CSE400 – Fundamentals of Probability in Computing

Lecture 9: Uniform, Exponential, Laplace and Gamma Random Variables

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Outline

- Types of Continuous Random Variables
- Uniform Random Variable: Example
- Exponential Random Variable: Example
- Laplace Random Variable: Example
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- Example
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Types of Continuous Random Variables

Uniform Random Variable

The probability density function (PDF) of a uniform random variable X is:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x < b, \\ 0, & \text{elsewhere.} \end{cases}$$

The cumulative distribution function (CDF) is:

$$F_X(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \le x < b, \\ 1, & x \ge b. \end{cases}$$

Figure 3.8:

- (a) Probability density function
- (b) Cumulative distribution function of a uniform random variable

Example 1: Uniform Random Variable

Problem

The phase of a sinusoid, Θ, is uniformly distributed over $[0, 2\pi)$. Therefore, the PDF of Θ is:

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

Find:

(a) $\Pr(\Theta > \frac{3\pi}{4})$

(b) $\Pr(\theta < \pi \mid \theta > \frac{3\pi}{4})$

(c) $\Pr(\cos(\theta) < \frac{1}{2})$

Solution

Given:

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

For a uniform random variable on $[0, 2\pi)$:

$$\Pr(a < \Theta < b) = \frac{b-a}{2\pi}$$

(a)

$$\Pr\left(\Theta > \frac{3\pi}{4}\right) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{5}{8}$$

(b)

Conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr\left(\frac{3\pi}{4} < \theta < \pi\right) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8}$$

$$\Pr(B) = \Pr\left(\theta > \frac{3\pi}{4}\right) = \frac{5}{8}$$

$$\Pr\left(\theta < \pi \mid \theta > \frac{3\pi}{4}\right) = \frac{1/8}{5/8} = \frac{1}{5}$$

(c)

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3},\ \frac{5\pi}{3}$$

$$\cos\theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \theta < \frac{5\pi}{3}$$

$$\Pr(\cos\theta < \frac{1}{2}) = \frac{\frac{5\pi}{3} - \frac{\pi}{3}}{2\pi} = \frac{4\pi/3}{2\pi} = \frac{2}{3}$$

Uniform Random Variable: Application Examples

- The phase of a sinusoidal signal, when all phase angles between 0 and 2π are equally likely.
- A random number generated by a computer between 0 and 1 for simulations.
- The arrival time of a user within a known time window, assuming no time preference.

Exponential Random Variable

The exponential random variable has PDF and CDF given by (for any $b > 0$):

$$f_X(x) = \frac{1}{b}\exp\left(-\frac{x}{b}\right)u(x)$$

$$F_X(x) = \left[1 - \exp\left(-\frac{x}{b}\right)\right]u(x)$$

Graphs shown:

- (a) PDF
- (b) CDF for $b = 2$

Example 2: Exponential Random Variable

Problem

Let X be an exponential random variable with PDF:

$$f_X(x) = e^{-x}u(x)$$

Find:

(a) $\Pr(3X < 5)$

(b) Generalize your answer to find $\Pr(3X < y)$ for some arbitrary constant y

Note

No solution is provided for Example 2 in the lecture slides.