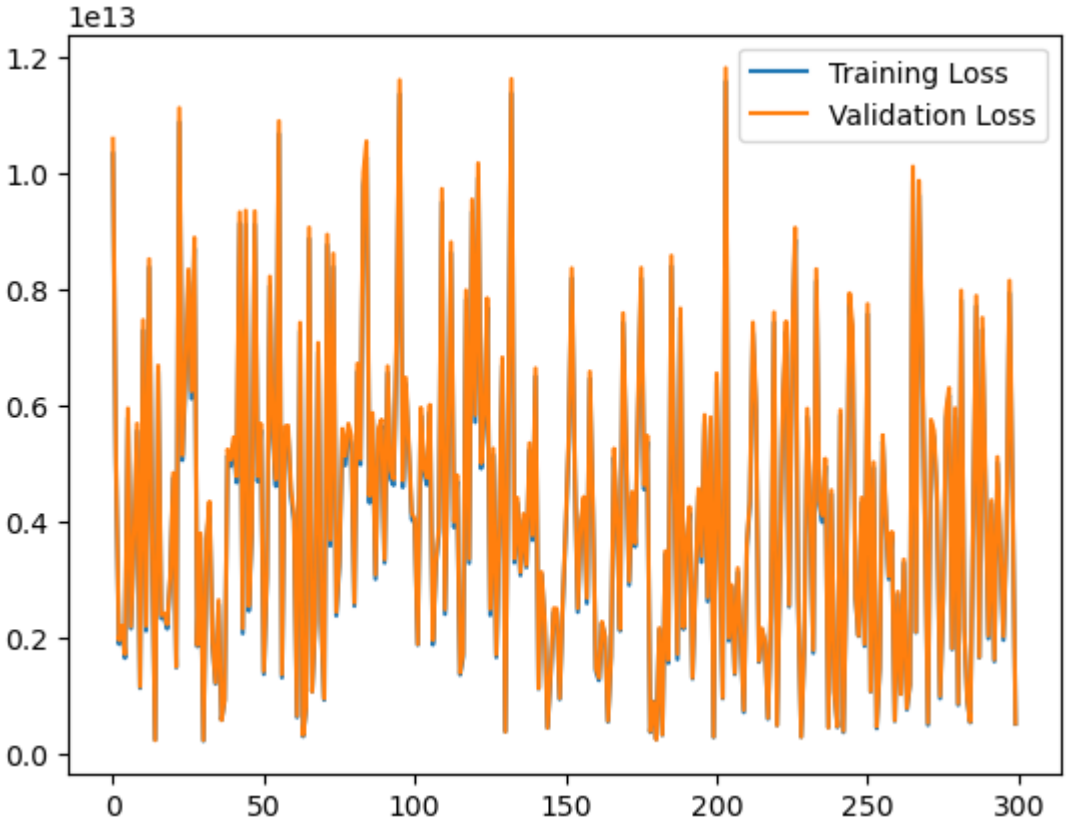


```
RMSE score on test data: 1.2971620554358054e+16
Polynomial degree =: 2
Penalty: l1
Learning Rate: 0.0001
```

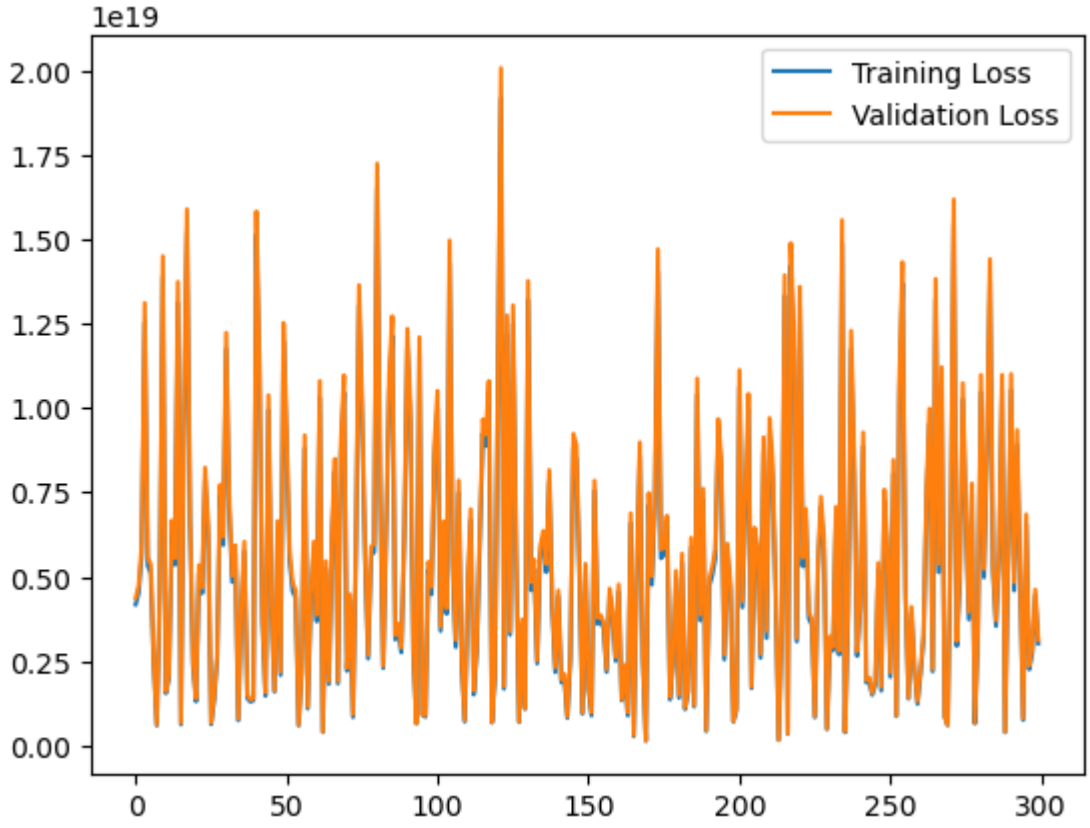

```
RMSE score on test data: 3.770076874012981
Polynomial degree =: 5
Penalty: l1
Learning Rate: 0.0001
```



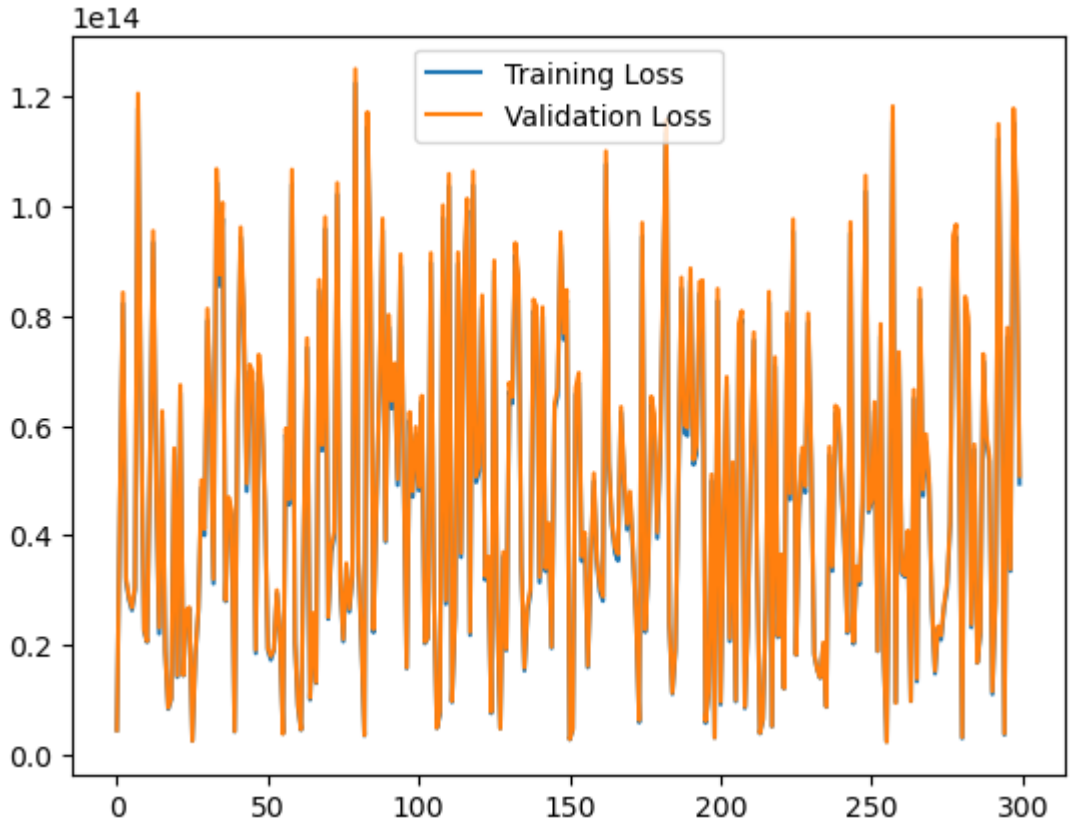

```
RMSE score on test data: 1.2534510663731338e+17
Polynomial degree =: 2
Penalty: l1
Learning Rate: 0.001
```



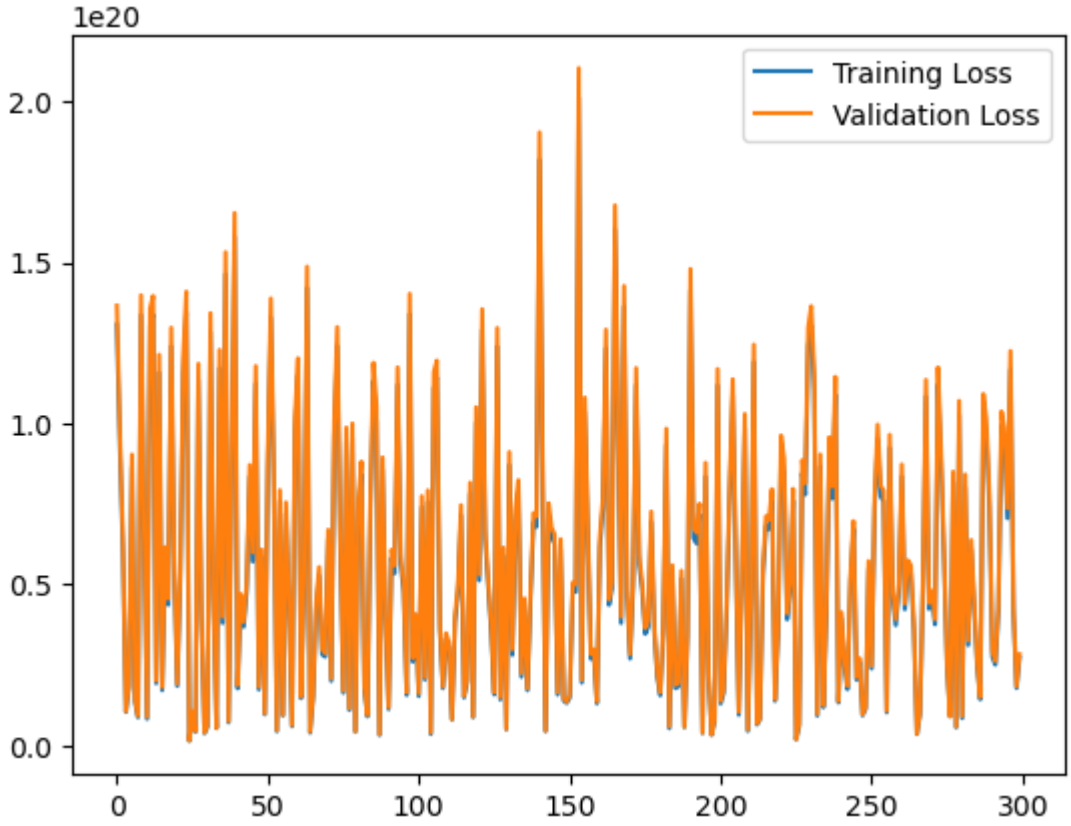
```
RMSE score on test data: 493769917474.58276
Polynomial degree =: 5
Penalty: l1
Learning Rate: 0.001
```



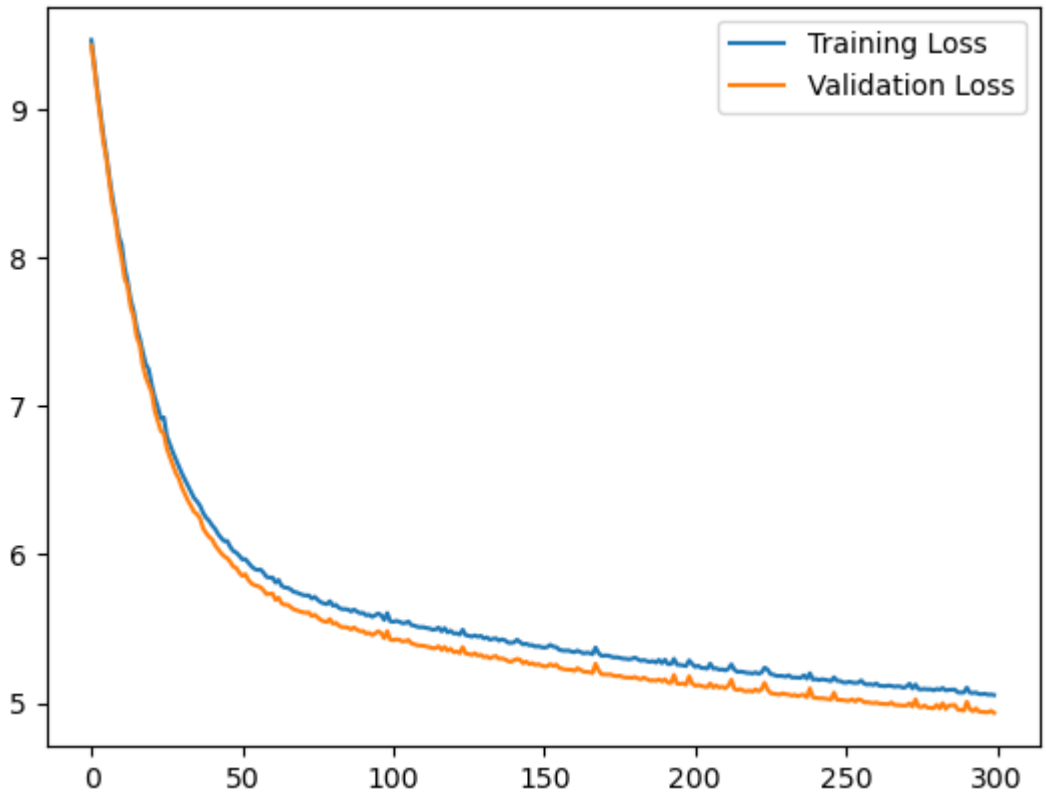

```
RMSE score on test data: 3.0020947762949115e+18
Polynomial degree =: 2
Penalty: l1
Learning Rate: 0.01
```



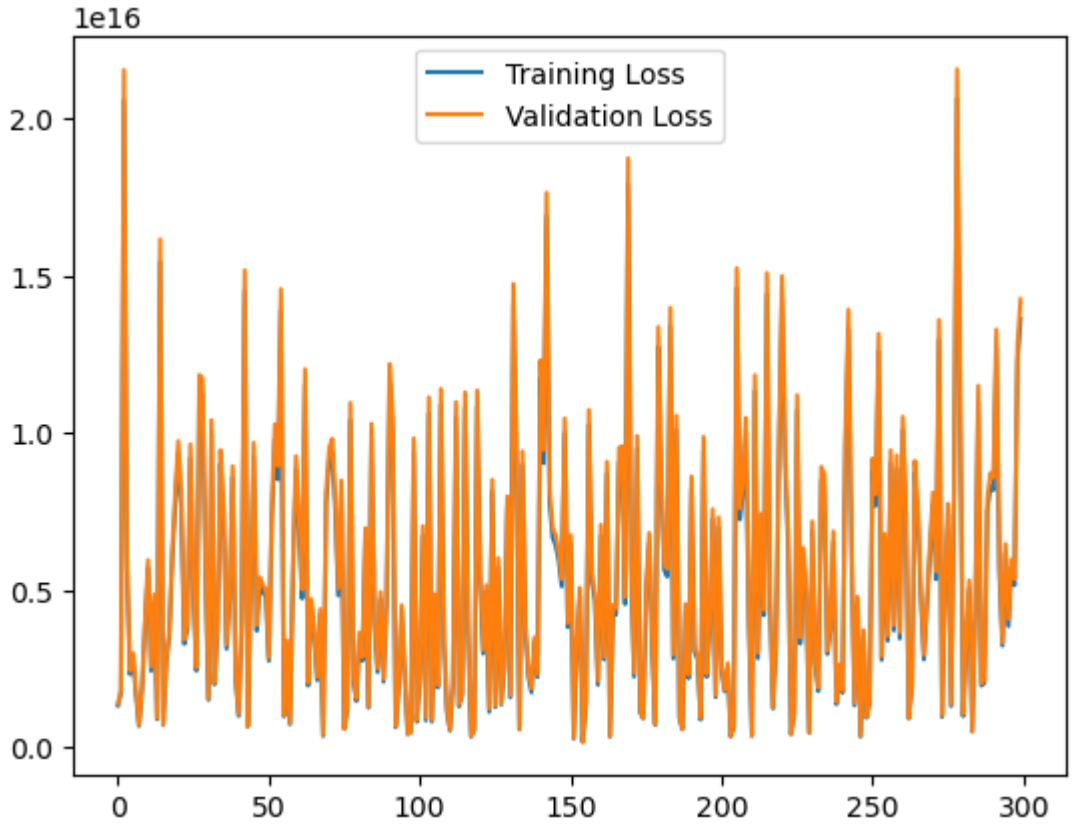

```
RMSE score on test data: 48963725981110.8
Polynomial degree =: 5
Penalty: l1
Learning Rate: 0.01
```



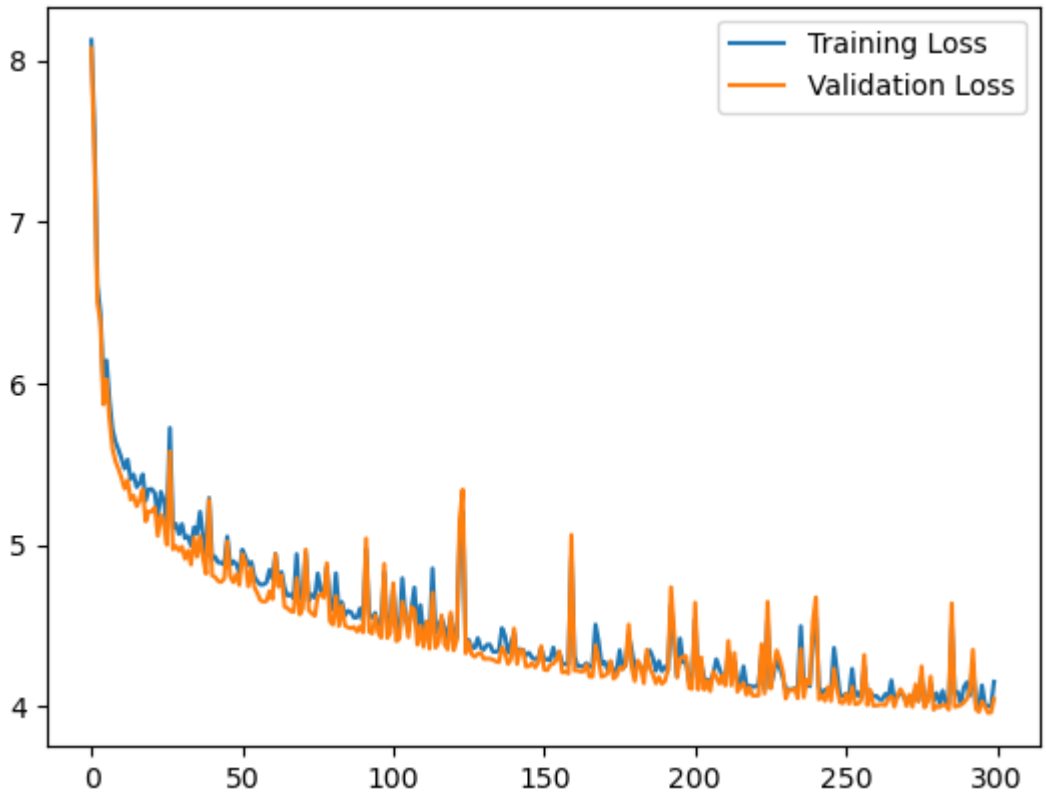
```
RMSE score on test data: 2.7161616109313794e+19
Polynomial degree =: 2
Penalty: l2
Learning Rate: 1e-06
```



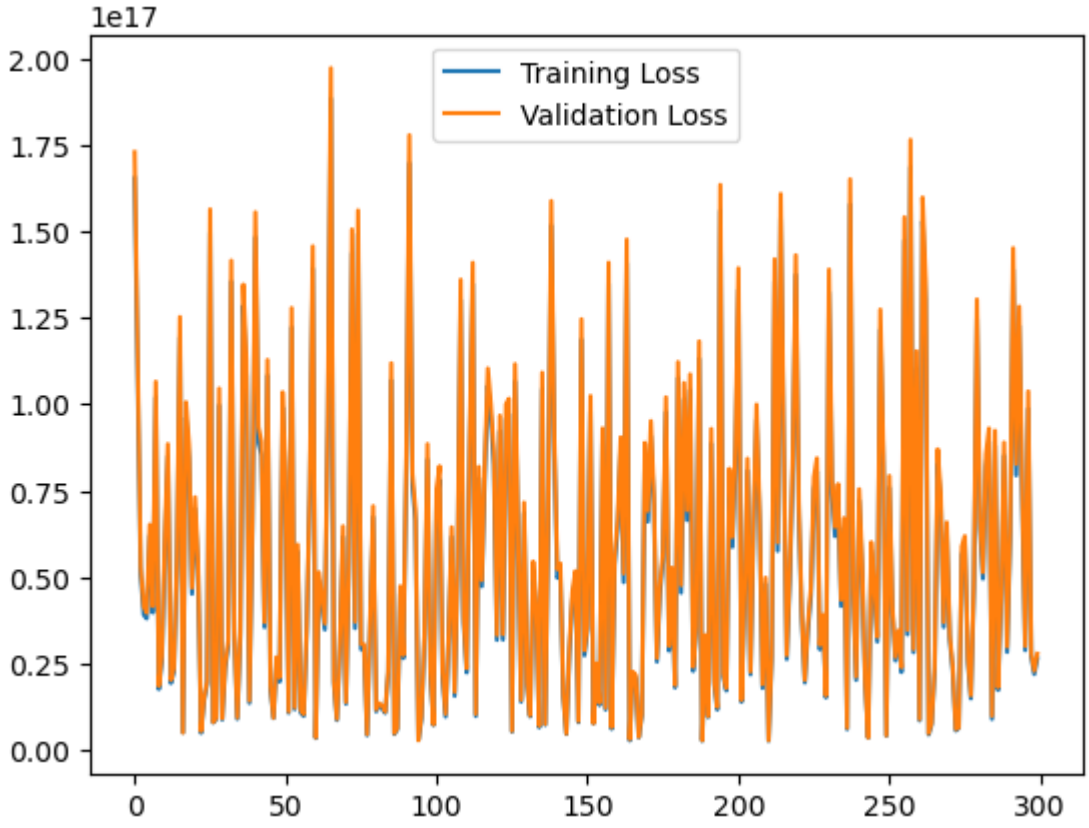

```
RMSE score on test data: 4.967026929673772
Polynomial degree =: 5
Penalty: l2
Learning Rate: 1e-06
```



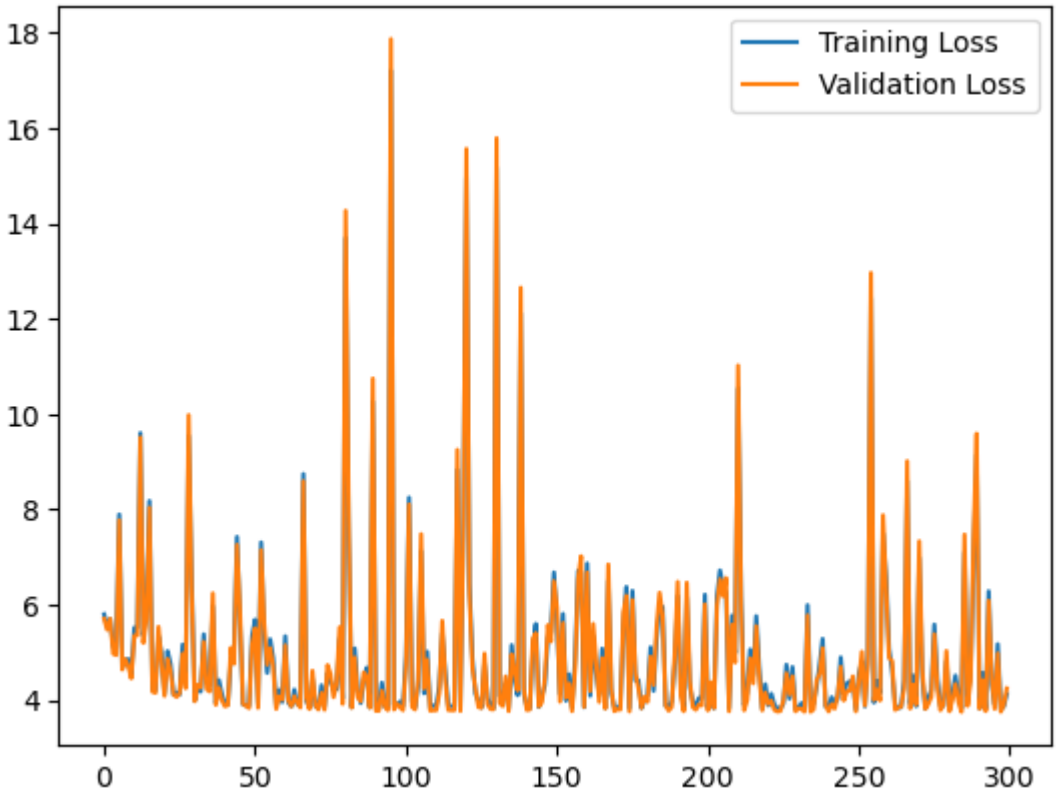

```
RMSE score on test data: 1.344293853433627e+16
Polynomial degree =: 2
Penalty: l2
Learning Rate: 1e-05
```



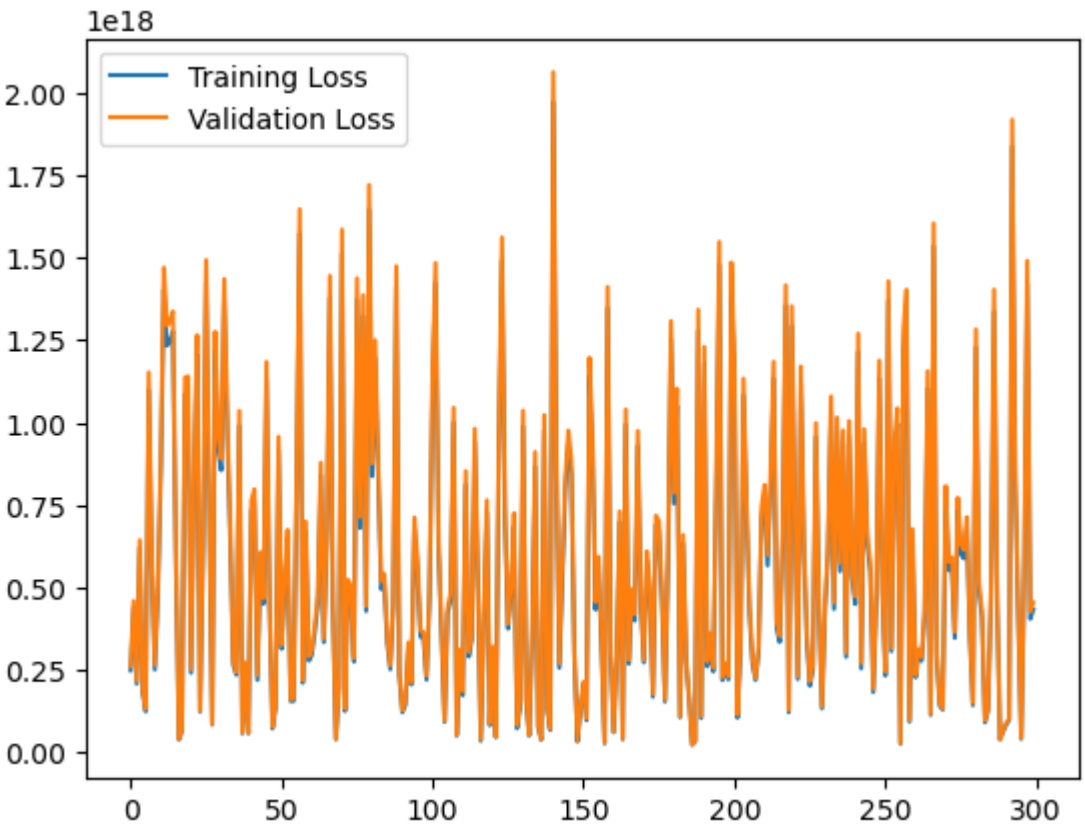
```
RMSE score on test data: 4.02519366708091
Polynomial degree =: 5
Penalty: l2
Learning Rate: 1e-05
```



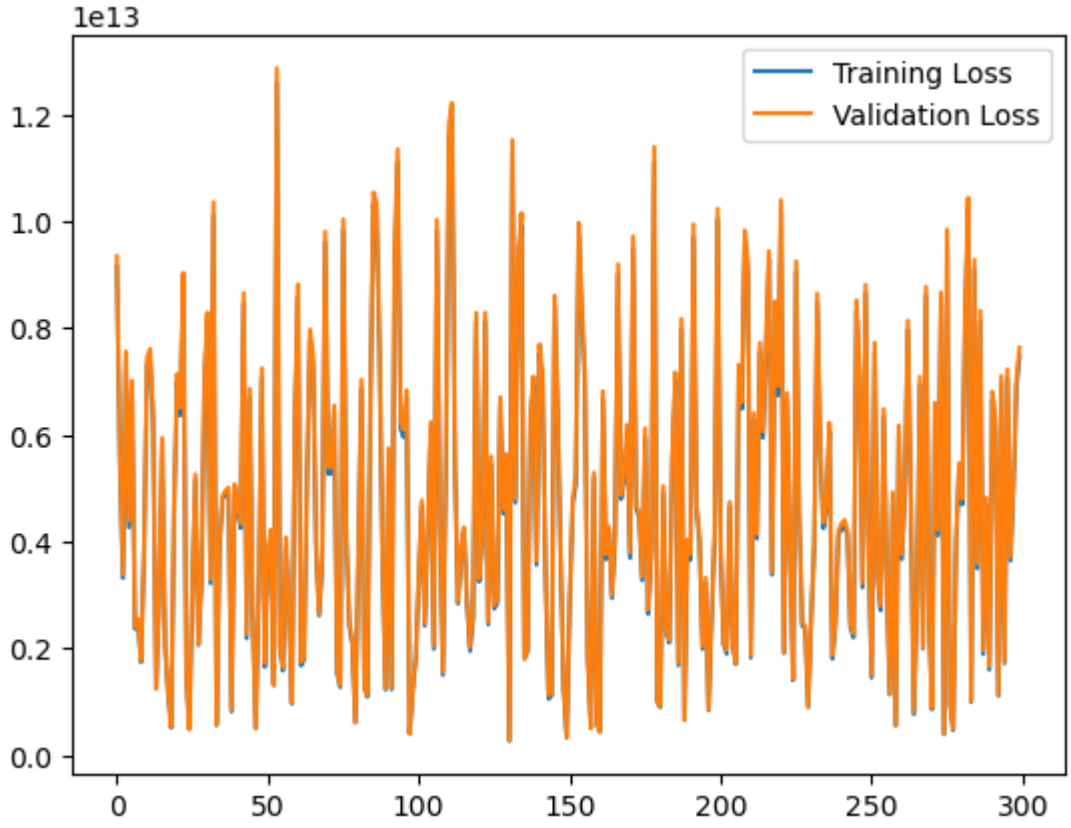

```
RMSE score on test data: 2.630551590131597e+16
Polynomial degree =: 2
Penalty: l2
Learning Rate: 0.0001
```



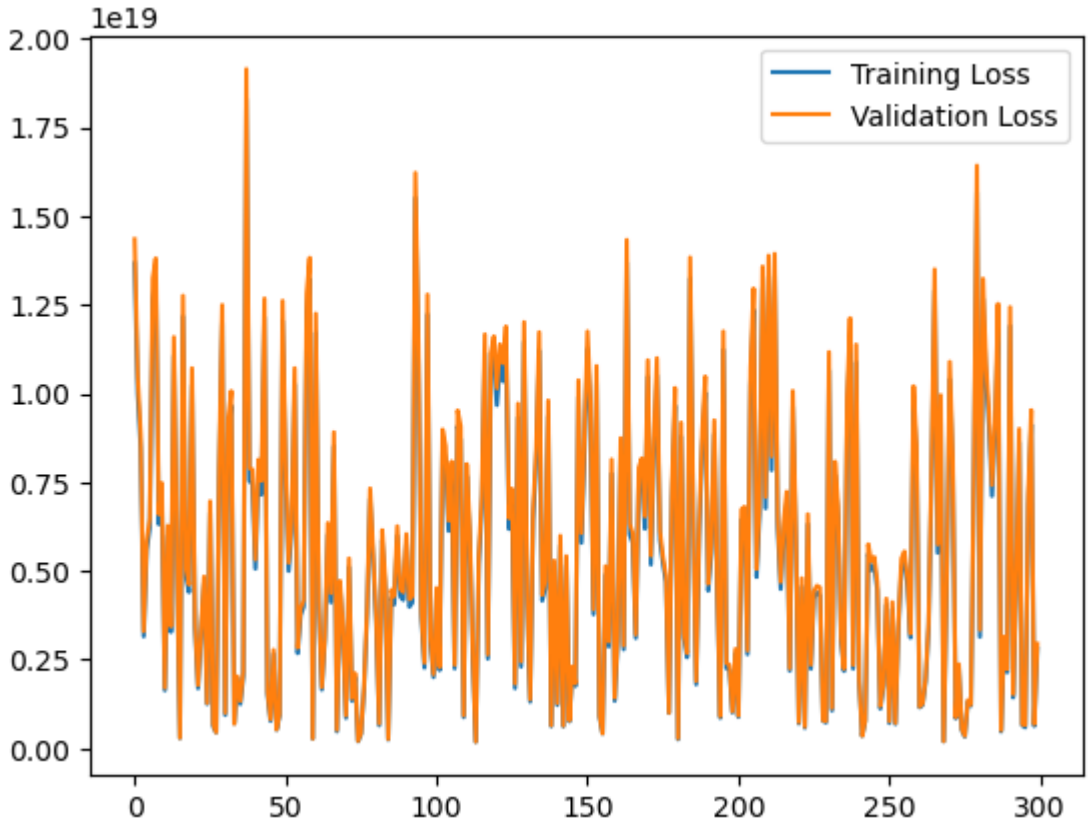

```
RMSE score on test data: 3.9898329295262225
Polynomial degree =: 5
Penalty: l2
Learning Rate: 0.0001
```



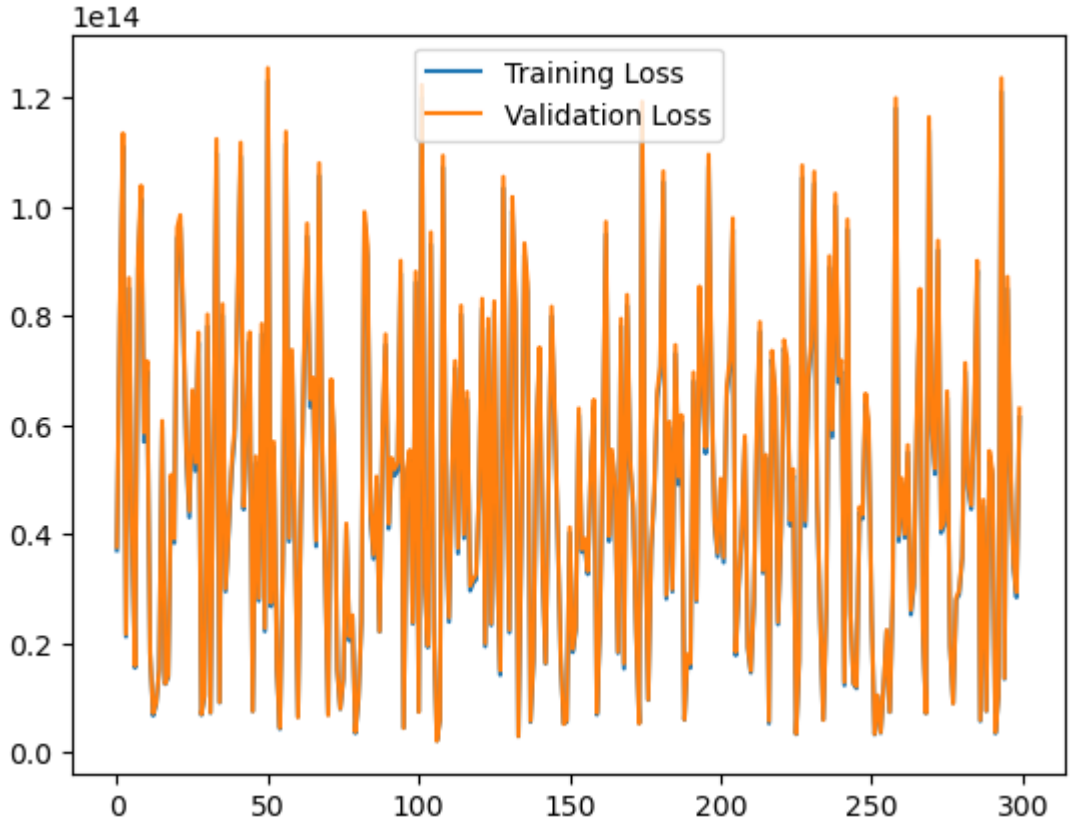
```
RMSE score on test data: 4.283540782282905e+17
Polynomial degree =: 2
Penalty: l2
Learning Rate: 0.001
```



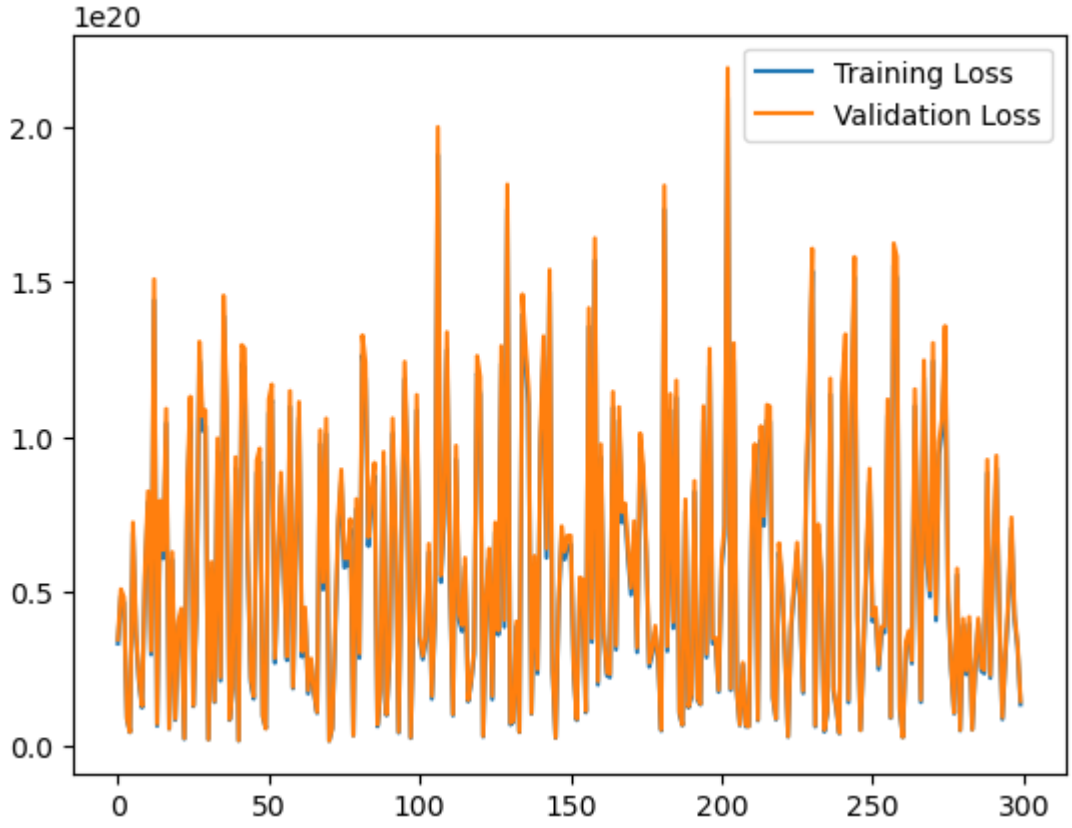

```
RMSE score on test data: 7430565722525.234
Polynomial degree =: 5
Penalty: l2
Learning Rate: 0.001
```




```
RMSE score on test data: 2.7507114710881167e+18
Polynomial degree =: 2
Penalty: l2
Learning Rate: 0.01
```



```
RMSE score on test data: 60868520316317.58
Polynomial degree =: 5
Penalty: l2
Learning Rate: 0.01
```

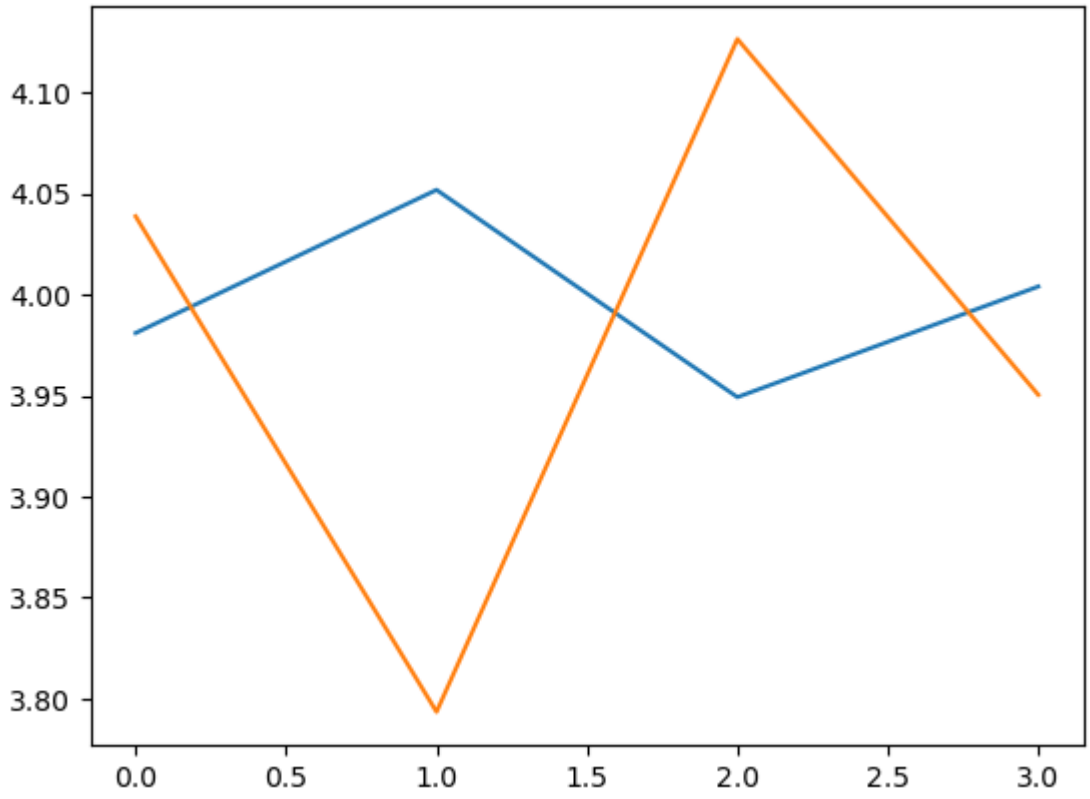



```
RMSE score on test data: 1.286544185438726e+19
```

In [33]:

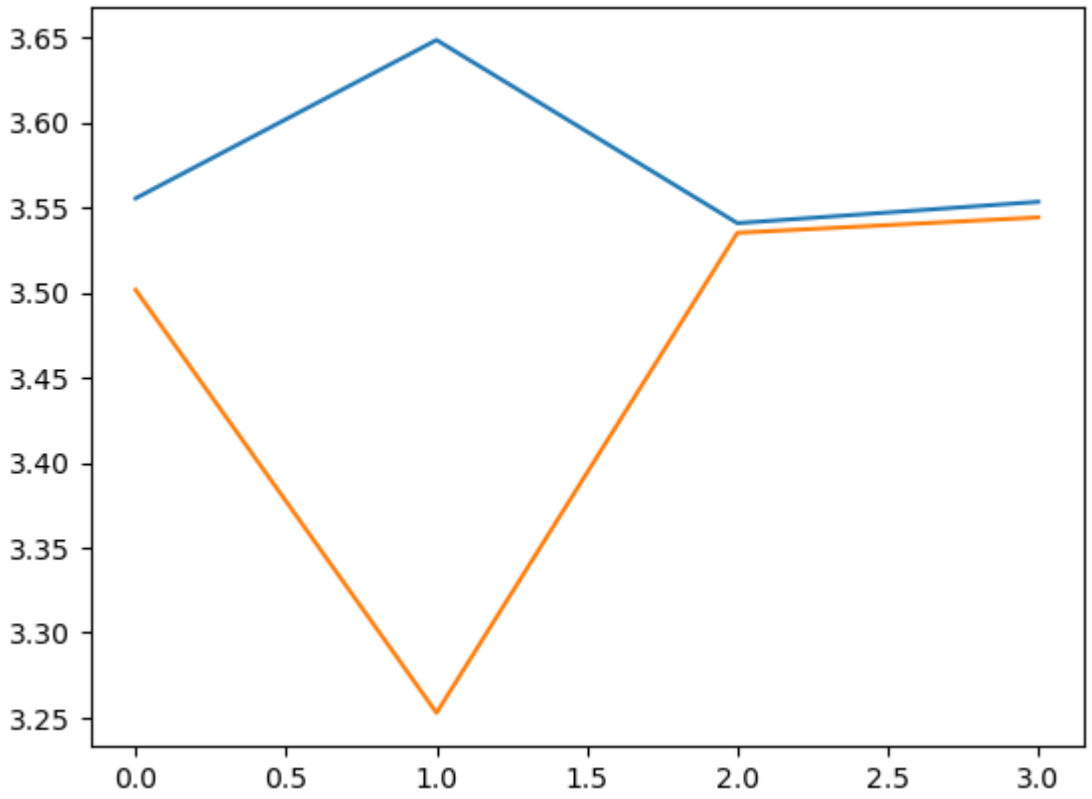
```
# Polynomial regression using elastic net

poly_degree = [2,5,10]

for r in poly_degree:
    print("Degree:", r)
    train_plot = []
    val_plot = []
    poly_features = PolynomialFeatures(degree=r, include_bias=False)
    el_net = ElasticNet()
    for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
        print(cnt)
        x = x_train_2[var[0],:]
        y = y_train_2[var[0]]
        x_val = x_train_2[var[1],:]
        y_val = y_train_2[var[1]]

        X_poly_1 = poly_features.fit_transform(x)
        el_net.fit(X_poly_1, y)
        y_pred_train=el_net.predict(X_poly_1)

        X_poly_2 = poly_features.fit_transform(x_val)
        el_net.fit(X_poly_2, y_val)
        y_pred_val=el_net.predict(X_poly_2)

        print("Train error:",np.sqrt(mse(y_pred_train,y)))
        print("Validation error:",np.sqrt(mse(y_pred_val,y_val)))
        train_plot.append(np.sqrt(mse(y_pred_train,y)))
        val_plot.append(np.sqrt(mse(y_pred_val,y_val)))

        X_test_poly = poly_features.fit_transform(x_test_2)
        y_pred_test = el_net.predict(X_test_poly)
        metric_dict['Polynomial degree for elastic net= '+str(r)] = np.sq
    plt.plot([0,1,2,3], train_plot, label = "Training Error")
    plt.plot([0,1,2,3], val_plot, label = "Val Error")
    plt.show()
    print('\n')
```

```
Degree: 2
0
Train error: 3.9809792086515667
Validation error: 4.038732445923404
1
Train error: 4.0518261122264905
Validation error: 3.793469635660633
2
Train error: 3.949243056023887
Validation error: 4.126506536817681
3
Train error: 4.003997486058499
Validation error: 3.9503997285724317
```



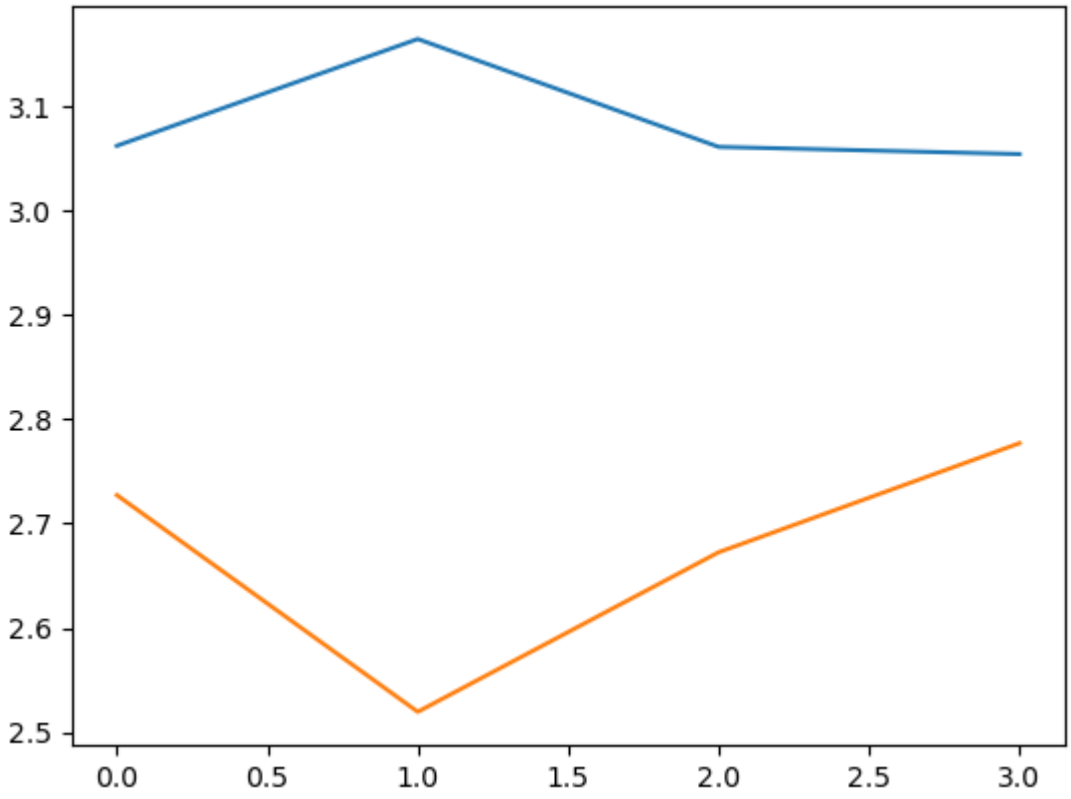

```
Degree: 5
0

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 6.673e+03, tolerance: 5.971e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.213e+03, tolerance: 2.052e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.5554164465245663
Validation error: 3.5015473359588265
1

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 6.990e+03, tolerance: 6.179e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.851e+03, tolerance: 1.870e+00
  model = cd_fast.enet_coordinate_descent(
```

```
Train error: 3.64850845473893
Validation error: 3.2528212536503998
2

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 6.639e+03, tolerance: 5.880e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.118e+03, tolerance: 2.163e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.5408035796202677
Validation error: 3.5351264253986843
3

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 6.552e+03, tolerance: 6.105e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.5533565411661088
Validation error: 3.5441836067983408

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 2.243e+03, tolerance: 1.902e+00
  model = cd_fast.enet_coordinate_descent(
```



```
Degree: 10
0

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.136e+03, tolerance: 5.971e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.436e+03, tolerance: 2.052e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.062281414024047
Validation error: 2.727345524226253
1

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.464e+03, tolerance: 6.179e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.232e+03, tolerance: 1.870e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.1646464802205694
Validation error: 2.5195633553373007
2

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.142e+03, tolerance: 5.880e+00
  model = cd_fast.enet_coordinate_descent(
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.406e+03, tolerance: 2.163e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.061226395047278
Validation error: 2.6725597341931024
3

C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 5.103e+03, tolerance: 6.105e+00
  model = cd_fast.enet_coordinate_descent(

Train error: 3.0543335150222743
Validation error: 2.7771198215282
```

```
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_d
escent.py:647: ConvergenceWarning: Objective did not converge. You might want
to increase the number of iterations, check the scale of the features or cons
ider increasing regularisation. Duality gap: 1.528e+03, tolerance: 1.902e+00
  model = cd_fast.enet_coordinate_descent(
```



Answer:- The results show that as the degree of the polynomial increases, the training error decreases, but the validation error may start to increase, indicating overfitting.

In the case of polynomial regression using ridge and lasso, regularization has been applied to prevent overfitting. The results show that both ridge and lasso help to reduce the validation error compared to polynomial regression without regularization.

It is worth noting that for some of the results, the validation error is much larger than the train error, which could indicate that the model is overfitting the training data. In such cases, it may be necessary to further tune the hyperparameters of the model or collect more data to improve the performance.

Part 7

Make predictions of the labels on the test data, using the trained model with chosen hyperparameters. Summarize performance using the appropriate evaluation metric. Discuss the results. Include thoughts about what further can be explored to increase performance.

In [34]:
```
# Making predictions on the test dataset. We have used all of our previusl
# calculated the RMSE scores.

ridge = Ridge()
for cnt,var in enumerate(kf.split(x_train_2,y_train_2)):
    x = x_train_2[var[0],:]
    y = y_train_2[var[0]]
    x_val = x_train_2[var[1],:]
    y_val = y_train_2[var[1]]
    ridge.fit(x, y)
    y_pred_train = ridge.predict(x)
    ridge.fit(x_val, y_val)
    y_pred_val = ridge.predict(x_val)
```

In [35]:
```
y_pred_test = ridge.predict(x_test_2)
print("Error:",np.sqrt(mse(y_pred_test,y_test_2)))
```

```
Error: 3.6959852293570363
```

In [36]:

```
# Now we will look at the RMSE scores for all the models on our test datas

metric_dict
```

Out[36]:

```
{'Linear regression': 3.693685608175782,
 'SGD with learning rate =1e-06': 3.96966749079844,
 'SGD with learning rate =1e-05': 3.7670963034078917,
 'SGD with learning rate =0.0001': 3.689297182286318,
 'SGD with learning rate =0.001': 3.9322983646063134,
 'SGD with learning rate =0.01': 4.374503043689119,
 'SGD with learning rate =1e-06 and penalty_factor =None': 3.969665182012894
5,
 'SGD with learning rate =1e-05 and penalty_factor =None': 3.767312827919267,
 'SGD with learning rate =0.0001 and penalty_factor =None': 3.69938914887747
7,
 'SGD with learning rate =0.001 and penalty_factor =None': 3.783067499048873
3,
 'SGD with learning rate =0.01 and penalty_factor =None': 3.836960346031612,
 'SGD with learning rate =1e-06 and penalty_factor =l1': 3.9696603920934166,
 'SGD with learning rate =1e-05 and penalty_factor =l1': 3.768378646001259,
 'SGD with learning rate =0.0001 and penalty_factor =l1': 3.7035761649960635,
 'SGD with learning rate =0.001 and penalty_factor =l1': 3.7274527732688942,
 'SGD with learning rate =0.01 and penalty_factor =l1': 3.727742795406936,
 'SGD with learning rate =1e-06 and penalty_factor =l2': 3.96968115196651,
 'SGD with learning rate =1e-05 and penalty_factor =l2': 3.768136952869328,
 'SGD with learning rate =0.0001 and penalty_factor =l2': 3.689837070793864,
 'SGD with learning rate =0.001 and penalty_factor =l2': 3.797161489890803,
 'SGD with learning rate =0.01 and penalty_factor =l2': 3.809220893603164,
 'Ridge lr=0': 3.704811795189289,
 'Ridge lr=0.001': 3.70477088748351,
 'Ridge lr=1': 3.6959852293570363,
 'Ridge lr=10': 3.752850778603473,
 'Ridge lr=100': 4.027764736515409,
 'Lasso lr=0': 3.7048117951892854,
 'Lasso lr=0.001': 3.7025921660801306,
 'Lasso lr=1': 3.9493545094080553,
 'Lasso lr=10': 7.815572369551083,
 'Lasso lr=100': 7.815572369551083,
 'ElasticNet lr=0': 3.7048117951892854,
 'ElasticNet lr=0.001': 3.698810421197111,
 'ElasticNet lr=1': 4.417471685851984,
 'ElasticNet lr=10': 7.446647805600527,
 'ElasticNet lr=100': 7.815572369551083,
 'Polynomial degree= 2': 3.859292154675337,
 'Polynomial degree= 5': 47.12077004638479,
 'Polynomial degree= 10': 44.19804017615544,
 'Polynomial degree for ridge= 2': 3.622974516903029,
 'Polynomial degree for ridge= 5': 4.11366726420557,
 'Polynomial degree for ridge= 10': 9.058475712119527,
 'Polynomial degree for Lasso = 2': 3.8232737973216584,
 'Polynomial degree for Lasso = 5': 3.5183674748612876,
 'Polynomial degree for Lasso = 10': 3.889818627513967,
 'Polynomial degree for SGD= 2 Learning_rate= 1e-06 and penalty_term= None':
5.002836638073316,
 'Polynomial degree for SGD= 5 Learning_rate= 1e-06 and penalty_term= None':
770328126591323.4,
 'Polynomial degree for SGD= 2 Learning_rate= 1e-05 and penalty_term= None':
3.9412323193397887,
 'Polynomial degree for SGD= 5 Learning_rate= 1e-05 and penalty_term= None':
3.1996231306365056e+16,
 'Polynomial degree for SGD= 2 Learning_rate= 0.0001 and penalty_term= None':
```

```
3.8693893503500916,
 'Polynomial degree for SGD= 5 Learning_rate= 0.0001 and penalty_term= None':
9.397934997545254e+17,
 'Polynomial degree for SGD= 2 Learning_rate= 0.001 and penalty_term= None':
1411997102619.439,
 'Polynomial degree for SGD= 5 Learning_rate= 0.001 and penalty_term= None':
4.524759715010909e+18,
 'Polynomial degree for SGD= 2 Learning_rate= 0.01 and penalty_term= None': 6
0805219622965.67,
 'Polynomial degree for SGD= 5 Learning_rate= 0.01 and penalty_term= None':
1.0193444668765428e+19,
 'Polynomial degree for SGD= 2 Learning_rate= 1e-06 and penalty_term= l1': 4.
966885500381611,
 'Polynomial degree for SGD= 5 Learning_rate= 1e-06 and penalty_term= l1': 1.
3761579679256478e+16,
 'Polynomial degree for SGD= 2 Learning_rate= 1e-05 and penalty_term= l1': 3.
8999474542505728,
 'Polynomial degree for SGD= 5 Learning_rate= 1e-05 and penalty_term= l1': 1.
2971620554358054e+16,
 'Polynomial degree for SGD= 2 Learning_rate= 0.0001 and penalty_term= l1':
3.770076874012981,
 'Polynomial degree for SGD= 5 Learning_rate= 0.0001 and penalty_term= l1':
1.2534510663731338e+17,
 'Polynomial degree for SGD= 2 Learning_rate= 0.001 and penalty_term= l1': 49
3769917474.58276,
 'Polynomial degree for SGD= 5 Learning_rate= 0.001 and penalty_term= l1': 3.
0020947762949115e+18,
 'Polynomial degree for SGD= 2 Learning_rate= 0.01 and penalty_term= l1': 489
63725981110.8,
 'Polynomial degree for SGD= 5 Learning_rate= 0.01 and penalty_term= l1': 2.7
161616109313794e+19,
 'Polynomial degree for SGD= 2 Learning_rate= 1e-06 and penalty_term= l2': 4.
967026929673772,
 'Polynomial degree for SGD= 5 Learning_rate= 1e-06 and penalty_term= l2': 1.
344293853433627e+16,
 'Polynomial degree for SGD= 2 Learning_rate= 1e-05 and penalty_term= l2': 4.
02519366708091,
 'Polynomial degree for SGD= 5 Learning_rate= 1e-05 and penalty_term= l2': 2.
630551590131597e+16,
 'Polynomial degree for SGD= 2 Learning_rate= 0.0001 and penalty_term= l2':
3.9898329295262225,
 'Polynomial degree for SGD= 5 Learning_rate= 0.0001 and penalty_term= l2':
4.283540782282905e+17,
 'Polynomial degree for SGD= 2 Learning_rate= 0.001 and penalty_term= l2': 74
30565722525.234,
 'Polynomial degree for SGD= 5 Learning_rate= 0.001 and penalty_term= l2': 2.
7507114710881167e+18,
 'Polynomial degree for SGD= 2 Learning_rate= 0.01 and penalty_term= l2': 608
68520316317.58,
 'Polynomial degree for SGD= 5 Learning_rate= 0.01 and penalty_term= l2': 1.2
86544185438726e+19,
 'Polynomial degree for elastic net= 2': 3.818430473841465,
 'Polynomial degree for elastic net= 5': 3.5010190728760713,
 'Polynomial degree for elastic net= 10': 4.004234525143996}
```

Answer: Polynomial degree for Elastic net= 5 works best on our test data, since they have the lowest RMSE scores.

We can try and add more data points so that we have a more robust understanding of the dataset. More amount of data always leads to better training and better results. We can preprocess the data further and try removing skewness or imputing null values. We can do in-depth EDA to find which features affect the target variable more and just feed those features into our model. These are some ways by which we can improve the efficiency of our models.

In [ ]:
```
1
```