PShaji_Assignment7

Priya Shaji

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Question 1

Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

Answer 1

Let us compute the function of Y

Given that:

Y = Min(X1, X2, . . . , Xn) = Xi

where Xi is the minimum value.

We can find the distribution of Y by applying binomial distribution.

Therefore,

$n!/(n-1)!1! * p^(1) * q^(n-1)$

$= n * p * q^(n-1)$

$= n * (1/k) * (n-1/k)^(n-1)$

$= n * (1/k) * (n-1)^(n-1)/k^(n-1)$

$= n * (n-1)^(n-1)/K^n$

Where,

$Xi = successfultrial(minimumvalue)$

$X1toXi-1, Xi+1toXn = failedtrials$

$n = numberoftrials$

$k = populationsize(totalpossibleoutcomespertrial)$

$p = 1/kprobabilityofsuccess$

$q = n-1/kprobabilityoffailure$

Question 2

Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

Answer 2

Ans a) Given: Expects one failure every ten yrs

Mean(machine_fails) = 10 yrs

Now, for geometric model distribution:

mean = 1/p

10 = 1/p

Probability of failure:

p = 1/10

1) Standard deviation:

```
p = 0.10
SD = sqrt((1-p)/(p^2))
SD
```

```
## [1] 9.486833
```

Probability that the machine will fail after 8 years:

Discrete:

```
p = 0.10
count = 0
for (i in 1:8)
{
   count = count + (1-p)^i*p
}
1 - count
```

```
## [1] 0.4874205
```

Ans b)

Mean(machine_fails) = 10 yrs

Exponential probability distribution mean :

= beta = 1/lambda = 1/10

standard deviation, for exponential distribution:

= 1/lamba = 1/10

The cummlative distribution fucntion (CDF) for an exponetial distribution:

$f(x, lambda) = 1 - e^(-lambdax), wherex >= 0 and 0 wherex < 0 f(8, 1/10) = 1 - e^(-1/10(8)), forx = 8$

Probability that the machine will fail after 8 years:

Continuous:

```
count = (1 - exp(1)^(-0.10*8))
1 - count
```

```
## [1] 0.449329
```

Ans c)

Probability of success in 8 years:

p = 1/10

Number of trials = 8

Standard deviation:

```
p = 0.10
n = 8
SD = sqrt(n*p*(1-p))
SD
```

```
## [1] 0.8485281
```

k = number of successes in n trials = 0 PMF

$Formula for binomial distribution - P(k, n.p) = p(X = 0) = nCrK * p^k * (1 - p)^n - k$

Probability of 8 straight failures without success

```
p = 0.10
n = 8
k =0
count = choose(n, k)*p^k*(1-p)^(n-k)
count
```

```
## [1] 0.4304672
```

Ans d)

Poisson distribution:

lambda = rate/unit time = 1

machine breakdown per 10 years = expected value

Standard deviation is sqrt(lambda) = sqrt(1) = 1

Probability of k = 0 breakdowns in years 1 through 8

Probability of 8 straight failures without success

```r
p = 0.10
n = 8
k =0
count = (1^k*exp(1)^(-1))/factorial(k)
count
```

```
## [1] 0.3678794
```