BANA 7020 - Optimization

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Final Project – Linear Mixed Integer Problem

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Problem Statement

A transportation company is evaluating a prototype system that combines trucks and drones for last-mile delivery services. The test run considers a set of orders that must be delivered to known locations. A delivery truck starts from a depot and visits "launch sites" corresponding to the customers locations. From each launch site, the truck deploys a series of drones that deliver the orders and return to meet the truck at the launch site. Once all the drones are recovered, the truck moves to the next launch site and repeats the process until all orders are delivered.

Abstract

The problem under consideration is a **Vehicle Routing** problem. Industry is experiencing surge in variety of delivery services due to availability of items and fulfilment over the internet. A prevalent model is to utilize a delivery truck with one or more unmanned vehicles, drones in our case to achieve the delivery purpose with trucks simultaneously.

Introduction and Approach

In our study for the travelling salesman problem with drones (TSP-D), we have modeled the design to find optimal joint route taken by truck and drones together to deliver the packages to each customer. Our objective is to minimize the total travel cost incurred at the same time by completing the task in the shortest time possible, hence selecting a minimum distance(optimal) route to cover all the customers.

All the considerations and constraints have been taken care in this study during the model formulation. For starting, we have chosen our elementary case where maximum number of drones that can be launched from a launch site is (k) = 2 and total customer orders = 15. One truck is fixed for all the iterations as per the problem statement. We will repeat the process for maximum number of drones, k= {3,4,5} and, also for more customer orders = {20,25} and showcase the results for these iterations. Sensitivity analysis has been performed on our model by varying the cost per distance unit of both truck and drones.

Model Implementation

1. Euclidean distance matrix[17,17] was generated using the random coordinates for depot, 15 customer, depot(clone) node. Depot coordinates were set to origin(0,0). Distance values were rounded up to the nearest integer.
2. The step 1 was repeated for 10 more test cases to run different iterations.
3. **Parameters** –
    a. NORDERS – Total Number of customer orders + 2 (node 1 is depot point and node n+2 is depot point again)
    b. ORDERS – Sequence of NORDERS
    c. K – Maximum number of drones
    d. M – A very large number

4. **Variables** –
    a. DIST[ORDERS, ORDERS] – Array of distance between customer locations
    b. X[ORDERS], Y[ORDERS] – Array of integer
    c. lambda[ORDERS] – Array for subtour elimination, explained later
    d. truck[ORDERS, ORDERS] – decision variable (1 if truck travels from i to j ; 0 otherwise, for all i,j ε (ORDERS))
    e. drone[ORDERS, ORDERS] – decision variable (1 if drone travels from i to j ; 0 otherwise, for all i,j ε (ORDERS))

    f. em_dr[ORDERS, ORDERS] – decision variable (1 if drone returns from i to j ; 0 otherwise, for all i,j ε (ORDERS))
    g. tr_n[ORDERS] – decision variable (1 if truck visit a node; 0 otherwise, for all i ε (ORDERS))

5. **Objective**

   For each unit of distance traveled by the truck there is a cost of $10, while for each unit of distance traveled by a drone there is a cost of $3, our objective is to minimize the cost.

   Objective: $\sum_{i,j \in ORDERS | i \neq j} DIST_{i,j} * truck_{i,j} * 10 + \sum_{i,j \in ORDERS | i \neq j} DIST_{i,j} * drone_{i,j} * 3 + \sum_{i,j \in ORDERS | i \neq j} DIST_{i,j} * em\_dr_{i,j} * 3$

6. **Constraints**

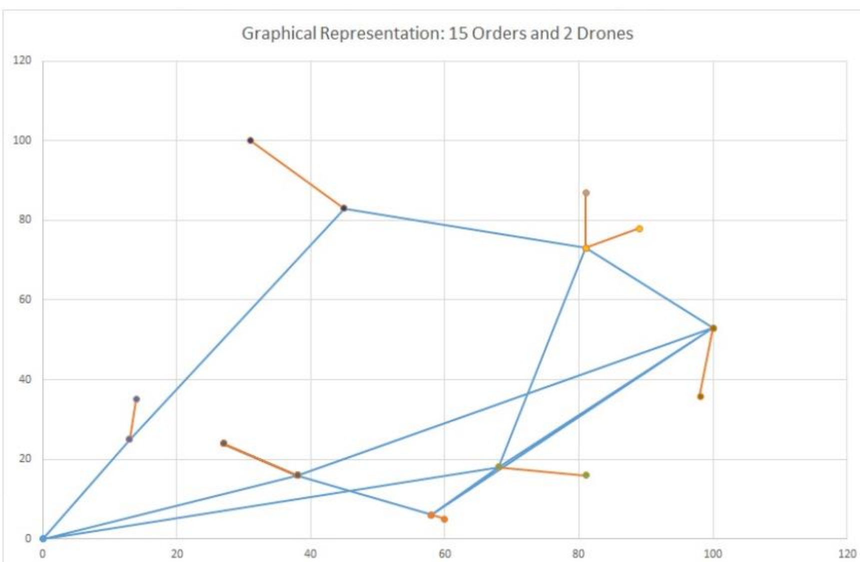
   - The truck must start and end at the depot (node 1 is depot (0,0) and node n+1 is duplicate of depot (0,0))
     $\sum_{j \in ORDERS} truck_{1,j} = 1$
     $\sum_{j \in ORDERS} truck_{j,n+2} = 1$
   - Truck should not come to start node from any other node and should not leave the end node i.e., duplicate node of depot
     $truck_{j,1} = 0, \forall\ j \in (ORDERS)$
     $truck_{n+2,j} = 0, \forall\ j \in (ORDERS)$
   - The truck should not follow the path – node 1 to n+1 node.
     $truck_{1,n+2} = 0$
   - The decision variable tr_n is active for node j, if the node is in optimal path of truck delivery and then the node will be active for launching drones too.
     $tr\_n_j = 1 * \sum_{i \in ORDERS} truck_{i,j}, \forall\ j \in (2..\ n+1)$
   - Truck should come to node(i) from anyone of the nodes and should leave node (i) to anyone of the nodes, given both nodes are in the optimal path for truck
     $\sum_{i \in ORDERS} truck_{i,j} = 1 * tr\_n_i, \forall\ i \in (2 \ldots n+1)$
     $\sum_{j \in ORDERS} truck_{i,j} = 1 * tr\_n_i, \forall\ j \in (2 \ldots n+1)$
   - Truck must leave from any node it has reached, other than depot.
     $\sum_{j \in ORDERS} truck_{i,j} = \sum_{j \in ORDERS} truck_{j,i} \ \forall\ i \in (2..\ n+1)$
   - The truck can deploy up to K drones at each stop in a launch site

$\sum_{j \in ORDERS} drone_{i,j}$ ≤ *K * tr_n*$_i$,$\forall$ *i ε (2.. n+1)*

- The drones have limited cargo capacity and as a result they can only visit 1 customer (not counting the starting point) before returning to the truck, i.e. if drone has been launched from any node, it must return to the same node
  em_dr$_{i,j}$ = *drone*$_{i,j}$,$\forall$ *i ε (2.. n+1)* , $\forall$ *j ε (2…n+1)*
- Each customer should be visited at least once by a drone. The customer locations used as launch sites will be visited by the truck and will serve as the starting and ending point of a drone tour
  drone$_{i,j}$ + *drone*$_{j,I}$ ≥ 1 ,$\forall$ *i ε (2.. n+1) & j ε (2…n+1)*
- Arc start and end nodes should be different
  drone$_{i,i}$ = 0, *truck*$_{i,i}$ = 0, $\forall$ *i ε (2.. n+1)*
- Subtour elimination Constraints
  lambda(1) = 1
  lambda(n+2) = n+2
  lambda(j) >= lambda(i) + 1 - M*(1 - *truck*$_{i,i}$)

7. <u>Executions Results and Conclusions –</u>

For 15 customer locations and 2 number of maximum drones launched from a launch site, we have got the following result. The blue line denotes the truck route and orange denotes the drone route.



```
Running model
Begin running model
Optimal tour cost is: 3422
TRUCK IS ON ROUTE 1 to 3
TRUCK IS ON ROUTE 2 to 16
TRUCK IS ON ROUTE 3 to 5
TRUCK IS ON ROUTE 5 to 8
TRUCK IS ON ROUTE 6 to 15
TRUCK IS ON ROUTE 8 to 2
TRUCK IS ON ROUTE 15 to 17
TRUCK IS ON ROUTE 16 to 6
DRONE IS ON ROUTE 2 to 12
DRONE IS ON ROUTE 3 to 14
DRONE IS ON ROUTE 5 to 9
DRONE IS ON ROUTE 6 to 10
DRONE IS ON ROUTE 8 to 4
DRONE IS ON ROUTE 15 to 13
DRONE IS ON ROUTE 16 to 7
DRONE IS ON ROUTE 16 to 11
DRONE RETURNING ON ROUTE 4 to 8
DRONE RETURNING ON ROUTE 7 to 16
DRONE RETURNING ON ROUTE 9 to 5
DRONE RETURNING ON ROUTE 10 to 6
DRONE RETURNING ON ROUTE 11 to 16
DRONE RETURNING ON ROUTE 12 to 2
DRONE RETURNING ON ROUTE 13 to 15
DRONE RETURNING ON ROUTE 14 to 3
End running model
```

The optimal cost came out to be 3422 per unit distance travelled by both truck and maximum 2 drones in parallel for 15 customer orders. As it can be interpreted from the routes taken, subtour eliminations point has been taken care well by our model.

Different Iterations

Cost per unit distance of truck and drones(each) are $10 and $3 respectively

- 1 Truck, maximum 3 drones in parallel for 15 customer orders

```
Running model
Begin running model
Optimal tour cost is: 3456
TRUCK IS ON ROUTE 1 to 4
TRUCK IS ON ROUTE 2 to 11
TRUCK IS ON ROUTE 3 to 15
TRUCK IS ON ROUTE 4 to 16
TRUCK IS ON ROUTE 5 to 2
TRUCK IS ON ROUTE 11 to 17
TRUCK IS ON ROUTE 15 to 5
TRUCK IS ON ROUTE 16 to 3
DRONE IS ON ROUTE 2 to 6
DRONE IS ON ROUTE 3 to 9
DRONE IS ON ROUTE 3 to 10
DRONE IS ON ROUTE 4 to 7
DRONE IS ON ROUTE 5 to 12
DRONE IS ON ROUTE 11 to 14
DRONE IS ON ROUTE 15 to 13
DRONE IS ON ROUTE 16 to 8
DRONE RETURNING ON ROUTE 6 to 2
DRONE RETURNING ON ROUTE 7 to 4
DRONE RETURNING ON ROUTE 8 to 16
DRONE RETURNING ON ROUTE 9 to 3
DRONE RETURNING ON ROUTE 10 to 3
DRONE RETURNING ON ROUTE 12 to 5
DRONE RETURNING ON ROUTE 13 to 15
DRONE RETURNING ON ROUTE 14 to 11
End running model
```

- 1 Truck, maximum 4 drones in parallel for 15 customer orders

```
Running model
Begin running model
Optimal tour cost is: 3398
TRUCK IS ON ROUTE 1 to 7
TRUCK IS ON ROUTE 4 to 10
TRUCK IS ON ROUTE 7 to 8
TRUCK IS ON ROUTE 8 to 15
TRUCK IS ON ROUTE 10 to 17
TRUCK IS ON ROUTE 13 to 4
TRUCK IS ON ROUTE 15 to 16
TRUCK IS ON ROUTE 16 to 13
DRONE IS ON ROUTE 4 to 12
DRONE IS ON ROUTE 7 to 5
DRONE IS ON ROUTE 8 to 5
DRONE IS ON ROUTE 10 to 3
DRONE IS ON ROUTE 13 to 14
DRONE IS ON ROUTE 15 to 2
DRONE IS ON ROUTE 15 to 6
DRONE IS ON ROUTE 16 to 9
DRONE IS ON ROUTE 16 to 11
DRONE RETURNING ON ROUTE 2 to 15
DRONE RETURNING ON ROUTE 3 to 10
DRONE RETURNING ON ROUTE 5 to 7
DRONE RETURNING ON ROUTE 5 to 8
DRONE RETURNING ON ROUTE 6 to 15
DRONE RETURNING ON ROUTE 9 to 16
DRONE RETURNING ON ROUTE 11 to 16
DRONE RETURNING ON ROUTE 12 to 4
DRONE RETURNING ON ROUTE 14 to 13
End running model
```

- 1 Truck, maximum 5 drones in parallel for 15 customer orders

```
Running model
Begin running model
Optimal tour cost is: 3694
TRUCK IS ON ROUTE 1 to 10
TRUCK IS ON ROUTE 4 to 17
TRUCK IS ON ROUTE 5 to 8
TRUCK IS ON ROUTE 7 to 5
TRUCK IS ON ROUTE 8 to 9
TRUCK IS ON ROUTE 9 to 4
TRUCK IS ON ROUTE 10 to 7
DRONE IS ON ROUTE 4 to 12
DRONE IS ON ROUTE 4 to 15
DRONE IS ON ROUTE 5 to 3
DRONE IS ON ROUTE 7 to 2
DRONE IS ON ROUTE 7 to 6
DRONE IS ON ROUTE 7 to 16
DRONE IS ON ROUTE 8 to 11
DRONE IS ON ROUTE 9 to 14
DRONE IS ON ROUTE 10 to 13
DRONE RETURNING ON ROUTE 2 to 7
DRONE RETURNING ON ROUTE 3 to 5
DRONE RETURNING ON ROUTE 6 to 7
DRONE RETURNING ON ROUTE 11 to 8
DRONE RETURNING ON ROUTE 12 to 4
DRONE RETURNING ON ROUTE 13 to 10
DRONE RETURNING ON ROUTE 14 to 9
DRONE RETURNING ON ROUTE 15 to 4
DRONE RETURNING ON ROUTE 16 to 7
End running model
```

- 1 Truck, maximum 3 drones in parallel for 20 customer orders

```
Running model
Begin running model
Optimal tour cost is: 4120
TRUCK IS ON ROUTE 1 to 16
TRUCK IS ON ROUTE 6 to 22
TRUCK IS ON ROUTE 8 to 11
TRUCK IS ON ROUTE 9 to 20
TRUCK IS ON ROUTE 10 to 6
TRUCK IS ON ROUTE 11 to 9
TRUCK IS ON ROUTE 13 to 8
TRUCK IS ON ROUTE 15 to 13
TRUCK IS ON ROUTE 16 to 15
TRUCK IS ON ROUTE 20 to 10
DRONE IS ON ROUTE 6 to 7
DRONE IS ON ROUTE 8 to 4
DRONE IS ON ROUTE 9 to 3
DRONE IS ON ROUTE 9 to 14
DRONE IS ON ROUTE 10 to 17
DRONE IS ON ROUTE 11 to 2
DRONE IS ON ROUTE 13 to 18
DRONE IS ON ROUTE 13 to 21
DRONE IS ON ROUTE 15 to 19
DRONE IS ON ROUTE 16 to 5
DRONE IS ON ROUTE 20 to 12
DRONE RETURNING ON ROUTE 2 to 11
DRONE RETURNING ON ROUTE 3 to 9
DRONE RETURNING ON ROUTE 4 to 8
DRONE RETURNING ON ROUTE 5 to 16
DRONE RETURNING ON ROUTE 7 to 6
DRONE RETURNING ON ROUTE 12 to 20
DRONE RETURNING ON ROUTE 14 to 9
DRONE RETURNING ON ROUTE 17 to 10
DRONE RETURNING ON ROUTE 18 to 13
DRONE RETURNING ON ROUTE 19 to 15
DRONE RETURNING ON ROUTE 21 to 13
End running model
```

➢ 1 Truck, maximum 2 drones in parallel for 25 customer orders

```
Optimal tour cost is: 4056
TRUCK IS ON ROUTE 1 to 6
TRUCK IS ON ROUTE 6 to 12
TRUCK IS ON ROUTE 7 to 15
TRUCK IS ON ROUTE 8 to 27
TRUCK IS ON ROUTE 9 to 19
TRUCK IS ON ROUTE 11 to 13
TRUCK IS ON ROUTE 12 to 9
TRUCK IS ON ROUTE 13 to 23
TRUCK IS ON ROUTE 15 to 8
TRUCK IS ON ROUTE 19 to 11
TRUCK IS ON ROUTE 23 to 24
TRUCK IS ON ROUTE 24 to 7
DRONE IS ON ROUTE 6 to 14
DRONE IS ON ROUTE 7 to 3
DRONE IS ON ROUTE 7 to 24
DRONE IS ON ROUTE 8 to 5
DRONE IS ON ROUTE 9 to 2
DRONE IS ON ROUTE 9 to 3
DRONE IS ON ROUTE 11 to 18
DRONE IS ON ROUTE 11 to 26
DRONE IS ON ROUTE 12 to 21
DRONE IS ON ROUTE 13 to 10
DRONE IS ON ROUTE 13 to 16
DRONE IS ON ROUTE 15 to 17
DRONE IS ON ROUTE 19 to 22
DRONE IS ON ROUTE 23 to 20
DRONE IS ON ROUTE 23 to 25
DRONE IS ON ROUTE 24 to 4
DRONE RETURNING ON ROUTE 2 to 9
DRONE RETURNING ON ROUTE 3 to 7
DRONE RETURNING ON ROUTE 3 to 9
DRONE RETURNING ON ROUTE 4 to 24
DRONE RETURNING ON ROUTE 5 to 8
DRONE RETURNING ON ROUTE 10 to 13
DRONE RETURNING ON ROUTE 14 to 6
DRONE RETURNING ON ROUTE 16 to 13
DRONE RETURNING ON ROUTE 17 to 15
DRONE RETURNING ON ROUTE 18 to 11
DRONE RETURNING ON ROUTE 20 to 23
DRONE RETURNING ON ROUTE 21 to 12
DRONE RETURNING ON ROUTE 22 to 19
DRONE RETURNING ON ROUTE 24 to 7
DRONE RETURNING ON ROUTE 25 to 23
DRONE RETURNING ON ROUTE 26 to 11
End running model
```

Sensitivity Analysis by varying cost per unit distance for truck and drones both

➢ **Cost per unit distance of truck and maximum 2 drones launched at a time are $10 and $6 (each) respectively, for 15 customer locations give the optimal solution as**

```
Begin running model
Optimal tour cost is: 4508
TRUCK IS ON ROUTE 1 to 4
TRUCK IS ON ROUTE 2 to 17
TRUCK IS ON ROUTE 4 to 7
TRUCK IS ON ROUTE 7 to 16
TRUCK IS ON ROUTE 9 to 2
TRUCK IS ON ROUTE 12 to 9
TRUCK IS ON ROUTE 15 to 12
TRUCK IS ON ROUTE 16 to 15
DRONE IS ON ROUTE 2 to 5
DRONE IS ON ROUTE 4 to 13
DRONE IS ON ROUTE 7 to 14
DRONE IS ON ROUTE 9 to 10
DRONE IS ON ROUTE 12 to 3
DRONE IS ON ROUTE 15 to 6
DRONE IS ON ROUTE 15 to 8
DRONE IS ON ROUTE 16 to 11
DRONE RETURNING ON ROUTE 3 to 12
DRONE RETURNING ON ROUTE 5 to 2
DRONE RETURNING ON ROUTE 6 to 15
DRONE RETURNING ON ROUTE 8 to 15
DRONE RETURNING ON ROUTE 10 to 9
DRONE RETURNING ON ROUTE 11 to 16
DRONE RETURNING ON ROUTE 13 to 4
DRONE RETURNING ON ROUTE 14 to 7
End running model
```

As compared the our earlier model, we can see the route for truck and drones is changed but the number of customer locations for drones and trucks remain the same.

➢ **Cost per unit distance of truck and maximum 2 drones launched at a time are $20 and $3 (each) respectively, for 15 customer locations give the optimal solution as**

```
Running model
Begin running model
Optimal tour cost is: 3910
TRUCK IS ON ROUTE 1 to 11
TRUCK IS ON ROUTE 2 to 5
TRUCK IS ON ROUTE 5 to 12
TRUCK IS ON ROUTE 9 to 16
TRUCK IS ON ROUTE 11 to 15
TRUCK IS ON ROUTE 12 to 17
TRUCK IS ON ROUTE 15 to 9
TRUCK IS ON ROUTE 16 to 2
DRONE IS ON ROUTE 2 to 14
DRONE IS ON ROUTE 5 to 7
DRONE IS ON ROUTE 5 to 13
DRONE IS ON ROUTE 9 to 8
DRONE IS ON ROUTE 11 to 3
DRONE IS ON ROUTE 12 to 4
DRONE IS ON ROUTE 15 to 10
DRONE IS ON ROUTE 16 to 6
DRONE RETURNING ON ROUTE 3 to 11
DRONE RETURNING ON ROUTE 4 to 12
DRONE RETURNING ON ROUTE 6 to 16
DRONE RETURNING ON ROUTE 7 to 5
DRONE RETURNING ON ROUTE 8 to 9
DRONE RETURNING ON ROUTE 10 to 15
DRONE RETURNING ON ROUTE 13 to 5
DRONE RETURNING ON ROUTE 14 to 2
End running model
```

In this case , we can see the route for truck and drones is changed and the number of customer locations for drones has increased and trucks has decreased.