

# Bayes' Theorem Scribe



L5\_A\_S1.pdf

PDF

This is Lecture 5 PDF on Bayes' Theorem, Random Variables, and Probability Mass Function. Please make a short lecture scribe for exam study using only this Lecture 5 PDF. Follow the same order as the slides. Write the main definitions, formulas, and example steps exactly as shown in the lecture. Do not add anything new, and don't leave anything. Do not explain extra. Do not use outside sources. Only use what is written in this PDF and keep it short and focused for exams.

Below is a **short, exam-focused lecture scribe** strictly compiled **only from Lecture 5 PDF**, following **the exact slide order**, keeping **definitions, formulas, and example steps as written**, with **no additions, no explanations, and no outside material**.

---

## Lecture 5: Bayes' Theorem, Random Variables, and Probability Mass Function

Bayes' Theorem – Weighted Average of Conditional Probabilities

Let A and B be events. We may express A as

$$A = AB \cup AB^c$$

As  $AB$  and  $AB^c$  are mutually exclusive, by Axiom 3,

$$\begin{aligned} \Pr(A) &= \Pr(AB) + \Pr(AB^c) \\ &= \Pr(A | B)\Pr(B) + \Pr(A | B^c)[1 - \Pr(B)] \end{aligned}$$

The probability of event A is a weighted average of the conditional probabilities with weights given as the probability of the event on which it is conditioned has of occurring.

Bayes' Theorem – Learning by Example

Example 3.1 (Part 1/2):

Accident-prone person: probability of accident = 0.4

Not accident-prone: probability of accident = 0.2

30% of population is accident prone

Let  $A_1$  = event that policyholder has an accident within a year

Let  $A$  = event that policyholder is accident prone

$$\Pr(A_1) = \Pr(A_1 | A)\Pr(A) + \Pr(A_1 | A^c)\Pr(A^c)$$

$$= (0.4)(0.3) + (0.2)(0.7) = 0.26$$

Example 3.1 (Part 2/2):

Given that a policyholder has an accident, find the probability that he or she is accident prone.

$$\Pr(A | A_1) = \Pr(A_1 | A)\Pr(A) / \Pr(A_1)$$

$$= \Pr(A)\Pr(A_1 | A) / \Pr(A_1)$$

$$= (0.3)(0.4) / 0.26 = 6/13$$

Bayes' Theorem – Formal Introduction

Law of Total Probability (Formula 3.4)

Using

$$\Pr(AB_i) = \Pr(B_i | A)\Pr(A)$$

Bayes Formula (Proposition 3.1)

$\Pr(B_i)$  is the **apriori probability**

$\Pr(B_i | A)$  is the **posteriori probability**

Bayes Formula – Learning by Example

Example 3.2:

Three cards: RR, BB, RB

One card selected at random and placed on ground

Upper side is red

Let  $R$  = event that upper side is red

Desired probability:  $\Pr(RB | R)$

$$\Pr(RB | R) = \Pr(RB \cap R) / \Pr(R)$$

$$= \Pr(R | RB)\Pr(RB) / [\Pr(R | RR)\Pr(RR) + \Pr(R | RB)\Pr(RB) + \Pr(R | BB)\Pr(BB)]$$

$$= (1/2 \cdot 1/3) / [(1 \cdot 1/3) + (1/2 \cdot 1/3) + (0 \cdot 1/3)] = 1/3$$

## Random Variables – Motivation and Concept

A random variable is a real-valued function defined on the sample space.

Values are determined by the outcomes of an experiment.

Probabilities are assigned to possible values of random variables.

Examples include sum of dice and number of heads in coin tosses.

The distribution of a random variable can be visualized as a bar diagram.

The x-axis represents values of the random variable.

The height of the bar at value  $a$  is  $\Pr[X = a]$ .

## Random Variables – Example

Tossing 3 fair coins.

Let  $Y$  = number of heads.

Possible values: 0, 1, 2, 3

$$\Pr\{Y = 0\} = 1/8$$

$$\Pr\{Y = 1\} = 3/8$$

$$\Pr\{Y = 2\} = 3/8$$

$$\Pr\{Y = 3\} = 1/8$$

Since  $Y$  must take one of these values, the probabilities sum to 1.

## Probability Mass Function – Concept

A random variable that can take at most a countable number of possible values is discrete.

Let  $X$  be a discrete random variable with range

$$R_X = X_1, X_2, X_3, \dots$$

The function

$$p(x) = \Pr\{X = x\}$$

is called the Probability Mass Function (PMF) of  $X$ .

Since  $X$  must take one of the values  $x_k$ ,

$$\sum p(x_k) = 1$$

## Probability Mass Function – Example

The PMF is given by

$$p(i) = c \lambda^i / i!, \quad i = 0, 1, 2, \dots$$

Find  $P\{X = 0\}$  and  $P\{X > 2\}$

Class Participation

Switch to Campuswire.

---

If you want, I can do the same **ultra-strict exam scribe** for your other lectures so everything matches perfectly in style and density.