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Lecture 5: Bayes’ Theorem, Random Variables, and Probability Mass Function

Bayes’ Theorem

Weighted Average of Conditional Probabilities

Let A and B be events.
We may express A as
$A = AB \cup AB^c$

As AB and AB^c are mutually exclusive, by Axiom 3,

$\Pr(A) = \Pr(AB) + \Pr(AB^c)$
$= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)[1 - \Pr(B)]$

The probability of event A is a weighted average of the conditional probabilities with weights given as the probability of the event on which it is conditioned has of occurring.

Bayes' Theorem

Learning by Example

Example #3.1 (Part 1/2)
An insurance company divides people into accident prone and not accident prone.
Pr(accident | accident prone) = 0.4
Pr(accident | not accident prone) = 0.2
Pr(accident prone) = 0.3

Let A_1 = event policyholder has an accident within a year
Let A = event policyholder is accident prone

$Pr(A_1) = Pr(A_1 \mid A)Pr(A) + Pr(A_1 \mid A^c)Pr(A^c)$
$= (0.4)(0.3) + (0.2)(0.7)$
$= 0.26$

Example #3.1 (Part 2/2)
Suppose a new policyholder has an accident within a year.
What is the probability that he or she is accident prone?

$Pr(A \mid A_1) = Pr(AA_1) / Pr(A_1)$
$= Pr(A)Pr(A_1 \mid A) / Pr(A_1)$
$= (0.3)(0.4) / 0.26$
$= 6 / 13$

Bayes' Theorem

Formal Introduction: Law of Total Probability and Bayes Theorem

This is known as the **Law of Total Probability** [Formula 3.4].

Using
$Pr(AB_i) = Pr(B_i \mid A)Pr(A)$

we get

This is known as the **Bayes Formula** [Proposition 3.1].

Here,
$Pr(B_i)$ is the **apriori probability**, and
$Pr(B_i \mid A)$ is the **posteriori probability** of event B_i given A.

Bayes Formula

Learning by Example

Example #3.2
Three cards:

- RR (both sides red)
- BB (both sides black)
- RB (one red, one black)

One card is selected randomly.
If the upper side is red, find the probability the other side is black.

Let RR, BB, RB denote the card type.
Let R be the event that the upturned side is red.

$Pr(RB \mid R) = Pr(RB \cap R) / Pr(R)$
$= Pr(R \mid RB)Pr(RB)$
$/ [Pr(R \mid RR)Pr(RR) + Pr(R \mid RB)Pr(RB) + Pr(R \mid BB)Pr(BB)]$

$= (1/2)(1/3) / [(1)(1/3) + (1/2)(1/3) + (0)(1/3)]$
$= 1/3$

Random Variables

Motivation and Concept

A random variable is a real-valued function defined on the sample space.

Values are determined by outcomes of an experiment.
Probabilities are assigned to possible values of random variables.

Examples:

- Dice tossing → sum of dice
- Coin flipping → number of heads

The distribution of a random variable can be visualized as a bar diagram.
Height at value a is Pr[X = a].

Random Variables

Example

Example 1
Toss 3 fair coins.
Let Y = number of heads.

Possible values: 0, 1, 2, 3

P{Y = 0} = 1/8
P{Y = 1} = 3/8
P{Y = 2} = 3/8
P{Y = 3} = 1/8

Since Y must take one of these values, the probabilities sum to 1.

Probability Mass Function (PMF)

Concept

A random variable that can take on at most a countable number of values is **discrete**.

Let X be a discrete random variable with range
$R_x = x_1, x_2, x_3, \ldots$

The function
$p(x_i) = Pr\{X = x_i\}$

is called the **Probability Mass Function (PMF)** of X.

Since X must take one of the values x_k,
$\Sigma\ p(x_k) = 1$

Probability Mass Function

Example

The PMF of X is given by

$p(i) = c \cdot \lambda^i / i!$, $i = 0, 1, 2, \ldots$

Find:
$P\{X = 0\}$
$P\{X > 2\}$

Class Participation

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CSE400 – Fundamentals of Probability in Computing

Lecture 5: Bayes' Theorem, Random Variables, and Probability Mass Function

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Bayes' Theorem

Weighted Average of Conditional Probabilities

Let A and B be events.
We may express A as
$A = AB \cup AB^c$

As AB and AB^c are mutually exclusive, by Axiom 3,

$\Pr(A) = \Pr(AB) + \Pr(AB^c)$
$= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)[1 - \Pr(B)]$

The probability of event A is a weighted average of the conditional probabilities with weights given as the probability of the event on which it is conditioned has of occurring.

Bayes' Theorem

Learning by Example

EXAMPLE #3.1 (Part 1/2)
An insurance company divides people into accident prone and not accident prone.
Pr(accident | accident prone) = 0.4
Pr(accident | not accident prone) = 0.2
Pr(accident prone) = 0.3

Let A_1 be the event that the policyholder has an accident within a year.
Let A be the event that the policyholder is accident prone.

$Pr(A_1) = Pr(A_1 \mid A)Pr(A) + Pr(A_1 \mid A^c)Pr(A^c)$
$= (0.4)(0.3) + (0.2)(0.7)$
$= 0.26$

EXAMPLE #3.1 (Part 2/2)
Suppose a new policyholder has an accident within a year.
What is the probability that he or she is accident prone?

$Pr(A \mid A_1) = Pr(AA_1) / Pr(A_1)$
$= Pr(A)Pr(A_1 \mid A) / Pr(A_1)$
$= (0.3)(0.4) / 0.26$
$= 6 / 13$

Bayes’ Theorem

Formal Introduction: Law of Total Probability and Bayes Theorem

This is known as the **Law of Total Probability** [Formula 3.4].

Using
$Pr(AB_i) = Pr(B_i \mid A)Pr(A)$

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This is known as the **Bayes Formula** [Proposition 3.1].

Here,
$Pr(B_i)$ is the **apriori probability**, and
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Bayes Formula

Learning by Example

EXAMPLE #3.2
Three cards:

- RR (both sides red)

- BB (both sides black)
- RB (one red, one black)

One card is selected at random.
If the upper side is red, find the probability the other side is black.

Let RR, BB, RB denote the events that the chosen card is all red, all black, or red–black.
Let R be the event that the upturned side is red.

$\Pr(RB \mid R) = \Pr(RB \cap R) / \Pr(R)$

$= \Pr(R \mid RB)\Pr(RB)$
$/ [\Pr(R \mid RR)\Pr(RR) + \Pr(R \mid RB)\Pr(RB) + \Pr(R \mid BB)\Pr(BB)]$

$= (1/2)(1/3) / [(1)(1/3) + (1/2)(1/3) + (0)(1/3)]$
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Random Variables

Motivation and Concept

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Height at value a is $\Pr[X = a]$.

Random Variables

Examples

Example 1

Toss 3 fair coins.

Let Y be the number of heads.

Possible values: 0, 1, 2, 3

P{Y = 0} = 1/8
P{Y = 1} = 3/8
P{Y = 2} = 3/8
P{Y = 3} = 1/8

Since Y must take one of the values 0 through 3, the probabilities sum to 1.

Probability Mass Function (PMF)

Concept

A random variable that can take on at most a countable number of possible values is **discrete**.

Let X be a discrete random variable with range
$R_x = x_1, x_2, x_3, \ldots$ (finite or countably infinite).

The function
$p(x_i) = Pr\{X = x_i\}$

is called the **Probability Mass Function (PMF)** of X.

Since X must take one of the values x_k,
$\Sigma\ p(x_k) = 1$.

Probability Mass Function

Example

The probability mass function of X is given by

$p(i) = c\,\lambda^i / i!, \quad i = 0, 1, 2, \ldots$

Find:
P{X = 0}
P{X > 2}

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CSE400 – Fundamentals of Probability in Computing

Lecture 5: Bayes' Theorem, Random Variables, and Probability Mass Function

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Bayes' Theorem

Weighted Average of Conditional Probabilities

Let A and B be events. We may express
$A = AB \cup AB^c$

As AB and AB^c are clearly mutually exclusive, by Axiom 3,

$Pr(A) = Pr(AB) + Pr(AB^c)$
$= Pr(A \mid B)Pr(B) + Pr(A \mid B^c)[1 - Pr(B)]$

The probability of event A is a weighted average of the conditional probabilities with weights given as the probability of the event on which it is conditioned has of occurring.

Bayes' Theorem

Learning by Example

EXAMPLE #3.1 (Part 1/2)
An insurance company divides people into accident prone and not accident prone.
An accident-prone person has an accident with probability 0.4.
A not accident-prone person has an accident with probability 0.2.
30 percent of the population is accident prone.

Let A_1 denote the event that the policyholder will have an accident within a year.
Let A denote the event that the policyholder is accident prone.

$Pr(A_1) = Pr(A_1 \mid A)Pr(A) + Pr(A_1 \mid A^c)Pr(A^c)$
$= (0.4)(0.3) + (0.2)(0.7)$
$= 0.26$

EXAMPLE #3.1 (Part 2/2)
Suppose that a new policyholder has an accident within a year.
What is the probability that he or she is accident prone?

$Pr(A \mid A_1) = Pr(AA_1) / Pr(A_1)$
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Learning by Example

EXAMPLE #3.2

Three cards:

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One card is selected at random.
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Let R be the event that the upturned side is red.

$Pr(RB \mid R) = Pr(RB \cap R) / Pr(R)$

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$/ [Pr(R \mid RR)Pr(RR) + Pr(R \mid RB)Pr(RB) + Pr(R \mid BB)Pr(BB)]$

$= (1/2)(1/3) / [(1)(1/3) + (1/2)(1/3) + (0)(1/3)]$
$= 1/3$

Random Variables

Motivation and Concept

When an experiment is performed, we are frequently interested mainly in some function of the outcome as opposed to the actual outcome itself.

These real-valued functions defined on the sample space are known as **Random Variables**.

Values are determined by the outcomes of an experiment.
Probabilities are assigned to possible values of random variables.

The distribution of a random variable can be visualized as a bar diagram.
The height of the bar at value *a* is Pr[X = a].

Random Variables

Examples

Example 1
Tossing 3 fair coins.
Let Y denote the number of heads.

Possible values: 0, 1, 2, 3

P{Y = 0} = 1/8
P{Y = 1} = 3/8
P{Y = 2} = 3/8
P{Y = 3} = 1/8

Since Y must take on one of the values 0 through 3, we must have:

Probability Mass Function (PMF)

Concept

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$R_x = x_1, x_2, x_3, \ldots$ (finite or countably infinite).

The function
$p(x_i) = Pr\{X = x_i\}$

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Example

The probability mass function of a random variable X is given by

$p(i) = c\,\lambda^i / i!, \quad i = 0, 1, 2, \ldots$

Find:
$P\{X = 0\}$
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CSE400 – Fundamentals of Probability in Computing

Lecture 5: Bayes' Theorem, Random Variables, and Probability Mass Function

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The probability of event A is a weighted average of the conditional probabilities with weights given as the probability of the event on which it is conditioned has of occurring.

Bayes' Theorem

Learning by Example

EXAMPLE #3.1 (Part 1/2)
An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not.

An accident-prone person has an accident with probability 0.4.
A not accident-prone person has an accident with probability 0.2.
30 percent of the population is accident prone.

Let A_1 denote the event that the policyholder will have an accident within a year.
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$= (0.4)(0.3) + (0.2)(0.7)$
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$\Pr(A \mid A_1) = \Pr(AA_1) / \Pr(A_1)$
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Let RR, BB, and RB denote the events that the chosen card is all red, all black, or red–black.
Let R be the event that the upturned side is red.

Pr(RB | R) = Pr(RB ∩ R) / Pr(R)

= Pr(R | RB)Pr(RB)
/ [Pr(R | RR)Pr(RR) + Pr(R | RB)Pr(RB) + Pr(R | BB)Pr(BB)]

= (1/2)(1/3) / [(1)(1/3) + (1/2)(1/3) + (0)(1/3)]
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Random Variables

Motivation and Concept

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Random Variables

Examples

Example – 1
Tossing 3 fair coins.

Let Y denote the number of heads.

$P\{Y = 0\} = 1/8$
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