440/540 Artificial Intelligence HW2

INSTRUCTIONS

- **Due: Wednesday, 08 Feb 2023 at 23:59 PM PDT** Remember that you have 5 slip days to use at your discretion BUT you can use no more than 1 per homework.
- **Format:** Submit the answer sheet containing your answers in PDF. You can provide either typed or handwritten answers to this homework. Handwritten answers must be legible and scanned into a PDF.
- **Note: Please DO NOT FORGET to include your name and WSU ID in your submission.**
- **How to submit:** Submit a PDF containing your answers on Canvas
- **Policy:** See the course website for homework policies and Academic Integrity.

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Q1. [20 pts] Propositional Logic

Determine using a truth table whether the following sentence is valid, satisfiable, or unsatisfiable:

- (a) $(P \wedge Q) \vee \neg Q$
- (b) $((P \wedge Q) \Rightarrow R) \Longleftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$

(a) [10 pts] Is (a) valid, satisfiable, or unsatisfiable?

Answer:

A sentence is supposed to be **valid** it is true in all models.
A sentence is supposed to be **satisfiable** if it is true in some of its models.
A sentence is supposed to be **unsatisfiable** if none of its models are true.
Here is the truth table for the sentence:

P	Q	¬Q	P ∧ Q	(P ∧ Q) ∨ ¬Q
T	T	F	T	T
T	F	T	F	T
F	T	F	F	F
F	F	T	F	T

From the truth table, we can see that the sentence is sometimes true (T), which means it is a satisfiable sentence and not a valid sentence.

(b) [10 pts] Is (a) valid, satisfiable, or unsatisfiable?

Answer:

A sentence is supposed to be **valid** it is true in all models.
A sentence is supposed to be **satisfiable** if it is true in some of its models.
A sentence is supposed to be **unsatisfiable** if none of its models are true.
Here is the truth table for the sentence:

P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$P \Rightarrow R$	$Q \Rightarrow R$	$((P \Rightarrow R) \vee (Q \Rightarrow R))$	$((P \wedge Q) \Rightarrow R) \Longleftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

From the truth table, we can see that the sentence is true (T), which means it is valid sentence.

Q2. [20 pts] Propositional Logic

Assume that a knowledge base KB contains the following rules:

- $P \Rightarrow \neg W$
- $R \Rightarrow S$
- $\neg R \Rightarrow P$

(a) [10 pts] Show that KB entails ($W \Rightarrow S$) using the truth table enumeration approach

Answer :

Truth table is as follows :-

P	R	S	W	¬R	¬W	P ⇒¬W	R ⇒ S	¬R ⇒ P	W ⇒ S
F	F	F	F	T	T	T	T	F	T
F	F	F	T	T	F	T	T	F	F
F	F	T	F	T	T	T	T	F	T
F	F	T	T	T	F	T	T	F	T
F	T	F	F	F	T	T	F	T	T
F	T	F	T	F	F	T	F	T	F
F	T	T	F	F	T	T	T	T	T
F	T	T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T	F	F
T	F	T	F	T	T	T	T	T	T
T	F	T	T	T	F	F	T	T	T
T	T	F	F	F	T	T	F	T	T
T	T	F	T	F	F	F	F	T	F
T	T	T	F	F	T	T	T	T	T
T	T	T	T	F	F	F	T	T	T

We can see that for every row where the statements in KB are all true (i.e., where $P \Rightarrow \neg W$, $R \Rightarrow S$, and $\neg R \Rightarrow P$ are all true), the statement ($W \Rightarrow S$) is also true.

Therefore, we can conclude that KB entails ($W \Rightarrow S$).

(b) [10 pts] Show that KB entails ($W \Rightarrow S$) using resolution

Answer :-

We know ($a \Rightarrow b$) $\equiv \neg a \vee b$ using implication elimination

- So, for clause $P \Rightarrow \neg W$, we use implication elimination,
- $\neg P \vee \neg W$ –(I)
- For , $R \Rightarrow S$, we use implication elimination,
- $\neg R \vee S$ –(II)
- For, $\neg R \Rightarrow P$, we use implication elimination,
- $\neg(\neg R) \vee P \equiv R \vee P$ –(III)

  We use disjunction on (I) and (III)

  $\neg P \vee \neg W \vee R \vee P$,

  $\neg P \vee P$, eliminate each other as they are complimentary, hence $\neg W \vee R$ –(IV)

  We do disjunction on (II) and (IV)

  $\neg W \vee R \neg R \vee S$

  $\neg R \vee R$, eliminate each other as they are complimentary, hence $\neg W \vee S$, which is $W \Rightarrow S$.

  From this, we can conclude that that KB entails ($W \Rightarrow S$).

Q3. [20 pts] Propositional Logic

Consider the following statement:

On either Saturday or Sunday, if I am free, I will go to the concert

Using propositional logic, we can represent it as:

$$(\text{Saturday} \lor \text{Sunday}) \Rightarrow (\text{free} \Rightarrow \text{concert})$$

(a) [10 pts] Convert the above sentence into conjunctive normal form, using the logical equivalence table in Fig. 1.

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

Figure 1: Logical equivalences.

Answer :-

Converting (Saturday ∨ Sunday) ⇒ (free ⇒ concert) to CNF :-

- We first use implication elimination, replacing implication with its equivalent form using negation ¬(Saturday ∨ Sunday) ∨ (free ⇒ concert)
- Then we use, implication elimination on the right side of disjunction, ¬(Saturday ∨ Sunday) ∨ (¬free ∨ concert)
- Applying De Morgan's laws to eliminate the negation,(¬Saturday ∧ ¬Sunday) ∨ (¬free ∨ concert)
- Distribute the disjunction operator (¬Saturday ∨ ¬free) ∧ (¬Saturday ∨ concert) ∧ (¬Sunday ∨ ¬free) ∧ (¬Sunday ∨ concert)
- Distribute the disjunction operator (¬Saturday ∨ ¬free ∨ Concert) ∧ (¬Sunday ∨ ¬free ∨ concert)

(b) [10 pts] Continue to convert the above into implication form of Horn clause.

Answer:

(¬Saturday ∨ ¬free ∨ Concert) ∧ (¬Sunday ∨ ¬free ∨ concert)

By substituting an implication for each literal conjunction, this can be changed into the Implication Form of Horn Clauses.

is equivalent to:
(¬Saturday ∨ ¬free) ∨ Concert
Apply De-Morgan's law
Saturday ∧ Free ⇒ Concert

And ,
(¬Sunday ∨ ¬free ∨ concert)
Apply De-Morgan's law
Sunday ∧ Free ⇒ Concert
So, the sentence would be ,
(Saturday ∧ Free ⇒ Concert) ∧ (Sunday ∧ Free ⇒ Concert)

Each clause is an example of a Horn clause, since each clause only contains one positive literal (a literal without a negative).

Q4. [20 pts] Propositional Logic

Given the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned

(a) [10 pts] Can we prove that the unicorn is mythical?

Answer:

S1 : Mythical ⇒ Immortal -- 1

S2 : ¬Mythical ⇒ ¬Immortal ∧ Mammal -- 2

S3 : Immortal ∨ Mammal ⇒ Horned -- 3

S4 : Horned ⇒ Magical -- 4

¬Immortal ⇒ ¬Mythical (Using contrapositive on S1)
So, ¬Immortal ⇒ ¬Immortal ∧ Mammal -- (hypothetical syllogism on S2)
So, Immortal ∨ (¬Immortal ∧ Mammal) --(Using implication definition)
So, (Immortal ∨ ¬Immortal) ∧ (Immortal ∨ Mammal) --(using Distributive)
So, Immortal ∨ Mammal --(Immortal ∨ ¬Immortal are complimentary of each other) -- 5
Modus Ponens on (3) and (5), we get Horned
Modus Ponens on (4) and (5), we get Magical
So, we can conclude that Unicorn is horned and magical. However, there is no way to show the unicorn is mythical.

(b) [5 pts] Can we prove that the unicorn is magical?

Answer:

S1 : Mythical ⇒ Immortal -- 1

S2 : ¬Mythical ⇒ ¬Immortal ∧ Mammal -- 2

S3 : Immortal ∨ Mammal ⇒ Horned -- 3

S4 : Horned ⇒ Magical -- 4

¬Immortal ⇒ ¬Mythical (Using contrapositive on S1)
So, ¬Immortal ⇒ ¬Immortal ∧ Mammal -- (hypothetical syllogism on S2)
So, Immortal ∨ (¬Immortal ∧ Mammal) --(Using implication definition)
So, (Immortal ∨ ¬Immortal) ∧ (Immortal ∨ Mammal) --(using Distributive)
So, Immortal ∨ Mammal --(Immortal ∨ ¬Immortal are complimentary of each other) -- 5
Modus Ponens on (3) and (5), we get Horned
Modus Ponens on (4) and (5), we get Magical

So, we can conclude that Unicorn is Magical.

(c) [5 pts] Can we prove that the unicorn is horned?

Answer:

S1 : Mythical $\Rightarrow$ Immortal -- 1

S2 : $\neg$Mythical $\Rightarrow$ $\neg$Immortal $\wedge$ Mammal -- 2

S3 : Immortal $\vee$ Mammal $\Rightarrow$ Horned -- 3

S4 : Horned $\Rightarrow$ Magical -- 4

$\neg$Immortal $\Rightarrow$ $\neg$Mythical (Using contrapositive on S1)
So, $\neg$Immortal $\Rightarrow$ $\neg$Immortal $\wedge$ Mammal -- (hypothetical syllogism on S2)
So, Immortal $\vee$ ($\neg$Immortal $\wedge$ Mammal) --(Using implication definition)
So, (Immortal $\vee$ $\neg$Immortal) $\wedge$ (Immortal $\vee$ Mammal) --(using Distributive)
So, Immortal $\vee$ Mammal --(Immortal $\vee$ $\neg$Immortal are complimentary of each other) -- 5
Modus Ponens on (3) and (5), we get Horned
So, we can conclude that Unicorn is horned.

Syntax	Meaning
Student(X)	X is a student
Takes(X, History)	X takes history
Takes(X, Biology)	X takes Biology
Fails(X, History)	X fails History
Fails(X, Biology)	X fails Biology
score(X, Biology)	X score Biology
score(X, History)	X score History
Person(X)	X is a person
Vegetarian(X)	X is a vegetarian
Dislikes(X, Y)	X dislikes Y
Likes(X, Y)	X likes Y
Smart(X)	X is smart
Woman(X)	X is a woman
Man(X)	X is a man
Town(X)	X in town
Barber(X)	X is a barber
Shaves(X, Y)	X shaves Y
Professor(X)	X is a professor
Politician(X)	X is a politician
Fools(X, Y, t)	X fools Y at time t

Figure 2: Vocabulary used in sentences

Q5. [20 pts] First-Order Logic

Given the vocabulary syntax below, translate each of the following sentences into the corresponding first-order logic statements:

(a) [2 pts] Not all students take both History and Biology

Answer:

$\neg\forall x\ [\text{student}(x) \Rightarrow (\text{takes}(x, \text{History}) \land \text{takes}(x, \text{Biology}))]$

(b) [2 pts] Only one student failed History

Answer:

$\exists\ x\ \forall y{:}\ [(\text{student}(x) \land \text{fails}(x, \text{History}) \land \text{student}(y) \land \text{fails}(y, \text{History})) \Rightarrow x = y]$

(c) [2 pts] Only one student failed both History and Biology

Answer:

$\exists$ x $\forall$y: [(student(x) $\wedge$ fails(x, History) $\wedge$ fails(x, Biology) $\wedge$ student(y) $\wedge$ fails(y, History) $\wedge$ fails(y, Biology)) $\Rightarrow$ x = y]

(d) [2 pts] The best score in History was better than the best score in Biology

Answer:

$\exists$ x $\forall$y: [score(x, History) > score(y, Biology)]

(e) [2 pts] Every person who likes all vegetarians is smart

Answer:

$\forall$x $\forall$y: [person(x) $\wedge$ (vegetarian(y) $\Rightarrow$ likes(x,y)) $\Rightarrow$ smart(y)]

(f) [2 pts] No person dislikes a smart vegetarian

Answer:

$\forall$x $\forall$y: [person(x) $\wedge$ smart(y) $\wedge$ vegetarian(y)) $\Rightarrow$ ¬dislikes(x,y))]

(g) [2 pts] There is a woman who likes all men who are not vegetarians

Answer:

$\exists$x Woman (x) $\wedge$ $\forall$y Man (y) $\wedge$ ¬ Vegetarian (y) $\Rightarrow$ Likes (x , y)

(h) [2 pts] There is a barber who shaves all men in town who do not shave themselves

Answer:

$(\exists x)(\text{Barber}(x) \wedge (\forall y)\ \text{Man}(y) \wedge \text{Shaves}(x,y) \Rightarrow \neg\ \text{Shaves}(y,y))$

(i) [2 pts] No person likes a professor unless the professor is smart

Answer:

$\forall x, y\ \text{Person}(x) \wedge \text{Professor}(y) \wedge \neg\text{Smart}(y) \Rightarrow \neg\text{Likes}(x, y)$

(j) [2 pts] Politicians can fool some of the people all the time, and they can fool all of the people some of the time, but they cannot fool all of the people all of the time

Answer:

$\forall x$: [politician(x) $\Rightarrow$ ($\exists y\ \forall t$: [person(y) $\wedge$ fools(x,y,t)) $\wedge$ ($\exists t\ \forall y$: [person(y) $\Rightarrow$ fools(x,y,t)]]) $\wedge$ $\neg$ ($\forall t\ \forall y$: [person(y) $\Rightarrow$ fools(x,y,t)]])]