Lecture Scribe in LaTeX

L5_A_S1.pdf
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Create a lecture scribe which functions as an examination reference. The scribe needs to process all definitions and assumptions and derivations and proofs and worked examples from the lecture slides according to their actual display in the slides. The scribe needs to present formal definitions and notation together with complete assumptions and conditions and demonstrate all content through complete derivations and example solutions that appear within the lecture. The context provides all necessary information. The lecture content contains no new explanations or intuitions or examples or different solution methods or materials that were not included in the lecture. give all in only latex code form author name as Name: Masoom Choksi AUID: AU2440263

latex

```latex
\documentclass[12pt]{article}
\usepackage{amsmath,amssymb,amsthm}
\usepackage{geometry}
\usepackage{enumitem}
\geometry{margin=1in}

\title{\textbf{CSE400: Fundamentals of Probability in Computing}\\
Lecture 5: Bayes' Theorem, Random Variables, and Probability Mass
Function}
\author{\\ \textbf{Name: Masoom Choksi} \\ \textbf{AUID: AU2440263}}
\date{}

\begin{document}
\maketitle

\section*{Bayes' Theorem}

\subsection*{Weighted Average of Conditional Probabilities}
```

```
Let $A$ and $B$ be events. We may express the event $A$ as
\[
A = AB \cup AB^c,
\]
since in order for an outcome to be in $A$, it must either be in both
$A$ and $B$, or be in $A$ but not in $B$.

Since $AB$ and $AB^c$ are mutually exclusive, by Axiom 3 of probability,
\begin{align*}
\Pr(A) &= \Pr(AB) + \Pr(AB^c) \\
&= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)\Pr(B^c) \\
&= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)\left[1 - \Pr(B)\right].
\end{align*}

Thus, the probability of event $A$ is a weighted average of the
conditional probabilities, with weights given by the probabilities of
the events on which it is conditioned.

\subsection*{Learning by Example}

\paragraph{Example 3.1 (Part 1)}

An insurance company classifies people into two categories:
\begin{itemize}
\item Accident-prone
\item Not accident-prone
\end{itemize}

The probability that an accident-prone person has an accident within a
1-year period is $0.4$, while this probability is $0.2$ for a person who
is not accident-prone. It is assumed that $30\%$ of the population is
accident-prone.

\textbf{Question:} What is the probability that a new policyholder will
have an accident within a year?

\textbf{Solution:}

Let
\begin{itemize}
\item $A_1$ denote the event that the policyholder has an accident
within a year,
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\item $A$ denote the event that the policyholder is accident-prone.
\end{itemize}

Then,
\begin{align*}
\Pr(A_1) &= \Pr(A_1 \mid A)\Pr(A) + \Pr(A_1 \mid A^c)\Pr(A^c) \\
&= (0.4)(0.3) + (0.2)(0.7) \\
&= 0.12 + 0.14 \\
&= 0.26.
\end{align*}

\paragraph{Example 3.1 (Part 2)}

Suppose that a new policyholder has an accident within a year. What is
the probability that the policyholder is accident-prone?

\textbf{Solution:}

We compute
\[
\Pr(A \mid A_1) = \frac{\Pr(A \cap A_1)}{\Pr(A_1)}.
\]
Using $\Pr(A \cap A_1) = \Pr(A)\Pr(A_1 \mid A)$,
\begin{align*}
\Pr(A \mid A_1) &= \frac{\Pr(A)\Pr(A_1 \mid A)}{\Pr(A_1)} \\
&= \frac{(0.3)(0.4)}{0.26} \\
&= \frac{6}{13}.
\end{align*}

\subsection*{Formal Introduction}

\subsubsection*{Law of Total Probability}

Let $B_1, B_2, \dots, B_n$ be mutually exclusive and exhaustive events
such that
\[
\bigcup_{i=1}^n B_i = S.
\]
Then for any event $A$,
\[
\Pr(A) = \sum_{i=1}^n \Pr(A \cap B_i).
\]
```

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This is known as the \textbf{Law of Total Probability}.

\subsubsection*{Bayes' Formula}

Using
\[
\Pr(A \cap B_i) = \Pr(B_i \mid A)\Pr(A),
\]
we obtain
\[
\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i)\Pr(B_i)}{\sum_{j=1}^n \Pr(A \mid
B_j)\Pr(B_j)}.
\]

This is known as \textbf{Bayes' Formula} (Proposition 3.1), where:
\begin{itemize}
\item $\Pr(B_i)$ is the \textit{a priori} probability,
\item $\Pr(B_i \mid A)$ is the \textit{a posteriori} probability after
observing event $A$.
\end{itemize}

\subsection*{Example 3.2}

Three cards are identical in form:
\begin{itemize}
\item One card has both sides red (RR),
\item One card has both sides black (BB),
\item One card has one red and one black side (RB).
\end{itemize}

One card is randomly selected and placed on the ground. If the upper
side is red, what is the probability that the other side is black?

\textbf{Solution:}

Let
\begin{itemize}
\item $RR, RB, BB$ denote the events that the selected card is all-red,
red-black, or all-black respectively,
\item $R$ denote the event that the upturned side is red.
\end{itemize}
```

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We compute
\[
\Pr(RB \mid R) = \frac{\Pr(R \mid RB)\Pr(RB)}{\Pr(R)}.
\]

Now,
\[
\Pr(R) = \Pr(R \mid RR)\Pr(RR) + \Pr(R \mid RB)\Pr(RB) + \Pr(R \mid
BB)\Pr(BB).
\]

Substituting values,
\begin{align*}
\Pr(RB \mid R) &= \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} \\
&= \frac{1/6}{1/2} \\
&= \frac{1}{3}.
\end{align*}

\section*{Random Variables}

\subsection*{Motivation and Concept}

In many experiments, we are interested in a function of the outcome
rather than the outcome itself.

\begin{itemize}
\item In tossing two dice, the sum is often of interest.
\item In flipping coins, the number of heads may be of interest.
\end{itemize}

A \textbf{random variable} is a real-valued function defined on the
sample space. Its value is determined by the outcome of an experiment,
and probabilities are assigned to its possible values.

\subsection*{Example}

Suppose three fair coins are tossed. Let $Y$ denote the number of heads
observed.

Then $Y$ takes values $\{0,1,2,3\}$ with probabilities:
\begin{align*}
\Pr(Y=0) &= \Pr(t,t,t) = \frac{1}{8}, \\
```

```
\Pr(Y=1) &= \Pr(t,t,h),(t,h,t),(h,t,t) = \frac{3}{8}, \\
\Pr(Y=2) &= \Pr(t,h,h),(h,t,h),(h,h,t) = \frac{3}{8}, \\
\Pr(Y=3) &= \Pr(h,h,h) = \frac{1}{8}.
\end{align*}

Since $Y$ must take one of these values,
\[
\sum_{k=0}^{3} \Pr(Y=k) = 1.
\]

\section*{Probability Mass Function}

\subsection*{Concept}

A random variable that takes at most a countable number of values is
called \textbf{discrete}.

Let $X$ be a discrete random variable with range
\[
R_X = \{x_1, x_2, x_3, \dots\}.
\]

The function
\[
p(x_k) = \Pr(X = x_k)
\]
is called the \textbf{Probability Mass Function (PMF)} of $X$.

Since $X$ must take one of the values in $R_X$,
\[
\sum_k p(x_k) = 1.
\]

\subsection*{Example}

The PMF of a random variable $X$ is given by
\[
p(i) = c \lambda^i, \quad i = 0,1,2,\dots,
\]
where $\lambda > 0$.

Since probabilities must sum to 1,
```

```
\[
\sum_{i=0}^{\infty} c \lambda^i = 1.
\]

This gives
\[
c \sum_{i=0}^{\infty} \lambda^i = c \frac{1}{1-\lambda} = 1,
\]
hence
\[
c = 1-\lambda.
\]

Therefore,
\[
\Pr(X=0) = p(0) = 1-\lambda,
\]
and
\begin{align*}
\Pr(X>2) &= 1 - \Pr(X \le 2) \\
&= 1 - \sum_{i=0}^{2} (1-\lambda)\lambda^i.
\end{align*}

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\author{\textbf{Name: Masoom Choksi}\\ \textbf{AUID: AU2440263}}
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\begin{document}
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\section*{Bayes' Theorem}

\subsection*{Weighted Average of Conditional Probabilities}

Let $A$ and $B$ be events. We may express $A$ as
\[
A = AB \cup AB^{c},
\]
since an outcome in $A$ must either occur with $B$ or with $B^{c}$.

The events $AB$ and $AB^{c}$ are mutually exclusive. Hence, by Axiom 3,
\begin{align*}
\Pr(A) &= \Pr(AB) + \Pr(AB^{c}) \\
&= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^{c})\Pr(B^{c}) \\
&= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^{c})[1-\Pr(B)].
\end{align*}

Thus, $\Pr(A)$ is a weighted average of the conditional probabilities.

\subsection*{Learning by Example}

\subsubsection*{Example 3.1 (Part 1)}
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An insurance company divides people into accident-prone and not
accident-prone. The probability of an accident within one year is $0.4$
for accident-prone individuals and $0.2$ otherwise. Assume $30\%$ of the
population is accident-prone.

Let $A_1$ be the event that a policyholder has an accident within one
year, and $A$ the event that the policyholder is accident-prone.

\[
\Pr(A_1) = \Pr(A_1 \mid A)\Pr(A) + \Pr(A_1 \mid A^c)\Pr(A^c)
\]
\[
= (0.4)(0.3) + (0.2)(0.7) = 0.26.
\]

\subsubsection*{Example 3.1 (Part 2)}

Given that a policyholder has an accident within a year, find the
probability that the policyholder is accident-prone.

\[
\Pr(A \mid A_1) = \frac{\Pr(A \cap A_1)}{\Pr(A_1)}
= \frac{\Pr(A)\Pr(A_1 \mid A)}{\Pr(A_1)}
= \frac{(0.3)(0.4)}{0.26}
= \frac{6}{13}.
\]

\subsection*{Formal Introduction}

\subsubsection*{Law of Total Probability}

Let $B_1, B_2, \dots, B_n$ be mutually exclusive and exhaustive events.
Then
\[
\Pr(A) = \sum_{i=1}^{n} \Pr(A \cap B_i).
\]
This is the Law of Total Probability (Formula 3.4).

\subsubsection*{Bayes' Formula}

Using
```

```
\[
\Pr(A \cap B_i) = \Pr(B_i \mid A)\Pr(A),
\]
we obtain
\[
\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i)\Pr(B_i)}
{\sum_{j=1}^{n}\Pr(A \mid B_j)\Pr(B_j)}.
\]

This is Bayes' Formula (Proposition 3.1). Here, $\Pr(B_i)$ is the
\emph{a priori} probability and $\Pr(B_i \mid A)$ is the \emph{a
posteriori} probability.

\subsection*{Example 3.2}

Three cards are identical: one with two red sides (RR), one with two
black sides (BB), and one with one red and one black side (RB). A card
is chosen at random and placed face up. If the visible side is red, find
the probability that the other side is black.

Let $R$ denote the event that the visible side is red.

\[
\Pr(RB \mid R) = \frac{\Pr(R \mid RB)\Pr(RB)}
{\Pr(R \mid RR)\Pr(RR) + \Pr(R \mid RB)\Pr(RB) + \Pr(R \mid BB)\Pr(BB)}.
\]

Substituting values,
\[
\Pr(RB \mid R) =
\frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)}
= \frac{1}{3}.
\]

\section*{Random Variables}

\subsection*{Motivation and Concept}

A random variable is a real-valued function defined on the sample space
of an experiment. Its value depends on the outcome, and probabilities
are assigned to its possible values.
```

```
\subsection*{Example}

Consider tossing three fair coins. Let $Y$ be the number of heads
observed.

\[
\Pr(Y=0) = \frac{1}{8}, \quad
\Pr(Y=1) = \frac{3}{8}, \quad
\Pr(Y=2) = \frac{3}{8}, \quad
\Pr(Y=3) = \frac{1}{8}.
\]

Since $Y$ must take one of these values,
\[
\sum_{k=0}^{3} \Pr(Y=k) = 1.
\]

\section*{Probability Mass Function}

\subsection*{Concept}

A random variable that takes at most a countable number of values is
called discrete.

Let $X$ be a discrete random variable with range
\[
R_X = \{x_1, x_2, x_3, \dots\}.
\]

The function
\[
p(x_k) = \Pr(X = x_k)
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is called the Probability Mass Function (PMF) of $X$. Since $X$ must
take one of these values,
\[
\sum_k p(x_k) = 1.
\]

\subsection*{Example}

The PMF of a random variable $X$ is given by
```

```latex
\[
p(i) = c\lambda^i, \quad i = 0,1,2,\dots,
\]
where $\lambda > 0$.

Since probabilities must sum to $1$,
\[
\sum_{i=0}^{\infty} c\lambda^i = c\frac{1}{1-\lambda} = 1,
\]
which implies
\[
c = 1-\lambda.
\]

Hence,
\[
\Pr(X=0) = 1-\lambda,
\]
and
\[
\Pr(X>2) = 1 - \sum_{i=0}^{2}(1-\lambda)\lambda^i.
\]

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\section*{Bayes' Theorem}

\subsection*{Weighted Average of Conditional Probabilities}

Let $A$ and $B$ be events. We may express $A$ as
\[
A = AB \cup AB^{c},
\]
since, in order for an outcome to be in $A$, it must either be in both
$A$ and $B$ or be in $A$ but not in $B$.

The events $AB$ and $AB^{c}$ are mutually exclusive. Hence, by Axiom 3,
\begin{align*}
\Pr(A) &= \Pr(AB) + \Pr(AB^{c}) \\
&= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^{c})\Pr(B^{c}) \\
&= \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^{c})[1-\Pr(B)].
\end{align*}

Thus, the probability of event $A$ is a weighted average of the
conditional probabilities, with weights given by the probabilities of
the events on which it is conditioned.
```

```
\subsection*{Learning by Example}

\subsubsection*{Example 3.1 (Part 1)}

An insurance company believes that people can be divided into two
classes: those who are accident prone and those who are not. The
probability that an accident-prone person will have an accident within a
fixed 1-year period is $0.4$, whereas this probability is $0.2$ for a
person who is not accident prone. Assume that $30\%$ of the population
is accident prone.

Let $A_1$ denote the event that the policyholder will have an accident
within a year of purchasing the policy, and let $A$ denote the event
that the policyholder is accident prone.

\begin{align*}
\Pr(A_1) &= \Pr(A_1 \mid A)\Pr(A) + \Pr(A_1 \mid A^{c})\Pr(A^{c}) \\
&= (0.4)(0.3) + (0.2)(0.7) \\
&= 0.26.
\end{align*}

\subsubsection*{Example 3.1 (Part 2)}

Suppose that a new policyholder has an accident within a year of
purchasing a policy. Find the probability that the policyholder is
accident prone.

\begin{align*}
\Pr(A \mid A_1) &= \frac{\Pr(A \cap A_1)}{\Pr(A_1)} \\
&= \frac{\Pr(A)\Pr(A_1 \mid A)}{\Pr(A_1)} \\
&= \frac{(0.3)(0.4)}{0.26} \\
&= \frac{6}{13}.
\end{align*}

\section*{Formal Introduction}

\subsection*{Law of Total Probability}

Let $B_1, B_2, \dots, B_n$ be mutually exclusive and exhaustive events.
Then
\[
\Pr(A) = \sum_{i=1}^{n} \Pr(A \cap B_i).
```

```
\]
This is known as the Law of Total Probability (Formula 3.4).

\subsection*{Bayes' Formula}

Using
\[
\Pr(A \cap B_i) = \Pr(B_i \mid A)\Pr(A),
\]
we obtain
\[
\Pr(B_i \mid A) =
\frac{\Pr(A \mid B_i)\Pr(B_i)}
{\sum_{j=1}^{n}\Pr(A \mid B_j)\Pr(B_j)}.
\]

This is known as Bayes' Formula (Proposition 3.1), where $\Pr(B_i)$ is
the \emph{a priori} probability and $\Pr(B_i \mid A)$ is the \emph{a
posteriori} probability.

\subsection*{Example 3.2}

Three cards are identical in form:
\begin{itemize}
\item One card has both sides red (RR),
\item One card has both sides black (BB),
\item One card has one red and one black side (RB).
\end{itemize}

The cards are mixed and one card is randomly selected and placed face
up. If the upturned side is red, find the probability that the other
side is black.

Let $R$ denote the event that the upturned side is red.

\begin{align*}
\Pr(RB \mid R) &=
\frac{\Pr(R \mid RB)\Pr(RB)}
{\Pr(R \mid RR)\Pr(RR) + \Pr(R \mid RB)\Pr(RB) + \Pr(R \mid BB)\Pr(BB)}
\\
&=
\frac{(1/2)(1/3)}
```

```latex
{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} \\
&= \frac{1}{3}.
\end{align*}

\section*{Random Variables}

\subsection*{Motivation and Concept}

A random variable is a real-valued function defined on the sample space
of an experiment. Values are determined by the outcomes of an
experiment, and probabilities are assigned to possible values of the
random variable.

\subsection*{Example}

Suppose the experiment consists of tossing three fair coins. Let $Y$
denote the number of heads that appear. Then $Y$ can take values
$0,1,2,$ and $3$ with probabilities:
\begin{align*}
\Pr(Y=0) &= \Pr(t,t,t) = \frac{1}{8}, \\
\Pr(Y=1) &= \Pr(t,t,h),(t,h,t),(h,t,t) = \frac{3}{8}, \\
\Pr(Y=2) &= \Pr(t,h,h),(h,t,h),(h,h,t) = \frac{3}{8}, \\
\Pr(Y=3) &= \Pr(h,h,h) = \frac{1}{8}.
\end{align*}

Since $Y$ must take one of these values,
\[
\sum_{k=0}^{3} \Pr(Y=k) = 1.
\]

\section*{Probability Mass Function}

\subsection*{Concept}

A random variable that can take on at most a countable number of
possible values is said to be discrete.

Let $X$ be a discrete random variable with range
\[
R_X = \{x_1, x_2, x_3, \dots\},
\]
where the range may be finite or countably infinite. The function
```

```
\[
p(x_k) = \Pr(X = x_k)
\]
is called the Probability Mass Function (PMF) of $X$. Since $X$ must
take one of the values $x_k$,
\[
\sum_k p(x_k) = 1.
\]

\subsection*{Example}

The probability mass function of a random variable $X$ is given by
\[
p(i) = c\lambda^{i}, \quad i = 0,1,2,\dots,
\]
where $\lambda > 0$.

Since probabilities must sum to $1$,
\[
\sum_{i=0}^{\infty} c\lambda^{i} = c\frac{1}{1-\lambda} = 1,
\]
which implies
\[
c = 1-\lambda.
\]

Hence,
\[
\Pr(X=0) = 1-\lambda,
\]
and
\[
\Pr(X>2) = 1 - \sum_{i=0}^{2}(1-\lambda)\lambda^{i}.
\]

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Do you like this personality?