

Lecture Scribe Latex



L8_Code.ipynb

File

Create a lecture scribe which functions as an examination reference. The scribe needs to process all definitions and assumptions and derivations and proofs and worked examples from the lecture slides according to their actual display in the slides. The scribe needs to present formal definitions and notation together with complete assumptions and conditions and demonstrate all content through complete derivations and example solutions that appear within the lecture. The context provides all necessary information. The lecture content contains no new explanations or intuitions or examples or different solution methods or materials that were not included in the lecture. give all in only latex code form author name as Name: Masoom Choksi AUID: AU2440263

latex

```
\documentclass[11pt]{article}
\usepackage{amsmath,amssymb}
\usepackage{geometry}
\geometry{margin=1in}

\title{Lecture Scribe: Gaussian Simulation and Density Estimation}
\author{Name: Masoom Choksi \\ AUID: AU2440263}
\date{ }

\begin{document}
\maketitle

\section*{Course Information}
\textbf{Course:} CSE400 -- Probability and Random Variables

\section*{In-Class Activity: Gaussian Simulation and Density Estimation}
```

This lecture covers:

```
\begin{itemize}
\item Simulation of Gaussian random variables
\item Visualization of probability distributions
\item Gaussian density estimation from data
\item Application of Gaussian density estimation to real-world problems
\end{itemize}
```

\section{Why Do We Need Probability Models?}

Real-world measurements are uncertain due to noise and randomness.

Examples include:

```
\begin{itemize}
\item Sensor measurements
\item Network packet delays
\item Image sensor noise
\end{itemize}
```

Such uncertainty is modeled using **Gaussian random variables**, and their distributions are estimated from data.

\section{Gaussian Random Variable Simulation (Known Parameters)}

\subsection{Assumptions}

```
\begin{itemize}
\item The random variable is Gaussian.
\item The mean and standard deviation are known.
\item Parameters are not estimated at this stage.
\end{itemize}
```

\subsection{Definition (Gaussian Random Variable)}

A random variable X is Gaussian with mean μ and standard deviation σ if its probability density function (PDF) is

$$\begin{aligned} f_X(x) = & \frac{1}{\sqrt{2\pi}\sigma} \\ & \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right). \end{aligned}$$

\subsection{Simulation Parameters}

```
\[
N = 100000, \quad \mu_{\text{true}} = 0.0, \quad \sigma_{\text{true}} = 4.0.
]
```

```
\subsection{Simulation}
Samples are generated as
\[
x_i = \sigma_{\text{true}} Z_i + \mu_{\text{true}},
\]
where  $Z_i \sim \mathcal{N}(0, 1)$ .
```

\section{Visualizing the Distribution}

```
\subsection{Histogram Construction}
\begin{itemize}
\item Bin range:  $[-15, 25]$ 
\item Bin width:  $1.0$ 
\item Histogram normalized to form a density
\end{itemize}
```

\subsection{True PDF}

For visualization, the true Gaussian PDF is

```
\[
f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_{\text{true}}} \exp\left(-\frac{(x-\mu_{\text{true}})^2}{2\sigma_{\text{true}}^2}\right).
```

\section{Reality Check: Unknown Parameters}

```
\subsection{Assumptions}
\begin{itemize}
\item The distribution is Gaussian.
\item The parameters (mean and variance) are unknown.
\item Only data samples are observed.
\end{itemize}
```

Gaussian density estimation is used to estimate the underlying distribution from data.

\section{Gaussian Density Estimation from Data}

```
\subsection{Simulation Parameters}
\[
N_e = 200, \quad \mu_{\text{true}} = 0.0, \quad \sigma_{\text{true}} =
```

4.0.

\]

```
\subsection{Observed Data Model}
\begin{itemize}
\item True samples:  $x_i \sim \mathcal{N}(\mu_{\text{true}}, \sigma_{\text{true}}^2)$ 
\item Measurement noise:  $n_i \sim \mathcal{N}(0, \sigma_n^2)$ , where  $\sigma_n = 1.5$ 
\end{itemize}
```

Observed data:

```
\[
x_i^{\text{obs}} = x_i + n_i.
\]
```

\subsection{Parameter Estimation}

The Gaussian parameters are estimated from observed data as

```
\[
\hat{\mu} = \frac{1}{N_e} \sum_{i=1}^{N_e} x_i^{\text{obs}},
\quad
\hat{\sigma} = \sqrt{\frac{1}{N_e} \sum_{i=1}^{N_e} (x_i^{\text{obs}} - \hat{\mu})^2}.
\]
```

\subsection{Estimated PDF}

```
\[
\hat{f}_X(x) = \frac{1}{\sqrt{2\pi}\hat{\sigma}} \exp\left(-\frac{(x-\hat{\mu})^2}{2\hat{\sigma}^2}\right).
```

\section{Application: Packet Delay in Networks}

\subsection{Assumptions}

\begin{itemize}
\item Packet delay is modeled as a single Gaussian random variable.
\item Delays are non-negative (physical constraint).
\end{itemize}

\end{itemize}

\subsection{Simulation Parameters}

```
\[
N = 150, \quad \mu = 45.0 \sim \text{ms}, \quad \sigma = 8.0 \sim \text{ms}.
```

\]

\subsection{Data Generation}

Packet delays are generated as

\[

 $D_i \sim \mathcal{N}(\mu, \sigma^2), \quad D_i > 0.$

\]

\subsection{Density Estimation}

Estimated parameters:

\[

 $\hat{\mu} = \frac{1}{N} \sum_{i=1}^N D_i,$

\quad

 $\hat{\sigma} = \sqrt{\frac{1}{N} \sum_{i=1}^N (D_i - \hat{\mu})^2}.$

\]

Estimated Gaussian density:

\[

 $\hat{f}_D(d) =$ $\frac{1}{\sqrt{2\pi}\hat{\sigma}}$ $\exp\left(-\frac{(d-\hat{\mu})^2}{2\hat{\sigma}^2}\right).$

\]

\section{Key Takeaways}

\begin{itemize}

- Gaussian simulation uses randomized algorithms.
- Density estimation learns distributions from data.
- The same method applies across different applications.
- The objective is to estimate **distributions**.

\end{itemize}

\end{document}



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\section*{In-Class Activity: Gaussian Simulation and Density Estimation}

\textbf{Course:} CSE400 -- Probability and Random Variables
```

This lecture contains the following components:

```
\begin{itemize}
\item Simulation of Gaussian random variables
\item Visualization of probability distributions
\item Gaussian density estimation from observed data
\item Application to packet delay modeling
\end{itemize}
```

```
\section{Why Do We Need Probability Models?}
```

Real-world measurements are uncertain due to noise and randomness.

```
\textbf{Examples:}
\begin{itemize}
\item Sensor measurements
\item Network packet delays
\item Image sensor noise
\end{itemize}
```

Such uncertainty is modeled using Gaussian random variables, and the goal is to estimate their distributions from data.

\section{Gaussian Random Variable Simulation (Known Parameters)}

At this stage, the Gaussian distribution parameters are known and not estimated.

\subsection{Simulation Parameters}

```
\[
N = 100000, \quad \mu_{\text{true}} = 0.0, \quad \sigma_{\text{true}} = 4.0
\]
```

\subsection{Gaussian Definition}

A random variable X is Gaussian with mean μ and standard deviation σ if its probability density function (PDF) is

```
\[
f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right).
\]
```

\subsection{Sample Generation}

Samples are generated using

```
\[
X_i = \sigma_{\text{true}} Z_i + \mu_{\text{true}},
\quad Z_i \sim \mathcal{N}(0,1),
\]
for $i = 1, 2, \dots, N$.
```

\section{Visualizing the Distribution}

\subsection{Histogram Construction}

The histogram is constructed using:

```
\begin{itemize}
\item Bin range: $[-15, 25]$
\item Bin width: $1.0$
\item Normalized histogram to approximate the PDF
\end{itemize}
```

Let the bin centers be denoted by $\$c_k$$, and the normalized counts by $\$\\hat{f}(c_k)$$.

\subsection{True Gaussian PDF}

For comparison, the true PDF is evaluated as

```
\[
f_X(x) = \\frac{1}{\\sqrt{2\\pi}}\\sigma_{\\text{true}}
\\exp\\left(-\\frac{(x - \\mu_{\\text{true}})^2}{2\\sigma_{\\text{true}}^2}\\right),
\\quad x \\in [-15, 25].
\]
```

\section{Reality Check: Unknown Parameters}

In practical scenarios:

```
\begin{itemize}
\item The mean and variance are unknown
\item Only data samples are observed
\end{itemize}
```

Gaussian density estimation is used to estimate the underlying distribution.

\section{Gaussian Density Estimation from Data}

\subsection{Simulation Parameters}

```
\[
N_e = 200, \quad \\mu_{\\text{true}} = 0.0, \quad \\sigma_{\\text{true}} =
4.0
\]
```

\subsection{Observed Data Model}

True samples:

```
\[
X_i \sim \mathcal{N}(\mu_{\text{true}}, \sigma_{\text{true}}^2)
\]
```

Measurement noise:

```
\[
N_i \sim \mathcal{N}(0, \sigma_n^2), \quad \sigma_n = 1.5
\]
```

Observed data:

```
\[
X_i^{\text{obs}} = X_i + N_i
\]
```

\subsection{Histogram Settings}

```
\begin{itemize}
\item Bin range: $[-20, 20]$
\item Bin width: $1.0$ 
\item Histogram normalized to density
\end{itemize}
```

\subsection{Parameter Estimation}

The estimated mean and standard deviation are

```
\[
\hat{\mu} = \frac{1}{N_e} \sum_{i=1}^{N_e} X_i^{\text{obs}},
\]
\[
\hat{\sigma} = \sqrt{\frac{1}{N_e} \sum_{i=1}^{N_e} (X_i^{\text{obs}} - \hat{\mu})^2}.
\]
```

\subsection{Estimated Gaussian Density}

The estimated PDF is

```
\[
\hat{f}_X(x) =
```

```
\frac{1}{\sqrt{2\pi}}\hat{\sigma}\exp\left(-\frac{(x - \hat{\mu})^2}{2\hat{\sigma}^2}\right).
```

\section{Application: Packet Delay in Networks}

\subsection{Assumptions}

```
\begin{itemize}
\item Packet delay is modeled as a single Gaussian random variable
\item Packet delays are physically constrained to be positive
\end{itemize}
```

\subsection{Simulation Parameters}

```
\[
N = 150, \quad \mu = 45.0 \text{ms}, \quad \sigma = 8.0 \text{ms}
]
```

\subsection{Data Generation}

Packet delays are generated as

```
\[
D_i \sim \mathcal{N}(\mu, \sigma^2),
]
```

with the constraint

```
\[
D_i > 0.
]
```

\subsection{Histogram Construction}

```
\begin{itemize}
\item Bin range: $[0, 70]$
\item Bin width: $2.5$
\item Normalized histogram
\end{itemize}
```

\subsection{Density Estimation}

Estimated parameters:

```
\[
```

```
\hat{\mu} = \frac{1}{N} \sum_{i=1}^N D_i,  
\quad  
\hat{\sigma} = \sqrt{\frac{1}{N} \sum_{i=1}^N  
(D_i - \hat{\mu})^2}.  
]
```

Estimated Gaussian density:

```
\[  
\hat{f}_D(d) =  
\frac{1}{\sqrt{2\pi}\hat{\sigma}}  
\exp\left(-\frac{(d - \hat{\mu})^2}{2\hat{\sigma}^2}\right),  
\quad d \geq 0.  
]
```

\section{Key Takeaways}

```
\begin{itemize}  
\item Gaussian simulation uses randomized algorithms  
\item Density estimation learns distributions from observed data  
\item The same estimation method applies across applications  
\item The objective is to estimate probability distributions  
\end{itemize}  
  
\end{document}
```