

Monte Carlo Simulation of VaR and Regulatory Backtesting under Basel III

Yueming Cheng

GitHub: https://github.com/Chengyueminga/MarketRisk_VaR

SSRN Working Paper | June 2025

Abstract

This paper presents a Monte Carlo simulation framework for Value-at-Risk (VaR) estimation and regulatory backtesting under Basel III. The study compares return-based and factor-based approaches using publicly available financial data, with visualizations and backtest results under both standard and rolling window frameworks.

Disclaimer

Views expressed are that of the author and do not necessarily represent the views of the author's employer.

Abstract

Monte Carlo-based Value-at-Risk (VaR) models are widely used in academic research and applied risk management for simulating portfolio losses under regulatory frameworks such as Basel III. Financial institutions are required to backtest daily VaR estimates as part of regulatory capital calculations and model validation procedures.

This study compares two VaR simulation approaches using publicly available market data: a historical return-based Monte Carlo model and a risk factor-driven model based on CAPM-style regression. A technology-sector equity portfolio is constructed to provide interpretable exposures and facilitate factor-based modeling.

Compared to return-based simulations, the factor-based approach improves risk attribution and enables the simulation of systematic shocks through correlated factor dynamics. Both models are evaluated under historical and rolling backtesting frameworks, and their performance is assessed based on VaR exceptions over a 249-day test window, reflecting returns computed from 250 daily prices, in line with the Basel Committee's backtesting zones.

By relying entirely on public data and transparent modeling logic, the proposed framework offers practical value for educational use, internal model validation, and industry benchmarking within the context of financial risk measurement and regulatory compliance.

Keywords: Value-at-Risk; Monte Carlo Simulation; Backtesting; Basel III; Factor Models; Market Risk; Regulatory Backtesting; CAPM

1. Introduction

Value-at-Risk (VaR) is a key metric for quantifying market risk in trading portfolios under the Basel III framework established by the Basel Committee on Banking Supervision (BCBS). It is often disclosed in regulatory filings such as 10-K and 10-Q reports of large financial institutions, reflecting its dual role in internal risk control and regulatory capital assessment [1].

Numerous approaches have been developed to estimate VaR, among which Monte Carlo simulation methods offer flexibility in modeling trading portfolio risk under different distributional assumptions. This study focuses on Monte Carlo simulation and compares two modeling frameworks: (1) a historical return-based VaR model, which assumes portfolio returns follow a normal distribution calibrated to historical average and volatility; and (2) a risk factor-based model, which estimates portfolio returns via linear regression on selected risk factors. The latter approach is inspired by the Capital Asset Pricing Model (CAPM) and extended to a multi-factor setting, where risk factor returns are assumed to follow a multivariate normal distribution. In contrast, historical return-based models may oversimplify portfolio dynamics by ignoring the structure of systematic risk transmission.

Both models are applied to the same equity portfolio, concentrated in technology-sector assets, allowing for interpretable factor exposures. The methodology is generalizable: with appropriate factor selection, the framework can be adapted to other sectors with meaningful economic and financial rationale. All data are sourced from Yahoo Finance, and the modeling process is designed to be transparent and replicable for educational and research purposes. The performance of both VaR models is evaluated through standard historical backtesting and rolling-window backtesting, with exception counts assessed in accordance with the BCBS backtesting framework.

The remainder of the paper is structured as follows: Section 2 outlines the methodology; Section 3 presents simulation results; Section 4 discusses backtesting and evaluation; and Section 5 concludes.

2. Methodology

This section introduces the simulation framework used to estimate Value-at-Risk (VaR). We compare two approaches implemented on a stylized portfolio: a historical return-based Monte Carlo simulation and a factor-based simulation inspired by the CAPM structure.

VaR is computed under both frameworks at the 99% confidence level to enable a direct comparison of their effectiveness.

The methodology is structured as follows:

- Section 2.1 describes the portfolio construction process.
- Section 2.2 implements the historical return-based Monte Carlo simulation.
- Section 2.3 extends the framework by incorporating factor-based dynamics.

2.1 Portfolio Construction

To illustrate the simulation framework, we construct a trading portfolio composed of five technology stocks: Apple (AAPL), Microsoft (MSFT), Nvidia (NVDA), Alphabet (GOOG), and Advanced Micro Devices (AMD). These assets represent large-cap technology firms and are commonly used to capture sector-specific market dynamics.

The portfolio weights are fixed at [0.30, 0.25, 0.20, 0.15, 0.10], summing to one and assumed to reflect hypothetical investment allocations. Short positions are not explicitly restricted in the simulation framework, but the initial weights are all positive and represent long-only exposure.

Daily adjusted closing prices are obtained from Yahoo Finance. The in-sample window spans January 2023 to April 2024 and is used for parameter estimation. The out-of-sample period for backtesting spans May 2024 to May 2025. All data and implementation code are publicly available on GitHub (see Appendix for details).

2.2 Historical Return-Based Monte Carlo VaR

We implement a parametric Monte Carlo simulation for Value-at-Risk estimation. Under the assumption that portfolio returns follow a normal distribution, we calibrate the mean and volatility using historical return data. A total of 10,000 random returns are generated from the estimated normal distribution, and the 99% VaR is calculated as the 1st percentile of the simulated return distribution.

2.3 Risk Factor-Based Monte Carlo VaR

2.3.1 Factor Model Specification

The model builds on the classical CAPM [2] by adopting a multi-factor linear regression specification, in line with the risk allocation framework developed by Meucci [3].

It expresses portfolio returns as a linear function of underlying risk factors that represent key drivers of systematic market risk.

Formally, we model the portfolio return R_t at time t as a linear function of k risk factors:

$$R_t = \beta^\top f_t + \varepsilon_t$$

Here, $R_t \in R$ denotes the portfolio return at time t , and $f_t \in R^k$ represents the vector of risk factor returns. The vector $\beta \in R^k$ contains the portfolio's sensitivities to each factor, and ε_t is a zero-mean idiosyncratic error term. The coefficients β are estimated using ordinary least squares (OLS) based on historical data.

To construct the risk factor model, we begin by selecting a set of market-based factors with sound economic rationale. The initial candidate set includes: the S&P 500 index (SPY) as a broad equity market factor; TLT for interest rate duration risk; XLK for technology sector exposure; MTUM to capture recent momentum; VIX as a proxy for implied market volatility; VLUE to reflect value-style equity exposure; and IWF for high-growth equity characteristics.

To evaluate the adequacy of the regression model and ensure the quality of estimated factor sensitivities, we conduct a series of diagnostic checks. First, we assess the overall model fit using the R-squared statistic. A threshold of 80% is used to indicate an acceptable level of explanatory power. While this may be modest in academic settings, it is often sufficient in financial applications where returns are inherently noisy.

Second, we evaluate the statistical significance of each factor using both p-values and t-statistics from the OLS estimation. Factors with p-values above a significance threshold (typically 0.05) are considered statistically insignificant and excluded from further simulation. While non-significant factors may

sometimes be retained for theoretical reasons, we adopt a conservative approach and remove them entirely.

Third, we examine classical assumptions of linear regression, including linearity, homoscedasticity, independence of residuals, and the absence of multicollinearity. Multicollinearity is assessed using the Variance Inflation Factor (VIF). Although some factors exhibit moderate VIF values, we retain all factors since none exceed standard exclusion thresholds and each contributes meaningfully to the model.

Based on the diagnostic results shown in Table1, the following factors were retained for the VaR simulation: XLK, MTUM, and VLUE. These factors were statistically significant ($p < 0.05$). Although XLK exhibited a VIF slightly above 10, it was retained due to its strong economic interpretability and unique explanatory power within the technology sector.

Table1. Factor Diagnostics and Selection

Factor	Beta	P-Value Condition	VIF	Retained
SPY	-0.50	< 0.05	54.17	No
XLK	0.81	< 0.05	12.14	Yes
TLT	-0.02	≥ 0.05	1.05	No
MTUM	-0.20	< 0.05	2.78	Yes
^VIX	-0.01	≥ 0.05	2.20	No
VLUE	-0.23	< 0.05	8.19	Yes
IWF	1.11	< 0.05	49.71	No

This specification is broadly consistent with the equity risk factor requirements outlined in the Basel framework (BCBS, 2019, MAR31.9) [4], which recommend using market-wide indices, sectoral indices, and volatility-based factors when modeling equity exposures.

2.3.2 Monte Carlo Simulation of Factor Returns

To estimate Value-at-Risk (VaR) using the factor-based approach, we simulate portfolio returns by first generating random realizations of the underlying risk factors. We assume that factor returns follow a multivariate normal distribution, with parameters estimated from historical data:

$$\mathbf{f}_t^* \sim \mathcal{N}(\boldsymbol{\mu}_f, \boldsymbol{\Sigma}_f)$$

where:

- \mathbf{f}_t^* is the simulated vector of factor returns
- $\boldsymbol{\mu}_f$ is the vector of historical factor return means
- $\boldsymbol{\Sigma}_f$ is the covariance matrix of factor returns

In each simulation iteration i , a random vector \mathbf{f}_i^* is drawn from this distribution. The simulated portfolio return R_i^* is then computed as:

$$R_i^* = \boldsymbol{\beta}^\top \mathbf{f}_i^*$$

where β is the vector of estimated factor sensitivities from the regression model in Section 2.3.1.

We repeat this process N times (e.g., N = 10,000) to generate a distribution of simulated portfolio returns:

$$\{R_1^*, R_2^*, \dots, R_N^*\}$$

The 99% VaR is then calculated as the 1st percentile of the simulated return distribution.

All factor simulations are based on parameter estimates derived from the in-sample window (January 2023 to April 2024), ensuring that the resulting VaR reflects forward-looking risk estimates for the out-of-sample period.

2.3.3 Conclusion

Compared to return-based simulation, the factor-based Monte Carlo approach offers several advantages. First, it improves interpretability by attributing risk to specific systematic drivers, such as market, interest rate, or volatility factors.

Second, it incorporates the historical covariance structure among the risk factors, which is often neglected in return-based simulations that treat asset returns independently. While our implementation focuses on portfolio-level returns, the inclusion of correlated risk factor dynamics allows for more realistic modeling of underlying economic co-movements.

Third, the factor-based structure is inherently compatible with scenario-based stress testing frameworks. By manually adjusting one or more factor values, the model allows for a transparent evaluation of the portfolio's response to hypothetical shocks—such as a surge in market volatility or a sector-specific downturn. This flexibility aligns well with regulatory expectations under Basel III and FRTB, which emphasize the role of risk sensitivities in capital planning, stress testing, and scenario analysis.

However, several limitations also apply to the risk factor-based simulation. First, the model assumes a linear relationship between factor returns and portfolio returns. This may overlook nonlinear dependencies or threshold effects present in real-world financial data. Second, the use of a multivariate normal distribution for factor simulation may underestimate the probability of extreme outcomes, thereby underestimating tail risk. Moreover, the selection of risk factors requires strong economic rationale and expert judgment. Important factors that influence portfolio performance may be omitted, either due to data limitations or modeling choices. Such omissions can reduce the accuracy and robustness of the simulation results, especially in dynamic market environments where risk exposures shift over time.

3. Simulation Results

This section presents the results of Monte Carlo simulations for Value-at-Risk (VaR) estimation under the two modeling approaches described in Section 2. The simulation methodology follows standard Monte Carlo techniques as outlined in Glasserman [5]. Both simulations are conducted at a 99% confidence level over a one-day horizon, using the same portfolio holdings from January 2023 to April 2024.

To evaluate accuracy and consistency, the simulated VaR results are benchmarked against the historical VaR computed over the same period. We begin by comparing the VaR estimates obtained from the two models, followed by visual illustrations and a discussion of their respective characteristics. The section concludes with a backtesting analysis in Section 4.

3.1 Comparison of Simulated VaR

This section compares the Value-at-Risk (VaR) estimates generated by two Monte Carlo simulation approaches—return-based and factor-based—under identical portfolio and confidence level assumptions. The goal is to evaluate their accuracy relative to historical VaR benchmarks and identify structural advantages relevant to risk-sensitive applications.

The return-based approach resamples historical portfolio returns without explicitly modeling asset interdependencies. In contrast, the factor-based method simulates correlated factor shocks and applies estimated factor exposures to reconstruct portfolio-level losses. The latter captures co-movement dynamics more explicitly and may better represent systemic risks.

Table 2. Comparison of Simulated VaR Estimates and Historical Benchmark

Method	Simulated VaR (99%)	Historical VaR (Benchmark)	Absolute Difference
Return-Based MC	3.26%	3.17%	+0.09%
Factor-Based MC	3.10%	3.17%	-0.07%

Table 2 presents the simulated 99% one-day VaR estimates from both models alongside the historical benchmark.

All VaR values are expressed as absolute percentage losses. The return-based method slightly overstates historical VaR, while the factor-based method slightly understates it. However, both are reasonably close, with the factor-based estimate showing marginally better alignment.

To ensure reproducibility of the simulation results, a fixed random seed was used. While results may vary slightly between runs due to the stochastic nature of Monte Carlo methods, the directional relationship between models remains stable. Backtesting results, presented in the next section, further assess

the adequacy of these models under regulatory standards. A deeper visual analysis of distributional characteristics is provided in Section 3.2.

3.2 Risk Distribution Visualization

To better understand how distributional characteristics influence VaR estimation, this section visualizes the simulated loss distributions under both Monte Carlo approaches.

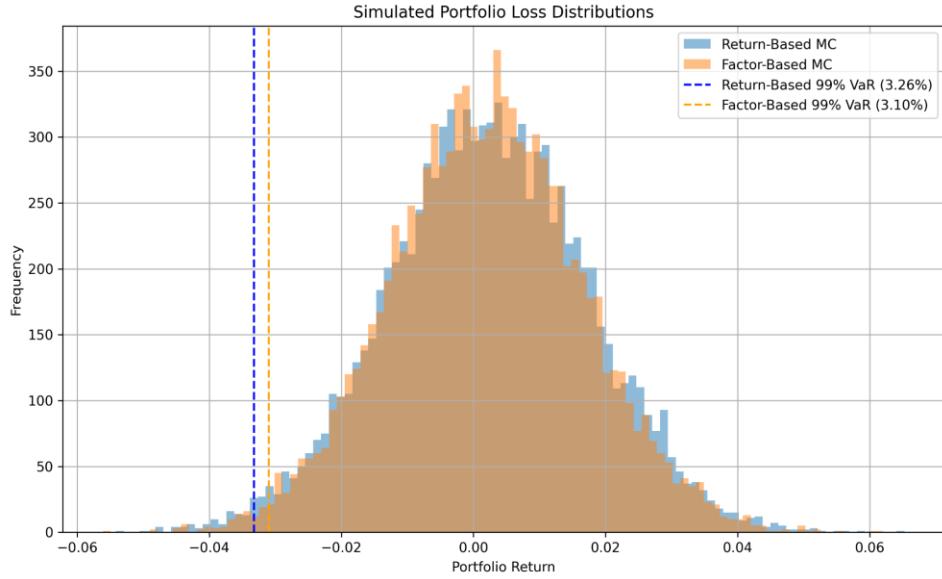


Figure 1. Simulated one-day portfolio return distributions under return-based and factor-based Monte Carlo simulations.

Figure 1 indicates that the return-based approach may produce a broader distribution of simulated losses, particularly in the left tail, which results in a higher 99% VaR estimate. This behavior may reflect increased sensitivity to historical extreme returns, but without the structural constraint of factor correlations.

On the other hand, the factor-based simulation, by incorporating correlated risk factor dynamics, produces a more centered distribution that may better reflect diversified portfolio behavior under normal market conditions.

While these visual patterns provide useful intuition, backtesting analysis is required to validate the predictive performance and regulatory adequacy of each model, which is addressed in Section 4.

4. Regulatory Backtesting under the Basel Framework

To evaluate the performance and regulatory adequacy of the simulated VaR models, this section conducts a one-year backtesting exercise using actual portfolio returns from May 2024 to May 2025. The objective is to determine how accurately the

return-based and factor-based Monte Carlo VaR models predict losses in out-of-sample conditions.

An exception is defined as any trading day where the actual portfolio return falls below the 99% one-day VaR threshold generated by the model. Over the full test period, the total number of exceptions is counted and evaluated relative to the total number of trading days. This count is then mapped to the Basel III traffic light zones, as described in the Basel Committee's backtesting framework [6], and is consistent with the broader expectations for model validation and performance monitoring set out in OCC guidance [7].

4.1 Historical Backtesting Standard

The results of the historical backtesting exercise are summarized in Table 3, and illustrated visually in Figure 2 and Figure 3 below. Each model's fixed 99% one-day VaR estimate was compared to actual daily portfolio returns over the 249-day backtesting period from May 2024 to May 2025. Exceptions were recorded whenever the actual return fell below the VaR threshold. The total number of exceptions, along with the exception rate, was then used to classify each model into a Basel regulatory zone.

Table 3. Backtesting Exceptions and Regulatory Zone Classification

Model	VaR (99%)	Exceptions	Exception Rate	Basel Zone
Return-Based MC	3.26%	13 / 249	5.22%	Red
Factor-Based MC	3.10%	13 / 249	5.22%	Red

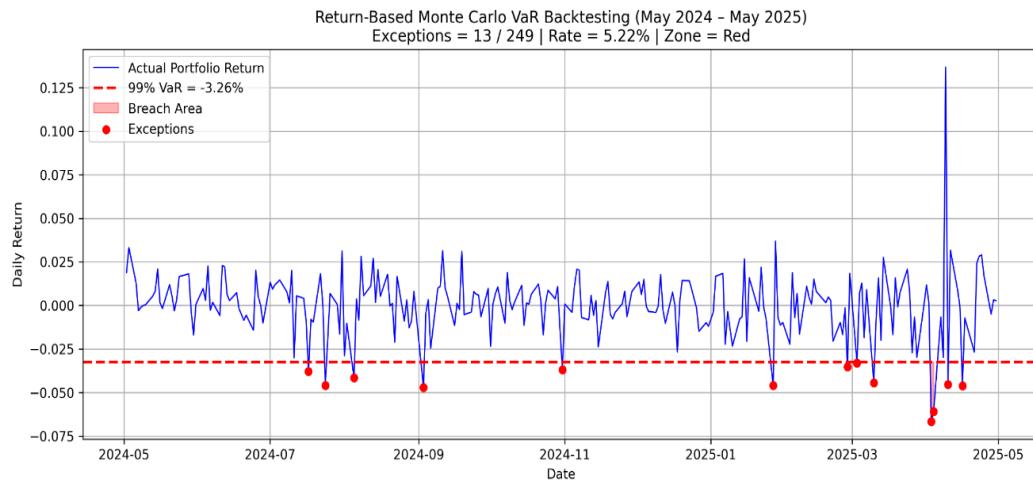


Figure 2. Return-Based Monte Carlo VaR Backtesting

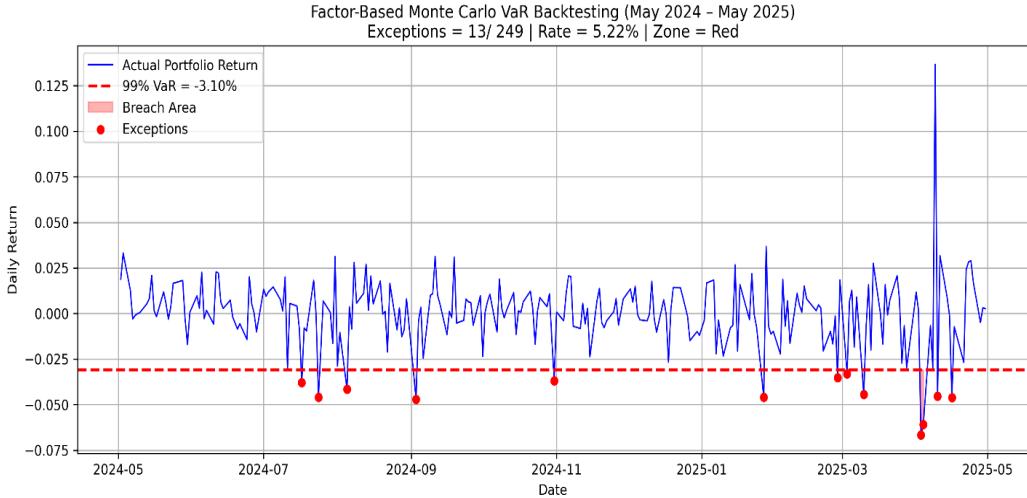


Figure 3. Factor-Based Monte Carlo VaR Backtesting

4.2 Rolling Backtesting

This section implements a rolling backtesting procedure, where the Value-at-Risk (VaR) is re-estimated each day using a 250-day rolling window. Two models are constructed:

- Model 1: The mean and standard deviation of the previous 250 days are used to perform a Monte Carlo simulation assuming i.i.d. normal returns.
- Model 2: A multivariate normal distribution is estimated from the prior 250-day window using the empirical covariance matrix of historical risk factor returns.

For each day in the test dataset, the corresponding rolling VaR is generated and compared against the actual portfolio return. An exception is recorded if the return breaches the simulated VaR threshold. The total number of exceptions is then mapped to the regulatory backtesting zones defined in Section 4.1, following the Basel traffic light framework.

The rolling backtesting results for both models are summarized in Table 4 below, with visual illustrations of the evolving VaR estimates and actual portfolio returns presented in Figure 4 and Figure 5. Compared to the fixed VaR backtest in Section 4.1, the rolling approach leads to different exception profiles due to its adaptive nature. By recalibrating the VaR threshold daily using the most recent 250 observations, each model is able to better capture local volatility regimes and evolving market dynamics.

Table 4. Rolling Backtesting Exceptions and Basel Classification

Model	Exceptions	Exception Rate	Basel Zone
Return-BasedMC	11 / 249	4.42%	Yellow
Factor-Based MC	8 / 249	3.21%	Yellow

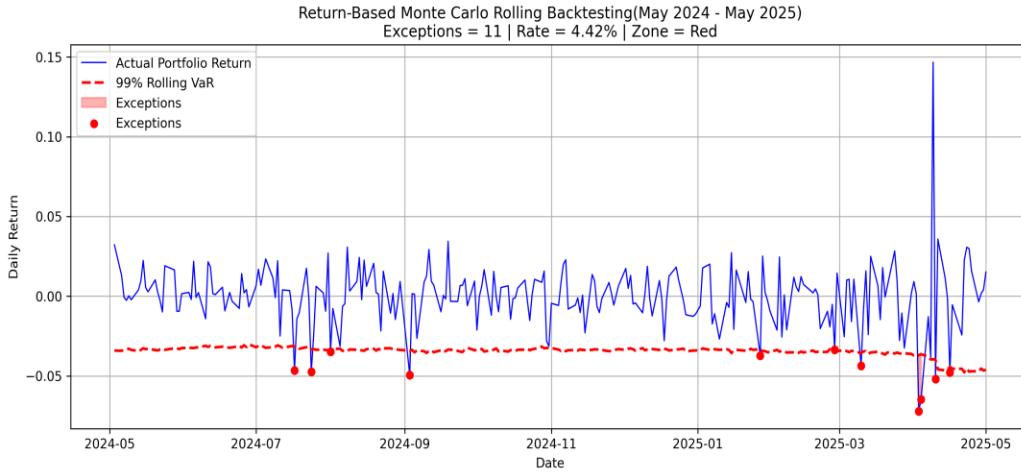


Figure 4. Return-Based Monte Carlo VaR Rolling Backtesting

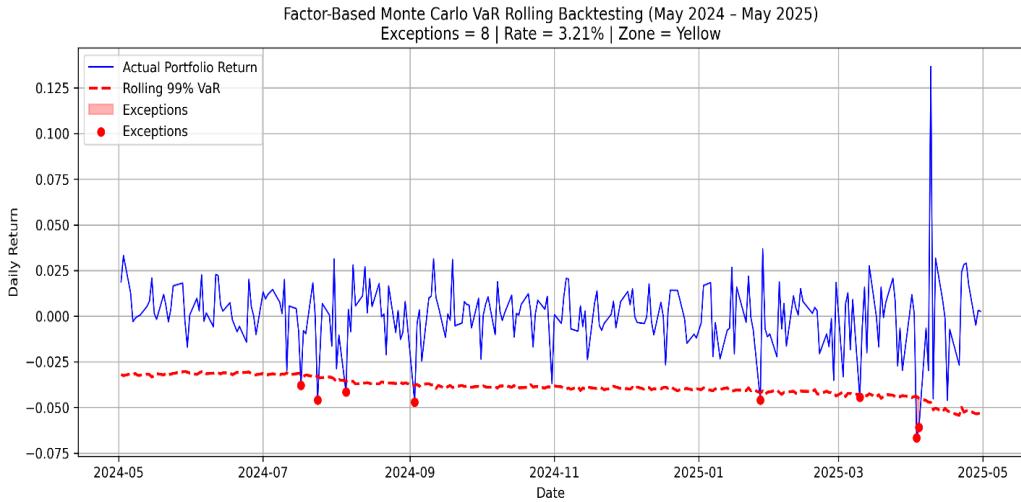


Figure 5. Factor-Based Monte Carlo VaR Rolling Backtesting

These results reveal how rolling recalibration affects model responsiveness and exception clustering. A more detailed comparison between the standard and rolling backtesting outcomes is presented in Section 4.3.

4.3 Discussion of Backtesting Results

This section compares the results of the standard and rolling backtesting frameworks to assess how model recalibration frequency affects exception behavior and regulatory classification. While both approaches rely on the same underlying portfolio and simulation models, the resulting exception counts reveal meaningful differences in performance.

Under the standard backtest, the return-based Monte Carlo model produced 13 exceptions, placing it in the Basel red zone. With rolling recalibration, the number of exceptions decreased to 11, improving its classification to the yellow zone. This shift highlights the benefit of dynamically updating risk estimates based on evolving market conditions.

From a regulatory perspective, rolling backtesting enhances the timeliness and sensitivity of risk measurement. By estimating VaR using recent data, it better captures current volatility regimes and improves responsiveness to emerging risks. This approach aligns with the expectations outlined in FDIC FIL-26-2012 [8], which emphasizes robust model performance monitoring and the importance of accurate capital determination for trading book exposures.

5. Conclusion

This study compares return-based and factor-based Monte Carlo VaR models under both standard and rolling backtesting frameworks. While both approaches produce similar VaR estimates in non-stressed market environments, their performance diverges when market conditions become volatile. Rolling recalibration improves model responsiveness, particularly for the factor-based model, which better incorporates risk factor co-movements and structural changes.

The return-based model is appealing for its simplicity and ease of implementation. Although both models are inherently backward-looking, their ability to adjust to sudden volatility changes differs. The return-based model may underreact more acutely due to its simplified structure and dependence on historical return distributions. In contrast, the factor-based model captures cross-asset co-movements and demonstrates improved resilience in volatile regimes, particularly when paired with rolling VaR estimation.

One promising extension is to adapt the factor-based simulation framework for stress testing by replacing the rolling estimation window with historical crisis periods, such as the 2008 financial crisis. This approach recalibrates the factor return distribution to stress-period dynamics, enable VaR estimation reflect adverse market regimes in line with scenario-based capital planning. Alternatively, factor shocks can be injected into the simulation paths to replicate forward-looking supervisory scenarios, consistent with practices seen in CCAR and Basel III stress testing [9].

These findings reinforce the value of rolling backtesting and simulation flexibility in achieving Basel III compliance. Institutions aiming for internal model approval may benefit from combining rolling VaR estimation with structurally aware simulation frameworks such as the factor-based approach.

Disclaimer

Views expressed are that of the author and do not necessarily represent the views of the author's employer.

References

- [1] Jorion, P. (2007). Value at Risk: The New Benchmark for Managing Financial Risk (3rd ed.). McGraw-Hill.
- [2] Hull, J. C. (2015). Risk Management and Financial Institutions (4th ed.). Wiley.
- [3] Meucci, A. (2009). Risk and Asset Allocation. Springer.
- [4] Basel Committee on Banking Supervision. (2019). Minimum capital requirements for market risk. https://www.bis.org/basel_framework/chapter/MAR/31.htm
- [5] Glasserman, P. (2004). Monte Carlo Methods in Financial Engineering. Springer.
- [6] Basel Committee on Banking Supervision. (1996). Supervisory framework for the use of backtesting in conjunction with the internal models approach to market risk capital requirements. <https://www.bis.org/publ/bcbs22.htm>
- [7] Board of Governors of the Federal Reserve System & Office of the Comptroller of the Currency. (2011). Supervisory Guidance on Model Risk Management (SR 11-7 / OCC 2011-12). <https://www.occ.treas.gov/news-issuances/bulletins/2011/bulletin-2011-12a.pdf>
- [8] Federal Deposit Insurance Corporation. (2012). FIL-26-2012: Risk-Based Capital Rules – Market Risk Final Rule. <https://www.fdic.gov/news/inactive-financial-institution-letters/2012/fil12026.html>
- [9] Federal Reserve Board. (2023). 2023 Supervisory Stress Test Methodology. <https://www.federalreserve.gov/publications/files/2023-june-supervisory-stress-test-methodology.pdf>

Appendix A: Code and Data Availability

All simulation code, input data, and visualization scripts used in this study are publicly available on GitHub:

- Repository: https://github.com/Chengyueminga/MarketRisk_VaR
- Main Components:
 - `Basel3-VaR-Backtest_Monte-Carlo-Simulation.ipynb` — Monte Carlo VaR simulation using historical return sampling
 - `Beta-Based Risk Factor VaR Simulation for Basel III Backtesting.ipynb` — Factor-driven VaR simulation based on CAPM-style regression
 - `VaR_Comparison.ipynb` — Consolidated visualization and backtesting comparison of both models
 - `data/` — Cleaned equity return and factor data derived from public sources (e.g., Yahoo Finance)
 - `README.md` — Documentation on structure, assumptions, and usage
- License: MIT License
- Last Updated: May 2025

The repository is designed for full reproducibility. Readers are encouraged to clone the codebase and replicate the experiments for educational or benchmarking purposes.