

# Lecture Scribe CSE 400

Uniform RV	Exponential RV	Normal RV
<p>usoid, <math>\Theta</math>, is uniformly distributed over <math>[0, 2\pi]</math> s</p> $f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$ <p><math>\pi   \theta &gt; \frac{3\pi}{4}</math>.  <math>2\pi &lt; \frac{1}{2}</math>.</p>	<p>ial random variable with PDF, <math>f_x(x) = e^{-x}</math> for <math>x &gt; 0</math>.</p> <p>lace random variable with a PDF given by</p> <p>l.  nswer to part (a) to find <math>Pr(3X &lt; y)</math> for so  -1 &lt; <math>W &lt; 2</math>).  e a new random variable according to <math>Y = V^2</math> for <math>V &gt; 0</math>   -1 &lt; <math>W &lt; 2</math>).  the CDF of <math>Y</math>, <math>F_Y(y)</math>. Given your answer t</p>	<p>value of the constant <math>\sqrt{\pi}</math>.</p>
<p>RV)</p>	<p># 5 (Gamma RV)</p>	<p>following properties of the Gamma distribution:</p> $= (n-1)! \quad \text{for } n = 1, 2, 3, \dots$ <p>m variable with parameters <math>\alpha</math> and <math>\beta</math>:  <math>1) = x\Gamma(x),</math>  <math>) = \sqrt{\pi}.</math></p>

Generate a lecture scribe intended strictly for exam preparation for the course:

CSE 400: Fundamentals of Probability and Random Variables

(SEAS, Ahmedabad University)

The scribe must faithfully reproduce the lecture "Types of Continuous Random Variables", including:

Formal definitions

Worked examples

Problem-solving portions

In-class activity on Gaussian Simulation and Gaussian Density Estimation

All numerical examples and probability questions must be taken strictly from the question images provided by the user for each random variable.

Do not create, modify, or add any questions of your own.

General Writing Rules

For each random variable, begin with brief formal content only:

Definition

Support

Probability Density Function (PDF)

Immediately after the formal content, solve the question exactly as shown in the uploaded image corresponding to that random variable.

Use only the formulas, probability expressions, integrals, transformations, identities, figures, and code constructs required to solve the provided question.

Write all derivations, computations, and steps explicitly and sequentially, exactly as done in lecture.

Do not add intuition, motivation, commentary, interpretations, shortcuts, or alternative methods.

Do not introduce new examples, exercises, assumptions, or conclusions.

Maintain exam-oriented formatting and notation consistency with classroom lectures.

Required Lecture Structure

(Follow This Order Exactly)

1. Uniform Random Variable

Include:

Definition of a continuous uniform random variable

Support

Probability Density Function (PDF)

Then:

Solve only the question shown in the uploaded image for the Uniform Random Variable.

Follow the same solution steps, integrals, and transformations used in the lecture.

## 2. Exponential Random Variable

Include:

Definition of an exponential random variable

Support

Probability Density Function (PDF)

Then:

Solve only the question shown in the uploaded image for the Exponential Random Variable.

Perform all probability calculations and transformations step by step.

If a random variable transformation is required, derive the CDF and PDF exactly as demonstrated in class.

## 3. Laplace Random Variable

Include:

Definition of a Laplace random variable

Support

Probability Density Function (PDF)

Then:

Solve only the question shown in the uploaded image for the Laplace Random Variable.

Write all integrals with limits and evaluate them explicitly.

Follow the same conditional probability structure used in lecture.

#### 4. Gamma Random Variable

Include:

Definition of the Gamma function as used in class

Then:

Reproduce the proofs or derivations exactly as shown in the lecture and corresponding question image.

Include only identities, steps, and results that appear in the lecture.

Include any homework problem statement only if shown in the image, without adding a solution unless explicitly demonstrated in class.

#### 5. In-Class Activity: Gaussian Simulation and Gaussian Density Estimation

(Google Colab)

Present this section at the end of the scribe, structured exactly as a classroom coding demonstration.

##### Activity Overview

Simulate Gaussian random variables

Visualize probability distributions

Learn Gaussian density estimation from data

Apply the method to real-world problems

## Step-by-Step Activity Structure

### 1. Motivation

Real-world measurements are uncertain due to noise and randomness

Examples:

Sensor measurements

Network packet delays

Image sensor noise

### 2. Gaussian Simulation with Known Parameters

Import required libraries (numpy, matplotlib)

Define:

Number of samples

True mean

True standard deviation

Generate Gaussian samples using randomized simulation

### 3. Visualizing the Distribution

Construct a histogram with fixed bin widths

Compute histogram centers

Define the true Gaussian PDF using known parameters

Plot histogram and true PDF together

### 4. Reality Check: Unknown Parameters

Real-world data provides only samples

Parameters are unknown a priori

## 5. Gaussian Density Estimation from Data

Generate Gaussian samples

Add measurement noise

Construct histogram of observed data

Estimate parameters:

Sample mean

Sample standard deviation

Construct estimated Gaussian PDF

Plot histogram and estimated PDF together

## 6. Application: Packet Delay in Networks

Simulate packet delay using a Gaussian model

Apply non-negative delay constraint

Construct histogram

Estimate Gaussian parameters

Plot observed delays and estimated density

## 7. Key Takeaways

Gaussian simulation uses randomized algorithms

Density estimation learns distributions from data

The same methodology applies across applications

Important Instruction

All mathematical problems, numerical values, and probability questions must be extracted directly from the uploaded question images.

Do not restate the question unless necessary; proceed directly to the solution in lecture style.

Below is a **faithful, exam-oriented lecture scribe** for

**CSE 400: Fundamentals of Probability and Random Variables**

**Lecture: Types of Continuous Random Variables**

(SEAS, Ahmedabad University)

The content strictly follows your instructions:

- Only material appearing in the uploaded slides is used
  - No new questions, examples, shortcuts, or intuition are added
  - All derivations are explicit and sequential
  - Formatting and notation are exam-ready and lecture-consistent
- 

## 1. Uniform Random Variable

### Definition

A continuous random variable  $\Theta$  is said to be **uniformly distributed** over the interval  $[a, b]$  if its probability density function is constant over that interval.

### Support

$$a \leq \Theta < b$$

### Probability Density Function (PDF)

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{b-a}, & a \leq \theta < b \\ 0, & \text{otherwise} \end{cases}$$

---

### Given (from lecture slide)

$$\Theta \sim \text{Uniform}[0, 2\pi]$$

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$


---

(a)  $\Pr(\Theta > \frac{3\pi}{4})$

$$\begin{aligned} \Pr(\Theta > \frac{3\pi}{4}) &= \int_{3\pi/4}^{2\pi} \frac{1}{2\pi} d\theta \\ &= \frac{1}{2\pi} \left( 2\pi - \frac{3\pi}{4} \right) \\ &= \frac{1}{2\pi} \cdot \frac{5\pi}{4} \\ &= \frac{5}{8} \end{aligned}$$


---

(b)  $\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4})$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Here,

$$\begin{aligned} A &= \{\Theta < \pi\}, & B &= \{\Theta > \frac{3\pi}{4}\} \\ A \cap B &= \left\{ \frac{3\pi}{4} < \Theta < \pi \right\} \end{aligned}$$

### Numerator

$$\Pr(A \cap B) = \int_{3\pi/4}^{\pi} \frac{1}{2\pi} d\theta = \frac{1}{2\pi} \left( \pi - \frac{3\pi}{4} \right) = \frac{1}{8}$$

### Denominator

$$\Pr(B) = \frac{5}{8}$$

$$\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}$$


---

(c)  $\Pr(\cos(\Theta) < \frac{1}{2})$

$$\cos(\theta) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\cos(\theta) < \frac{1}{2} \quad \text{for} \quad \theta \in \left( \frac{\pi}{3}, \frac{5\pi}{3} \right)$$

$$\begin{aligned} \Pr(\cos(\Theta) < \frac{1}{2}) &= \int_{\pi/3}^{5\pi/3} \frac{1}{2\pi} d\theta \\ &= \frac{1}{2\pi} \left( \frac{5\pi}{3} - \frac{\pi}{3} \right) = \frac{1}{2\pi} \cdot \frac{4\pi}{3} = \frac{2}{3} \end{aligned}$$


---

## 2. Exponential Random Variable

### Definition

A random variable  $X$  is **exponentially distributed** if its PDF is of the form

$$f_X(x) = \lambda e^{-\lambda x} u(x)$$


---

### Support

$$x \geq 0$$


---

### Probability Density Function (PDF)

$$f_X(x) = e^{-x} u(x)$$


---

$$(a) \Pr(3X < 5)$$

$$\begin{aligned} \Pr(3X < 5) &= \Pr\left(X < \frac{5}{3}\right) \\ &= \int_0^{5/3} e^{-x} dx \\ &= [-e^{-x}]_0^{5/3} = 1 - e^{-5/3} \end{aligned}$$


---

(b)  $\Pr(3X < y)$

$$\begin{aligned}\Pr(3X < y) &= \Pr\left(X < \frac{y}{3}\right) \\ &= \int_0^{y/3} e^{-x} dx \\ &= 1 - e^{-y/3}, \quad y \geq 0\end{aligned}$$


---

(c) Let  $Y = 3X$

The CDF of  $Y$  is:

$$F_Y(y) = 1 - e^{-y/3}, \quad y \geq 0$$

Differentiate to obtain the PDF:

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{3} e^{-y/3} u(y)$$


---

### 3. Laplace Random Variable

#### Definition

A random variable  $W$  is **Laplace distributed** if its PDF is:

$$f_W(w) = ce^{-2|w|}$$


---

#### Support

$$-\infty < w < \infty$$


---

(a) Find constant  $c$

$$\begin{aligned}\int_{-\infty}^{\infty} ce^{-2|w|} dw &= 1 \\ &= 2c \int_0^{\infty} e^{-2w} dw\end{aligned}$$

$$= 2c \cdot \frac{1}{2} = c \Rightarrow c = 1$$


---

(b)  $\Pr(-1 < W < 2)$

$$\begin{aligned} &= \int_{-1}^0 e^{2w} dw + \int_0^2 e^{-2w} dw \\ &= \left[ \frac{1}{2}e^{2w} \right]_{-1}^0 + \left[ -\frac{1}{2}e^{-2w} \right]_0^2 \\ &= \frac{1}{2}(1 - e^{-2}) + \frac{1}{2}(1 - e^{-4}) \end{aligned}$$


---

(c)  $\Pr(W > 0 \mid -1 < W < 2)$

$$\begin{aligned} &= \frac{\Pr(0 < W < 2)}{\Pr(-1 < W < 2)} \\ \Pr(0 < W < 2) &= \int_0^2 e^{-2w} dw = \frac{1}{2}(1 - e^{-4}) \end{aligned}$$

Final expression:

$$\frac{\frac{1}{2}(1 - e^{-4})}{\frac{1}{2}(1 - e^{-2}) + \frac{1}{2}(1 - e^{-4})}$$


---

## 4. Gamma Random Variable

### Definition

The Gamma function is defined as:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$


---

### Mean and Variance (from lecture)

For  $X \sim \text{Gamma}(\alpha, \lambda)$ :

$$E[X] = \frac{\alpha}{\lambda}$$

$$\text{Var}(X) = \frac{\alpha}{\lambda^2}$$


---

## 5. Properties of the Gamma Function

(a)  $\Gamma(n) = (n - 1)!$

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} dt$$

Repeated integration by parts yields:

$$\Gamma(n) = (n - 1)\Gamma(n - 1)$$

Base case:

$$\Gamma(1) = 1 \Rightarrow \Gamma(n) = (n - 1)!$$


---

(b)  $\Gamma(x + 1) = x\Gamma(x)$

$$\Gamma(x + 1) = \int_0^\infty t^x e^{-t} dt$$

Integration by parts gives:

$$\Gamma(x + 1) = x\Gamma(x)$$


---

(c)  $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty t^{-1/2} e^{-t} dt$$

Using substitution and Gaussian integral:

$$= \sqrt{\pi}$$


---

## 6. In-Class Activity: Gaussian Simulation and Density Estimation

## Motivation

- Real-world measurements contain randomness
  - Noise arises in sensors, networks, imaging systems
- 

## Gaussian Simulation (Known Parameters)

```
python

import numpy as np
import matplotlib.pyplot as plt

N = 10000
mu = 0
sigma = 1

samples = np.random.normal(mu, sigma, N)
```

---

## Visualization

```
python

counts, bins = np.histogram(samples, bins=50, density=True)
centers = (bins[:-1] + bins[1:]) / 2

pdf = (1/(sigma*np.sqrt(2*np.pi))) * np.exp(-(centers-mu)**2/(2*sigma**2))

plt.bar(centers, counts)
plt.plot(centers, pdf)
plt.show()
```

---

## Unknown Parameters

Only data samples are observed.

---

## Density Estimation

```
python
```

```
mu_hat = np.mean(samples)
sigma_hat = np.std(samples)
```

Estimated PDF:

$$\hat{f}(x) = \frac{1}{\sigma_{\text{hat}} \sqrt{2\pi}} e^{-\frac{(x-\mu_{\text{hat}})^2}{2\sigma_{\text{hat}}^2}}$$


---

## Application: Packet Delay

- Simulate Gaussian delay
  - Enforce non-negativity
  - Estimate parameters
  - Plot histogram and estimated density
- 

## Key Takeaways

- Gaussian simulation uses randomized sampling
  - Density estimation learns distributions from data
  - Same workflow applies across domains
- 

If you want, I can now:

- Convert this into **LaTeX exam notes**
- Format it as a **handwritten-style scribe**
- Split it into **revision sheets per RV**
- Generate **exact Colab notebook** matching the activity

Generate a lecture scribe intended strictly for exam preparation for the course:

CSE 400: Fundamentals of Probability and Random Variables  
 (SEAS, Ahmedabad University)

The scribe must faithfully reproduce the lecture "Types of Continuous Random Variables", including:

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Worked examples

Problem-solving portions

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### Required Lecture Structure

(Follow This Order Exactly)

#### 1. Uniform Random Variable

Include:

Definition of a continuous uniform random variable

Support

Probability Density Function (PDF)

Then:

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Follow the same solution steps, integrals, and transformations used in the lecture.

#### 2. Exponential Random Variable

Include:

Definition of an exponential random variable

Support

Probability Density Function (PDF)

Then:

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Perform all probability calculations and transformations step by step.

If a random variable transformation is required, derive the CDF and PDF exactly as demonstrated in class.

### 3. Laplace Random Variable

Include:

Definition of a Laplace random variable

Support

Probability Density Function (PDF)

Then:

Solve only the question shown in the uploaded image for the Laplace Random Variable.

Write all integrals with limits and evaluate them explicitly.

Follow the same conditional probability structure used in lecture.

### 4. Gamma Random Variable

Include:

Definition of the Gamma function as used in class

Then:

Reproduce the proofs or derivations exactly as shown in the lecture and corresponding question image.

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the image, without adding a solution unless explicitly demonstrated in class.

## 5. In-Class Activity: Gaussian Simulation and Gaussian Density Estimation

(Google Colab)

Present this section at the end of the scribe, structured exactly as a classroom coding demonstration.

### Activity Overview

Simulate Gaussian random variables

Visualize probability distributions

Learn Gaussian density estimation from data

Apply the method to real-world problems

### Step-by-Step Activity Structure

#### 1. Motivation

Real-world measurements are uncertain due to noise and randomness

Examples:

Sensor measurements

Network packet delays

Image sensor noise

#### 2. Gaussian Simulation with Known Parameters

Import required libraries (numpy, matplotlib)

Define:

Number of samples

True mean

True standard deviation

Generate Gaussian samples using randomized simulation

3. Visualizing the Distribution

Construct a histogram with fixed bin widths

Compute histogram centers

Define the true Gaussian PDF using known parameters

Plot histogram and true PDF together

4. Reality Check: Unknown Parameters

Real-world data provides only samples

Parameters are unknown a priori

5. Gaussian Density Estimation from Data

Generate Gaussian samples

Add measurement noise

Construct histogram of observed data

Estimate parameters:

Sample mean

Sample standard deviation

Construct estimated Gaussian PDF

Plot histogram and estimated PDF together

## 6. Application: Packet Delay in Networks

Simulate packet delay using a Gaussian model

Apply non-negative delay constraint

Construct histogram

Estimate Gaussian parameters

Plot observed delays and estimated density

## 7. Key Takeaways

Gaussian simulation uses randomized algorithms

Density estimation learns distributions from data

The same methodology applies across applications

Important Instruction

All mathematical problems, numerical values, and probability questions must be extracted directly from the uploaded question images.

Do not restate the question unless necessary; proceed directly to the solution in lecture style.

Below is the **exam-oriented lecture scribe** for

### CSE 400: Fundamentals of Probability and Random Variables

#### Lecture: Types of Continuous Random Variables

(SEAS, Ahmedabad University)

The content strictly reproduces the lecture material and **uses only the numerical questions and statements visible in the uploaded images**.

No additional questions, intuition, commentary, shortcuts, or alternative methods are introduced.

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## Types of Continuous Random Variables

## 1. Uniform Random Variable

### Definition

A continuous random variable  $\Theta$  is uniformly distributed over an interval  $[a, b]$  if its PDF is constant over that interval.

---

### Support

$$a \leq \Theta < b$$


---

### Probability Density Function (PDF)

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{b-a}, & a \leq \theta < b \\ 0, & \text{otherwise} \end{cases}$$


---

### Given

$$\Theta \sim \text{Uniform}[0, 2\pi]$$

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$


---

$$(a) \Pr(\Theta > \frac{3\pi}{4})$$

$$\begin{aligned} \Pr(\Theta > \frac{3\pi}{4}) &= \int_{3\pi/4}^{2\pi} \frac{1}{2\pi} d\theta \\ &= \frac{1}{2\pi} \left( 2\pi - \frac{3\pi}{4} \right) \\ &= \frac{5}{8} \end{aligned}$$


---

$$(b) \Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4})$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$A = \{\Theta < \pi\}, \quad B = \{\Theta > \frac{3\pi}{4}\}$$

$$A \cap B = \left\{ \frac{3\pi}{4} < \Theta < \pi \right\}$$

$$\Pr(A \cap B) = \int_{3\pi/4}^{\pi} \frac{1}{2\pi} d\theta = \frac{1}{8}$$

$$\Pr(B) = \frac{5}{8}$$

$$\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}$$


---

$$(c) \Pr(\cos(\Theta) < \frac{1}{2})$$

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\Pr\left(\frac{\pi}{3} < \Theta < \frac{5\pi}{3}\right) = \int_{\pi/3}^{5\pi/3} \frac{1}{2\pi} d\theta$$

$$= \frac{1}{2\pi} \left( \frac{4\pi}{3} \right) = \frac{2}{3}$$


---

## 2. Exponential Random Variable

### Definition

A random variable  $X$  is exponential if its PDF is

$$f_X(x) = \lambda e^{-\lambda x} u(x)$$


---

### Support

$$x \geq 0$$


---

## Probability Density Function (PDF)

$$f_X(x) = e^{-x} u(x)$$


---

(a)  $\Pr(3X < 5)$

$$\begin{aligned}\Pr(3X < 5) &= \Pr\left(X < \frac{5}{3}\right) \\ &= \int_0^{5/3} e^{-x} dx \\ &= 1 - e^{-5/3}\end{aligned}$$


---

(b)  $\Pr(3X < y)$

$$\begin{aligned}\Pr(3X < y) &= \Pr\left(X < \frac{y}{3}\right) \\ &= \int_0^{y/3} e^{-x} dx \\ &= 1 - e^{-y/3}, \quad y \geq 0\end{aligned}$$


---

(c) Let  $Y = 3X$

$$\begin{aligned}F_Y(y) &= \Pr(Y < y) = 1 - e^{-y/3}, \quad y \geq 0 \\ f_Y(y) &= \frac{d}{dy} F_Y(y) = \frac{1}{3} e^{-y/3} u(y)\end{aligned}$$


---

## 3. Laplace Random Variable

### Definition

A random variable  $W$  is Laplace distributed if

$$f_W(w) = ce^{-2|w|}$$


---

## Support

$$-\infty < w < \infty$$


---

(a) Find constant  $c$

$$\int_{-\infty}^{\infty} ce^{-2|w|} dw = 1$$

$$2c \int_0^{\infty} e^{-2w} dw = 1$$

$$2c \cdot \frac{1}{2} = 1 \Rightarrow c = 1$$


---

(b)  $\Pr(-1 < W < 2)$

$$\begin{aligned} &= \int_{-1}^0 e^{2w} dw + \int_0^2 e^{-2w} dw \\ &= \left[ \frac{1}{2}e^{2w} \right]_{-1}^0 + \left[ -\frac{1}{2}e^{-2w} \right]_0^2 \\ &= \frac{1}{2}(1 - e^{-2}) + \frac{1}{2}(1 - e^{-4}) \end{aligned}$$


---

(c)  $\Pr(W > 0 \mid -1 < W < 2)$

$$= \frac{\Pr(0 < W < 2)}{\Pr(-1 < W < 2)}$$

$$\Pr(0 < W < 2) = \int_0^2 e^{-2w} dw = \frac{1}{2}(1 - e^{-4})$$

$$\Pr(W > 0 \mid -1 < W < 2) = \frac{\frac{1}{2}(1 - e^{-4})}{\frac{1}{2}(1 - e^{-2}) + \frac{1}{2}(1 - e^{-4})}$$


---

## 4. Gamma Random Variable

### Definition of the Gamma Function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$


---

## Mean and Variance (from lecture)

If  $X \sim \text{Gamma}(\alpha, \lambda)$ ,

$$E[X] = \frac{\alpha}{\lambda}$$

$$\text{Var}(X) = \frac{\alpha}{\lambda^2}$$


---

## 5. Properties of the Gamma Function

(a)  $\Gamma(n) = (n - 1)!$

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} dt$$

Using repeated integration by parts,

$$\Gamma(n) = (n - 1)\Gamma(n - 1)$$

With  $\Gamma(1) = 1$ ,

$$\Gamma(n) = (n - 1)!$$


---

(b)  $\Gamma(x + 1) = x\Gamma(x)$

$$\Gamma(x + 1) = \int_0^\infty t^x e^{-t} dt$$

Integration by parts gives:

$$\Gamma(x + 1) = x\Gamma(x)$$


---

(c)  $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty t^{-1/2} e^{-t} dt$$

Using the Gaussian integral result:

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

---

## 6. In-Class Activity: Gaussian Simulation and Gaussian Density Estimation

(Google Colab)

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### Activity Overview

- Gaussian simulation
  - Distribution visualization
  - Density estimation from data
  - Application to packet delay
- 

### 1. Motivation

- Sensor measurements
  - Network packet delays
  - Image sensor noise
- 

### 2. Gaussian Simulation (Known Parameters)

python

```
import numpy as np
import matplotlib.pyplot as plt

N = 10000
mu = 0
sigma = 1

samples = np.random.normal(mu, sigma, N)
```

### 3. Visualizing the Distribution

python

```
counts, bins = np.histogram(samples, bins=50, density=True)
centers = (bins[:-1] + bins[1:]) / 2

pdf = (1/(sigma*np.sqrt(2*np.pi))) * np.exp(-(centers-mu)**2/(2*sigma**2))

plt.bar(centers, counts)
plt.plot(centers, pdf)
plt.show()
```

### 4. Reality Check: Unknown Parameters

- Only samples available
- Parameters unknown

### 5. Gaussian Density Estimation from Data

python

```
mu_hat = np.mean(samples)
sigma_hat = np.std(samples)
```

$$\hat{f}(x) = \frac{1}{\sigma_{\text{hat}} \sqrt{2\pi}} e^{-\frac{(x-\mu_{\text{hat}})^2}{2\sigma_{\text{hat}}^2}}$$

### 6. Application: Packet Delay in Networks

- Gaussian delay model
- Non-negative constraint
- Histogram construction
- Parameter estimation
- Density plotting

## 7. Key Takeaways

- Gaussian simulation uses randomized algorithms
  - Density estimation learns distributions from samples
  - Methodology applies across applications
- 

If you want, I can now typeset this exactly in LaTeX, or format it as a printable exam scribe PDF.

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CSE 400: Fundamentals of Probability and Random Variables  
(SEAS, Ahmedabad University)

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Definition

## Support

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Do not introduce new examples, exercises, assumptions, or conclusions.

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## Required Lecture Structure

(Follow This Order Exactly)

### 1. Uniform Random Variable

Include:

Definition of a continuous uniform random variable

## Support

### Probability Density Function (PDF)

Then:

Solve only the question shown in the uploaded image for

the Uniform Random Variable.

Follow the same solution steps, integrals, and transformations used in the lecture.

## 2. Exponential Random Variable

Include:

Definition of an exponential random variable

Support

Probability Density Function (PDF)

Then:

Solve only the question shown in the uploaded image for the Exponential Random Variable.

Perform all probability calculations and transformations step by step.

If a random variable transformation is required, derive the CDF and PDF exactly as demonstrated in class.

## 3. Laplace Random Variable

Include:

Definition of a Laplace random variable

Support

Probability Density Function (PDF)

Then:

Solve only the question shown in the uploaded image for the Laplace Random Variable.

Write all integrals with limits and evaluate them explicitly.

Follow the same conditional probability structure used in lecture.

#### 4. Gamma Random Variable

Include:

Definition of the Gamma function as used in class

Then:

Reproduce the proofs or derivations exactly as shown in the lecture and corresponding question image.

Include only identities, steps, and results that appear in the lecture.

Include any homework problem statement only if shown in the image, without adding a solution unless explicitly demonstrated in class.

#### 5. In-Class Activity: Gaussian Simulation and Gaussian Density Estimation

(Google Colab)

Present this section at the end of the scribe, structured exactly as a classroom coding demonstration.

Activity Overview

Simulate Gaussian random variables

Visualize probability distributions

Learn Gaussian density estimation from data

Apply the method to real-world problems

Step-by-Step Activity Structure

## 1. Motivation

Real-world measurements are uncertain due to noise and randomness

Examples:

Sensor measurements

Network packet delays

Image sensor noise

## 2. Gaussian Simulation with Known Parameters

Import required libraries (numpy, matplotlib)

Define:

Number of samples

True mean

True standard deviation

Generate Gaussian samples using randomized simulation

## 3. Visualizing the Distribution

Construct a histogram with fixed bin widths

Compute histogram centers

Define the true Gaussian PDF using known parameters

Plot histogram and true PDF together

## 4. Reality Check: Unknown Parameters

Real-world data provides only samples

Parameters are unknown a priori

## 5. Gaussian Density Estimation from Data

Generate Gaussian samples

Add measurement noise

Construct histogram of observed data

Estimate parameters:

Sample mean

Sample standard deviation

Construct estimated Gaussian PDF

Plot histogram and estimated PDF together

## 6. Application: Packet Delay in Networks

Simulate packet delay using a Gaussian model

Apply non-negative delay constraint

Construct histogram

Estimate Gaussian parameters

Plot observed delays and estimated density

## 7. Key Takeaways

Gaussian simulation uses randomized algorithms

Density estimation learns distributions from data

The same methodology applies across applications

Important Instruction

All mathematical problems, numerical values, and

probability questions must be extracted directly from the uploaded question images.  
Do not restate the question unless necessary; proceed directly to the solution in lecture style.

Below is the strict exam-preparation lecture scribe for

### CSE 400: Fundamentals of Probability and Random Variables

#### Lecture: Types of Continuous Random Variables

(SEAS, Ahmedabad University)

This scribe **faithfully reproduces the lecture** using **only** the material, numerical values, and questions visible in the uploaded images.

No additional examples, interpretations, shortcuts, or assumptions are introduced.

All steps are written **explicitly and sequentially**, following classroom notation and structure.

---

## Types of Continuous Random Variables

---

### 1. Uniform Random Variable

#### Definition

A continuous random variable  $\Theta$  is said to be uniformly distributed over an interval  $[a, b]$  if its probability density function is constant over that interval.

---

#### Support

$$a \leq \Theta < b$$

---

#### Probability Density Function (PDF)

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{b-a}, & a \leq \theta < b \\ 0, & \text{otherwise} \end{cases}$$

---

**Given**

$$\Theta \sim \text{Uniform}[0, 2\pi]$$

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$


---

**(a)**

$$\begin{aligned} \Pr(\Theta > \frac{3\pi}{4}) &= \int_{3\pi/4}^{2\pi} \frac{1}{2\pi} d\theta \\ &= \frac{1}{2\pi} \left( 2\pi - \frac{3\pi}{4} \right) \\ &= \frac{5}{8} \end{aligned}$$


---

**(b)**

$$\begin{aligned} \Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) &= \Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} \\ A &= \{\Theta < \pi\}, \quad B = \{\Theta > \frac{3\pi}{4}\} \\ A \cap B &= \left\{ \frac{3\pi}{4} < \Theta < \pi \right\} \\ \Pr(A \cap B) &= \int_{3\pi/4}^{\pi} \frac{1}{2\pi} d\theta = \frac{1}{8} \\ \Pr(B) &= \frac{5}{8} \\ \Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) &= \frac{1/8}{5/8} = \frac{1}{5} \end{aligned}$$


---

**(c)**

$$\Pr(\cos(\Theta) < \frac{1}{2})$$

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\begin{aligned}\Pr\left(\frac{\pi}{3} < \Theta < \frac{5\pi}{3}\right) &= \int_{\pi/3}^{5\pi/3} \frac{1}{2\pi} d\theta \\ &= \frac{1}{2\pi} \left(\frac{4\pi}{3}\right) = \frac{2}{3}\end{aligned}$$


---

## 2. Exponential Random Variable

### Definition

A random variable  $X$  is exponential if its PDF is

$$f_X(x) = \lambda e^{-\lambda x} u(x)$$


---

### Support

$$x \geq 0$$


---

### Probability Density Function (PDF)

$$f_X(x) = e^{-x} u(x)$$


---

(a)

$$\Pr(3X < 5)$$

$$\begin{aligned}&= \Pr\left(X < \frac{5}{3}\right) \\ &= \int_0^{5/3} e^{-x} dx \\ &= 1 - e^{-5/3}\end{aligned}$$


---

(b)

$$\begin{aligned}
 & \Pr(3X < y) \\
 &= \Pr\left(X < \frac{y}{3}\right) \\
 &= \int_0^{y/3} e^{-x} dx \\
 &= 1 - e^{-y/3}, \quad y \geq 0
 \end{aligned}$$


---

(c) Let  $Y = 3X$ 

$$\begin{aligned}
 F_Y(y) &= \Pr(Y < y) = 1 - e^{-y/3}, \quad y \geq 0 \\
 f_Y(y) &= \frac{d}{dy}F_Y(y) = \frac{1}{3}e^{-y/3}u(y)
 \end{aligned}$$


---

### 3. Laplace Random Variable

#### Definition

A random variable  $W$  is Laplace distributed if

$$f_W(w) = ce^{-2|w|}$$


---

#### Support

$$-\infty < w < \infty$$


---

(a)

$$\begin{aligned}
 \int_{-\infty}^{\infty} ce^{-2|w|} dw &= 1 \\
 2c \int_0^{\infty} e^{-2w} dw &= 1 \\
 2c \cdot \frac{1}{2} &= 1 \Rightarrow c = 1
 \end{aligned}$$

(b)

$$\begin{aligned}
 & \Pr(-1 < W < 2) \\
 &= \int_{-1}^0 e^{2w} dw + \int_0^2 e^{-2w} dw \\
 &= \left[ \frac{1}{2} e^{2w} \right]_{-1}^0 + \left[ -\frac{1}{2} e^{-2w} \right]_0^2 \\
 &= \frac{1}{2}(1 - e^{-2}) + \frac{1}{2}(1 - e^{-4})
 \end{aligned}$$


---

(c)

$$\begin{aligned}
 & \Pr(W > 0 \mid -1 < W < 2) \\
 &= \frac{\Pr(0 < W < 2)}{\Pr(-1 < W < 2)} \\
 & \Pr(0 < W < 2) = \int_0^2 e^{-2w} dw = \frac{1}{2}(1 - e^{-4}) \\
 & \Pr(W > 0 \mid -1 < W < 2) = \frac{\frac{1}{2}(1 - e^{-4})}{\frac{1}{2}(1 - e^{-2}) + \frac{1}{2}(1 - e^{-4})}
 \end{aligned}$$


---

## 4. Gamma Random Variable

### Definition of the Gamma Function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$


---

### Mean and Variance

If  $X \sim \text{Gamma}(\alpha, \lambda)$ ,

$$E[X] = \frac{\alpha}{\lambda}$$

$$\text{Var}(X) = \frac{\alpha}{\lambda^2}$$

## 5. Properties of the Gamma Function

(a)

$$\Gamma(n) = (n - 1)!$$

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t} dt$$

Using integration by parts repeatedly,

$$\Gamma(n) = (n - 1)\Gamma(n - 1)$$

With  $\Gamma(1) = 1$ ,

$$\Gamma(n) = (n - 1)!$$


---

(b)

$$\Gamma(x + 1) = x\Gamma(x)$$

$$\Gamma(x + 1) = \int_0^\infty t^x e^{-t} dt$$

Integration by parts gives:

$$\Gamma(x + 1) = x\Gamma(x)$$


---

(c)

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty t^{-1/2} e^{-t} dt$$

Using the Gaussian integral,

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$


---

## 6. In-Class Activity: Gaussian Simulation and Gaussian Density Estimation

(Google Colab)

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### Activity Overview

- Gaussian simulation
  - Distribution visualization
  - Density estimation
  - Packet delay modeling
- 

### 1. Motivation

- Sensor measurements
  - Network packet delays
  - Image sensor noise
- 

### 2. Gaussian Simulation (Known Parameters)

python

```
import numpy as np
import matplotlib.pyplot as plt

N = 10000
mu = 0
sigma = 1

samples = np.random.normal(mu, sigma, N)
```

---

### 3. Visualizing the Distribution

python

```
counts, bins = np.histogram(samples, bins=50, density=True)
centers = (bins[:-1] + bins[1:]) / 2
```

```
pdf = (1/(sigma*np.sqrt(2*np.pi))) * np.exp(-(centers-mu)**2/(2*sigma**2))

plt.bar(centers, counts)
plt.plot(centers, pdf)
plt.show()
```

---

## 4. Reality Check: Unknown Parameters

- Only samples are available
  - Parameters unknown
- 

## 5. Gaussian Density Estimation from Data

```
python
```

```
mu_hat = np.mean(samples)
sigma_hat = np.std(samples)
```

$$\hat{f}(x) = \frac{1}{\sigma_{\text{hat}} \sqrt{2\pi}} e^{-\frac{(x-\mu_{\text{hat}})^2}{2\sigma_{\text{hat}}^2}}$$

---

## 6. Application: Packet Delay in Networks

- Gaussian delay simulation
  - Non-negative constraint
  - Histogram construction
  - Parameter estimation
  - Density plotting
- 

## 7. Key Takeaways

- Gaussian simulation uses randomized algorithms
  - Density estimation learns distributions from samples
  - Same methodology applies across applications
-

If you want next:

- **Exact LaTeX exam notes**
- **Handwritten-style PDF**
- **One-page revision sheets**
- **Colab notebook identical to lecture**

Just say the word.