Small observations on Unbiased Hadamard matrices of order 256

July 4, 2014

I tried using the method you described earlier, that is using 8 unbiased Hadamard matrices of order 16(which included the three Bush type matrices) to find 120 Hadamard matrices of order 256 and then to check if they were unbiased.
Out of the 120 matrices, 60 of them were not unbiased with any of the remaining 120, so I removed them. Out of the remaining 60,I could find a maximum set of 4 mutually unbiased Hadamard matrices of order 256.

I printed the incidence matrix for unbiasedness, with 60 rows and 60 columns, the i^{th} row and column representing the i^{th} matrix.
$a_{ij} = 1$ implies that Matrix i is unbiased with Matrix j, $a_{ij} = 0$ implies that Matrix i is not unbiased with Matrix j.
The variable C at the end of each row counts the number of one's in each row.All the rows with the same value of C form a set in which every pair is not mutually unbiased with each other, but is mutually unbiased with every other of the $60 - C$ matrices.
Since there were only 4 different values of C, I guess there is a maximum of 4 mutually unbiased Hadamard matrices of order 256 from this set!

$$
\begin{pmatrix}
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
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0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
1111111111111111110110111001111010111110010101111111001001110 & C = 45 \\
1111111111111111110111111111101101101111111011111111110111111 & C = 54 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
1111111111111111110110111001111010111110010101111111001001110 & C = 45 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
1111111111111111111111110111110111010111111111110101111111101 & C = 53 \\
1111111111111111110110111001111010111110010101111111001001110 & C = 45 \\
1111111111111111110110111001111010111110010101111111001001110 & C = 45 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
0000000000000000001101001110011111111001101110010101111110011 & C = 28 \\
1111111111111111110111111111101101101111111011111111110111111 & C = 54
\end{pmatrix}
$$

$$
\begin{pmatrix}
111111111111111111111111011111011101011111111110101111111101 & C = 53 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
111111111111111111101111111110110110111111101111111110111111 & C = 54 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
111111111111111111111111011111011101011111111110101111111101 & C = 53 \\
111111111111111111101111111110110110111111101111111110111111 & C = 54 \\
111111111111111111111111011111011101011111111110101111111101 & C = 53 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
111111111111111111101111111110110110111111101111111110111111 & C = 54 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
111111111111111111111111011111011101011111111110101111111101 & C = 53 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
111111111111111111111111011111011101011111111110101111111101 & C = 53 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
111111111111111111101111111110110110111111101111111110111111 & C = 54 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
000000000000000000110100111001111111001101110010101111110011 & C = 28 \\
111111111111111111111111011111011101011111111110101111111101 & C = 53 \\
111111111111111111011011100111101011111001010111111001001110 & C = 45
\end{pmatrix}
$$

I tried a similar thing with the 8 unbiased matrices that Sara had computed, and got this matrix:

$$
\begin{pmatrix}
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
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000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
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000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
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000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111111110111111111011111111111111101001111 & C = & 55 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45
\end{pmatrix}
$$

$$
\begin{pmatrix}
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52 \\
111111111111111111111111110111111111011111111111111101001111 & C = & 55 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111111111110111111111011111111111111101001111 & C = & 55 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111111110111111111011111111111111101001111 & C = & 55 \\
111111111111111111111111110111111111011111111111111101001111 & C = & 55 \\
000000000000000000110101001101011101101010111110101110110111 & C = & 28 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52 \\
111111111111111111001110111010110011111101001001110011111101 & C = & 45 \\
111111111111111111111011111111101110110111110111011111111010 & C = & 52
\end{pmatrix}
$$

$$HH = \begin{pmatrix} 12 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8 & 12 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12 & -8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 & 12 \end{pmatrix}$$ If my program has not erred greviously,then I think there must be only 4 unbiased Hadamard matrices from this set of order 256.