LECTURE 4

Subtraction with Complements (1's and 2's)

Subtraction with Complements

- The direct method of subtraction taught in elementary schools uses the borrow concept.
- In this method, we borrow a 1 from a higher significant position when the minuend digit is smaller than the subtrahend digit.
- The method works well when people perform subtraction with paper and pencil.
- However, when subtraction is implemented with digital hardware, the method is less efficient than the method that uses complements.

Subtraction of two *n-digit unsigned numbers M - N in base r*

- **The subtraction of two *n-digit unsigned numbers M - N in base r can be done as* follows:**

1. Add the minuend *M to the r's complement of the subtrahend N. Mathematically,*

   $M + (r^n - N) = M - N + r^n.$

2. If $M \geq N$, *the sum will produce an end carry* r^n, *which can be discarded; what is* left is the result *M - N.*

Subtraction of two *n-digit unsigned numbers M - N in base r*

3. If M < N, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r’s complement of (N - M).

   To obtain the answer in a familiar form, take the r’s complement of the sum and place a negative sign in front.

Subtraction of two *n-digit unsigned numbers M - N in base r*

- Both numbers must have the same number of digits

1.Calculate the r's complement of the subtrahend N. Add the minuend M to the r's complement of the subtrahend N.

Inspect the result obtained in step (1) for an end carry:

(a) If an end carry occurs, discard it.

(b) If an end carry does not occur, take the *r*'s complement of the number obtained in step (1) and place a negative sign in front.

Subtraction Using 10's complement

Using 10's complement, subtract 72532 – 3250.

$$
\begin{aligned}
M &= 72532 \\
\text{10's complement of } N &= +\ \underline{96750} \\
\text{Sum} &= 169282 \\
\text{Discard end carry } 10^5 &= -\,\underline{100000} \\
\textit{Answer} &= 69282
\end{aligned}
$$

Subtraction Using 10's complement

Using 10's complement, subtract 3250 − 72532.

$$\begin{aligned} M &= \ 03250 \\ \text{10's complement of } N &= +\ 27468 \\ \text{Sum} &= \ 30718 \end{aligned}$$

There is no end carry. Therefore, the answer is −(10's complement of 30718) = −69282.

Subtraction Using 2's complement

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

(a)

$$
\begin{array}{rr}
X = & 1010100 \\
\text{2's complement of } Y = + & \underline{0111101} \\
\text{Sum} = & 10010001 \\
\text{Discard end carry } 2^7 = - & \underline{10000000} \\
\textit{Answer: } X - Y = & 0010001
\end{array}
$$

(b)

$$
\begin{array}{rr}
Y = & 1000011 \\
\text{2's complement of } X = + & \underline{0101100} \\
\text{Sum} = & 1101111
\end{array}
$$

There is no end carry. Therefore, the answer is $Y - X = -(\text{2's complement of } 1101111) = -0010001$.

Subtraction with r's Complement

The subtraction of two positive numbers ($M - N$), both of base r, may be done as follows:

(1) Add the minuend M to the r's complement of the subtrahend N.

(2) Inspect the result obtained in step (1) for an end carry:
   (a) If an end carry occurs, discard it.
   (b) If an end carry does not occur, take the r's complement of the number obtained in step (1) and place a negative sign in front.

Subtraction with (r-1)'s Complement

Both numbers must have the same number of digits

(1) Add the minuend M to the $(r - 1)$'s complement of the subtrahend N.

(2) Inspect the result obtained in step (1) for an end carry.

   (a) If an end carry occurs, add 1 to the least significant digit (end around carry).

   (b) If an end carry does not occur, take the $(r - 1)$'s complement of the number obtained in step (1) and place a negative sign in front.

Subtraction Using 9's complement

(a) $M = 72532$
$N = 03250$

$$\begin{array}{r} 72532 \\ +\ 96749 \\ \hline 1\ \ 69281 \\ +\ \ \ \ \ \ \ 1 \\ \hline 69282 \end{array}$$

end around carry

9's complement of $N = 96749$

ANSWER: 69282

(b) $M = 03250$
$N = 72532$

$$\begin{array}{r} 03250 \\ +\ 27467 \\ \hline 30717 \end{array}$$

no carry

9's complement of $N = 27467$

ANSWER: $-69282 = -$ (9's complement of 30717)

Subtraction Using 1's complement

(a) $X - Y = 1010100 - 1000011$

$$\begin{aligned} X &= 1010100 \\ \text{1's complement of } Y &= +\ 0111100 \\ \text{Sum} &= 10010000 \\ \text{End-around carry} &= +\ \underline{1} \\ \textit{Answer: } X - Y &= 0010001 \end{aligned}$$

(b) $Y - X = 1000011 - 1010100$

$$\begin{aligned} Y &= 1000011 \\ \text{1's complement of } X &= +\ \underline{0101011} \\ \text{Sum} &= 1101110 \end{aligned}$$

There is no end carry. Therefore, the answer is $Y - X = -(\text{1's complement of } 1101110) = -0010001$.

Some additional problems:

- Perform subtraction on the given unsigned numbers using the **10's** and **9's** complement of the subtrahend. Where the result should be negative, find its **10's/9's** complement and affix a minus sign. Verify your answers.

  (a) 4,637 - 2,579 (b) 125 - 1,800

  (c) 2,043 - 4,361 (d) 1,631 – 745

Additional Problems:

- Perform subtraction on the given unsigned binary numbers using the **2's and 1's** complement of the subtrahend. Where the result should be negative, find its **2's/1's** complement and affix a minus sign.

  (a) 10011 - 10010

  (b) 100010 - 100110

  (c) 1001 - 110101

  (d) 101000 - 10101