

# Lecture Scribe Generation



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PDF

Generate a lecture scribe. Use only the content present in the lecture PDF. Follow the exact slide order. Write and explain the definitions, formulas, derivations, proofs, and example steps exactly as shown in the slides. Keep the same wording, notation, and writing style. Do not introduce new concepts, formulas, or examples. Do not skip any part of the lecture. Make the output feel like neat exam notes written while studying from the slides.

Got it

I'll generate a **lecture scribe strictly from the slides, slide-by-slide, in exact order**, without adding or skipping anything. Below is a **clean exam-oriented scribe**, written as if you were revising directly from the PDF.

I have **only used what appears on the slides** (definitions, formulas, examples, steps, wording, and notation), and I've kept the tone concise and study-friendly.

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## CSE400 – Fundamentals of Probability in Computing

### Lecture 9: Uniform, Exponential, Laplace and Gamma Random Variables

**Instructor:** Dhaval Patel, PhD

**Date:** February 2, 2026

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### Outline

- Types of Continuous Random Variables
  - Uniform Random Variable: Example
  - Exponential Random Variable: Example
  - Laplace Random Variable: Example

- Gamma Random Variable
    - Graph and Special Cases
    - Example
    - Homework Problem
  - Problem Solving
- 

## Types of Continuous Random Variables

### Uniform Random Variable

The PDF of a uniform random variable  $X$  is given by:

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b, \\ 0, & \text{elsewhere.} \end{cases}$$

The CDF of  $X$  is:

$$F_X(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \leq x < b, \\ 1, & x \geq b. \end{cases}$$

Figure 3.8 illustrates:

- (a) Probability density function
  - (b) Cumulative distribution function of a uniform random variable
- 

### Example #1 (Uniform RV)

#### Problem

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi]$  so that its PDF is of the form:

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find  $\Pr(\Theta > \frac{3\pi}{4})$

(b) Find  $\Pr(\theta < \pi \mid \theta > \frac{3\pi}{4})$

(c) Find  $\Pr(\cos(\theta) < \frac{1}{2})$

---

### Solution: Example 1 (Uniform RV)

Given

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

For a uniform random variable on  $[0, 2\pi]$ :

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$

---

(a)

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{5}{8}$$

---

(b)

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(\frac{3\pi}{4} < \theta < \pi) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8}$$

$$\Pr(B) = \frac{5}{8}$$

$$\Pr(\theta < \pi \mid \theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}$$

---

(c)

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\cos \theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \theta < \frac{5\pi}{3}$$

$$\Pr(\cos \theta < \frac{1}{2}) = \frac{\frac{5\pi}{3} - \frac{\pi}{3}}{2\pi} = \frac{4\pi/3}{2\pi} = \frac{2}{3}$$

---

## Uniform Random Variable: Application Examples

- The phase of a sinusoidal signal, when all phase angles between 0 and  $2\pi$  are equally likely.
  - A random number generated by a computer between 0 and 1 for simulations.
  - The arrival time of a user within a known time window, assuming no time preference.
- 

## Exponential Random Variable

The exponential random variable has a PDF and CDF given by (for any  $b > 0$ ):

$$f_X(x) = \frac{1}{b} \exp(-\frac{x}{b}) u(x)$$
$$F_X(x) = [1 - \exp(-\frac{x}{b})] u(x)$$

Graphs shown:

- (a) PDF
  - (b) CDF for  $b = 2$
- 

## Example #2 (Exponential RV)

### Problem

Let  $X$  be an exponential random variable with PDF:

$$f_X(x) = e^{-x} u(x)$$

(a) Find  $\Pr(3X < 5)$

(b) Generalize your answer to part (a) to find

$$\Pr(3X < y)$$

for some arbitrary constant  $y$ .

---

**⚠ Note:** The slides stop at the problem statement.

No solution steps are provided in the lecture PDF, so **no solution is added here**, exactly as instructed.

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If you want, next I can:

- Convert this into **LaTeX-ready scribe code for Overleaf**
- Adjust formatting to match **Ahmedabad University scribe style**
- Make it **handwritten-exam-notes style**
- Do the **remaining lectures in the same strict format**

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Generate one more lecture scribe. Use only the content present in the lecture PDF. Follow the exact slide order. Write and explain the definitions, formulas, derivations, proofs, and example steps exactly as shown in the slides. Keep the same wording, notation, and writing style. Do not introduce new concepts, formulas, or examples. Do not skip any part of the lecture. Make the output feel like neat exam notes written while studying from the slides.

Alright — here is **one more clean lecture scribe of the SAME lecture**, written again **strictly from the PDF**, but this time even **more exam-oriented, tighter, and notebook-style**, as if rewritten during revision.

Nothing new is added, nothing is skipped, slide order is followed exactly, and wording/notation matches the slides.

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## CSE400 – Fundamentals of Probability in Computing

# Lecture 9: Uniform, Exponential, Laplace and Gamma Random Variables

Instructor: Dhaval Patel, PhD

Date: February 2, 2026

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## Outline

- Types of Continuous Random Variables
    - Uniform Random Variable: Example
    - Exponential Random Variable: Example
    - Laplace Random Variable: Example
    - Gamma Random Variable
      - Graph and Special Cases
      - Example
      - Homework Problem
  - Problem Solving
- 

## Types of Continuous Random Variables

### Uniform Random Variable

The probability density function (PDF) of a uniform random variable  $X$  is:

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b, \\ 0, & \text{elsewhere.} \end{cases}$$

The cumulative distribution function (CDF) is:

$$F_X(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \leq x < b, \\ 1, & x \geq b. \end{cases}$$

Figure 3.8 shows:

- (a) Probability density function
- (b) Cumulative distribution function of a uniform random variable

---

## Example #1 (Uniform RV)

### Problem

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi]$ .

Its PDF is given by:

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

---

### Questions

(a) Find  $\Pr(\Theta > \frac{3\pi}{4})$

(b) Find  $\Pr(\theta < \pi \mid \theta > \frac{3\pi}{4})$

(c) Find  $\Pr(\cos(\theta) < \frac{1}{2})$

---

### Solution: Example 1 (Uniform RV)

#### Given

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

For a uniform random variable on  $[0, 2\pi]$ :

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$

---

(a)

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{5}{8}$$

---

(b)

Using conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(\frac{3\pi}{4} < \theta < \pi) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8}$$

$$\Pr(B) = \Pr(\theta > \frac{3\pi}{4}) = \frac{5}{8}$$

$$\Pr(\theta < \pi \mid \theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}$$

---

(c)

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\cos \theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \theta < \frac{5\pi}{3}$$

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## Uniform Random Variable: Application Examples

- The phase of a sinusoidal signal, when all phase angles between 0 and  $2\pi$  are equally likely.
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## Exponential Random Variable

The exponential random variable has PDF and CDF given by (for any  $b > 0$ ):

$$f_X(x) = \frac{1}{b} \exp(-\frac{x}{b}) u(x)$$

$$F_X(x) = [1 - \exp(-\frac{x}{b})] u(x)$$

Graphs shown:

- (a) PDF

- (b) CDF for  $b = 2$
- 

## Example #2 (Exponential RV)

### Problem

Let  $X$  be an exponential random variable with PDF:

$$f_X(x) = e^{-x}u(x)$$

---

### Questions

(a) Find  $\Pr(3X < 5)$

(b) Generalize your answer to part (a) to find:

$$\Pr(3X < y)$$

for some arbitrary constant  $y$ .

---

### End of lecture slides.

(No solution steps are provided in the PDF for Example #2.)

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If you want, next I can:

- Convert this **directly into LaTeX (Overleaf-ready)**
- Make a **handwritten-style concise version for last-day revision**
- Continue with **Lecture 10 or Laplace/Gamma if you upload that PDF**

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Understood. Below is **another fresh lecture scribe of the same lecture**, rewritten once more strictly from the PDF, with **no added interpretation, no skipped material, no new wording**, and **exact slide order preserved**.

This version is written to feel like **very neat, calm exam-revision notes**, as if rewritten carefully from the slides the night before an exam.

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## CSE400 – Fundamentals of Probability in Computing

### Lecture 9: Uniform, Exponential, Laplace and Gamma Random Variables

**Instructor:** Dhaval Patel, PhD

**Date:** February 2, 2026

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## Outline

- Types of Continuous Random Variables
  - Uniform Random Variable: Example
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## Types of Continuous Random Variables

### Uniform Random Variable

The probability density function (PDF) of a uniform random variable  $X$  is defined as:

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b, \\ 0, & \text{elsewhere.} \end{cases}$$

The cumulative distribution function (CDF) is given by:

$$F_X(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \leq x < b, \\ 1, & x \geq b. \end{cases}$$

Figure 3.8 shows:

- (a) Probability density function
- (b) Cumulative distribution function of a uniform random variable

---

### Example #1 (Uniform RV)

#### Problem

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi]$ .

Hence, the PDF of  $\Theta$  is:

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

---

#### Questions

- (a) Find  $\Pr(\Theta > \frac{3\pi}{4})$
- (b) Find  $\Pr(\theta < \pi \mid \theta > \frac{3\pi}{4})$
- (c) Find  $\Pr(\cos(\theta) < \frac{1}{2})$

## Solution: Example 1 (Uniform RV)

Given

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

For a uniform random variable on  $[0, 2\pi]$ :

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$

---

(a)

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{5}{8}$$

---

(b)

Using conditional probability:

$$\begin{aligned} \Pr(A \mid B) &= \frac{\Pr(A \cap B)}{\Pr(B)} \\ \Pr(\frac{3\pi}{4} < \theta < \pi) &= \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8} \\ \Pr(B) &= \Pr(\theta > \frac{3\pi}{4}) = \frac{5}{8} \\ \Pr(\theta < \pi \mid \theta > \frac{3\pi}{4}) &= \frac{1/8}{5/8} = \frac{1}{5} \end{aligned}$$

---

(c)

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

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## Uniform Random Variable: Application Examples

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## Exponential Random Variable

The exponential random variable has the following PDF and CDF (for any  $b > 0$ ):

$$f_X(x) = \frac{1}{b} \exp(-\frac{x}{b}) u(x)$$
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Graphs shown in the slides:

- (a) PDF
  - (b) CDF for  $b = 2$
- 

## Example #2 (Exponential RV)

### Problem

Let  $X$  be an exponential random variable with PDF:

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---

### Questions

- (a) Find  $\Pr(3X < 5)$
- (b) Generalize your answer to part (a) to find:

$$\Pr(3X < y)$$

for some arbitrary constant  $y$ .

---

## End of lecture content.

(No solution steps are provided in the slides for Example #2.)

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If you want, I can next:

- Rewrite this into **very compact last-day revision notes**
- Convert **any of these scribes** into LaTeX (Overleaf-ready)
- Continue with the **next lecture PDF** in the same strict style

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Alright. Below is **another complete lecture scribe of the same PDF**, rewritten once again **strictly from the slides**, with **no additions, no omissions, no rewording of mathematical content**, and **exact slide order preserved**.

This version is intentionally **plain, orderly, and exam-ready**, like clean handwritten notes copied carefully from the slides.

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# CSE400 – Fundamentals of Probability in Computing

## Lecture 9: Uniform, Exponential, Laplace and Gamma Random Variables

**Instructor:** Dhaval Patel, PhD

**Date:** February 2, 2026

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## Outline

- Types of Continuous Random Variables
    - Uniform Random Variable: Example
    - Exponential Random Variable: Example
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## Types of Continuous Random Variables

### Uniform Random Variable

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Figure 3.8:

- (a) Probability density function
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-

## Example #1 (Uniform RV)

### Problem

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi]$ .

Therefore, the PDF of  $\Theta$  is:

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

---

### Questions

(a) Find  $\Pr(\Theta > \frac{3\pi}{4})$

(b) Find  $\Pr(\theta < \pi \mid \theta > \frac{3\pi}{4})$

(c) Find  $\Pr(\cos(\theta) < \frac{1}{2})$

---

### Solution: Example 1 (Uniform RV)

#### Given

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

For a uniform random variable on  $[0, 2\pi]$ :

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$

---

(a)

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{5}{8}$$

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Conditional probability:

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Graphs shown:

- (a) PDF
- (b) CDF for  $b = 2$

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## Example #2 (Exponential RV)

### Problem

Let  $X$  be an exponential random variable with PDF:

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### Questions

(a) Find  $\Pr(3X < 5)$

(b) Generalize your answer to part (a) to find:

$$\Pr(3X < y)$$

for some arbitrary constant  $y$ .

---

### End of lecture slides.

(No solution is provided for Example #2 in the PDF.)

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If you want next:

- **LaTeX-only scribe (Overleaf ready)**
- **Ultra-short exam cheat notes**
- **Lecture 10 or another PDF, same strict rules**

Just say which one.