HW 14

Exercise 8.8,Page 496, Question 6

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Verify the formula given in the Key Idea by finding the first few terms of the Taylor series of the given function and identifying a pattern.

$f(x) = tan^{-1}(x); c = 0$

Answer

Key Idea 32 of Section 8.8 states:

$\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/2n + 1$

We need to find the derivatives for the first few n's and evaluate them at x=0 to find a trend.

$f(x) = tan^{-1}(x), f(0) = 0$

$f'(x) = 1/x^2 + 1, f'(0) = 1$

$f''(x) = -(2x/(x^2 + 1)^2), f''(0) = 0$

$f'''(x) = -2(-3x^2 + 1/(x^2 + 1)^3), f'''(0) = -2$

$f''''(x) = 24x(-x^2 + 1/(x^2 + 1)^4), f''''(0) = 0$

$f'''''(x) = 24(5x^4 - 10x^2 + 1)/(x^2 + 1)^5, f'''''(0) = 24$

$f''''''(x) = 240x(-3x^4 + 10x^2 - 3)/(x^2 + 1)^6, f''''''(0) = 0$

$f'''''''(x) = 720(7x^6 - 35x^4 + 21x^2 - 1)/(x^2 + 1)^7, f'''''''(0) = -720$

Therefore, when we substitute this value into tylor's formula we get:

$tan^1(x) = \sum_{n=0}^{\infty}(f^n(0)x^n/n!)$

$= 0x^0/0! + 1x^1/1! + 0x^2/2! - 2x^3/3! + 0x^4/4! + 24x^5/5! + 0x^6/6! - 720x^7/7! + \ldots.$

$0 + x^1/1 + 0 - x^3/3 + 0 + x^5/5 + 0 - x^7/7 + \ldots.$

$x^1/1 - x^3/3 + x^5/5 - x^7/7$

$\sum_{n=0}^{\infty}(-1)^n * x^{2n+1}/2n + 1$

Therefore, it is verified.