Module - 2

Continuous Distributions.

1) Uniform Distribution [Rectangular Distribution]

X is said to follow uniform distribution in the interval (a, b), if its pdf is

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

Eg: For a uniform distribution in the interval (1, 4)

$$f(x) = \frac{1}{3}, \quad 1 < x < 4$$

Mean

$$E(x) = \int_x x \cdot f(x) \cdot dx$$

$$= \int_a^b x \cdot \frac{1}{b-a} \cdot dx$$

$$= \frac{1}{b-a} \cdot \int_a^b x \cdot dx$$

$$= \frac{1}{b-a} \cdot \left[\frac{x^2}{2}\right]_a^b = \frac{1}{\cancel{b-a}} \cdot \frac{\overset{b+a}{\cancel{b^2-a^2}}}{2} = \frac{b+a}{2}$$

Variance

$$v(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \int_x x^2 \cdot f(x) \cdot dx$$

$$= \int_a^b x^2 \cdot \frac{1}{b-a} \cdot dx$$

$$= \frac{1}{b-a} \left[\times \frac{b^3-a^3}{3} \right.$$

$$= \frac{b^3-a^3}{3(b-a)}$$

$$= \frac{a^2+ab+b^2}{3} \quad \left[\because b^3-a^3 = (b-a)(a^2+ab+b^2) \right]$$

$$v(x) = E(x^2) - [E(x)]^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{b^2+2ab+a^2}{4}$$

$$= \frac{4a^2+4ab+4b^2-3b^2-6ab-3a^2}{12}$$

$$= \frac{a^2-2ab+b^2}{12} = \frac{(b-a)^2}{12}$$

$t(b-a)$

1) Busses in certain root run every half an hour. What is the probability that a man coming to the bus stand at a random time will have to wait for at least 20 minutes

Ans. Let X denote the waiting time of the ~~[illegible]~~ person. X can be considered as uniform random variable in the interval ~~(0, 30)~~. $(0, 30)$

The pdf is.

$$f(x) = \frac{1}{b-a} \quad a < x < b.$$

$$= \frac{1}{30} \; , \; 0 < x < 30.$$

$$P(x \geq 20) = \int_{20}^{30} dx \; \frac{1}{30} \cdot dx$$

$$= \frac{1}{3\cancel{0}} \times 1\cancel{0}$$

$$= \frac{1}{3}$$

* The time measured in minutes required by a person to travel from his home to the railway station is a random variable obeying uniform distribution in the interval (20, 25). If he leaves the home at 8.00 am. What is the probability that he will catch the train which leaves the station at, 8.23.am

Ans. Let x denote the travelling time of a person from home to railway station. X can be considered as a uniform random variable in the interval (20, 25).

The pdf is.

$$f(x) = \frac{1}{b-a} \quad ; a \leq x \leq b.$$

$$= \frac{1}{25-20} \quad ; 20 \leq x \leq 25$$

$$= \frac{1}{5}$$

$$P(X \leq 23) = \int_{20}^{23} \frac{1}{5} \, dx$$

$$= \frac{(23-20)}{5} = \frac{3}{5}$$

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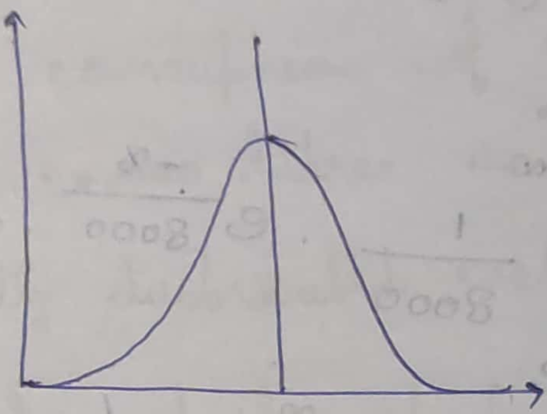
Normal distribution

A symmetrical data distribution, where most of the results lie near the mean.



Distribution formula.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{x_i - \mu}{\sigma}\right]^2}$$

where $-\infty < x < \infty$; $-\infty < \mu < \infty$; $\sigma > 0$

μ = Mean of distribution

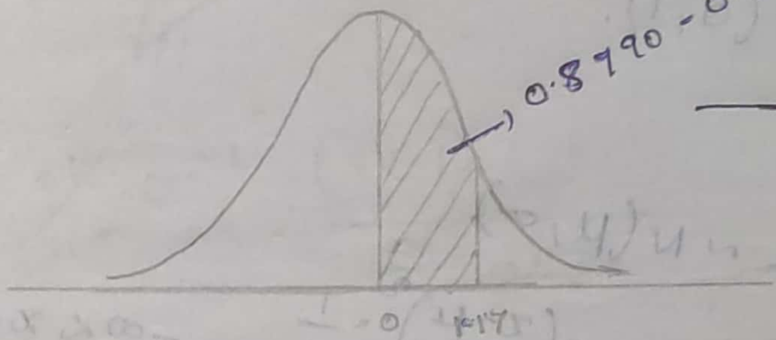
σ = Standard deviation of distribution.

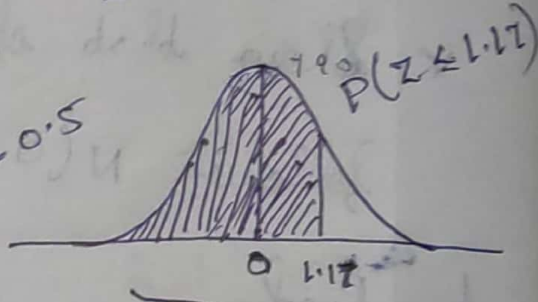
π = 3.14159

Standard normal distributions.

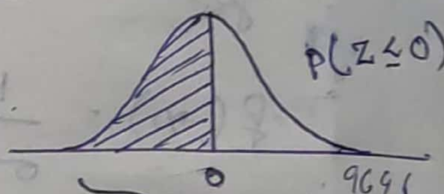
Normal distribution with mean 0 and standard deviation 1

2) $P(0 < Z < 1.17)$





$0.8790 \rightarrow$ Table value.



$0.5 \rightarrow$ Table value.

$P(0 < Z < 1.17) = \underline{0.3790}$

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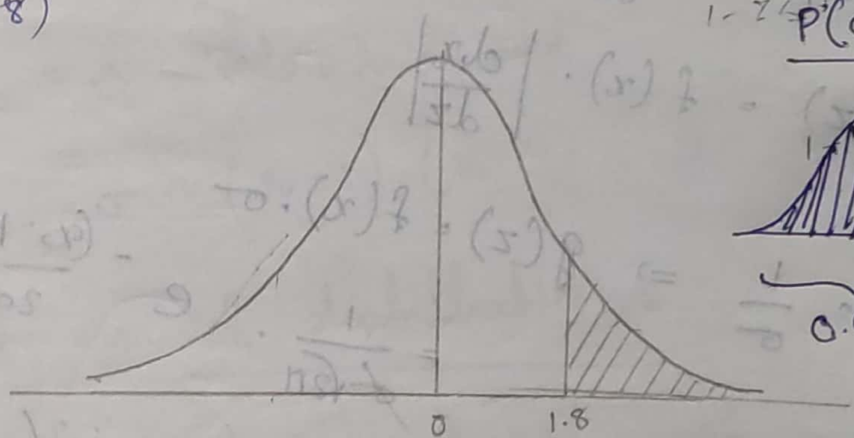
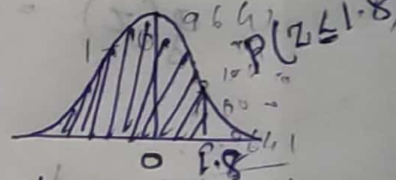
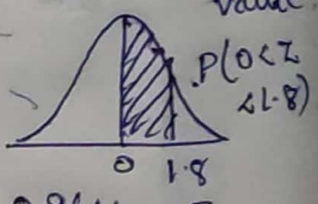
3) $P(Z > 1.8)$

$\underline{P(0 < Z < 1.8)}$





$0.9641 \rightarrow$ Table value.



Total area from 0 to right is 0.5

$P(Z > 1.8) = 0.5 - P(0 < Z < 1.8)$

$= 0.5 - 0.4641$

$= \underline{0.0359}$

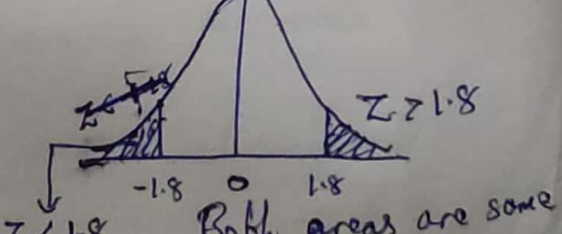
$0.9641 - 0.5$

$= \underline{0.4641}$

$P(Z < -1.8) = 0.5 - P(-1.8 < Z < 0)$

$= 0.5 - 0.4641$

$= \underline{0.0359}$



Both areas are same

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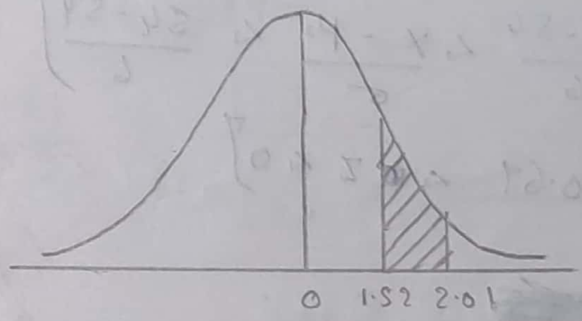
4) $P(1.52 < Z < 2.01)$



$P(1.52 < Z < 2.01) = P(0 < Z < 2.01) - P(0 < Z < 1.52)$

$= 0.4778 - 0.4357$

$= \underline{\underline{0.0421}}$

If x follows normal distribution with mean 54 and standard deviation 6, then find.

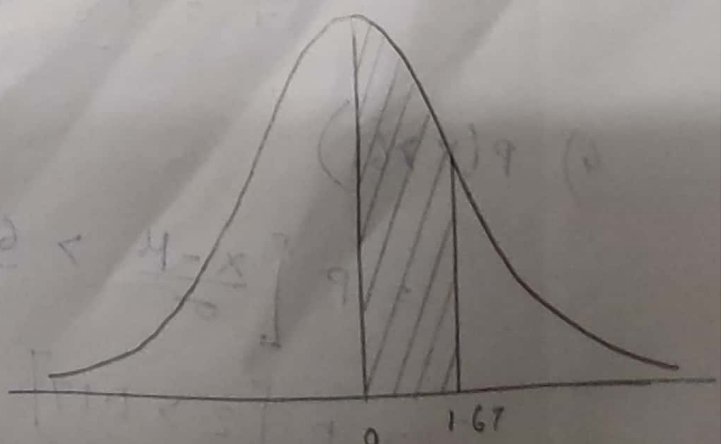
1) $P(54 < x < 64)$.

$= P\left[\frac{54 - \mu}{\sigma} < x < \frac{64 - \mu}{\sigma}\right]$

$= P\left[\frac{54 - 54}{6} < x < \frac{64 - 54}{6}\right]$

$= P[0 < x < 1.67]$

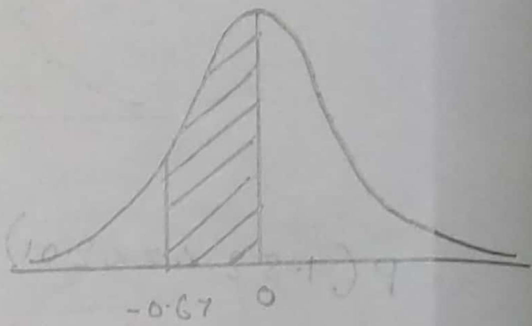
$= \underline{\underline{0.4525}}$



2) $P(50 < x < 54)$

$= P\left[\frac{50-54}{6} < \frac{x-\mu}{\sigma} < \frac{54-54}{6}\right]$

$= P[-0.67 < Z < 0]$



$= 0.2486$

3) $P(48 < x < 62)$

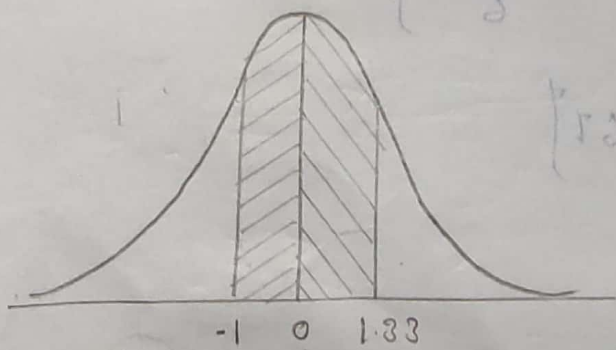
$= P\left[\frac{48-54}{6} < \frac{x-\mu}{\sigma} < \frac{62-54}{6}\right]$

$= P[-1 < Z < 1.33]$

$= P[0 < Z < 1.33] + P[-1 < Z < 0]$

$= 0.4082 + 0.8413$

$= 0.7428$



4) $P(X > 61)$

$= P\left[\frac{X-\mu}{\sigma} > \frac{61-54}{6}\right]$

$= P[Z > 1.17]$

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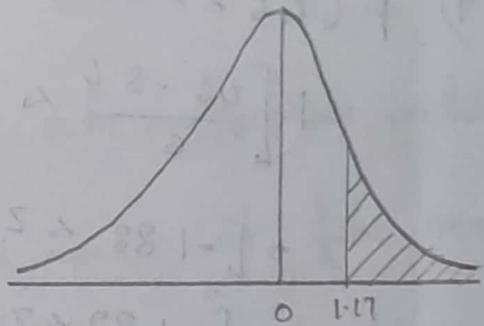
$= 0.5 - P[0 < Z < 1.17]$

$= 0.5 - 0.3790$

$= \underline{0.121}$



5) $P(X < 42)$

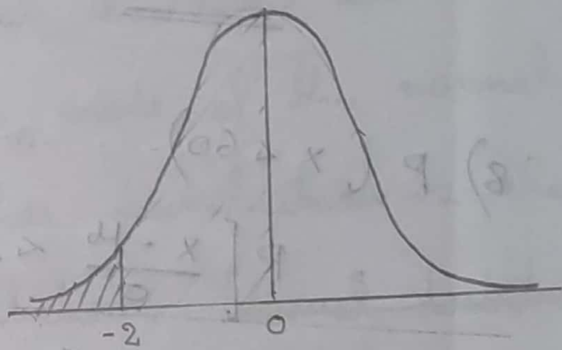
$= P\left[\frac{X-\mu}{\sigma} < \frac{42-54}{6}\right]$

$= P[Z < -2.0]$

$= 0.5 - P[-2 < Z < 0]$

$= 0.5 - 0.4772$

$= \underline{0.0228}$



6) $P(54 < X < 64)$

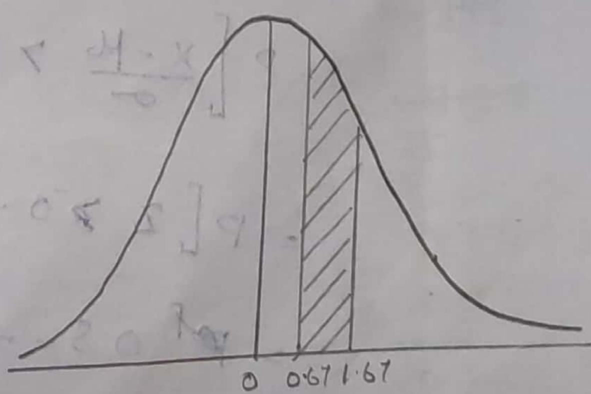
$= P\left[\frac{58-54}{6} < \frac{X-\mu}{\sigma} < \frac{64-54}{6}\right]$

$= P[0.67 < Z < 1.67]$

$= P[0 < Z < 1.67] - P[0 < Z < 0.67]$

$= 0.4525 - 0.2486$

$= \underline{0.2034}$



7) $P(46 < x < 53)$

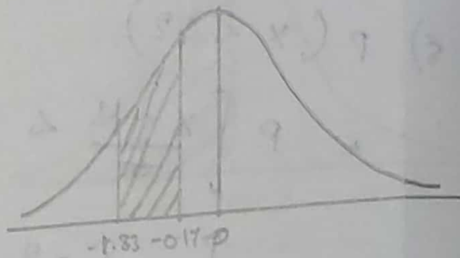
$= P\left[\frac{46-54}{6} < \frac{x-\mu}{\sigma} < \frac{53-54}{6}\right]$

$= P[-1.83 < Z < -0.17]$

$= P[-1.83 < Z < 0] - P[-0.17 < Z < 0]$

$= 0.4082 - 0.0675$

$= \underline{\underline{0.3407}}$



8) $P(x < 60)$

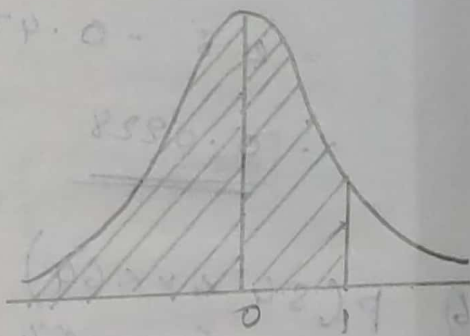
$= P\left[\frac{x-\mu}{\sigma} < \frac{60-54}{6}\right]$

$= P[Z < 1]$

$= 0.5 + P[0 < Z < 1]$

$= 0.5 + 0.3413$

$= \underline{\underline{0.8413}}$



9) $P(x > 50)$

$= P\left[\frac{x-\mu}{\sigma} > \frac{50-54}{6}\right]$

$= P[Z > -0.67]$

$= 0.5 + P[-0.67 < Z < 0]$

$= 0.5 + 0.2486$

$= \underline{\underline{0.7486}}$