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Analysis

- The hitting rate of info particles can be given as:

$$f_{hit}^{3D}(t) = \frac{r(d-r)}{d\sqrt{4\pi D t^3}} e^{-\frac{(d-r)^2}{4Dt}}$$

where d = distance betn R_x & T_x
r = Radius of receiver
D = Diffusion constant

- Probability to absorb 1 particle after t seconds can be given as

$$P_{hit}(t) = \int_0^t f_{hit}(t)\,dt$$

$$P_{hit}(t) = \int_0^t \frac{r(d-r)}{d\sqrt{4\pi D t^3}} e^{-\frac{(d-r)^2}{4Dt}}$$

$$= \frac{r}{d} \operatorname{erfc}\left(\frac{d-r}{\sqrt{4Dt}}\right)$$

where $\operatorname{erf}(x) = \int_0^x \frac{2}{\sqrt{\pi}} e^{-y^2} dy$

- For the $(i-1)^{th}$ time slot

$$\therefore P_{i-1} = \int_{(i-1)T}^{iT} f_{hit}(t)\, dt$$

- $$P_{i-1} = \frac{r}{d}\left\{ \operatorname{erfc}\left(\frac{d-r}{\sqrt{4DiT}}\right) - \operatorname{erfc}\left(\frac{d-r}{\sqrt{4D(i-1)T}}\right)\right\}$$

- $\boxed{C_j = N_{Tx} P_j}$ Avg no. of particles at j^{th} time slot if N_{Tx} particles are released

- No. of received particles (r_i) follows Poissons distribution.

- $I_i = \lambda_0 T + \sum_{j=1}^{\infty} s_{i-j} C_j$ (sum of ISI & background)

λ_0 - background noise power per unit time

- Probability of receiving r_i particles

$$P(r_i \mid I_i + s_i C_0) = \frac{e^{-(I_i + s_i C_0)} (I_i + s_i C_0)^{r_i}}{r_i!}$$

- SNR can be defined as

$$SNR = 10 \log_{10} \frac{C_0}{2\lambda_0 T}$$

- $N_{Tx} = \frac{2\lambda_0 T 10^{SNR/10}}{P_0}$ (No of particles released with given SNR)

- Optimal zero - bit memory receiver

$$\hat{s}_i = \begin{cases} 0, & r_i \le \tau \\ 1, & r_i > \tau \end{cases}$$

estimated symbol

r_i - No. of received particles
τ - Threshold

To find τ

$$P(r_i = \tau \mid s_i = 0) = P(r_i = \tau \mid s_i = 1)$$

Probability of r_i given s_i

$$P(r_i \mid s_i) = \frac{e^{-\lambda/s_i} (\lambda/s_i)^{r_i}}{r_i!}$$

$$\lambda_{s_i} = \lambda_0 T + C_0 s_i + \frac{\sum_{j=1} c_j}{2}$$

Finding T $\quad P(r_i = T \mid s_i = 0) = P(r_i = T \mid s_i = 1)$

$$\therefore \frac{e^{-\lambda|_{s_i=0}} \left(\lambda/_{s_i=0}\right)^{r_i}}{r_i!} = \frac{e^{-\lambda/_{s_i=1}} \left(\lambda/_{s_i=1}\right)^{r_i}}{r_i!}$$

$$\therefore e^{-\left(C_0 + \frac{\sum c_j}{2} + \lambda_0 T\right)} \cdot \left(\frac{\sum c_j}{2} + \lambda_0 T\right)^{r_i}$$

$$= e^{-\left(C_0 + \frac{\sum c_j}{2} + \lambda_0 T\right)} \cdot \left(C_0 + \frac{\sum c_j}{2} + \lambda_0 T\right)^{r_i}$$

$$\therefore \frac{e^{-\left(\frac{\sum c_j}{2} + \lambda_0 T\right)}}{e^{-\left(\frac{\sum c_j}{2} + \lambda_0 T + C_0\right)}} = \left[\frac{\left(C_0 + \frac{\sum c_j}{2} + \lambda_0 T\right)}{\left(\frac{\sum c_j}{2} + \lambda_0 T\right)}\right]^{r_i}$$

$\therefore$ here $r_i = T$

$$e^{C_0} = \left[\frac{\left(C_0 + \frac{\sum c_j}{2} + \lambda_0 T\right)}{\left(\frac{\sum c_j}{2} + \lambda_0 T\right)}\right]^{T}$$

$$\therefore C_0 = T \ln\left(\frac{C_0}{\frac{\sum c_j}{2} + \lambda_0 T} + 1\right)$$

$$\therefore \boxed{T = \frac{C_0}{\ln\left(1+\left(\frac{C_0}{\sum_2 C_j + \lambda_0 T}\right)\right)}}$$ sub optimal Threshold

- Optimal Threshold

$$(T^*, P_e^*) = \arg\min_{T} P_e(T)$$

where $P_e(T) = \frac{1}{2^L} \sum_{S_{i-1}} P_e(S_{i-1}, T)$

Total possibilities for length L

summing over all permutation

$$P_e(S_{i-1}, T) = \frac{1}{2}\Big[Q\big(\lambda_0 T + \sum_{j=1}^{L} S_{i-j} C_j, \lceil T \rceil\big) + 1 - Q\big(\lambda_0 T + \sum_{j=1}^{L} S_{i-j} C_j + C_0, \lceil T \rceil\big)\Big]$$

- For one bit receiver

$$\bar{s}_i = \begin{cases} 0, & r_i \leq T \mid s_{i-1} \\ 1, & r_i > T \mid s_{i-1} \end{cases}$$

- For optimal

$$T^* \mid_{s_{i-1}} = \arg\min_{T} P_e(T, s_{i-1})$$

where $P_e(T, s_{i-1})$

$$= \frac{1}{2^{L-1}} \sum_{s_{i-2}, \dots, s_{i-L}} P_e(s_{i-1}, T)$$

$$= \frac{m+n}{2}$$

where

$$m = \frac{1}{2^L} \sum \sum$$