

Heap (Algorithms)

Procedure used in heapsort:-

→ Max-Heapify →  $O(\log n)$  → maintain max-heap property

→ Build-max-Heap → produce max heap from unordered list:

→ Heapsort - ] -  $O(n \log n)$

10 MAX-HEAPIFY(A, i)

$l = \text{Left}(i) \rightarrow 2i + 1$  } Index in array

$r = \text{Right}(i) \rightarrow 2i + 2$

11 if  $l < A.\text{heap-size}$  and  $A[l] > A[i]$

largest = l

01 else largest = i

12 if  $r < A.\text{heap-size}$  and  $A[r] > A[\text{largest}]$

largest = r

02 if largest ≠ i

03 exchange  $A[i]$  with  $A[\text{largest}]$  } Index

MAX-HEAPIFY(A, largest)

$$T(n) \leq T(2n/3) + O(1)$$

- Time complexity  
 $O(\log n)$

↓  
See the  
tree structure.

04

11 BUILD-MAX-HEAP(A) → A[]

(Important)

05  $A.\text{heap-size} = A.\text{length}$  (length of array)

for  $i$  from  $\lfloor A.\text{length}/2 \rfloor$  to 1

06 MAX-HEAPIFY(A, i)

Important

12 HEAPSORT(A)

## BUILD-MAX-HEAP(A)

08 for  $i = A.\text{length}$  down to 2

09 Exchange  $A[1]$  with  $A[i]$

A.heap-size = A.heap-size - 1

max-heapify(A, 1)

end for anyone, anywhere, can make a positive difference. -Mark Sanborn

MARCH '22

S	M	T	W	T	F	S
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Wk 09  
25February  
Friday

priority queue implementation in Heapdata structure.

↓  
[The highest priority is always  
at the root of the heap]

let's see for maxheap

9 ↗ Element highest priority → Dequeue

5  
4  
3  
2  
enqueue → 1

priority queue operations:-

↳ Inserting into priority Queue :-

MaxHeapInsert ( $A$ , element, priority):  
     $e$        $p$ 

A.append(element, priority)

 $i = A.length - 1$  (position of element)Now length =  $A.length$ .BUILD-MAX-HEAP ( $A$ );II. Tremove ( $i$ )

III. extractMax ()

IV. getMax ()

V. changePriority ( $i, p$ )

2022

056-309

06  
28February  
MondayDFS pseudocode :-

DFS(G)

For each vertex  $u \in G \cdot V$      $u \cdot \text{color} = \text{White}$      $u \cdot \text{f} = \text{NIL}$      $\text{time} = 0$     for each vertex  $v \in G \cdot V$         if  $v \cdot \text{color} == \text{White}$             DFS-VISIT( $G, v$ )DFS-VISIT( $G, u$ )     $\text{time} = \text{time} + 1$      // white vertex  $u$  just been discovered.     $u \cdot d = \text{time}$      $u \cdot \text{color} = \text{Gray}$     For each  $v \in G \cdot \text{adjacent}[u]$      // explore edge  $(u, v)$         if  $v \cdot \text{color} == \text{White}$              $v \cdot \text{parent} = u$             DFS-VISIT( $G, v$ )     $u \cdot \text{color} = \text{BLACK}$      // Blacken  $u$ ; it is done.     $\text{time} = \text{time} + 1$      $u \cdot f = \text{time}$ .

FEBRUARY '22

Never let the fear of striking out keep you from playing the game. -Babe Ruth

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Bipartite using BFS :-Find set pseudocode :-Find( $x$ )    if  $x \cdot \text{parent} \neq x$          $x \cdot \text{parent} := \text{Find}(x \cdot \text{parent})$         return  $x \cdot \text{parent}$ 

else

        return  $x$ 

end function.

makeSet :-Find( $x$ )    if  $x \cdot \text{parent} = x$         return  $x$ 

else

 $x \cdot \text{parent} = \text{Find}(x \cdot \text{parent})$         return  $x \cdot \text{parent}$ 

end function

MST-Kruskal( $G, w$ )H =  $\emptyset$ Unionset pseudocode :-function union( $x, y$ )     $x := \text{Find}(x)$      //  $x$  and  $y$  are replaced by root.     $y := \text{Find}(y)$     if  $x = y$  then

return

end if

    if  $x \cdot \text{size} < y \cdot \text{size}$          $(x, y) := (y, x)$ 

end if

 $y \cdot \text{parent} := x$     // update the size of  $x$      $x \cdot \text{size} := x \cdot \text{size} + y \cdot \text{size}$ 

end function

A happy family is but an earlier heaven. -George Bernard Shaw

060-305

March  
Tuesday01  
WK 105

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Wk  
02March  
WednesdayMGT-Kruskal(G, w)  
 $A = \emptyset$ for each vertex  $v \in G \cdot V$   
makeSet( $v$ )Create a simple list of the edges  $G \cdot E$  sort the edges  
(monotonically by w)for each edge  $(u, v)$  form the sorted list.if  $f\text{iset}(u) \neq f\text{iset}(v)$  $A = A \cup \{(u, v)\}$ Union( $u, v$ )return  $A$ .

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062-303

March  
Thursday03  
Wk 103

DFS pseudocode-II (Important)

→  $S$  Initiative stack  $\emptyset$  with one element  $s$   
while  $S$  is not empty09 Take a node  $u$  from  $S$  (pop)10 If  $\text{visited}[u] = \text{false}$ 11 Set  $\text{visited}[u] = \text{true}$ 12 for each edge  $(u, v)$  from  $u$ add  $v$  to  $S$  (push)

end for

13 end if

end while

01 DFS( $u$ )Set  $\text{visited}[u] = \text{true}$ 02 add  $u$  to tree  $R$ 03 for edge  $(u, v)$  from  $u$ 14 If  $\text{visited}[v] = \text{false}$ ,15 DFS( $v$ )

16 end if

end for

05

06

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08

→ DFS-VISIT( $s$ )↓  
using stack

MARCH '22

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The family is one of nature's masterpieces. George Santayana



The measure of intelligence is the ability to change.- Albert Einstein

APRIL '22

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W  
04 March Friday

### Picim's algorithm:-

MST-picimisnlg ( $G_1, w, \pi_C$ )

for each vertex  $v \in G_1 \cdot V$

$v \cdot key = \infty$

$v \cdot \pi_C = \text{NIL}$

$\pi_C \cdot key = 0$

$Q = \emptyset$  // primary queue

for each vertex  $v \in G_1 \cdot V$

Insert ( $Q, v$ )

while  $Q \neq \emptyset$

$u = \text{Extract-MIN}(Q)$

for each vertex  $v$  in adjacency [u]

if  $v \in Q$  and  $w(u, v) < v \cdot key$

$v \cdot \pi_C = u$

$v \cdot key = w(u, v)$

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March Saturday

05 WK 10  
064-301

### Dijkstra's Algorithm

DIJKSTRA ( $G, w, s$ )

09 for each vertex  $v \in G \cdot V - \{s\}$

$v \cdot d = \infty$

$v \cdot \pi_C = \text{NIL}$

end for

11  $s \cdot d = 0$

$S = \emptyset$

12  $Q = S \cdot V$

while  $Q \neq \emptyset$

01  $u = \text{Extract-MIN}(Q)$

$S = S \cup \{u\}$

02 for each vertex  $v \in G \cdot \text{Adj}[u]$

if  $v \cdot d > u \cdot d + w(u, v)$

$v \cdot d = u \cdot d + w(u, v)$

$v \cdot \pi_C = u$

03 end for

04 end while

Sunday 06

### Dijkstra's Algorithm:-

DIJKSTRA ( $G, w, s$ )

Before that,

→ INITIALIZE-SINGLE-SOURCE ( $G, s$ )

for each vertex  $v \in G \cdot V$

20  $v \cdot d = \infty$

30  $v \cdot \pi_C = \text{NIL}$

40  $S \cdot d = 0$

It is not that I'm so smart. But I stay with the questions much longer.- Albert Einstein

MARCH '22

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Start each day with a positive thought and a grateful heart.- Roy T. Bennett,



APRIL '22

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March  
Monday

Ad-1

 $\Rightarrow [\text{Priority Queue. } Q]$  $\rightarrow \text{Max-Heapsy}(A, i)$  $l = \text{left}(i)$  $r = \text{right}(i)$  $\text{largest} = 0$ if  $i < A.\text{heapsize}$  and  $A[i] > A[\text{largest}]$  $\text{largest} = i$ 

else

 $\text{largest} = i$ if  $i < A.\text{heapsize}$  and  $A[l] > A[\text{largest}]$ 

else

 $\text{largest} = l$ if  $\text{largest} \neq i$ swap  $\text{largest } A[i]$  and  $A[\text{largest}]$  $\text{max-Heapsy}(A, \text{largest})$ 

04

 $\rightarrow \text{min-Heapsy}(A, i)$  $l = \text{left}(i)$  $r = \text{right}(i)$  $\text{minimum} = 0$ if  $i < A.\text{heapsize}$  and  $A[l] < A[i]$  $\text{minimum} = l$ else  $\text{minimum} = i$ if  $i < A.\text{heapsize}$  and  $A[r] < A[\text{minimum}]$  $\text{minimum} = r$ if  $\text{minimum} \neq i$ swap  $A[i]$  and  $A[\text{minimum}]$ 

What I'm looking for is not out there, it is in me. - Helen Keller

 $\text{min-Heapsy}(A, \text{minimum})$ 

MARCH '22

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 $\rightarrow \text{BUILD-MAX-HEAP}(A, i)$  $A.\text{heapsize} = A.\text{length}$ for  $i$  from  $\lfloor \frac{A.\text{length}}{2} \rfloor$  to 1 $\text{MAX-Heapsy}(A, i)$ 

end for

end function

+ BUILD-min-Heap(A, i)

 $A.\text{heapsize} = A.\text{length}$ for  $i$  from  $\lfloor \frac{A.\text{length}}{2} \rfloor$  to 1 $\text{min-Heapsy}(A, i)$ 

end for

end function

In[2], p[2]

Extract-min(A)

BUILD-min-Heap(A)

 $A.\text{heapsize} = A.\text{length}$ swap  $A[1]$  and  $A[A.\text{length}]$  and  $i = A.\text{length}$ // Now, Remove the element  $A[A.\text{length}]$  from array $A.\text{heapsize} = A.\text{length} - 1$  or  $\text{Delete}(A, i)$  (deletes its element)

BUILD-min-Heap(A)

end process.

05 Priority Queue in heaps

Create Heap(H)  $\rightarrow$  empty heap HInsert(H, V)  $\rightarrow$  Insert element V in HMin-Heapsy(H, i)  $\rightarrow$  O(logn)Find in (H)  $\rightarrow$  (find the highest element or min key element)Delete(H, i)  $\rightarrow$  Delete i-th element[Extract-min]  $\rightarrow$  To extract min key element.

Never bend your head. Hold it high. Look the world straight in the eye. - Helen Keller



March

Tuesday



APRIL '22

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					1	2
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10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

## BFS :- (Using Queue)

BFS ( $G_1, V$ )for all vertices  $v \in G_1.V - \{r\}$  $v.\text{color} = \text{white}$  or  $\text{visited}(v) = \text{none}$  $v.d = \infty \rightarrow (\text{distance})$  $v.\pi = \text{nil} \rightarrow (\text{predecessors})$  $r.\text{color} = \text{gray}$  or  $r.\text{visited}(r) = \text{True}$  $r.d = 0$  $r.\pi = \text{nil}$  $Q = \emptyset$ enqueue( $Q, r$ )while  $Q \neq \emptyset$ dequeue( $Q$ )for all vertices  $v$  adjacent [ $r$ ]if  $v.\text{color} = \text{white}$  or  $\text{visited}[v] = \text{False}$  $v.\text{color} = \text{gray}$  $v.d = r.d + 1$  $v.\pi = r$ enqueue( $Q, v$ )

end if

end for

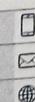
 $v.\text{color} = \text{black}$ 

end while

end process

S	M	T	W	T	F	S
1	2	3	4	5		
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13	14	15	16	17	18	19
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27	28	29	30	31		

You never fail until you stop trying. Albert Einstein



## DFS :- (Using stack)

DFS ( $G_1, s$ )Let  $S$  be a stack and  $S = \emptyset$  $S.\text{push}(s)$  $\text{visited}[s] = \text{True}$ while  $S \neq \emptyset$  $v = S.\text{top}()$  $S.\text{pop}()$ for all neighbour 'v' of vertex  $s$ if  $v$  is not visited $S.\text{push}(v)$  $\text{visited}[v] = \text{True}$ 

end for

end while

OR

DFS ( $G_1, s$ )Let  $S = \emptyset$   $S.\text{length} = G_1.V.\text{size}$ for all vertices  $v \in G_1.V - \{s\}$  $\text{visited}[v] = \text{false}$  and  $v.\pi = \text{nil}$  $\text{time} = 0$  $S.\text{push}(s)$ while  $S$  is not equal to  $\emptyset$ while for  $v \in G_1.\text{adjacent}[S.\text{top}()]$ if  $\text{visited}[v] = \text{false}$  and  $v = \text{left}(s)$  $S.\text{push}(v)$ if  $\text{visited}[v] = \text{true}$  and  $v = \text{right}(s)$  $S.\text{push}(S.\text{left})$ 

if both left and right child visited

Gravitation is not responsible for people falling in love. - Albert Einstein

 $S.\text{pop}()$ 

end for

end while

end process

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Topological ordering →

Topo (G)

find a node  $v$  with no incoming edges

Order it as first

Delete  $v$  from G

Topo (G - {v})

append this after v

Bipartiteness

Bipartite (G, S)

Let  $Q$  be an empty Q  
 $Q = \emptyset$

Suppose for all  $u \in G \cdot V - \{s\}$

$u \cdot \text{color} = \text{red/tied}$

$u \cdot p = 0$

$u \cdot \pi = \text{nil}$

$s \cdot \text{color} = \text{blue}$

enqueue ( $Q, s$ )

while  $Q \neq \emptyset$

dequeue ( $Q, s$ ) and  $s \cdot \text{color} = \text{blue}$

for each  $v$  adjacent [S]

if ( $v \cdot \text{color} = \text{red}$ )

$q \cdot \text{enqueue}(v)$

else if ( $v \cdot \text{color} = s \cdot \text{color}$ )

return not bipartite

return bipartite

Huffman coding

Message :- BCCHABBDDAECCBBAAEDDCC

length = 20

[ASCII - 8 bit] we know that

	Character	count / frequency	Code (3 bits)	here [total bits for msg]
10	A	3	000	
11	B	5	001	
12	C	6	010	
13	D	4	011	
14	E	2	100	
15		20	1	$20 \times 3 = 60$ bits

use huffman coding :-

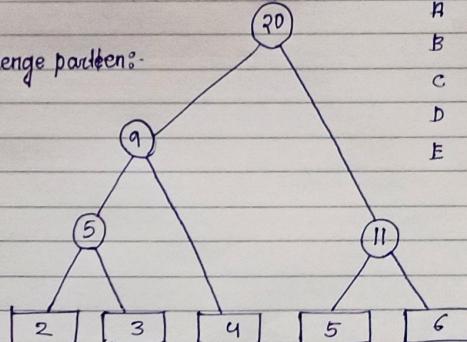
↳ optimal merge pattern

here

2	3	4	5	6
A	D	B	C	

Now, see.

optimum merge pattern :-

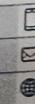


Char	Count	Code
A	3	001
B	5	10
C	6	11
D	4	01
E	2	000

Show me a family of readers, and I will show you the people who move the world. - Napoleon Bonaparte

The most pathetic person in the world is some one who has sight but no vision.-Helen Keller

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