CONTENT

Divisibility Rule

(2, 3, 4, 5, 6, 7, 8, 9, 10, 11)

2

All the number divisible by 2 if the unit digit of the number is

0, 2, 4, 6, 8

4567 — Unit digit

Number	Place Value	Face Value
47398	7000	7
98	90	9

2 Family - unit digit

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

∴ In 4 we need to check if last 2 digit is divisible by 4

In 8 → last 3 digits divisible by 8

In 16 → last 4 digits divisible by 16

3

Rule 1.

All the number divisible by 3 if sum of the digit completed divisible by 3.

→ 231 → 2+3+1 = 6, Divisible by 3.
→ 111, 222, 444, 777

Note 3 digit number, having same digit is divisible by 3.

Rule 2

Sum of number's digit when divided by 3 the remainder we get = the remainder when the number is divided by 3

→ 475
  ↳ 4+7+5 = 16/3 → 1 Remainder
  ↳ 475/3 = 1 Remainder

Family of 3 follows the same sum rule.

$3^2 = 9$ → Sum of digit / 9

$3^3 = 27$ → Sum of digit / 27

Q) 32x53, how many possible values of x, so that the number will be divisible by 3.

A) 1 B) 2 C) 3 D) 4

32x53 → 3+2+x+5+3 = 13+x
→ 13+2 = 15
→ 13+5 = 18
→ 13+8 = 21

3 possible values

∴ x = 3, Option C

Note.

1) If Sum of digit/3, excluding x
→ is divisible by 3, then always total possible cases is 4
→ is not divisible by 3, then always total possible cases is 3

473x1 → $15/3$ = x = 4 (0,3,6,9) ← possible values

7328x → $20/3$ = x = 3 (20+1, 20+4, 20+7)

73x9984371 → 51+x
↳ Divisible by 3 → (0, 3, 6, 9)

Divisibility by 4

All the number divisible by 4, if last two digit of the number completely divisible by 4.

$(2^2) = 4 \rightarrow$ Therefore last 2 digits divisible by 4

(Family of 4)

Divisibility by 5

All the number divisible by 5, if unit digit of the number is 0, 5

Family of 5.

$(5^1) = 5 \rightarrow$ If unit digit is divisible by 5

$(5^2) = 25 \rightarrow$ If last 2 digits, divisible by 25

$(5^3) = 125 \rightarrow$ If last 3 digits, divisible by 125

Divisibility by 6

All the number divisible by 6, if the number is completely divisible 2 & 3 both.

$\rightarrow 222 \rightarrow$ (Divisible by 2) AND (Divisible by 3) $\rightarrow$ Divisible by 6

Divisibility by 7

→ Double the last digit and subtract with the remaining digit, if that value is divisible by 7 then divisible

$441 \rightarrow 44 - (2) = 42$ ✓

$4004 \rightarrow 400 - (8) = 392$ ✓

$456456 \rightarrow 45645 - 12 = 45633$

$45633 \rightarrow 4563 - 6 = 4557$

$4557 \rightarrow 455 - 14 = 441$

$441 \rightarrow 44 - 2 = 42$ ✓

Divisibility by 8

As the family of 2, follow the same principle was 2 & 4 followed

$(2^3) = 8 \Rightarrow$ Last 3 digits divisible by 8, then the number is divisible by 8.

Divisibility by 9 (9 is called as a magic number)

Rule 1

All the number divisible by 9 if sum of the digit completed divisible by 9.

Rule 2

Sum of number's digit when divided by 9 the remainder we get = the remainder when the number is divided by 9

Rule 3

Decimal remaining after division with 9, repeats infinitely

$41/9 \rightarrow 4.5555... \rightarrow 4.\overline{5}$ (— ← Bar Bracket)

Rule 4.

Any number divisible by 9, will still be divisible if the position of the digits are interchanged.

Q A number XYZ is divisible by 9
YXZ is divisible by 9? Ans Yes

Divisibility by 11

If difference of sum of odd & even positions of a number is 0, is a multiple of 11.

→ 73453
↳ Sum of Odd position - $7+4+3 = 14$
Even position - $3+5 = 8$
$6 \neq 0, 11, 22..$

→ 479314
↳ Odd - $4+9+1 \rightarrow 14$
Even - $7+3+4 \rightarrow 14$
0, Divisible by 11.

No of Zero

Q How many zero will be at the end of this calculation
$25 \times 24 \times 120 \times 30 \times 70 \times 480 \times 55$

To find out how many 0 will form, two things to be noted.

① → 0's present at the end
② → No of pairs of 2×5

∴ From Rule 1.
120, 30, 70, 480 → 4 (0s)

∴ From Rule 2.
$25 \times 24 \times 12 \times 3 \times 7 \times 48 \times 55$
↳ $(5 \times 5) \times (3 \times 2 \times 2 \times 2) \times (3 \times 2 \times 2) \times 3 \times 7 \times (3 \times 2 \times 2 \times 2 \times 2) \times (11 \times 5)$

Combining all 5 and 2, discarding others
$(5 \times 5 \times 5), (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$

∴ Total 3 (0's)

Rule ① + ② → $4+3 = 7$ (0's)

Q) $40 \times 50 \times 44 \times 75 \times 850$

Rule 1 $\rightarrow$ 40, 50, 850 $\rightarrow$ 3(0's)

Rule 2 $\rightarrow$ $4 \times 5 \times 44 \times 75 \times 85$

$\rightarrow (2\times2) \times 5 \times (2\times2\times11) \times (3\times5\times5) \times (5\times17)$

$\rightarrow (5\times5\times5\times5) \times (2\times2\times2\times2)$

4(0's)

$\therefore$ $3+4 = 7$(0's)

Q) $10\times20\times30\times40\times50\times60\times70\times80\times90\times100$

Rule 1 $\rightarrow$ 11(0's)

Rule 2 $\rightarrow$ Considering only multiples of 2 & 5

$\rightarrow 2\times(2\times2)\times5\times(2\times3)\times(2\times2\times2)$

$\rightarrow (2\times2\times2\times2\times2\times2\times2) \times (5)$

1(0's)

$11+1 = 12$(0's)

Q) $20\times40\times60\times80\times100$

Rule 1 $\rightarrow$ 6(0s)

Rule 2 $\rightarrow$ $2\times(2\times2)\times(2\times3)\times(2\times2\times2)$,

$\rightarrow$ 0(0's)

$\rightarrow$ 6(0s)

Finding zero from factorial of a number

40!

Find out the occurance of numbers divisible by 5

Because occurance of 5 is less than occurance of 2.

$40! = (40\times35\times30\times25\times20\times15\times10\times5)\times(\ldots\ldots)$

$(5\times5\times5\times5\times5\times5\times5\times5\times5) = 9$(5's)

$\therefore$ 9(0's)

Another formula.

$136! =$

5	136
5	27
5	5
	1

Add them $\rightarrow$ 33(0's)

Actual occurance of 5

$40! =$

5	40
5	8
5	1

$\rightarrow$ 9(0's)

Q) $100!$

$$\begin{array}{r|l} 5 & 100 \\ \hline 5 & 20 \\ \hline 5 & 4 \end{array} \Big\} \rightarrow 24\,(0's)$$

Q) $500!$

$$\begin{array}{r|l} 5 & 500 \\ \hline 5 & 100 \\ \hline 5 & 20 \\ \hline & 4 \end{array} \Big\} \rightarrow 124\,(0's)$$

Q) $248!$

$$\begin{array}{r|l} 5 & 248 \\ \hline 5 & 49 \\ \hline 5 & 9 \\ \hline & 1 \end{array} \Big\} \rightarrow 59\,(0's)$$

Q) $\dfrac{33!}{3^x}$, Find the maximum power of 3 which will completely divide 33!

Follow the same approach

$$\begin{array}{r|l} 3 & 33 \\ \hline 3 & 11 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \Big\} \rightarrow 15 \Rightarrow \frac{33!}{3^{15}}$$

Q) $\dfrac{60!}{4^x} =$

$$\begin{array}{r|l} 4 & 60 \\ \hline 4 & 15 \\ \hline & 3 \end{array} \Big\} \rightarrow 18$$

$\therefore \dfrac{60!}{4^{18}}$

Q) $15 \times 14 \times 13 \times 12 \times \dots = 3^m \times x \times xyz$

Find maximum power of m to divide 15!

$\dfrac{15!}{3^m}$

$$\begin{array}{r|l} 3 & 15 \\ \hline 3 & 5 \\ \hline & 1 \end{array} \Big\} \rightarrow 6 \Rightarrow \frac{15!}{3^6}$$

Q) $\dfrac{80!}{6^x} =$

$$\begin{array}{r|l} 6 & 80 \\ \hline 6 & 13 \\ \hline & 2 \end{array} \Big\} \rightarrow 15$$

$\therefore \dfrac{80!}{6^{15}}$

Power of Numbers

1 Million - 0.1 Cr - 10 Lakh = 10^6

1 Billion - 100 cr — 1000 M = 10^9 = 1 अरब

1 Trillion - 100000 cr - 1000 B = 1000000 M = 10^{12}

1000 अरब

Unit Digit Concept

4371<u>9</u>

129<u>7</u>

Unit Digit

While solving unit digit problem, only unit digit is important

Q 47x27x32x71x83x744x348498 =

Consider only unit digit of each number

7x7x2x1x3x4x8

49 x 2 x 12 x 8

18 x 16

48

8 is the unit digit of the product

Q) 2x22x222x2222x22222x222222 =

2x2x2x2x2x2

4 x 4 x 4

16 x 4

24 = 4

Q) 4x73x148x3484x79244x379899

4x3x8x4x4x9

12 x 32 x 36

4 x 6 = 24

4

Q) 1x2x3x4x5.... x99

1x2x3x4x5 = 20, if unit digit is 0, then further results will show unit digit 0 as well.

Q) 1x3x5x7x9x11x .... 99 =

1x3x5 = 15, if 5 is in place of the unit digit, further multiplying with odd number will result unit digit as 5.

Q) $2 \times 4 \times 6 \times 8 \times 12 \times 14 \times 16 \times \ldots 98 =$ [0 not used]

$\underbrace{2 \times 4 \times 6 \times 8}_{0's} \times \underbrace{2 \times 4 \times 6 \times 8}_{10's} \times \underbrace{\quad}_{20's} \ldots \underbrace{\quad}_{90's}$

$64 \times 64 \times 64 \ldots$

$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$16 \quad 16 \quad 16 \quad 16 \quad 16$

$36 \times 36 \times 6$

$36 \times 6 = 36 \Rightarrow 6$

2, 3, 7, 8 $\longrightarrow$ Unit Digit repeats after every 4th power

4 & 9 $\longrightarrow$ Unit Digit repeats after every 2nd power

0, 1, 5, 6 $\longrightarrow$ fix unit digit

Q) $2^{7346} =$

As 2's unit digit repeat after every 4th power so divide 7346 by 4.

Apply Divisibility rule of 4 = $4 \overline{)46}$ (quotient 11, remainder 2) $\rightarrow 2^2 = \underline{\underline{4}}$

Q) $3^{70259} =$

For 3, unit digit repeats after every 4th power

Divisibility rule of 4 to divide 70259

$4 \overline{)59}$ (quotient 14, remainder 3) $\rightarrow 3^3 = 27 = 7$

Q) $(7937)^{7937} =$

Consider unit digit only $(7937)^{7937} = (7)^{7937}$

7's unit digit repeats after every 4th power

Divisibility rule of 4 upon 7937

$4 \overline{)37}$ (quotient 9, remainder 1) $\rightarrow 7^1 = \underline{\underline{7}}$

Q) $(127346)^{237943} =$

6's unit digit never changes on any power

$\therefore \underline{\underline{6}}$

Q) $(213)^{482} \times (1384)^{2317} \times (17987)^{31482} =$

$\rightarrow 213 \rightarrow$ repeats after every 4th power

$482/4 \Rightarrow 4\overline{)82}$, 80, remainder $2 \rightarrow 3^2 = \underline{9}$

$\rightarrow 1384 \rightarrow$ repeats after every 2nd power

$2\overline{)2317}$ = 1158, 2316, remainder $1 \rightarrow 4^1 = \underline{4}$

$\rightarrow 17987 \rightarrow$ repeats after every 4th power

$31482/4$: $4\overline{)82}$ = 20, 80, remainder $2 \rightarrow 7^2 = 4\underline{9}$

$\therefore 9 \times 4 \times 9$

$36 \times 9 \rightarrow 54 \rightarrow \underline{4}$

Q) $4^{61} + 4^{62} + 4^{63} + 4^{64}$

4's unit digit repeat after every 2nd power

$4^{61/2} + 4^{62/2} + 4^{63/2} + 4^{64/2}$

$4^1 + 4^0 + 4^1 + 4^0 = 4+1+4+1 = 1\underline{0} \rightarrow 0$

Q) $(2791)^{2791} + (3795)^{3795} + (68496)^{7137987} =$

$1 + 5 + 6 = 1\underline{2} \rightarrow 2$

Q) $(61)^{37} - (32)^{41} =$

1's unit place always 1

2's unit place repeat every 4 time

$\therefore$ $4\overline{)41}$ = 10, 40, remainder $1 \rightarrow 1 \Rightarrow 2^1$

$\therefore 1 - 2^1 =$

Consider $11 - 2 = 9$

Prime & Composite Number

Prime Number

Such numbers which have only 2 factors

These 2 factors are 1 and the number itself

* 2 is the only even prime number
* 25 prime numbers between 1 to 100
  2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53
  59, 61, 67, 71, 73, 79, 83, 89, 97
* 3, 5, 7 - only prime number with a difference of two
* Any prime number other than 2, 3 when divide by 6, the remainder will be either 1 or 5

But its vice versa not true

$\frac{x}{6}$ = 1 or 5 as remainder, where x is a prime number

But if remainder 1 or 5 is coming then it is not always that number is a prime number 25/6 = 1 remainder
Not prime

Identification of prime number

Steps to find

1) If there is a number(n), which is to be identified prime or not. Find the nearest larger square value to it.

2) Then write down the prime number less than that number and divide it with the number(n)

101 → 121, nearest square number
↓
11

11 → 2, 3, 5, 7 (less than 11 prime numbers)

2, 3, 5, 7 no one can divide 101

Hence 101 is a prime number

197 → $(15)^2 = 225$

Prime Number less than 15 = 2, 3, 5, 7, 11, 13

No it is not divisible by any, hence it is a prime number

259 → $(17)^2 = 289$ → 2, 3, 5, 7, 11, 13, 17

Divisible by 7

Not prime

Composite Number

Such number which have atleast 3 factors
1, itself & other....

Determination of number of factors

240.

Step 1: Find out the Canonical form of the number

Step 2: Add 1 to each power of the prime number

Step 3: Multiply each power

Canonical form

2	240
2	120
2	60
2	30
3	15
5	5
	1

$= 2^4 \times 3^1 \times 5^1$

$= (4+1) \times (1+1) \times (1+1)$

$= 5 \times 2 \times 2 = 20$

$\therefore$ 240 has total 20 factors

Formula for no. of factors

$N = a^p \times b^q \times c^r$

$\therefore (p+1) \times (q+1) \times (r+1)$

Q 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$= 2^3 \times 5^3$

$= (3+1) \times (3+1)$

$= 16$

Q 360

2	360
2	180
2	90
3	45
3	15
5	5
	1

$= 2^3 \times 3^2 \times 5$

$= (3+1) \times (2+1) \times (1+1)$

$= 4 \times 3 \times 2 = 24$

Q 1600

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$= 2^6 \times 5^2$

$= (6+1) \times (2+1) = 7 \times 3$

$= 21$

Even factor

$N = 2^p \times x^q \times y^r \Rightarrow p \times (q+1) \times (r+1)$

Odd factor

$N = 2^p \times x^q \times y^r \Rightarrow (q+1) \times (r+1)$

Also

$$\boxed{\begin{array}{l}\text{Total factor} - \text{Even} = \text{Odd factors} \\ \text{Total factor} - \text{Odd} = \text{Even factors}\end{array}}$$

Q 360

Total $= 24$, Even $= 3 \times 3 \times 2 = 18$

Odd $= 24 - 18 = 6$

Sum of factors

$N = a^p \times b^q \times c^r$

Steps.

1) Find out the canonical form
2) Add 1 to the power & deduct 1 from the number and divide by the product of each (number − 1)

$$= \frac{(a^{(p+1)}-1)\times(b^{(q+1)}-1)\times(c^{(r+1)}-1)}{(a-1)\times(b-1)\times(c-1)}$$

Q 360

2	360
2	180
2	90
3	45
3	15
5	5
	1

$$= 2^3 \times 3^2 \times 5$$

$$= \frac{(2^{(3+1)}-1)\times(3^{(2+1)}-1)\times(5^{(1+1)}-1)}{(2-1)\times(3-1)\times(5-1)}$$

$$= \frac{(16-1)\times(27-1)\times(25-1)}{1\times2\times4}$$

$$= \frac{15\times26\times\cancel{24}^{3}}{\cancel{2\times4}} = 1170$$

Q 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$= 2^3 \times 5^3$$

$$= \frac{(2^{(3+1)}-1)\times(5^{(3+1)}-1)}{(2-1)\times(5-1)}$$

$$= \frac{(2^4-1)\times(5^4-1)}{1\times4}$$

$$= \frac{15\times\cancel{624}^{156}}{\cancel{4}} = 2340$$

Q 1500

2	1500
2	750
3	375
5	125
5	25
5	5
	1

$$= 2^2\times3^1\times5^3 = \frac{(2^3-1)\times(3^2-1)\times(5^4-1)}{(2-1)\times(3-1)\times(5-1)}$$

$$= \frac{7\times\cancel{8}\times624}{\cancel{2\times4}} = 4368$$

Q 6000

2	6000
2	3000
2	1500
2	750
3	375
5	125
5	25
5	5
	1

$$= 2^4\times3^1\times5^3$$

$$= \frac{(2^{(4+1)}-1)\times(3^{(1+1)}-1)\times(5^{(3+1)}-1)}{(2-1)\times(3-1)\times(5-1)}$$

$$= \frac{31\times\cancel{8}\times624}{\cancel{2\times4}} = 19344$$

Multiple of factors

Steps

1) Find out the Number of factors
2) Then divide it by 2 to the power of the number

$$= (N)^{n/2}$$

N = number
n = Number of factors

Q Determine the product of all factors of 600.

2	600
2	300
2	150
3	75
5	25
5	5
	1

$= 2^3 \times 3 \times 5^2$

No. of factor $= (3+1) \times (1+1) \times (2+1)$

$= 4 \times 2 \times 3$

$= 24$

$\therefore$ Multiple of factor $= (600)^{(24/2)}$

$= 600^{12}$

Q $\frac{7x - 96}{x}$, how many possible values of x

-UPSC 2023

In this question we need to find no of factors of 96

Because any value of x will divide $7x$

$\therefore 96 = 2^5 \times 3 \Rightarrow (5+1) \times (1+1) = 6 \times 2 = 12$

Q A number 720, find out its

1. Total factor
2. Odd factor
3. Even factor
4. Sum of factor
5. Product of factor

2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

$= 2^4 \times 3^2 \times 5$

① Total factor $= (4+1) \times (2+1) \times (1+1)$

$= 5 \times 3 \times 2 = 30$

② Even factor: $4 \times (2+1) \times (1+1)$

$= 4 \times 3 \times 2 = 24$

③ Odd factor = Total factor - Even factor

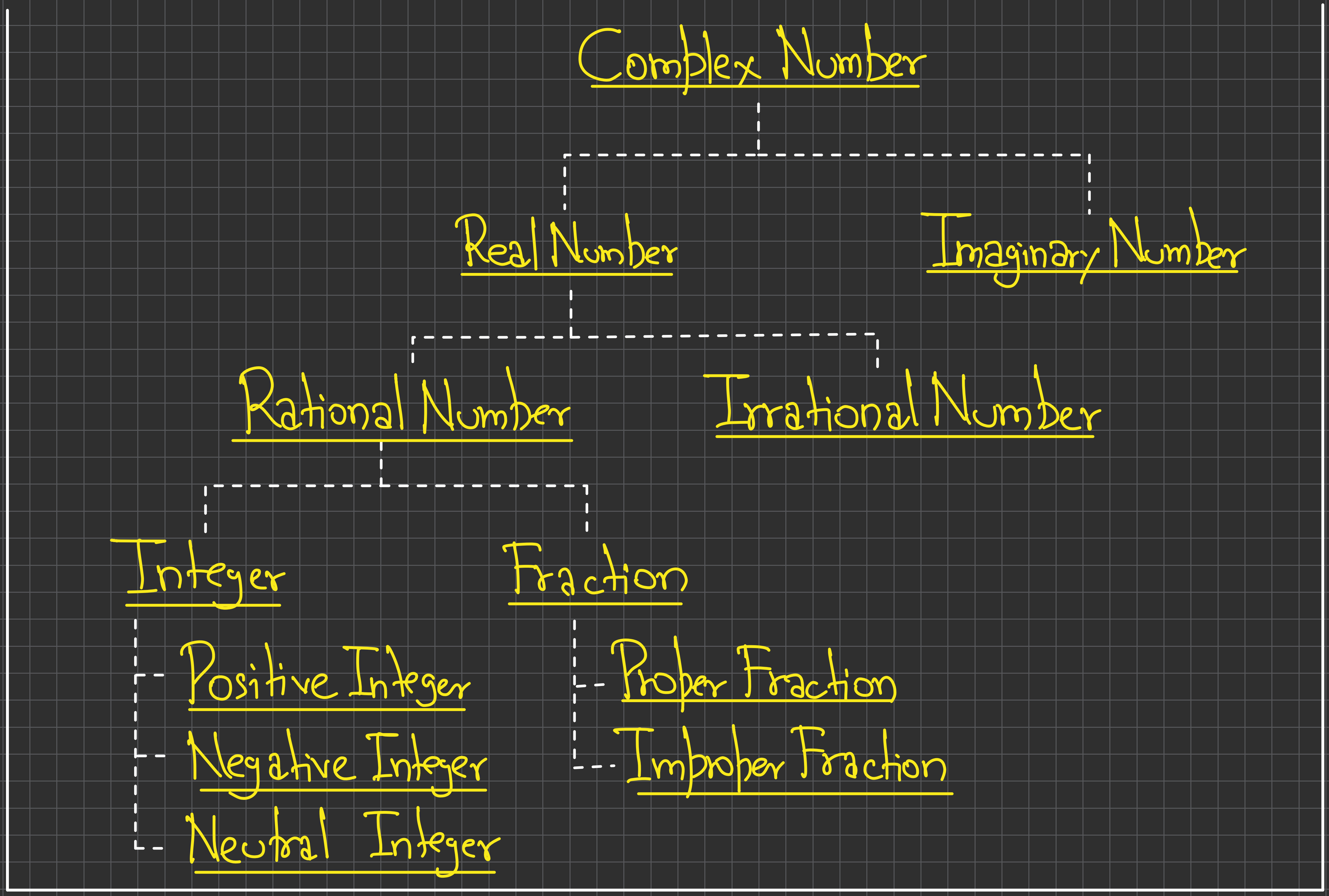
$= 30 - 24 = 6$

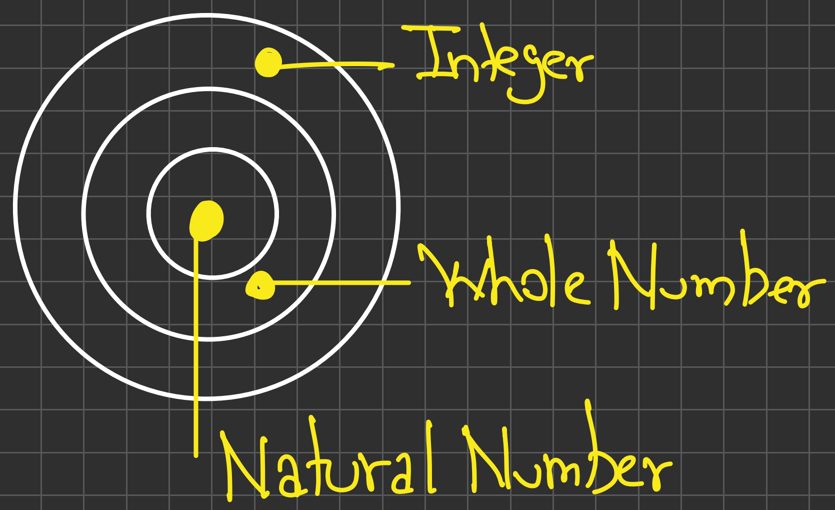
④ Sum of factor:

$$\frac{(2^{(4+1)}-1) \times (3^{(2+1)}-1) \times (5^{(1+1)}-1)}{(2-1)(3-1)(5-1)}$$

$$= \frac{(2^5-1) \times (3^3-1) \times (5^2-1)}{2 \times 4} = \frac{31 \times 26 \times 24^{3}}{\cancel{2} \times \cancel{4}} = 2418$$

⑤ Product of factor $= (720)^{30/2} = (720)^{15}$


Complex Number
Real Number
Imaginary Number
Rational Number
Irrational Number
Integer
Fraction
Positive Integer
Negative Integer
Neutral Integer
Proper Fraction
Improper Fraction


Integer
Whole Number
Natural Number

Classification of Number

* All the numbers are called Complex number
* Complex number is divided into
  - → Real number
  - → Imaginary number

Imaginary Number

* It is represented by (i)
* Such numbers which can't be represented on number line
* Such numbers which underroot have negative sign.
* Example: $\sqrt{-1}, \sqrt{-2}, \sqrt{-3}$

Real Number

* All the number other than Imaginary Number are known as Real Number
* Such number which can represent on numberline is called Real Number
* Example: $2, 3, -5, 0, \sqrt{24}, 3/5,$
* Real Number is classified into two categories
  - → Rational Numbers
  - → Irrational Numbers

Irrational Number

* Such numbers which cannot be represented in the form of p/q.
* Such number which is non-repetative
* Example: 0.43799131783347983l.....
  $\pi = 22/7 = 3.142433791841.....$
  $\sqrt{2}, \sqrt{3}, \sqrt{5}...$

Rational Number

* All the number which can represent in the form p/q.
* Example. $4, 1/3, \sqrt{4}, -20/7$
* Rational number is divided into two types
  - → Integer
  - → Fraction

Fraction

* If in the p/q, and $q \neq 1$ & other than 0, then it is a fraction
* Example- $4/3$, $11/100$
* Fractions are of two type
  - → Proper Fraction
  - → Improper Fraction

Improper Fraction

* If in the p/q, $p > q$, it is called improper fraction
* Its value is always > 1.
* Example: $7/3$, $40/11$, 2.71, 1.11

Proper Fraction

* If in the p/q, $p < q$, it is called proper fraction
* Its value is always < 1
* Example: $5/7$, $11/40$, 0.4, 0.81

Integer

* All the number of p/q, if the value of q is always 1.
* Integers are of 3 types
  - → Positive Integers (+1, +2, ...)
  - → Negative Integers (-1, -2, ...)
  - → Neutral Integers (0)

Whole number- 0, 1, 2, 3....

Natural number- 1, 2, 3, 4....

* 0 is not a natural number, as we cannot represent it in nature

Conversion from decimal to fraction (p/q)

- $0.1 \Rightarrow \frac{1}{10}$ • $0.002 \Rightarrow \frac{2}{1000} = \frac{1}{500}$ • $0.0048 \Rightarrow \frac{48}{10000} = \frac{3}{625}$

- $0.2222.... \Rightarrow \frac{2}{9}$

When a number repeats infinite time, divide the number with that number of 9.

- $0.484848... \Rightarrow \frac{48}{99} = \frac{16}{33}$

- $0.473147314731.... \Rightarrow \frac{4731}{9999}$

Only 4 repeating

- $6\overline{74} \Rightarrow \frac{674-74}{900} = \frac{607}{900}$

non repeating

900 — No of not repeating numbers

No of repeating number

- $0.4\overline{73} \Rightarrow \frac{473-4}{990} = \frac{469}{990}$

- $0.8\overline{4} \Rightarrow \frac{84-8}{90} = \frac{76}{90} = \frac{38}{45}$

- $0.73\overline{45} \Rightarrow \frac{7345-73}{9900} = \frac{7272}{9900} = \frac{808}{1100} = \frac{202}{225}$

- $2.4\overline{8} \Rightarrow \frac{48-4}{90} = \frac{44}{90}\left(\frac{22}{45}\right) + 2 = \frac{112}{45}$

Q Which is a irrational number

A) 2.731459864

B) 3.1431431431....

C) 8.479812793....

D) 3/11

Option C, as it cannot be represented in p/q form

Roman Numbers

1	I
5	V
10	X
50	L
100	C
500	D
1000	M

Rules

* The Roman Number are written only in decreasing order - MDC,

Only one symbol is allowed to be written smaller than the preceding one

IX, it means $10-1=9$

CD, it means $500-100=400$

Ex. 2024 - $\underbrace{MM}_{2}\underbrace{XX}_{2}\underbrace{IV}_{4} \rightarrow$ MMXXIV

1900 - $\underbrace{M}_{1000}\underbrace{C}_{100}\underbrace{M}_{1000} \rightarrow 1000 + (1000-100) = \underline{1900}$

1995 - MVM | 2145 - MMCVL

Remainder Theorm

$$\begin{array}{r|l} & \ \ 3 \\ \hline 11 & \ \ 40 \\ & \ \ 33 \\ \hline & \ \ 07 \end{array}$$

3 — Quotient

40 — Dividend

11 — Divisor

07 — Remainder

Concept 1 - In the Remainder theorm question, Simplification is not allowed

Ex. $\dfrac{400}{150} = \dfrac{40\cancel{0}}{15\cancel{0}} = 8/3$ (NOT ALLOWED)

Directly divide $150 \overline{)400}$ quotient 2, 300, 100 ← 100 is the remainder

Even if you simplified it, multiply those values to get the original remainder

$\dfrac{40\cancel{0}}{15\cancel{0}} = \dfrac{\cancel{40}^{8}}{\cancel{15}3} = \dfrac{8}{3} \Rightarrow 2$ as remainder

10   5

∴ Now $2 \times 10 \times 5 \Rightarrow$ **100** it will also give the the original remainder

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Q) $\dfrac{91 \times 92 \times 93 \times 94 \times 95 \times 96}{100}$

$\dfrac{91 \times 92 \times 93 \times 94 \times \cancel{95}^{19} \times \cancel{96}^{24}}{\cancel{100}\ \cancel{20}\ 5} = \dfrac{91 \times 92 \times 93 \times 94 \times 19 \times 24}{5}$

4   5

Separate remainder

$91/5 = 1$

$92/5 = 2$

$93/5 = 3$

$94/5 = 4$

$19/5 = 4$

$24/5 = 4$

$\dfrac{24 \times 16}{5}$ Again individual remainder

$24/5$ — 4

$16/5$ — 1

$= 4 \times 1 = 4$

∴ $4 \times 5 \times 4 = $ **80** - Remainder

Q) $1000/300$:

$\dfrac{100\cancel{0}}{30\cancel{0}} = 10/3 = 1$ - remainder

10, 10

$\Rightarrow 1 \times 10 \times 10 = $ **100** - Remainder

Concept 2 – If the numerator is less than denominator then remainder is always equal to numerator

$\frac{4}{9} =$ 4 is the remainder

$\frac{500}{1000} =$ 500 is the remainder

Concept 3 – Remainder can never be a negative value

$\frac{-40}{7} =$ Ans = $+2$

As -5 cannot be the remainder

$\therefore 7-5 = \underline{\underline{2}}$

$\frac{-70}{11} = -4 \Rightarrow 11-4 = \underline{\underline{+7}}$

$\frac{-100}{11} = -1 \Rightarrow 11-1 = \underline{\underline{+10}}$

Concept 4 – If $\frac{a \times b \times c}{N}$ then remainder of

$a = r_a$

$b = r_b$

$c = r_c$

$\therefore \frac{r_a \times r_b \times r_c}{N}$

Q) $\frac{73 \times 74 \times 75 \times 76}{9} =$

$= \frac{1 \times 2 \times 3 \times 4}{9} = \frac{24}{9} = \underline{\underline{6}}$ ← Remainder

Q) $\frac{91 \times 92 \times 93 \times 94 \times 95}{11}$

$= \frac{3 \times 4 \times 5 \times 6 \times 7}{11} = \frac{60 \times 42}{11}$

$\frac{5 \times 9}{11} = \frac{45}{11} = 1$ Ans ← Remainder

Q) $\frac{100 \times 200 \times 300 \times 400}{11}$

$= \frac{1 \times 2 \times 3 \times 4}{11} = \frac{24}{11} = 2$ – Ans ← Remainder

Q) $\frac{90 \times 91 \times 92 \times 93 \times 94}{100}$

We have to simplify

$\frac{9\cancel{0} \times 91 \times \cancel{92} \times 93 \times 94}{10\cancel{0}} \Rightarrow \frac{9 \times 1 \times 2 \times 3 \times 4}{10} = \frac{9 \times 24}{10} =$

$= \frac{9 \times 4}{10} = \frac{36}{10} = \underline{\underline{6}} \rightarrow$ Multiply with $10 \Rightarrow \underline{60 \text{ – Remainder}}$

Q) $\frac{24\times55\times66\times78\times94\times95}{100}$

Using Concept 1

$$\frac{6\,\cancel{24}\times\overset{11}{\cancel{55}}\times66\times78\times94\times\overset{19}{\cancel{95}}}{\cancel{100}\,\cancel{25}\,\cancel{5}}$$

4

5

5

The base gets completely divisible ∴ remainder will be 0

Concept 5 - If $\frac{(a+1)^n}{a}$ Denominator = a
Numerator = a+1
Any power of (a+1)

the remainder will be always 1

$\frac{(4)^{7000}}{3}$ = Remainder 1

$\frac{(25)^{798}}{24}$ = 1 - Remainder

Q) $\frac{3^{240}}{8}$

Try to make it $\frac{9}{8}$

$\frac{(3^2)^{120}}{8} = \frac{(9)^{120}}{8} = 1$ - Remainder

Q) $\frac{3^{33}}{8} = \frac{3\times(3)^{32}}{8} = \frac{3\times(3^2)^{16}}{8}$

$= 3\times1 = 3$ - Remainder

Q) $\frac{4^{41}}{15}$

$= \frac{4\times(4^2)^{20}}{15} = \frac{4\times(16)^{20}}{15} = 4\times1 = 4$

Remainder

Q) $\frac{2^{27}}{7} =$

$\frac{2\times(2^3)^{23}}{7} = \frac{2\times(8)^{23}}{7} = 2\times1 = 2$ - Remainder