

# PShaji\_Assignment14

Priya Shaji

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$$\int 4e^{-7x}dx$$

This week, we'll work out some Taylor Series expansions of popular functions.

$$f(x) = 1/(1 - x)$$

$$f(x) = e^x$$

$$f(x) = \ln(1 + x)$$

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as a R-Markdown document.

**Answer**

**1)  $f(x) = 1/(1 - x)$**

$$f(a) = 1/1 - a, f(0) = 1$$

$$f'(a) = 1/(1 - a)^2, f'(0) = 1$$

$$f''(a) = 2/(1 - a)^3, f''(0) = 2$$

$$f'''(a) = 6/(1 - a)^3, f'''(0) = 6$$

$$f''''(a) = 24/(1 - a)^5, f''''(0) = 24$$

Plug in the relevant expressions into formula for Taylor Series expansion:

$$f(a) + f'(a)(x - a) + f''(x - a)/2! + f'''(x - a)/3! + f''''(x - a)/4! \dots$$

$$1 + 1x + 2x^2/2! + 6x^3/3! + 24x^4/4! + \dots$$

$$1 + x + x^2 + x^3 + x^4 + x^5 \dots$$

$$\sum_{n=0}^{\infty} x^n$$

**2)  $f(x) = e^x$**

$$f(a) = e^a, f(0) = 1$$

$$f'(a) = e^a, f'(0) = 1$$

$$f''(a) = e^a, f''(0) = 1$$

$$f'''(a) = e^a, f'''(0) = 1$$

$$f''''(a) = e^a, f''''(0) = 1$$

Plug in the relevant expressions into formula for Taylor Series expansion:

$$f(a) + f'(a)(x - a) + f''(x - a)/2! + f'''(x - a)/3! + f''''(x - a)/4! + \dots$$

$$= 1 + x^2/2 + x^3/3 + x^4/4 + \dots$$

$$= \sum_{n=0}^{\infty} x^n/n!$$

**3)  $f(x) = \ln(1 + x)$**

$$f(a) = \ln(1 + a), \quad f(0) = 0$$

$$f'(a) = 1/1 + a, \quad f'(0) = 1$$

$$f''(a) = -1/(1 + a)^2, \quad f''(0) = -1$$

$$f'''(a) = 2/(1 + a)^3, \quad f'''(0) = 2$$

$$f''''(a) = -6/(1 + a)^4, \quad f''''(0) = -6$$

Plug in the relevant expressions into formula for Taylor Series expansion:

$$f(a) + f'(a)(x - a) + f''(x - a)/2! + f'''(x - a)/3! + f''''(x - a)/4! + \dots$$

$$= x - x^2/2 + x^3/3 - x^4/4 + \dots$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} x^n/n!$$