

1 GH(2³,1)

Product table of D_{2^3}

*	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	4	3	6	5	8	7
3	3	4	1	2	7	8	5	6
4	4	3	2	1	8	7	6	5
5	5	6	7	8	1	2	3	4
6	6	5	8	7	2	1	4	3
7	7	8	5	6	3	4	1	2
8	8	7	6	5	4	3	2	1

Generalized Hadamard matrix of order 2^3

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 6 & 4 & 2 & 5 & 7 \\ 1 & 4 & 6 & 7 & 8 & 5 & 3 & 2 \\ 1 & 5 & 4 & 8 & 3 & 7 & 2 & 6 \\ 1 & 6 & 2 & 5 & 7 & 4 & 8 & 3 \\ 1 & 7 & 5 & 3 & 2 & 8 & 6 & 4 \\ 1 & 8 & 7 & 2 & 6 & 3 & 4 & 5 \end{pmatrix}$$

Product of pairs of rows of $GH(2^3)$

Row i*Row j
1*2:1 2 3 4 5 6 7 8
1*3:1 3 8 6 4 2 5 7
1*4:1 4 6 7 8 5 3 2
1*5:1 5 4 8 3 7 2 6
1*6:1 6 2 5 7 4 8 3
1*7:1 7 5 3 2 8 6 4
1*8:1 8 7 2 6 3 4 5
2*3:1 4 6 7 8 5 3 2
2*4:1 3 8 6 4 2 5 7
2*5:1 6 2 5 7 4 8 3
2*6:1 5 4 8 3 7 2 6
2*7:1 8 7 2 6 3 4 5
2*8:1 7 5 3 2 8 6 4
3*4:1 2 3 4 5 6 7 8
3*5:1 7 5 3 2 8 6 4
3*6:1 8 7 2 6 3 4 5
3*7:1 5 4 8 3 7 2 6
3*8:1 6 2 5 7 4 8 3
4*5:1 8 7 2 6 3 4 5

4*6:1 7 5 3 2 8 6 4
 4*7:1 6 2 5 7 4 8 3
 4*8:1 5 4 8 3 7 2 6
 5*6:1 2 3 4 5 6 7 8
 5*7:1 3 8 6 4 2 5 7
 5*8:1 4 6 7 8 5 3 2
 6*7:1 4 6 7 8 5 3 2
 6*8:1 3 8 6 4 2 5 7
 7*8:1 2 3 4 5 6 7 8

2 GH($2^4,1$)

Product table of D_{2^4}

*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
3	3	4	1	2	7	8	5	6	11	12	9	10	15	16	13	14
4	4	3	2	1	8	7	6	5	12	11	10	9	16	15	14	13
5	5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
7	7	8	5	6	3	4	1	2	15	16	13	14	11	12	9	10
8	8	7	6	5	4	3	2	1	16	15	14	13	12	11	10	9
9	9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
10	10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
11	11	12	9	10	15	16	13	14	3	4	1	2	7	8	5	6
12	12	11	10	9	16	15	14	13	4	3	2	1	8	7	6	5
13	13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
14	14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3
15	15	16	13	14	11	12	9	10	7	8	5	6	3	4	1	2
16	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Generalized Hadamard matrix of order 2^4

$$GH(16, 1) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ 1 & 3 & 5 & 7 & 9 & 11 & 13 & 15 & 4 & 2 & 8 & 6 & 12 & 10 & 16 & 14 \\ 1 & 4 & 7 & 6 & 13 & 16 & 11 & 10 & 12 & 9 & 14 & 15 & 8 & 5 & 2 & 3 \\ 1 & 5 & 9 & 13 & 4 & 8 & 12 & 16 & 7 & 3 & 15 & 11 & 6 & 2 & 14 & 10 \\ 1 & 6 & 11 & 16 & 8 & 3 & 14 & 9 & 15 & 12 & 5 & 2 & 10 & 13 & 4 & 7 \\ 1 & 7 & 13 & 11 & 12 & 14 & 8 & 2 & 6 & 4 & 10 & 16 & 15 & 9 & 3 & 5 \\ 1 & 8 & 15 & 10 & 16 & 9 & 2 & 7 & 14 & 11 & 4 & 5 & 3 & 6 & 13 & 12 \\ 1 & 9 & 4 & 12 & 7 & 15 & 6 & 14 & 13 & 5 & 16 & 8 & 11 & 3 & 10 & 2 \\ 1 & 10 & 2 & 9 & 3 & 12 & 4 & 11 & 5 & 14 & 6 & 13 & 7 & 16 & 8 & 15 \\ 1 & 11 & 8 & 14 & 15 & 5 & 10 & 4 & 16 & 6 & 9 & 3 & 2 & 12 & 7 & 13 \\ 1 & 12 & 6 & 15 & 11 & 2 & 16 & 5 & 8 & 13 & 3 & 10 & 14 & 7 & 9 & 4 \\ 1 & 13 & 12 & 8 & 6 & 10 & 15 & 3 & 11 & 7 & 2 & 14 & 16 & 4 & 5 & 9 \\ 1 & 14 & 10 & 5 & 2 & 13 & 9 & 6 & 3 & 16 & 12 & 7 & 4 & 15 & 11 & 8 \\ 1 & 15 & 16 & 2 & 14 & 4 & 3 & 13 & 10 & 8 & 7 & 9 & 5 & 11 & 12 & 6 \\ 1 & 16 & 14 & 3 & 10 & 7 & 5 & 12 & 2 & 15 & 13 & 4 & 9 & 8 & 6 & 11 \end{pmatrix}$$

Product of pairs of rows of $GH(2^4)$

*Row i * Rowj*

$$\begin{aligned} 1 * 2 &= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \\ 1 * 3 &= [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14] \\ 1 * 4 &= [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3] \\ 1 * 5 &= [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10] \\ 1 * 6 &= [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7] \\ 1 * 7 &= [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5] \\ 1 * 8 &= [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12] \\ 1 * 9 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2] \\ 1 * 10 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15] \\ 1 * 11 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13] \\ 1 * 12 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4] \\ 1 * 13 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9] \\ 1 * 14 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\ 1 * 15 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6] \\ 1 * 16 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\ 2 * 3 &= [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3] \\ 2 * 4 &= [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14] \\ 2 * 5 &= [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7] \end{aligned}$$

$$\begin{aligned}
2 * 6 &= [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10] \\
2 * 7 &= [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12] \\
2 * 8 &= [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5] \\
2 * 9 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15] \\
2 * 10 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2] \\
2 * 11 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4] \\
2 * 12 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13] \\
2 * 13 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\
2 * 14 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9] \\
2 * 15 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\
2 * 16 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6]
\end{aligned}$$

$$\begin{aligned}
3 * 4 &= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \\
3 * 5 &= [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5] \\
3 * 6 &= [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12] \\
3 * 7 &= [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10] \\
3 * 8 &= [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7] \\
3 * 9 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13] \\
3 * 10 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4] \\
3 * 11 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2] \\
3 * 12 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15] \\
3 * 13 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6] \\
3 * 14 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\
3 * 15 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9] \\
3 * 16 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\
3 * 5 &= [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12]
\end{aligned}$$

$$\begin{aligned}
4 * 6 &= [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5] \\
4 * 7 &= [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7] \\
4 * 8 &= [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10] \\
4 * 9 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4] \\
4 * 10 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13] \\
4 * 11 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15]
\end{aligned}$$

$$\begin{aligned}
4 * 12 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2] \\
4 * 13 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\
4 * 14 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6] \\
4 * 15 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\
4 * 16 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9]
\end{aligned}$$

$$\begin{aligned}
5 * 6 &= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \\
5 * 7 &= [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14] \\
5 * 8 &= [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3] \\
5 * 9 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9] \\
5 * 10 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\
5 * 11 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6] \\
5 * 12 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\
5 * 13 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2] \\
5 * 14 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15] \\
5 * 15 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13] \\
5 * 16 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4]
\end{aligned}$$

$$\begin{aligned}
6 * 7 &= [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3] \\
6 * 8 &= [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14] \\
6 * 9 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\
6 * 10 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9] \\
6 * 11 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\
6 * 12 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6] \\
6 * 13 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15] \\
6 * 14 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2] \\
6 * 15 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4] \\
6 * 16 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13]
\end{aligned}$$

$$\begin{aligned}
7 * 8 &= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \\
7 * 9 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6] \\
7 * 10 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\
7 * 11 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9]
\end{aligned}$$

$$\begin{aligned}
7 * 12 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\
7 * 13 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13] \\
7 * 14 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4] \\
7 * 15 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2] \\
7 * 16 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15]
\end{aligned}$$

$$\begin{aligned}
8 * 9 &= [1, 16, 14, 3, 10, 7, 5, 12, 2, 15, 13, 4, 9, 8, 6, 11] \\
8 * 10 &= [1, 15, 16, 2, 14, 4, 3, 13, 10, 8, 7, 9, 5, 11, 12, 6] \\
8 * 11 &= [1, 14, 10, 5, 2, 13, 9, 6, 3, 16, 12, 7, 4, 15, 11, 8] \\
8 * 12 &= [1, 13, 12, 8, 6, 10, 15, 3, 11, 7, 2, 14, 16, 4, 5, 9] \\
8 * 13 &= [1, 12, 6, 15, 11, 2, 16, 5, 8, 13, 3, 10, 14, 7, 9, 4] \\
8 * 14 &= [1, 11, 8, 14, 15, 5, 10, 4, 16, 6, 9, 3, 2, 12, 7, 13] \\
8 * 15 &= [1, 10, 2, 9, 3, 12, 4, 11, 5, 14, 6, 13, 7, 16, 8, 15] \\
8 * 16 &= [1, 9, 4, 12, 7, 15, 6, 14, 13, 5, 16, 8, 11, 3, 10, 2]
\end{aligned}$$

$$\begin{aligned}
9 * 10 &= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \\
9 * 11 &= [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14] \\
9 * 12 &= [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3] \\
9 * 13 &= [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10] \\
9 * 14 &= [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7] \\
9 * 15 &= [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5] \\
9 * 16 &= [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12]
\end{aligned}$$

$$\begin{aligned}
10 * 11 &= [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3] \\
10 * 12 &= [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14] \\
10 * 13 &= [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7] \\
10 * 14 &= [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10] \\
10 * 15 &= [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12] \\
10 * 16 &= [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5]
\end{aligned}$$

$$\begin{aligned}
11 * 12 &= [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \\
11 * 13 &= [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5]
\end{aligned}$$

$$11 * 14 = [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12]$$

$$11 * 15 = [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10]$$

$$11 * 16 = [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7]$$

$$12 * 13 = [1, 8, 15, 10, 16, 9, 2, 7, 14, 11, 4, 5, 3, 6, 13, 12]$$

$$12 * 14 = [1, 7, 13, 11, 12, 14, 8, 2, 6, 4, 10, 16, 15, 9, 3, 5]$$

$$12 * 15 = [1, 6, 11, 16, 8, 3, 14, 9, 15, 12, 5, 2, 10, 13, 4, 7]$$

$$12 * 16 = [1, 5, 9, 13, 4, 8, 12, 16, 7, 3, 15, 11, 6, 2, 14, 10]$$

$$13 * 14 = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]$$

$$13 * 15 = [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14]$$

$$13 * 16 = [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3]$$

$$14 * 15 = [1, 4, 7, 6, 13, 16, 11, 10, 12, 9, 14, 15, 8, 5, 2, 3]$$

$$14 * 16 = [1, 3, 5, 7, 9, 11, 13, 15, 4, 2, 8, 6, 12, 10, 16, 14]$$

$$15 * 16 = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]$$