SPRING 2019

# CE 311S : PROBABILITY AND STATISTICS

Discussion session

M 1:00 – 2:00 PM

**PRIYADARSHAN PATIL**
Teaching Assistant, The University of Texas at Austin

# Administrative Stuff

- Homework 2 due Friday 11:59 PM
- Exam 1 is on March 8th, and will definitely cover everything till discrete distributions, so make sure you are comfortable with what's been covered already

# Week 5: Question

A coin of diameter 1 inch is thrown on a table covered with a grid of lines, each two inches apart. P is the probability that the coin lands inside a square without touching any of the lines of the grid? You can assume that the person throwing has no skill in throwing the coin and is throwing it randomly.

Is P greater than or equal to 0.33?

Hint: think about where the center of the coin can land.

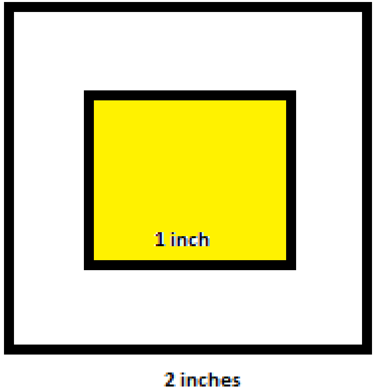
# Week 5: Question

Please turn in your answer (Yes/No) with your wager and name.

Current standandings: Link

# Week 5: Answer

- The probability is 1/4 or 0.25



# Any questions so far?

- About this specific problem, course material covered so far, etc.
- Quick 5 minute survey: **https://tinyurl.com/311S-survey**
- **Please be honest!**

# Solve the following

**Group 1**

Let A= No. of ways Ann, Tom, Leslie, Jerry, and Ron can sit on chairs marked A, B and C in a classroom **such that Ann and Tom never sit together**
**Find P(A)**

**n(A)= 48 P(A)= 48/60= 4/5**

**Group 2**

A= No. of ways a team of 4 can be chosen from a group of 4 senior and 4 junior players **such that the team consists of at least one junior and senior. Find P(A)**

**n(A)= 68; P(A)= 68/70= 34/35**

**Group 3**

Let A= No. of ways a 5 digit number can be framed using numbers 2, 4, 5 and 7 **such that the number is greater than 52,000. Find P(A)**

**n(A)= 448; P(A)= 448/1024= 7/16**

**Group 4**

Your friends visit Austin for 7 days. Each evening you can take them to one of A, B, C, or D restaurant for dinner. No. of ways of picking restaurants **such that B is visited atleast twice**.

**n(A)= 9094(Ordered); P(A)= 9094/$4^7$= 4547/8192**

# PMF/CDF Example

A discrete random variable X has the following probability distribution

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | $k$ | $2k$ | $3k$ |

What is the value of $k$?

1/6

Find $P(X \leq 2.5)$

1/2

Two independent observations, $X_1$ and $X_2$ are made of X

Find the pmf of $X_1 + X_2$

$$P(X_1 + X_2 = 2) = \frac{1}{36}; P(X_1 + X_2 = 3) = \frac{1}{9}; P(X_1 + X_2 = 4) = \frac{5}{18}$$

$$P(X_1 + X_2 = 5) = \frac{1}{3}; P(X_1 + X_2 = 6) = \frac{1}{4}; P(X_1 + X_2 = otherwise) = 0$$

**Practice:** Find $P(1.5 < X_1 + X_2 \leq 3.5)$ (=5/36)

| **Name of special distribution (X)** | **Properties** | **What X represents and its range ($R_X$)** | **Parameters** | **Probability** $P(X=x)$ | **Expected value** $E(X)$ | **Variance** $Var(X)$ |
|---|---|---|---|---|---|---|
| Binomial distribution | ○ You perform $n$ __________ experiments and you know $n$ before starting<br>○ Each experiment or trial ___________ of each other<br>○ The probability of success, $p$ (or seeing a particular event) is ______ for every trial | The number of times __________ is observed<br><br>$R_X$: | | $\binom{n}{x} p^x (1-p)^{n-x}$ | | |
| Hypergeometric distribution | ○ I sample $k$ objects from a ________ population with $b$ success and $r$ failures<br>○ Each sample of $k$ objects is ______________ to be chosen | The number of times success is observed<br><br>$R_X$: | | $\frac{\binom{b}{x}\binom{r}{k-x}}{\binom{b+r}{k}}$ | | |
| Negative binomial distribution | ○ You perform identical experiments and you ___________ $n$ before starting, rather you keep performing experiments until you observe ____________<br>○ Each experiment or trial independent of each other<br>○ The probability of success, $p$ is same for every trial | The number of ______________ before $m$ –th success is observed<br><br>$R_X$: | | $\begin{cases} \binom{x-1}{m-1} p^m (1-p)^{x-m} \\ 0 \end{cases}$ | | |
| Poisson distribution | ○ An event occurs at an average rate of $\lambda$ such that:<br>○ Occurrences of the event are ____________ of each other<br>○ More than one of these events _________ occur simultaneously | The number of events<br><br>$R_X$: | | $\frac{e^{-\lambda}\boldsymbol{\lambda}^x}{x!}$ | | |

| **Name of special distribution (X)** | **Properties** | **What X represents and its range ($R_X$)** | **Parameters** | **Probability $P(X=x)$** | **Expected value $E(X)$** | **Variance $Var(X)$** |
|---|---|---|---|---|---|---|
| Binomial distribution | ○ You perform $\boldsymbol{n}$ identical experiments and you know $\boldsymbol{n}$ before starting<br>○ Each experiment or trial independent of each other<br>○ The probability of success, $\boldsymbol{p}$ (or seeing a particular event) is same for every trial | The number of times success is observed<br><br>$\boldsymbol{R_X}$: {0,1,2…,n} | $n, p$ | $\binom{n}{x} p^x (1-p)^{n-x}$ | $np$ | $np(1-p)$ |
| Hypergeometric distribution | ○ I sample $k$ objects from a finite population with $b$ success and $r$ failures<br>○ Each sample of $k$ objects is equally likely to be chosen | The number successes observed<br><br>$\boldsymbol{R_X}$: $\{0,1,2,..min\{b,k\}\}$ | $k, b, r$ | $\frac{\binom{b}{x}\binom{r}{k-x}}{\binom{b+r}{k}}$ | $\frac{kb}{b+r}$ | $k * \frac{b}{b+r} * \frac{r}{b+r} * \frac{b+r-k}{b+r-1}$ |
| Negative binomial distribution | ○ You perform identical experiments and you do not know $\boldsymbol{n}$ before starting, rather you keep performing experiments until you observe m-th success<br>○ Each experiment or trial independent of each other<br>○ The probability of success, $p$ is same for every trial | The number of trials before $m$ –th success is observed<br><br>$\boldsymbol{R_X}$: $\{m, m+1, \ldots \infty\}$ | $m, p$ | $\begin{cases} \binom{x-1}{m-1} p^m (1-p)^{x-m} \\ 0 \end{cases}$ | $\frac{m}{p}$ | $\frac{m(1-p)}{p^2}$ |
| Poisson distribution | ○ An event occurs at an average rate of $\lambda$ such that:<br>○ Occurrences of the event are independent of each other<br>○ More than one of these events cannot occur simultaneously | The number of events<br><br>$\boldsymbol{R_X}$: $\{0,1,2 \ldots \infty\}$ | $\lambda$ | $\frac{e^{-\lambda}\boldsymbol{\lambda}^x}{x!}$ | $\lambda$ | $\lambda$ |

# Identify the type of variable and its parameters

1. You are single this Valentine' and want to meet an Oracle to ask what will be the future of your romantic relationship.

2. The Oracle is very famous and it is **only 40% likely that you will get an appointment with her** in the next month.

3. You keep trying every month until your appointment is scheduled. Let **X be the number of times you try before your appointment is finally scheduled**.
4. **Negative Binomial (m=1, p=0.4)**

# Identify the type of variable and its parameters

1. You finally get an appointment scheduled and she asks to play a game with you to decide your future.

2. She hands you a bag containing 100 red pills and 40 blue pills and asks you to **select 10 pills one by one (replacing the previously drawn pill in the bag)**. Based on how many red pills, you get she will name your future date!

3. Let Y be the number of red pills you end up choosing.
4. **Binomial (n=10; p=5/7)**

# Identify the type of variable and its parameters

1. Your date is chosen. It is Pat who you have always adored as a crush. The Oracle then asks you what will be your pickup line and **you offer her 6 choices as below**. She marks each choice okay or bad.
    1. You fascinate me more than the Fundamental Theorem of Calculus (BAD)
    2. Are you a 90 degree angle? 'Cause you are looking right! (OKAY)
    3. I am equivalent to the Empty Set when you are not with me. (OKAY)
    4. My love for you is like dividing by zero... you cannot define it. (OKAY)
    5. Let's take each other to the limit to see if we converge (OKAY)
    6. You are the solution to my homogeneous system of linear equations. (BAD)

2. Regardless of what she ranked, **you select 3 pick up lines at random** to try on Pat. **Let Y be the number of okay pickup lines you try**.
3. **Hypergeometric (b=4,r=2 and k=3)**

# Identify the type of variable and its parameters

1. You try your pick up lines and ah, Pat finally says yes. Both of you go to a coffee shop on the Drag, but with midterms approaching it is full with singles studying for their tests and there is a huge line.

2. You notice that the **front desk serves customers at rate of 3 customers per minute**.

3. Let X be the number of customers served.
4. **Poisson (lambda= 3 customers/min)**

# Thank you

Any Questions?