HW 10

Chapter 11,Page 424, Question 15

Priya Shaji

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Consider the game of tennis when deuce is reached. If a player wins the next point, he has advantage. On the following point, he either wins the game or the game returns to deuce. Assume that for any point, player A has probability .6 of winning the point and player B has probability .4 of winning the point. (a) Set this up as a Markov chain with state 1: A wins; 2: B wins; 3: advantage A; 4: deuce; 5: advantage B. (b) Find the absorption probabilities. (c) At deuce, find the expected duration of the game and the probability that B will win.

Answer (a)

a. **Set this up as a Markov chain with state 1: A wins; 2: B wins; 3: advantage A; 4: deuce; 5: advantage B.**

Step 1) Compose transition matrix

```
Markov_chain_state_names <- c("S1: Game_Win A", "S2: Game_Win B", "S3: Advan A", "S4: Deuce", "S5: Advan B")
Markov_chain<- matrix(c(1, 0, 0, 0, 0,
                        0, 1, 0, 0, 0,
                        .6, 0, 0, .4, 0,
                        0, 0, .6, 0, .4,
                        0, .4, 0, .6, 0),
                      nrow = 5, byrow = TRUE)
colnames(Markov_chain) <- Markov_chain_state_names
rownames(Markov_chain) <- Markov_chain_state_names
Markov_chain
```

```
##                 S1: Game_Win A S2: Game_Win B S3: Advan A S4: Deuce
## S1: Game_Win A             1.0            0.0         0.0        0.0
## S2: Game_Win B             0.0            1.0         0.0        0.0
## S3: Advan A                0.6            0.0         0.0        0.4
## S4: Deuce                  0.0            0.0         0.6        0.0
## S5: Advan B                0.0            0.4         0.0        0.6
##                 S5: Advan B
## S1: Game_Win A          0.0
## S2: Game_Win B          0.0
## S3: Advan A             0.0
## S4: Deuce               0.4
## S5: Advan B             0.0
```

Answer (b)

b. **Find the absorption probabilities.**

Step 2) Change the transition matrix to canonical form

```
Markov_chain <- Markov_chain[, c(3:5, 1:2)]
Markov_chain <- Markov_chain[c(3:5, 1:2),]
Markov_chain
```

```
##                 S3: Advan A S4: Deuce S5: Advan B S1: Game_Win A
## S3: Advan A             0.0       0.4         0.0             0.6
## S4: Deuce               0.6       0.0         0.4             0.0
## S5: Advan B             0.0       0.6         0.0             0.0
## S1: Game_Win A          0.0       0.0         0.0             1.0
## S2: Game_Win B          0.0       0.0         0.0             0.0
##                 S2: Game_Win B
## S3: Advan A                0.0
## S4: Deuce                  0.0
## S5: Advan B                0.4
## S1: Game_Win A             0.0
## S2: Game_Win B             1.0
```

Step 3) Subset matrix Q (transient to transient)

Step 4) Subset matrix R (transient to absorbing)

Step 5) Compose an identity matrix I with same dimensions as Q

Step 6) Compute the fundamental matrix by solving the set of linear equations

Step 7) Compute absorption probabilities

```
Q <- Markov_chain[1:3, 1:3]
R <- Markov_chain[1:3, 4:5]
I <- diag(3)
N <- solve(I - Q)
M <- N %*% R
M
```

```
##             S1: Game_Win A S2: Game_Win B
## S3: Advan A      0.8769231      0.1230769
## S4: Deuce        0.6923077      0.3076923
## S5: Advan B      0.4153846      0.5846154
```

Answer (c)

c. **At deuce, find the expected duration of the game and the probability that B will win**

Step 8) Compute expected steps to absorption

Step 9) Column vector of 1s

Step 10) Calculate expected steps to absorption

Step 11) Expected duration of the game in steps

```
c<- c(rep(1, 3))
Nc <- N %*% c


Nc[2,1]
```

```
## S4: Deuce
##  3.846154
```

Step 12) Probability that B will win

```
M[2,2]
```

```
## [1] 0.3076923
```