1a) Data = { (1.15, 0.1), -1,
(-1.1, 0.7), +1,
(1, -0.2), -1,
(-1.05, 0.4), +1,
(0.8, 1.1), -1,
(-1.35, 1.9), +1,
(1.1, 1.1), (-1),
(-2, 1.5), +1,
(1.8, 0.5), -1,
(-1.65, 1.9), +1 }

$C_1 = -1$
$C_2 = +1$

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Constrained optimization problem:

$$\max_{\hat{\gamma}, w, b} \frac{\hat{\gamma}}{\|w\|}$$

$$\text{s.t} \quad y^{(i)}(w^T x^{(i)} + b) \geq \hat{\gamma} \quad i = 1, \dots n.$$

its harder to solve this because it gonna give infinity Solution.
it gives lot of solution which is infinity & aren't legal
Hence it can be convert
by Since we can scale functional margin, we can make it 1 and replace maximizing $\frac{q}{\|x\|}$ with minimizing $\|x^2\|$

$$\therefore \min_{\gamma, w, b} \frac{1}{2}\|w\|^2$$

$$\text{s.t} \quad y^{(i)}(w^T x^{(i)} + b) \geq 1, \quad i = 1, \dots n.$$

Constraints are as follows.

$(-1)(1.15 w_0 + 0.1 w_1 + b) \geq 1$
$(+1)(-1.1 w_0 + 0.7 w_1 + b) \geq 1$
$(-1)(1 w_0 + (-0.2) w_1 + b) \geq 1$
$(+1)(-1.05 w_0 + 0.4 w_1 + b) \geq 1$
$(-1)(0.8 w_0 + 1.1 w_1 + b) \geq 1$
$(+1)(-1.35 w_0 + 1.9 w_1 + b) \geq 1$
$(-1)(1.1 w_0 + 1.1 w_1 + b) \geq 1$
$(+1)(-2 w_0 + (+1.5) w_1 + b) \geq 1$
$(-1)(1.8 w_0 + 0.5 w_1 + b) \geq 1$
$(+1)(-1.65 w_0 + 1.9 w_1 + b) \geq 1$

$\Rightarrow$ which can be converted to Lagrangian. ②

$$\mathcal{L}(w,b,\alpha) = \frac{1}{2}\|w\|^2 - \sum_{i=1}^{m} \alpha_i \left[y^{(i)}(w^T x^{(i)} + b) - 1\right]$$

$$= \frac{1}{2}\|w\|^2 - \sum_{i=1}^{10} \alpha_i \left[y^{(i)}(w^T x^{(i)} + b) - 1\right]$$

$$= \frac{1}{2}\|w\|^2 - \alpha_1\left[(-1)(1.15w_0 + 0.1w_1 + b) - 1\right] - \alpha_2\left[(+1)(-1.1w_0 + 0.7w_1 + b) - 1\right] - \alpha_3\left[(-1)(1w_0 - 0.2w_1 + b) - 1\right]$$
$$- \alpha_4\left[(+1)(-1.05w_0 + 0.4w_1 + b) - 1\right] - \alpha_5\left[(-1)[0.8w_0 + 1.1w_1 + b) - 1\right] - \alpha_6\left[(+1)[-1.35w_0 + 1.9w_1 + b] - 1\right] - \alpha_7$$
$$\left[(-1)(-1.1w_0 + 1.1w_1 + b) - 1\right] - \alpha_8\left[+1(-2w_0 + 1.5w_1 + b) - 1\right] - \alpha_9\left[(-1)(+1.8w_0 + 0.5w_1 + b) - 1\right] - \alpha_{10}$$
$$\left[(+1)(-1.65w_0 + 1.9w_1 + b) - 1\right]$$

$\rightarrow$ to optimize, where the constraint become part of function. $\frac{1}{2}\|w\|^2 \rightarrow$ minimize the square of parameter vector.

1 penalty term per datapoint in training set.

$$\sum_{i=1}^{m} \alpha_i (y^{(i)}(w^T x^{(i)} + b) - 1 \geq 0$$

Given α^* we get w^* & b^*.

$$w = \sum_{i=1}^{m} \alpha_i y^{(i)} x^{(i)} \qquad \sum_{i=1}^{m} \alpha y^{(i)} = 0$$

$$b^* = -\frac{\max_{i:y^{(i)}=-1} w^{*T} x^{(i)} + \min_{i:y^{(i)}=1} w^{*T} x^{(i)}}{2}$$

So α^* can be solved using Lagrangian duality

$$L(w,b,\alpha) = \max_{\alpha} W(\alpha) = \sum_{i=1}^{m} \alpha_i - \frac{1}{2}\sum_{i,j=1}^{m} y^{(i)} y^{(j)} \alpha_i \alpha_j \langle x^i, x^j \rangle$$

$$\text{st } \alpha_i \geq 0 \quad i = 1 \ldots m. \quad \sum_{i=1}^{m} \alpha_i y^{(i)} = 0$$

Optimization Function.

$$:\max_\alpha \sum_{i=1}^{10} \alpha_i - \frac{1}{2} \sum_{i=j=1}^{10} y^{(i)} y^{(j)} \alpha_i \alpha_j (x^i, x^j)$$

(Qa)

$$\Rightarrow \max_\alpha \sum_{i=1}^{10} \alpha_i - \frac{1}{2} \sum_{i=j=1}^{10} y^{(i)} y^{(j)} \alpha_i \alpha_j (x^i x^j)$$

~~Consider~~ Consider term $\sum_{i=j=1}^{10} y^{(i)} y^{(j)} \alpha_i \alpha_j (x^i x^j)$ for Calculation.

for $i=j=1$

$(-1)(-1)\alpha_1\alpha_1 (1.15, 0.1)(1.15, 0.1). = \alpha_1^2 1.3325$

for $i=1$ $j=2$

$(-1)(+1)\alpha_1\alpha_2 (1.15, 0.1)(-1.1, 0.7) = +\alpha_1\alpha_2 1.195$

for $i=1$ $j=3$

$(-1)(-1). \alpha_1\alpha_3 (1.15, 0.1)(1, -0.2) = \alpha_1\alpha_3 1.13$

for $i=1$ $j=4$

$(-1)(+1). \alpha_1\alpha_4 (1.15, 0.1)(-1.05, 0.4) = \alpha_1\alpha_4 (-1.1675)$

for $i=1$ $j=5$

$(-1)(-1) \alpha_1\alpha_5 (1.15, 0.1)(0.8, 1.1) = -\alpha_1\alpha_5 1.03$

for $i=1$ $j=6$

$(-1)(+1) \alpha_1\alpha_6 (1.15, 0.1)(-1.35, 1.9) = \alpha_1\alpha_6 1.3625$

for $i=1$ $j=7$

$(-1)(-1) \alpha_1\alpha_7 (1.15, 0.1)(1.1, 1.1) = 1.375 \alpha_1\alpha_7$

for $i=1$ $j=8$

$(-1)(+1) \alpha_1\alpha_8 (1.15, 0.1)(-2, 1.5) = \alpha_1\alpha_8 2.15$

for $i=1$ $j=9.$

$(-1)(-1) \alpha_1\alpha_9 (1.15, 0.1)(1.8, 0.5) = \alpha_1\alpha_9. 2.12$

for $i=1$ $j=10$.
$(-1)(+1)\alpha_1\alpha_{10}.(1.15,0.1)(-1.65,1.9) = \alpha_1\alpha_{10}\ 1.7075$

for $i=2$ $j=2$
$(+1)(+1)\alpha_2^2(-1.1,0.7)(-1.1,0.7) = 1.7\alpha_2^2$

for. $i=3$ $j=3$
$(-1)(-1)\alpha_3^2(1,-0.2)(1,-0.2) = 1.04\alpha_3^2$

for $i=4$ $j=4$.
$(+1)(+1)\alpha_4^2(-1.05,0.4)(-1.05,0.4) = 1.2625\alpha_4^2$

for $i=5$ $j=5$
$(-1)(-1)\alpha_5^2(0.8,1.1)(0.8,1.1) = 1.85\alpha_5^2$

for $i=6$ $j=6$
$(+1)(+1).(\alpha_6)^2(-1.35,1.9)(-1.35,1.9) = 5.4325\alpha_6^2$

for $i=7$ $j=7$
$(-1)(-1)\alpha_7^2(1.1,1.1)(1.1,1.1) = 2.42\alpha_7^2$

for $i=8$ $j=8$
$(+1)(+1)\alpha_8^2(-2,1.5)(-2,1.5) = 6.25\alpha_8^2$

for $i=9$ $j=9$
$(-1)(-1)\alpha_9^2(1.8,0.5)(1.8,0.5) = 3.49\alpha_9^2$

for $i=10$ $j=10$
$(+1)(+1)\alpha_{10}^2(-1.65,1.9)(-1.65,1.9) = 6.3325\alpha_{10}^2$

for $i=2$ $j=3$

$(+1)(-1)\ \alpha_2\alpha_3\ (-1.1, 0.7)(1, -0.2) = \alpha_2\alpha_3\ 1.24$

for $i=2$ $j=4$

$(+1)(+1)\ \alpha_2\alpha_4\ (-1.1, 0.7)(-1.05, 0.4) = \alpha_2\alpha_4\ 1.435$

for $i=2$ $j=5$

$(+1)(-1)\ \alpha_2\alpha_5\ (-1.1, 0.7)(0.8, 1.1) = \alpha_2\alpha_5\ 0.11$

for $i=2$ $j=6$

$(+1)(+1)\ \alpha_2\alpha_6\ (-1.1, 0.7)(-1.35, 1.9) = 2.115\ \alpha_2\alpha_6$

for $i=2$ $j=7$

$(+1)(-1)\ \alpha_2\alpha_7\ (-1.1, 0.7)(1.1, 1.1) = 0.44\ \alpha_2\alpha_7$

for $i=2$ $j=8$

$(+1)(+1)\ \alpha_2\alpha_8\ (-1.1, 0.7)(-2, 1.5) = 3.25\ \alpha_2\alpha_8$

for $i=2$ $j=9$

$(+1)(-1)\ \alpha_2\alpha_9\ (-1.1, 0.7)(1.8, 0.5) = 1.63\ \alpha_2\alpha_9$

for $i=2$ $j=10$

$(+1)(+1)\ \alpha_2\alpha_{10}\ (-1.1, 0.7)(-1.65, 1.9) = 3.145\ \alpha_2\alpha_{10}$

for $i=3$ $j=4$

$(-1)(+1)\ \alpha_3\alpha_4\ (1, -0.2)(-1.05, 0.4) = \alpha_3\alpha_4\ (+1.13)$

for $i=3$ $j=5$

$(-1)(-1)\ \alpha_3\alpha_5\ (1, -0.2)(0.8, 1.1) = 0.58\ \alpha_3\alpha_5$

for $i=3$ $j=6$

$(-1)(+1)\ \alpha_3\alpha_6\ (1, -0.2)(-1.35, 1.9) = 1.73\ \alpha_3\alpha_6$

for $i=3$ $j=7$

$(-1)(-1)\ \alpha_3\alpha_7\ (1, -0.2)(1.1, 1.1) = 0.88\ \alpha_3\alpha_7$

for $i=3$ $j=8$

$(-1)(+1)\ \alpha_3\alpha_8\ (1, -0.2)(-2, 1.5) = \alpha_3\alpha_8\ 2.3$

for $i=3$ $j=9$

$(-1)(-1)\ \alpha_3\alpha_9\ (1, -02)(1.8, 0.5) = \alpha_3\alpha_9\ 1.7$

for $i=3$ $j=10$

$(-1)(+1)\ \alpha_3\alpha_{10}\ (1, -0.2)(-1.65, 1.9) = +2.03\ \alpha_3\alpha_{10}$

for $i=4$ $j=5$

$(+1)(-1)\ \alpha_4\alpha_5\ (-1.05, 0.4)(0.8, 1.1) = 0.4\ \alpha_4\alpha_5$

for $i=4$ $j=6$

$(+1)(+1)\ \alpha_4\alpha_6\ (-1.05, 0.4)(-1.35, 1.9) = 2.177\ \alpha_4\alpha_6$

for $i=4$ $j=7$

$(+1)(-1)\ \alpha_4\alpha_7\ (-1.05, 0.4)(1.1, 1.1) = 0.715\ \alpha_4\alpha_7$

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for $i=4$ $j=8$
$(+1)(+1)\ \alpha_4\alpha_8\ (-1.05, 0.4)(-2, 1.5) = 2.7\alpha_4\alpha_8$

for $i=4$ $j=9$
$(+1)(+1)\ \alpha_4\alpha_9\ (-1.05, 0.4).(1.8, 0.5) = +1.69\alpha_4\alpha_9$

for $i=4$ $j=10$
$(+1)(+1)\ \alpha_4\alpha_{10}\ (-1.05, 0.4)(-1.65, 1.9) = 2.4925\alpha_4\alpha_{10}$

for $i=5$ $j=6$
$(-1)(+1)\ \alpha_5\alpha_6\ (0.8, 1.1)(-1.35, 1.9) = 1.01\alpha_5\alpha_6$

for $i=5$ $j=7$
$(-1)(-1)\ \alpha_5\alpha_7\ (0.8, 1.1)(1.1, 1.1) = 2.09\alpha_5\alpha_7$

for $i=5$ $j=8$
$(-1)(+1)\ \alpha_5\alpha_8\ (0.8, 1.1)(-2, 1.5) = -0.05\alpha_5\alpha_8$

for $i=5$ $j=9$
$(-1)(-1)\ \alpha_5\alpha_9\ (0.8, 1.1)(1.8, 0.5) = 1.99\alpha_5\alpha_9$

for $i=5$ $j=10$
$(-1)(+1)\ \alpha_5\alpha_{10}\ (0.8, 1.1)(-1.65, 1.9) = 2.255\alpha_5\alpha_{10}$

for $i=6$ $j=7$
$(+1)(-1)\ (-1.35, 1.9)(1.1, 1.1) = -0.605\alpha_7\alpha_6$

for $i=6$ $j=8$
$(+1)(+1)\ (-1.35, 1.9)(-2, 1.5)\alpha_6\alpha_8 = 5.55\alpha_6\alpha_8$

for $i=6$ $j=9$
$(+1)(-1)\ (-1.35, 1.9)(1.8, 0.5)\alpha_6\alpha_9 = 1.48\alpha_6\alpha_9$

for $i=6$ $j=10$
$(+1)(+1)\ (-1.35, 1.9)(-1.65, 1.9)\alpha_6\alpha_{10} = 5.8375\alpha_6\alpha_{10}$

for $i=7$ $j=8$
$(-1)(+1)\ (1.1, 1.1)(-2, 1.5)\alpha_7\alpha_8 = 0.55\alpha_7\alpha_8$

for $i=7$ $j=9$
$(-1)(-1)\ (1.1, 1.1)(1.8, 0.5)\alpha_7\alpha_9 = 2.53\alpha_7\alpha_9$

for $i=7$ $j=10$
$(-1)(+1)\ (1.1, 1.1)(-1.65, 1.9)\alpha_7\alpha_{10} = -0.275\alpha_7\alpha_{10}$

for $i=8$ $j=9$
$(+1)(-1)\ (-2, 1.5)(1.8, 0.5)\alpha_8\alpha_9 = 2.85\alpha_8\alpha_9$

for $i=8$ $j=10$
$(+1)(+1)\ (-2, 1.5)(-1.65, 1.9)\alpha_8\alpha_{10} = 6.15\alpha_8\alpha_{10}$

for $i=9$ $j=10$
$(-1)(+1)\ (1.8, 0.5)(-1.65, 1.9)\alpha_9\alpha_{10} = 2.02\alpha_9\alpha_{10}$.

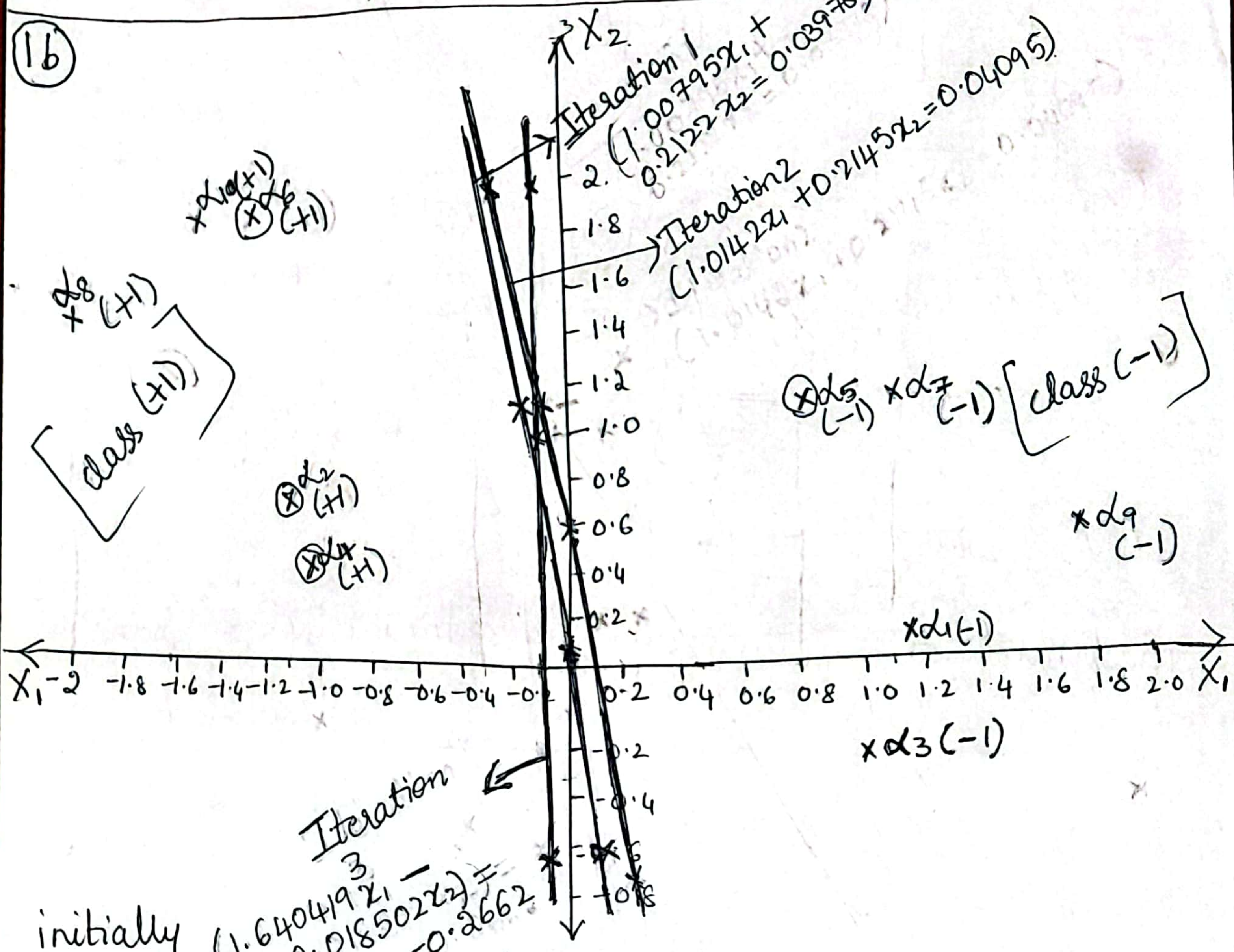
$\therefore \max_\alpha \sum_{i=1}^{10} \alpha_i - \frac{1}{2} \Big[$

$$
\begin{aligned}
&\sum_{i,j=1}^{10} \alpha_1 \big(\alpha_1 1.3325 + \alpha_2 1.195 + 1.13\alpha_3 + \alpha_4(-1.1675) - \alpha_5 1.03 \\
&\quad + \alpha_6 1.3625 + \alpha_7 1.375 + \alpha_8 2.15 + \alpha_9 2.12 + \alpha_{10} 1.7075\big) \\
&+ \alpha_2 \big(\alpha_1 1.195 + \alpha_2 1.7 + 1.24\alpha_3 + 1.435\alpha_4 + \alpha_5 0.11 + \\
&2.115\alpha_6 + 0.44\alpha_7 + 3.25\alpha_8 + 1.63\alpha_9 + 3.145\alpha_{10}\big) + \alpha_3 \big(\\
&\alpha_1 1.13 + \alpha_2 1.24 + \alpha_3 1.04 + 1.13\alpha_4 + 0.58\alpha_5 + 1.73\alpha_6 + \\
&0.88\alpha_7 + 2.3\alpha_8 + 1.7\alpha_9 + 2.03\alpha_{10}\big) + \alpha_4 \big(-1.1675\alpha_1 + 1.435\alpha_2 \\
&+ 1.13\alpha_3 + 1.2625\alpha_4 + 0.4\alpha_5 + 2.177\alpha_6 + 0.715\alpha_7 + 2.7\alpha_8 \\
&+ 1.69\alpha_9 + 2.4925\alpha_{10}\big) + \alpha_5 \big(-1.03\alpha_1 + 0.11\alpha_2 + 0.58\alpha_3 + \\
&0.4\alpha_4 + 1.85\alpha_5 + 1.01\alpha_6 + 2.09\alpha_7 + (-0.05\alpha_8) + 1.99\alpha_9 \\
&+ 2.255\alpha_{10}\big) + \alpha_6 \big(1.3625\alpha_1 + 2.115\alpha_2 + 1.73\alpha_6 + 2.177\alpha_4 + \\
&1.01\alpha_5 + 5.4325\alpha_6 + (-0.605\alpha_7) + 5.55\alpha_8 + 1.48\alpha_9 + \\
&5.8375\alpha_{10}\big) + \alpha_7 \big(1.375\alpha_1 + 0.44\alpha_2 + 0.88\alpha_3 + 0.715\alpha_4 + \\
&2.09\alpha_5 + (-0.605\alpha_6) + 2.42\alpha_7 + 0.55\alpha_8 + 2.53\alpha_9 - 0.275\alpha_{10} \\
&\big) + \alpha_8 \big(2.15\alpha_1 + 3.25\alpha_2 + 2.3\alpha_3 + 2.7\alpha_4 + (-0.05\alpha_5) + \\
&5.55\alpha_6 + 0.55\alpha_7 + 6.25\alpha_8 + 2.85\alpha_9 + 6.15\alpha_{10}\big) + \alpha_9 \big(2.12\alpha_1 \\
&+ 1.63\alpha_2 + 1.7\alpha_3 + 1.69\alpha_4 + 1.99\alpha_5 + 1.48\alpha_6 + 2.53\alpha_7 + 2.85\alpha_8 \\
&+ 3.49\alpha_9 + 2.02\alpha_{10}\big) + \alpha_{10} \big(1.7075\alpha_1 + 3.145\alpha_2 + 2.03\alpha_3 \\
&+ 2.4925\alpha_4 + 2.255\alpha_5 + 5.8375\alpha_6 - 0.275\alpha_7 + 6.15\alpha_8 \\
&+ 2.02\alpha_9 + 6.3325\alpha_{10}\big)
\end{aligned}
$$

$$\sum_{i=1}^{10} \alpha_i y^i = 0$$

$$\therefore -\alpha_1 + \alpha_2 - \alpha_3 + \alpha_4 - \alpha_5 + \alpha_6 - \alpha_7 + \alpha_8 - \alpha_9 + \alpha_{10} = 0$$

(1b)



initially all $\alpha_i = 0 \Rightarrow w = 0 \quad b = 0$

Iteration 1:

Choosing two datapoints α_2 and α_5 because they are nearer.

We know that

$$\alpha_5 y^{(5)} + \alpha_2 y^{(2)} = -\sum_{i=3}^{m} \alpha_i y^{(i)} = \zeta$$

$$\max_\alpha W(\alpha) = \sum_{i=1}^{m} \alpha_i - \frac{1}{2}\left[\sum_{i,j=1}^{m} y^{(i)} y^{(j)} \alpha_i \alpha_j \langle x^{(i)}, x^{(j)} \rangle\right]$$

$\alpha_2 y^{(2)} + \alpha_5 y^{(5)} = 0$ $\rightarrow$ all α are 0 except $\alpha_2 \alpha_5$

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$\alpha_5 = [0 - \alpha_2 y^{(2)}] y^{(5)}$

$\alpha_5 = [0 - \alpha_2 (+1)](-1)$

$\boxed{\alpha_5 = \alpha_2}$

$$w(\alpha_2) = \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{m} y^{(i)} y^{(j)} \alpha_i \alpha_j \langle x^{(i)}, x^{(j)} \rangle$$

$$= \alpha_2 + \alpha_2 - \frac{1}{2}\left[\alpha_2^2 (1.7) + 1.85\,\alpha_2^2 + 2.(-0.11)\alpha_2^2 (-1)\right]$$

$$= 2\alpha_2 - \frac{1}{2}\left[3.77\,\alpha_2^2\right]$$

$$w(\alpha_2) = 2\alpha_2 - \frac{3.77}{2}\alpha_2^2$$

maximize α_2 using analytic optimization.

$$\frac{\partial w(\alpha_2)}{\partial \alpha_2} = 2 - \frac{3.77 \times \cancel{2}}{\cancel{2}}\alpha_2 \rightarrow \text{equate to } 0$$

$$0 = 2 - 3.77\alpha_2$$

$$\alpha_2 = \frac{2}{3.77}$$

$$\boxed{\alpha_2 = 0.5305} = \alpha_5$$

WKT $\quad w = \sum_{i=1}^{m} \alpha_i y^{(i)} x^{(i)}$

$$= \alpha_2 y^{(2)} x^{(2)} + \alpha_5 y^{(5)} x^{(5)}$$

$$= 0.5305 (+1)(-1.1, 0.7) + 0.5305 (-1)(0.8, 1.1)$$

$$= 0.5305 (-1.9, -0.4)$$

$$w \Rightarrow ((-1.00795), (-0.2122))$$

$$b = -\frac{\max_{i:y^{(i)}=-1} w^{*T}x^{(i)} + \min_{i:y^{(i)}=1} w^{*T}x^{(i)}}{2}$$ ⑧

$$= -\left[\frac{(-1.00795, -0.2122)^T(0.8, 1.1) + w^{*T}(-1.1, 0.7)}{2}\right]$$

$$= -\left(\frac{-1.03978 + 0.960205}{2}\right) = -\left(\frac{-1.03978 + 0.960205}{2}\right)$$

$$= 0.079575/2 = \underline{0.03978}$$

Iteration 2:

Choosing α_4 and α_5

WKT $\alpha_2 = 0.5305$

$$S = -\sum_{i=3}^{m} \alpha_i y^{(i)} = -\alpha_2 y^{(2)}$$

$$= -(0.5305)(+1)$$

$$\boxed{S = -0.5305}$$

$$\alpha_4 = [S - \alpha_5 y^{(5)}]\, y^{(4)}$$

$$\alpha_4 = [-0.5305 - \alpha_5(-1)](+1)$$

$$\alpha_4 = [-0.5305 + \alpha_5] = +\alpha_5 - 0.5305$$

$$w(\alpha_4) = \alpha_4 + \alpha_2 + \alpha_5 - \frac{1}{2}\left[\sum_{i,j=1}^{m} y^{(i)}y^{(j)}\alpha_i\alpha_j \langle x^{(i)} x^{(j)}\rangle\right]$$

$$= \alpha_4 + 0.5305 + (0.5305 + \alpha_4) - \frac{1}{2}\Big[\alpha_2^2(1.7) + \alpha_4^2(1.2625) + \alpha_5^2(1.85) + 2(-1)\alpha_4\alpha_5(-0.4) + 2(-1)\alpha_2\alpha_5(-0.11) + 2(+1)\alpha_4\alpha_2(1.435)\Big]$$

$$= 2\alpha_4 + 1.061 - \frac{1}{2}\Big[0.2814 \times 1.7 + \alpha_4^2(1.2625) + (0.5305 + \alpha_4)^2(1.85) + 0.8\,\alpha_4(\alpha_4 + 0.5305) + 0.22 \times 0.5305(\alpha_4 + 0.5305) + 1.5225\alpha_4\Big]$$

$$= 2\alpha_4 + 1.061 - \frac{1}{2}\Big[\underline{0.47838} + 1.2625\alpha_4^2 + \underline{0.2814 \times 1.85} + \alpha_4^2(1.85) + 1.96285\alpha_4 + 0.8\alpha_4^2 + 0.4244\alpha_4 + 0.11671\alpha_4 + \underline{0.061914} + 1.5225\alpha_4\Big]$$

CamScanner

$= 2\alpha_4 + 1.061 - \frac{1}{2}\left[1.06088 + 3.9125\alpha_4^2 + 4.02646\alpha_4\right]$ (4)

$= 2\alpha_4 + 1.061 - 0.53044 - 1.9562\alpha_4^2 - 2.01323\alpha_4$

$w(\alpha_4) = -0.01323\alpha_4 + 0.53056 - 1.9562\alpha_4^2$

$$\frac{\partial w(\alpha_4)}{\partial \alpha_4} = -0.01323 - 1.9562 \times 2\alpha_4$$

$$-0.01323 = 3.9124\alpha_4$$

$$\boxed{\alpha_4 = -0.00338}$$

$$\boxed{\alpha_5 = 0.52712}$$

Since $\alpha_i \geq 0$

$$\alpha_4(+1) + \alpha_5(-1) = \int$$

$$\alpha_4 - \alpha_5 = \int$$

$$\alpha_4 = \int + \alpha_5$$

$$-\alpha_4 = -\int - \alpha_5$$

$$= -(-0.5305) - 0.52712$$

$$\boxed{\alpha_4 = 0.00338}$$

$$\alpha_5 = \alpha_4 - \int$$

$$-\alpha_5 = -\alpha_4 + \int$$

$$\alpha_5 = 0.00338 + 0.5305$$

$$\boxed{\alpha_5 = 0.53388}$$

WKT $w = \sum_{i=1}^{m} \alpha_i y^{(i)} x^{(i)}$

$= \alpha_2 y^{(2)} x^{(2)} + \alpha_5 y^{(5)} x^{(5)} + \alpha_4 y^{(4)} x^{(4)}$

$= 0.5305 \times (+1)(-1.1, 0.7) + 0.53388(-1)((0.8), (1.1))$

$+ 0.00338\,(+1)(-1.05, 0.4)$

$= (-0.58355, 0.37135) + (-0.427104, -0.587268)$ (10)

$+ (-0.003549, 0.001352)$

$$\underline{w^* = (-1.0142, -0.2145)} = \cancel{(-0.4306, -0.5859)}$$

$$b = -\frac{\max_{i:y^{(i)}=(-1)} w^{*T} x^{(i)} + \min_{i:y^{(i)}=(1)} w^{*T} x^{(i)}}{2}$$

$$= -\frac{[-0.04797 + 0.9654]}{2} = \cancel{0.46272}$$

$$b = \boxed{0.040955}$$

Iteration 3 :

Choosing α_6 and α_5

Keeping $\alpha_2 = 0.5305$ and $\alpha_4 = 0.00338$.

$$\zeta = -\sum_{i=3}^{m} \alpha_i y^{(i)} = -[\alpha_2 y^{(2)} + \alpha_4 y^{(4)}]$$

$$= -[(0.5305)(+1) + (0.00338)(+1)]$$

$$\boxed{\zeta = -0.53388}$$

$$\alpha_6 = [\zeta - \alpha_5 y^{(5)}] y^{(6)}$$

$$= [\zeta - \alpha_5(-1)](+1)$$

$$\alpha_6 = [-0.53388 + \alpha_5]$$

$$\alpha_6 = \alpha_5 - 0.53388$$

$$w(\alpha_6) = \alpha_6 + \alpha_4 + \alpha_2 + \alpha_5 - \frac{1}{2}\sum_{i,j=1}^{m} y^{(i)} y^{(j)} \alpha_i \alpha_j \langle x^{(i)}, x^{(j)} \rangle$$

$= 0.53388 + \alpha_6 + \alpha_6 + 0.53388 - \frac{1}{2}[\alpha_6^2 (5.4325) +$
$\alpha_4^2 (1.2625) + \alpha_2^2 (1.7) + \alpha_5^2 (1.85) + 2(+1)\alpha_4\alpha_2(1.435)$
$+ 2(-1)\alpha_4\alpha_5(-0.4) + 2(-1)\alpha_2\alpha_5(-0.11) + 2(+1)$
$\alpha_6\alpha_4(2.1775) + 2(+1)\alpha_6\alpha_2(2.815) + 2(-1)\alpha_6\alpha_5$

$$(1.01)\Big]$$

$$= 1.06776 + 2\alpha_6 - \frac{1}{2}\Big[5.4325\alpha_6^2 + 0.0000144 + 0.4784 + (\alpha_6 + 0.53388)^2(1.85) + 0.005158 + 0.00271(\alpha_6 + 0.53388) + 0.000743(\alpha_6 + 0.53388) + 0.01475(\alpha_6) + 0.01907\alpha_6 + 2.02\alpha_6(\alpha_6 + 0.53388)\Big]$$

$$= 1.06776 + 2\alpha_6 - \frac{1}{2}\Big[5.4325\alpha_6^2 + 0.0000144 + 0.4784 + \alpha_6^2 1.85 + 0.52730 + 1.975356\alpha_6 + 0.005158 + 0.00271\alpha_6 + 0.001446 + 0.000743\alpha_6 + 0.0003967 + 0.01475\alpha_6 + 0.01907\alpha_6 + 2.02\alpha_6^2 + 1.0784\alpha_6\Big]$$

$$= 1.06776 + 2\alpha_6 - \frac{1}{2}\Big[1.0127151 + 5.2625\alpha_6^2 + 0.934229\alpha_6\Big]$$

$$= 1.06776 + 2\alpha_6 - 0.5063 - 2.6312\alpha_6^2 - 0.467114\alpha_6$$

$$= 1.5328\alpha_6 + 0.56146 - 2.6312\alpha_6^2$$

$$\frac{\partial w(\alpha_6)}{\partial(\alpha_6)} = 1.5328 - 2 \times 2.6312\alpha_6$$

$$0 = 1.5328 - 5.2624\alpha_6$$

$$\alpha_6 = \frac{1.5328}{5.2624}$$

$$\boxed{\alpha_6 = 0.29127}$$

$$\boxed{\alpha_5 = 0.82515}$$

WKT $\quad w = \sum_{i=1}^{m} \alpha_i y^{(i)} x^{(i)}$

$$= \alpha_5 y^{(5)} x^{(5)} + \alpha_6 y^{(6)} x^{(6)} + \alpha_2 y^{(2)} x^{(2)} + \alpha_4 y^{(4)} x^{(4)}$$

(1a)

$$w = 0.82515 \times (-1)(0.8, 1.1) + 0.29127\,(+1)(-1.35, 1.9) + (-0.58355, 0.37135) + (-0.003549, 0.001352)$$

$$= (-1.640419, 0.018502)$$

$$b = -\frac{\max_{i:y^{(i)}=(-1)} w^{*T} x^{(i)} + \min_{i:y^{(i)}} w^{*T} x^{(i)}}{2}$$

$$= -\left[\frac{-1.222728 + 1.755040}{2}\right] = \underline{-0.2662}$$

$$\boxed{b = -0.2662}$$

(3a) The dataset

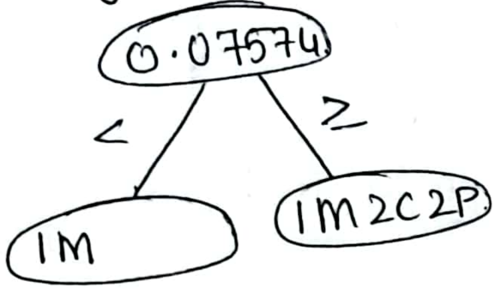
height	diameter	weight	class
0.11782005,	0.13836528,	0.37683831,	Plastic
0.149398845,	0.1378846,	0.458635784,	Plastic
0.09284698,	0.0894648,	0.39811590,	metal
0.06349317,	0.03,	4.29058345,	metal
0.13438655,	0.15,	0.75,	Ceramic
0.08809672,	0.0935788,	0.5143918151,	Ceramic

Consider height:

Sorting the height value.

0.06349317 – Metal
0.08809672 – Ceramic
0.09284698 – Metal
0.11782005 – Plastic
0.13438655 – Ceramic
0.149398845 – Plastic

Calculating thershold.: $\dfrac{0.06349317 + 0.08809672}{2} = 0.075745$



Calculate the entropy

(ie). $E_1 = -\frac{1}{1}\log\frac{1}{1} = 0$

$$E_2 = -\frac{1}{5}\log\frac{1}{5} - \frac{2}{5}\log\frac{2}{5} - \frac{2}{5}\log\frac{2}{5}$$

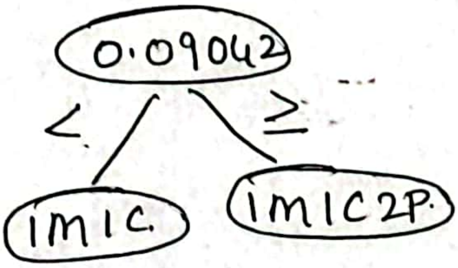
$$= 0.458094$$

weighted entropy $= \frac{1}{6}(0) + \frac{5}{6}(0.458094) = 0.38174$

Information gain $= 1 - 0.38174 = \underline{0.6182}$.

② $\frac{0.08809672 + 0.09284698}{2} = 0.09042 \rightarrow$ thershold

Calculate entropy



$E_1 = -\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2} = 0.30102$

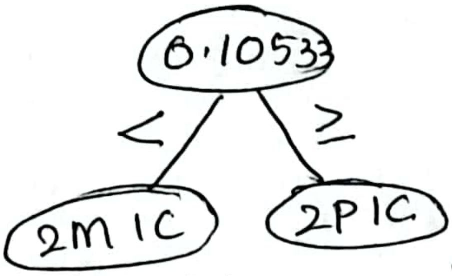
$E_2 = -\frac{1}{4}\log\frac{1}{4} - \frac{1}{4}\log\frac{1}{4} - \frac{2}{4}\log\frac{2}{4}$
$= 0.4515$

weighted Entropy $= \frac{2}{6}(0.30102) + \frac{4}{5}(0.4515)$
$= 0.40134$

Information gain $= 1 - 0.40134 = \underline{0.59866}$

③ $\frac{0.09284698 + 0.11782005}{2} = 0.10533$

$E_1 = -\frac{2}{3}\log\frac{2}{3} - \frac{2}{3}\log\frac{2}{3} = 0.27643$



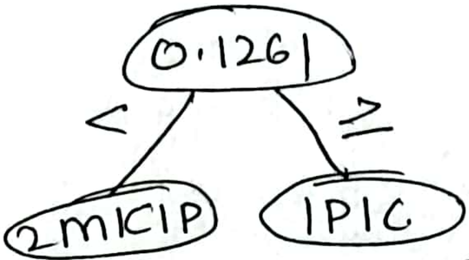
$E_2 = -\frac{2}{3}\log\frac{2}{3} - \frac{2}{3}\log\frac{2}{3} = 0.27643$

weighted Entropy $= \frac{3}{6}(0.27643) + \frac{3}{6}(0.27643)$
$= 0.55286$

Information gain $= 1 - 0.55286 = \underline{0.44714}$

④ $\frac{0.11782005 + 0.13438655}{2} = 0.1261$

$E_1 = -\frac{2}{4}\log\frac{2}{4} - \frac{1}{4}\log\frac{1}{4} - \log\frac{1}{4} \times \frac{1}{4}$
$= 0.4515$



$E_2 = -\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2} = 0.30102$

weighted Entropy: $\frac{2}{6} \times 0.30102 + \frac{4}{6}(0.4515)$
$= 0.40134$.

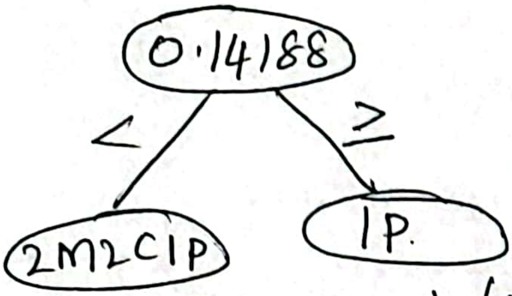
Information gain = $\underline{0.59866}$

(5) $\dfrac{0.13438655 + 0.1493988}{2} = 0.14188$

$E_1 = -\frac{1}{5}\log\frac{1}{5} - \frac{2}{5}\log\frac{2}{5} - \frac{2}{5}\log\frac{2}{5}$

$= 0.458094$

$E_2 = -\frac{1}{1}\log\frac{1}{1} = 0$



Weighted Entropy $= \frac{1}{6}(0) + \frac{5}{6}(0.458094) = 0.38174$

Information gain $= 1 - 0.38174 = \underline{0.6184}$

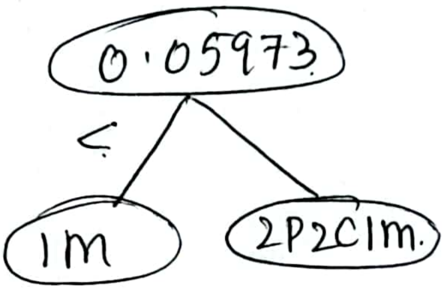
Height with $I_g = 0.6184$. Of thershold $= 0.14188$ or 0.075745 Can be choosen.

Consider Diameter:

Sorting diameter.

0.03	Metal
0.0894648	Metal
0.093578	Ceramic
0.1378846	Plastic
0.1383652.	plastic
0.15	Ceramic

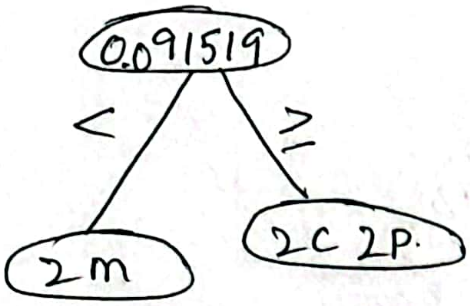
(1) $\dfrac{0.03 + 0.0894648}{2} = 0.05973$



Calculating entropy and information gain

$\boxed{I_g = 0.6182}$

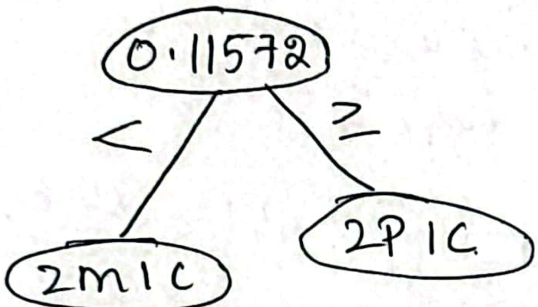
(2) $\dfrac{0.0894648 + 0.093578}{2} = 0.091519.$



Entropy Calculation & Information gain is equal to $\underline{0.79932} = Ig$ (15)

(3) $\dfrac{0.0935788 + 0.1378846}{2} = 0.115726$

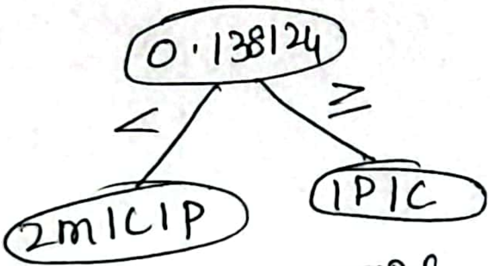
Calculating Entropy & Information gain is $\underline{0.44714}$



(4) $\dfrac{0.1378846 + 0.13836528}{2} = 0.1381245$

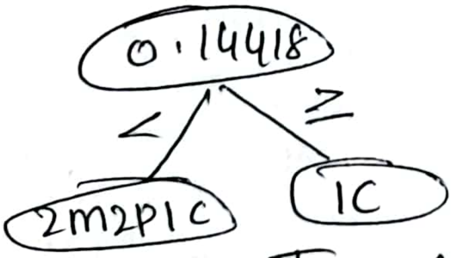
Calculating Entropy &

$\boxed{Ig = 0.59866}$



(5) $\dfrac{0.15 + 0.13836528}{2} = 0.14418.$

$\boxed{Ig = 0.6182}$



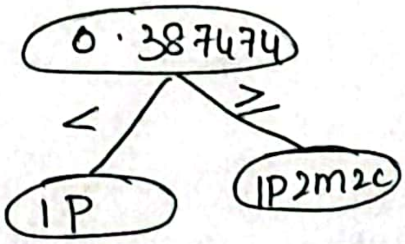
Diameter with $Ig = 0.79932$ of thershold. 0.091519 can be chosen.

Consider weight

Sort the weight values:

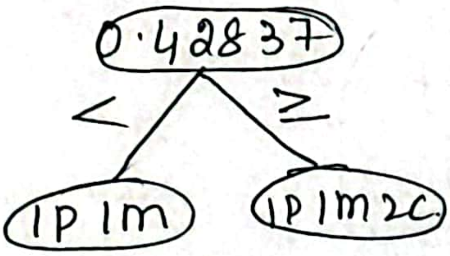
0.37683831	Plastic	0.514391815	Ceramic
0.39811590	Metal.	0.75	Ceramic
0.458635784	Plastic.	4.29058345	metal

① $\frac{0.37683831+0.39811590}{2} = 0.387474.$



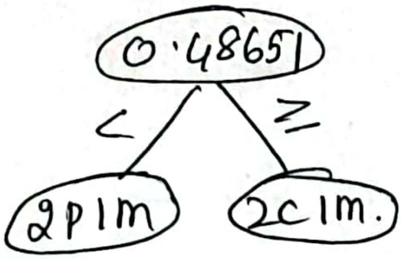
$\boxed{Ig = 0.458094.}$

② $\frac{0.39811590+0.4586357}{2} = 0.4283725$



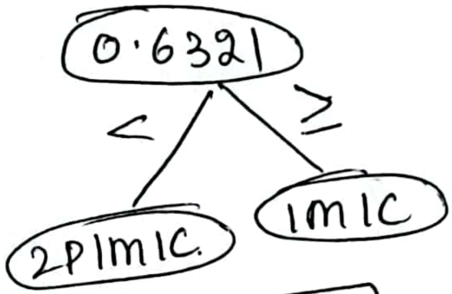
$\boxed{Ig = 0.59866}$

③ $0.4586357+0.5143918/2 = 0.48651.$



$\boxed{Ig = 0.44714}$

④ $\frac{0.514391+0.75}{2.} = 0.6321$



$\boxed{Ig = 0.59866.}$

⑤ $\frac{0.75+4.29058}{2} = 2.520$

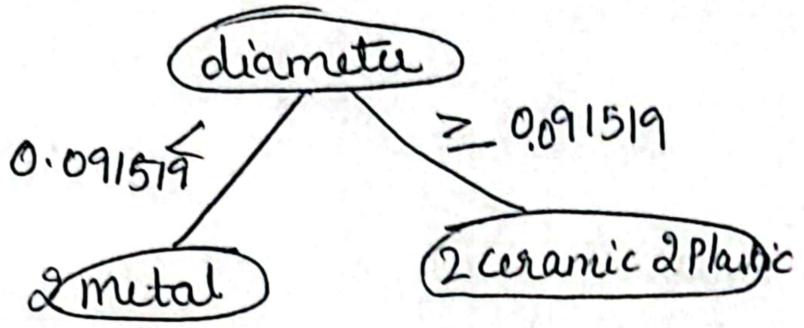
$\boxed{Ig = 0.6182}$

Weight with Ig = 0.6182 of thershold = 2.520 can be. Choosen.

Choosing the best feature among weight, diameter and height

→ height ~~to~~ and weight has lesser Ig value compared to diameter.

Hence the higher Ig value diameter is chosen as root. (17)



Consider 2 ceramic & 2 Plastic dataset.

height		weight		class
0.117820005	,	0.37683531	,	Plastic
0.149398845	,	0.4586355	,	Plastic
0.13438655	,	0.75	,	Ceramic
0.08809672	,	0.51439188	,	Ceramic

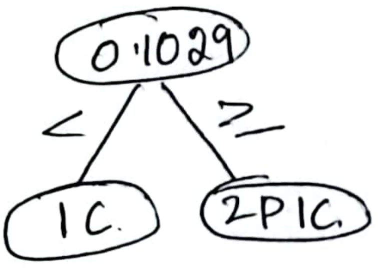
Consider height

Sort height.

0.08809672 → Ceramic
0.117820005 → Plastic
0.13438655 → Ceramic
0.149398845 → Plastic.

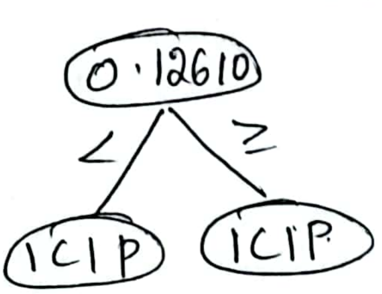
(1) $\frac{0.08809672+0.11782005}{2} = 0.1029.$

$E_1 = 0 \quad E_2 = 0.2764.$



$\boxed{Ig = 0.7927}$

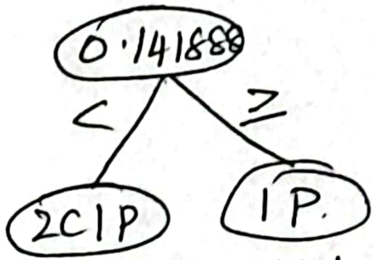
(2) $\frac{0.117820005+0.13438655}{2} = 0.12610.$



$\boxed{Ig = 0.69897}$

#

(3) $\frac{0.13438655 + 0.14939884}{2} = 0.141888$



$\boxed{Ig = 0.7927}$

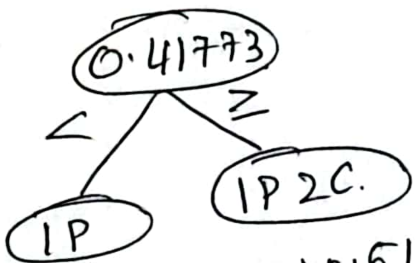
~~Co~~ We can consider. height with $Ig = 0.7927$ of thershold. 0.1029 or 0.141888

Consider weight

Sort the data.

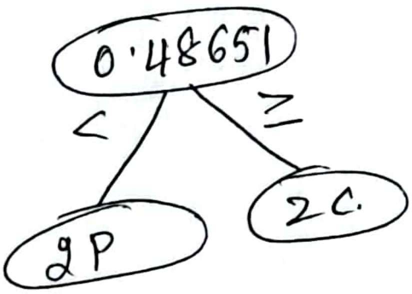
~~0.0985788~~ ~~Ceramic~~
~~0.75~~
0.37683831 , Plastic
0.4586355 , Plastic
0.51439181 , Ceramic
0.75 , Ceramic.

(1) $\frac{0.37683831 + 0.4586355}{2} = 0.417736$



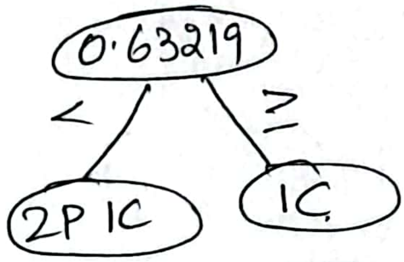
$\boxed{Ig = 0.7927}$

(2) $\frac{0.4586355 + 0.51439181}{2} = 0.48651$



$\boxed{Ig = 1}$

(3) $\frac{0.51439181 + 0.75}{2}$
$= 0.632195$



$\boxed{Ig = 0.7927}$

Weight with $Ig = 1$ of thershold. 0.63219 can be considered.

Final decision tree looks like.