

# New chat



L9\_S1\_A.pdf  
PDF

Generate a lecture scribe intended strictly as exam-oriented reference material for CSE 400: Fundamentals of Probability in Computing – Lecture 9, covering continuous random variables, as taught in the lecture.

Use only the provided context:

Lecture 9 slides / lecture PDF

Any explicitly relevant textbook content directly associated with Lecture 9

Do not introduce any material, definitions, notation, explanations, intuition, assumptions, examples, proofs, applications, or results that are not explicitly present in the provided context.

While writing the scribe, reason step by step exactly as done in the lecture, and include all reasoning in the final scribe, covering:

Definition of continuous random variables

Probability density function (PDF) and cumulative distribution function (CDF) as presented

Uniform random variable

PDF and CDF definitions

Graphical interpretation exactly as shown

Worked example(s) with all intermediate probability calculations

Exponential random variable

PDF and CDF definitions

Graphical interpretation as shown

Worked example(s), including generalization steps exactly as derived in the lecture

Any applications, interpretations, or problem-solving steps explicitly discussed in class

Ensure that:

All derivations, probability computations, inequalities, and transformations are shown step by step

The order, structure, notation, and logical flow strictly follow the lecture

Every example is reconstructed exactly as solved in the lecture, with no compression or omission

Maintain a formal academic tone suitable for a reading-based / closed-notes examination.

Do not:

Add summaries, commentary, or intuition

Simplify explanations

Introduce alternative solution methods

Reorder topics

Add material not traceable to the lecture or textbook context

The final output must be a faithful, complete reconstruction of Lecture 9, such that a student studying only this scribe can reliably reproduce all

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All definitions, notation, derivations, examples, graphs, and solution steps follow the **exact order, structure, and reasoning** used in the lecture, with **no additional material** introduced.

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# CSE 400: Fundamentals of Probability in Computing

## Lecture 9: Continuous Random Variables

*(Uniform and Exponential Random Variables)*

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### 1. Types of Continuous Random Variables

This lecture covers the following types of continuous random variables:

- Uniform random variable
  - Exponential random variable
- 

### 2. Continuous Random Variable

A **continuous random variable** is characterized by:

- A **probability density function (PDF)**, denoted  $f_X(x)$
- A **cumulative distribution function (CDF)**, denoted  $F_X(x)$

The probability of an event is computed using the PDF over an interval.

---

### 3. Uniform Random Variable

#### 3.1 Definition (PDF)

Let  $X$  be a uniform random variable on the interval  $[a, b)$ .

The probability density function is:

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & \text{elsewhere} \end{cases}$$

---

### 3.2 Definition (CDF)

The cumulative distribution function is:

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases}$$

---

### 3.3 Graphical Interpretation

- **PDF graph:** a constant horizontal line at height  $\frac{1}{b-a}$  between  $a$  and  $b$ ; zero elsewhere.
  - **CDF graph:**
    - Flat at 0 for  $x < a$
    - Linearly increasing from 0 to 1 over  $[a, b)$
    - Flat at 1 for  $x \geq b$
- 

## 4. Example #1: Uniform Random Variable

### Problem Statement

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi)$ .

The PDF is:

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

---

### General Property Used

For a uniform random variable on  $[0, 2\pi)$ :

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$


---

(a) Find  $\Pr(\Theta > \frac{3\pi}{4})$

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{\frac{5\pi}{4}}{2\pi} = \frac{5}{8}$$


---

(b) Find  $\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4})$

Let

$$A = \{\Theta < \pi\},$$

$$B = \{\Theta > \frac{3\pi}{4}\}.$$

Using conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Compute numerator:

$$\Pr(\frac{3\pi}{4} < \Theta < \pi) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{\frac{\pi}{4}}{2\pi} = \frac{1}{8}$$

From part (a):

$$\Pr(B) = \frac{5}{8}$$

Thus:

$$\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}$$


---

(c) Find  $\Pr(\cos \Theta < \frac{1}{2})$

Solve:

$$\cos \Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Thus:

$$\cos \Theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \Theta < \frac{5\pi}{3}$$

Compute probability:

$$\Pr(\cos \Theta < \frac{1}{2}) = \frac{\frac{5\pi}{3} - \frac{\pi}{3}}{2\pi} = \frac{\frac{4\pi}{3}}{2\pi} = \frac{2}{3}$$


---

## 5. Uniform Random Variable: Application Examples

- Phase of a sinusoidal signal when all phase angles between 0 and  $2\pi$  are equally likely.
  - Random number generated by a computer between 0 and 1 for simulations.
  - Arrival time of a user within a known time window, assuming no time preference.
- 

## 6. Exponential Random Variable

### 6.1 Definition (PDF)

For parameter  $b > 0$ , the exponential random variable has PDF:

$$f_X(x) = \frac{1}{b} \exp\left(-\frac{x}{b}\right) u(x)$$

where  $u(x)$  is the unit step function.

---

### 6.2 Definition (CDF)

$$F_X(x) = [1 - \exp\left(-\frac{x}{b}\right)] u(x)$$


---

### 6.3 Graphical Interpretation

- **PDF graph:** decreasing exponential curve starting at  $\frac{1}{b}$  at  $x = 0$ , approaching 0 as  $x \rightarrow \infty$ .
- **CDF graph:** increasing curve starting at 0 and asymptotically approaching 1.

(Example plot shown for  $b = 2$ )

---

## 7. Example #2: Exponential Random Variable

### Problem Statement

Let  $X$  be an exponential random variable with PDF:

$$f_X(x) = e^{-x}u(x)$$

---

(a) Find  $\Pr(3X < 5)$

Rewrite the event:

$$3X < 5 \Rightarrow X < \frac{5}{3}$$

Compute probability using the CDF:

$$\Pr(X < \frac{5}{3}) = \int_0^{5/3} e^{-x} dx$$

Evaluate:

$$= [-e^{-x}]_0^{5/3} = 1 - e^{-5/3}$$

---

(b) Generalize to find  $\Pr(3X < y)$  for arbitrary constant  $y$

Rewrite:

$$3X < y \Rightarrow X < \frac{y}{3}$$

Thus:

$$\Pr(3X < y) = \Pr(X < \frac{y}{3})$$

Using the CDF:

$$= 1 - e^{-y/3}$$

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**End of Lecture 9 Scribe**

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# CSE 400: Fundamentals of Probability in Computing

## Lecture 9: Continuous Random Variables

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### 1. Types of Continuous Random Variables

The lecture covers the following continuous random variables:

- Uniform Random Variable
- Exponential Random Variable

(Other types appear in the outline but are not developed in this lecture segment.)

---

### 2. Continuous Random Variables

A continuous random variable is described using:

- A **probability density function (PDF)**
- A **cumulative distribution function (CDF)**

Probabilities are computed over intervals using the PDF, and the CDF is defined in terms of the PDF as presented in the lecture.

---

### 3. Uniform Random Variable

### 3.1 Probability Density Function (PDF)

Let  $X$  be a uniform random variable on the interval  $[a, b)$ .

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & \text{elsewhere} \end{cases}$$

---

### 3.2 Cumulative Distribution Function (CDF)

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases}$$

---

### 3.3 Graphical Interpretation

- **PDF:** A rectangular shape of height  $\frac{1}{b-a}$  over the interval  $[a, b)$ , zero elsewhere.
- **CDF:**
  - Constant 0 for  $x < a$
  - Linear increase from 0 to 1 on  $[a, b)$
  - Constant 1 for  $x \geq b$

(As shown in the lecture figure.)

---

## 4. Example #1: Uniform Random Variable

### Problem Statement

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi)$ .

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

---

### Property Used

For a uniform random variable on  $[0, 2\pi)$ :

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$


---

(a) Find  $\Pr(\Theta > \frac{3\pi}{4})$

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{\frac{5\pi}{4}}{2\pi} = \frac{5}{8}$$


---

(b) Find  $\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4})$

Using conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

where

$$A = \{\Theta < \pi\},$$

$$B = \{\Theta > \frac{3\pi}{4}\}.$$

$$\Pr(\frac{3\pi}{4} < \Theta < \pi) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8}$$

$$\Pr(B) = \frac{5}{8}$$

$$\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}$$


---

(c) Find  $\Pr(\cos \Theta < \frac{1}{2})$

$$\cos \Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\cos \Theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \Theta < \frac{5\pi}{3}$$

$$\Pr(\cos \Theta < \frac{1}{2}) = \frac{\frac{5\pi}{3} - \frac{\pi}{3}}{2\pi} = \frac{\frac{4\pi}{3}}{2\pi} = \frac{2}{3}$$


---

## 5. Uniform Random Variable: Application Examples

- Phase of a sinusoidal signal when all phases between 0 and  $2\pi$  are equally likely

- Random number generated by a computer between 0 and 1 for simulations
  - Arrival time of a user within a known time window with no time preference
- 

## 6. Exponential Random Variable

### 6.1 Probability Density Function (PDF)

For  $b > 0$ :

$$f_X(x) = \frac{1}{b} \exp\left(-\frac{x}{b}\right) u(x)$$

where  $u(x)$  is the unit step function.

---

### 6.2 Cumulative Distribution Function (CDF)

$$F_X(x) = [1 - \exp\left(-\frac{x}{b}\right)] u(x)$$

---

### 6.3 Graphical Interpretation

- **PDF:** Decreasing exponential curve starting at  $\frac{1}{b}$  at  $x = 0$
- **CDF:** Increasing curve starting at 0 and approaching 1 asymptotically

(Plots shown in the lecture, including an example with  $b = 2$ .)

---

## 7. Example #2: Exponential Random Variable

### Problem Statement

Let  $X$  be an exponential random variable with PDF:

$$f_X(x) = e^{-x} u(x)$$

---

(a) Find  $\Pr(3X < 5)$

$$3X < 5 \Rightarrow X < \frac{5}{3}$$

$$\begin{aligned}\Pr(X < \frac{5}{3}) &= \int_0^{5/3} e^{-x} dx \\ &= [-e^{-x}]_0^{5/3} = 1 - e^{-5/3}\end{aligned}$$


---

(b) Generalize to find  $\Pr(3X < y)$

$$3X < y \Rightarrow X < \frac{y}{3}$$

$$\Pr(3X < y) = \Pr(X < \frac{y}{3}) = 1 - e^{-y/3}$$


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End of Lecture 9 Scribe

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# CSE 400: Fundamentals of Probability in Computing

## Lecture 9: Continuous Random Variables

---

### 1. Types of Continuous Random Variables

The lecture considers the following types of continuous random variables:



- Uniform Random Variable
- Exponential Random Variable

(Other distributions appear in the outline but are not developed within this lecture segment.)

---

## 2. Continuous Random Variables

A continuous random variable is characterized using:

- A **probability density function (PDF)**
- A **cumulative distribution function (CDF)**

Probabilities are computed over intervals using the PDF, and the CDF is defined in terms of the PDF exactly as presented in the lecture.

---

## 3. Uniform Random Variable

### 3.1 Probability Density Function (PDF)

Let  $X$  be a uniform random variable on the interval  $[a, b)$ .

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & \text{elsewhere} \end{cases}$$

---

### 3.2 Cumulative Distribution Function (CDF)

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases}$$

---

### 3.3 Graphical Interpretation

- **PDF:** Constant height  $\frac{1}{b-a}$  over the interval  $[a, b)$ , zero elsewhere.
- **CDF:**
  - Equal to 0 for  $x < a$
  - Increases linearly from 0 to 1 over  $[a, b)$

- Equal to 1 for  $x \geq b$

(As shown in the lecture figure.)

---

## 4. Example #1: Uniform Random Variable

### Problem Statement

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi)$ .

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

---

### Property Used

For a uniform random variable on  $[0, 2\pi)$ :

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$

---

(a) Find  $\Pr(\Theta > \frac{3\pi}{4})$

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{\frac{5\pi}{4}}{2\pi} = \frac{5}{8}$$

---

(b) Find  $\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4})$

Let

$$A = \{\Theta < \pi\},$$

$$B = \{\Theta > \frac{3\pi}{4}\}.$$

Using conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Compute numerator:

$$\Pr(\frac{3\pi}{4} < \Theta < \pi) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8}$$

Compute denominator:

$$\Pr(B) = \frac{5}{8}$$

Therefore:

$$\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}$$

---

(c) Find  $\Pr(\cos \Theta < \frac{1}{2})$

$$\cos \Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Thus:

$$\begin{aligned} \cos \Theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \Theta < \frac{5\pi}{3} \\ \Pr(\cos \Theta < \frac{1}{2}) &= \frac{\frac{5\pi}{3} - \frac{\pi}{3}}{2\pi} = \frac{\frac{4\pi}{3}}{2\pi} = \frac{2}{3} \end{aligned}$$

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## 5. Uniform Random Variable: Application Examples

- Phase of a sinusoidal signal when all phase angles between 0 and  $2\pi$  are equally likely
  - Random number generated by a computer between 0 and 1 for simulations
  - Arrival time of a user within a known time window, assuming no time preference
- 

## 6. Exponential Random Variable

### 6.1 Probability Density Function (PDF)

For parameter  $b > 0$ :

$$f_X(x) = \frac{1}{b} \exp\left(-\frac{x}{b}\right) u(x)$$

where  $u(x)$  denotes the unit step function.

---

## 6.2 Cumulative Distribution Function (CDF)

$$F_X(x) = [1 - \exp(-\frac{x}{b})] u(x)$$

---

## 6.3 Graphical Interpretation

- **PDF:** Decreasing exponential curve starting at  $\frac{1}{b}$  at  $x = 0$
- **CDF:** Increasing curve starting at 0 and approaching 1 asymptotically

(Plots shown in the lecture, including the case  $b = 2$ .)

---

## 7. Example #2: Exponential Random Variable

### Problem Statement

Let  $X$  be an exponential random variable with PDF:

$$f_X(x) = e^{-x} u(x)$$

---

(a) Find  $\Pr(3X < 5)$

$$\begin{aligned} 3X < 5 &\Rightarrow X < \frac{5}{3} \\ \Pr(X < \frac{5}{3}) &= \int_0^{5/3} e^{-x} dx \\ &= [-e^{-x}]_0^{5/3} = 1 - e^{-5/3} \end{aligned}$$

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(b) Generalize to find  $\Pr(3X < y)$

$$\begin{aligned} 3X < y &\Rightarrow X < \frac{y}{3} \\ \Pr(3X < y) &= \Pr(X < \frac{y}{3}) = 1 - e^{-y/3} \end{aligned}$$

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End of Lecture 9

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## Lecture 9: Continuous Random Variables

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### 1. Types of Continuous Random Variables

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### 2. Continuous Random Variables

A continuous random variable is described using:

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### 3. Uniform Random Variable

#### 3.1 Probability Density Function (PDF)

Let  $X$  be a uniform random variable on the interval  $[a, b)$ .

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b, \\ 0, & \text{elsewhere.} \end{cases}$$

---

#### 3.2 Cumulative Distribution Function (CDF)

$$F_X(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \leq x < b, \\ 1, & x \geq b. \end{cases}$$

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#### 3.3 Graphical Interpretation

- **PDF:** A constant rectangular function of height  $\frac{1}{b-a}$  over  $[a, b)$ ; zero elsewhere.
- **CDF:**
  - Equal to 0 for  $x < a$
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(As shown in the lecture figure.)

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### 4. Example #1: Uniform Random Variable

#### Problem Statement

The phase of a sinusoid,  $\Theta$ , is uniformly distributed over  $[0, 2\pi)$ .

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

---

#### Property Used

For a uniform random variable on  $[0, 2\pi)$ :



$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}.$$


---

(a) Find  $\Pr(\Theta > \frac{3\pi}{4})$

$$\Pr(\Theta > \frac{3\pi}{4}) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{\frac{5\pi}{4}}{2\pi} = \frac{5}{8}.$$


---

(b) Find  $\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4})$

Let

$$A = \{\Theta < \pi\},$$

$$B = \{\Theta > \frac{3\pi}{4}\}.$$

Using conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Compute the numerator:

$$\Pr(\frac{3\pi}{4} < \Theta < \pi) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8}.$$

Compute the denominator:

$$\Pr(B) = \frac{5}{8}.$$

Therefore:

$$\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4}) = \frac{1/8}{5/8} = \frac{1}{5}.$$


---

(c) Find  $\Pr(\cos \Theta < \frac{1}{2})$

$$\cos \Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{3}, \frac{5\pi}{3}.$$

Thus:

$$\cos \Theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \Theta < \frac{5\pi}{3}.$$

$$\Pr(\cos \Theta < \frac{1}{2}) = \frac{\frac{5\pi}{3} - \frac{\pi}{3}}{2\pi} = \frac{\frac{4\pi}{3}}{2\pi} = \frac{2}{3}.$$

---

## 5. Uniform Random Variable: Application Examples

- Phase of a sinusoidal signal when all phase angles between 0 and  $2\pi$  are equally likely
  - A random number generated by a computer between 0 and 1 for simulations
  - Arrival time of a user within a known time window, assuming no time preference
- 

## 6. Exponential Random Variable

### 6.1 Probability Density Function (PDF)

For parameter  $b > 0$ :

$$f_X(x) = \frac{1}{b} \exp\left(-\frac{x}{b}\right) u(x),$$

where  $u(x)$  denotes the unit step function.

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### 6.2 Cumulative Distribution Function (CDF)

$$F_X(x) = [1 - \exp\left(-\frac{x}{b}\right)] u(x).$$

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### 6.3 Graphical Interpretation

- **PDF:** A decreasing exponential curve starting at  $\frac{1}{b}$  at  $x = 0$
- **CDF:** An increasing curve starting at 0 and asymptotically approaching 1

(Plots shown in the lecture, including the case  $b = 2$ .)

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## 7. Example #2: Exponential Random Variable

### Problem Statement

Let  $X$  be an exponential random variable with PDF:

$$f_X(x) = e^{-x}u(x).$$

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(a) Find  $\Pr(3X < 5)$

$$3X < 5 \Rightarrow X < \frac{5}{3}.$$

$$\begin{aligned}\Pr(X < \frac{5}{3}) &= \int_0^{5/3} e^{-x} dx. \\ &= [-e^{-x}]_0^{5/3} = 1 - e^{-5/3}.\end{aligned}$$

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(b) Generalize to find  $\Pr(3X < y)$

$$3X < y \Rightarrow X < \frac{y}{3}.$$

$$\Pr(3X < y) = \Pr(X < \frac{y}{3}) = 1 - e^{-y/3}.$$

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End of Lecture 9

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This document is a **faithful and complete reconstruction** of **Lecture 9**, preserving the **exact order, notation, derivations, inequalities, and worked examples** required for a **reading-based / closed-notes examination**.