PShaji_Assignment9

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The price of one share of stock in the Pilsdorff Beer Company (see Exer- cise 8.2.12) is given by Yn on the nth day of the year. Finn observes that the differences Xn = Yn+1 − Yn appear to be independent random variables with a common distribution having mean μ = 0 and variance σ2 = 1/4. If Y1 = 100, estimate the probability that Y365 is (a) ≥ 100. (b) ≥ 110. (c) ≥ 120.

Answer

(a) ≥ 100

```
mean <- 0
var <- 1/4
sd <- sqrt(var)
n <- 364
x <- 0/sqrt(n)
pnorm(x, mean, sd, lower.tail = FALSE)
```

```
## [1] 0.5
```

(b) ≥ 110

```
x <- 10/sqrt(n)
pnorm(x, mean, sd, lower.tail = FALSE)
```

```
## [1] 0.1472537
```

(b) ≥ 120

```
x <- 20/sqrt(n)
pnorm(x, mean, sd, lower.tail = FALSE)
```

```
## [1] 0.01801584
```

Calculate the expected value and variance of the binomial distribution using the moment generating function.

Answer

Binomial moment generating function:

$$M(t) = (pe^t + q)^n$$

Expected value => 1st derivative

$$M‘(t) = n(pe^t + q)^{n-1} * pe^t$$

$$E(x) = M‘(0) = np$$

Variance => 2nd derivative

$$M‘‘(t) = n[1 - p + pe^t]^{n-1}(pe^t)n(n-1)(1 - p + pe^t)^{n-2}(pe^t)$$

$$E(x^2) = M‘‘(0) = n(n-1)p^2 + np$$

$$var(x) = E(x^2) - E(x)^2$$

$$var(x) = n(n-1)p^2 + np - (np)^2$$

$$var(x) = (n^2p^2 - 1np^2) + np - (np)^2$$

$$var(x) = (np^2) - np^2 + np - (np)^2$$

$$var(x) = np - np^2$$

$$var(x) = np(1-p)$$

Calculate the expected value and variance of the exponential distribution using the moment generating function.

Answer

Exponential moment generating function:

$$M(t) = \lambda/\lambda - t$$

Expected value => 1st derivative

$$M‘(t) = \lambda/(\lambda - t)^2$$

$$E(x) = M‘(0) = \lambda/\lambda^2 = 1/\lambda$$

Variance => 2nd derivative

$$M‘‘(t) = 2\lambda/(\lambda - t)^3$$

$$E(x^2) = M‘‘(0) = 2\lambda/\lambda^3 = 2/\lambda^2$$

$$var(x) = E(x^2) - E(x)^2$$

$$var(x) = 2/\lambda^2 - 1/\lambda^2 = 1/\lambda^2$$