Landscape Analysis of 2-qubit System

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Abstract: Quantum control is aiming at steering a quantum system governed by a Hamiltonian H such that a specific target quantum gate G (a unitary matrix) can be reached within time T. This can be formulated as an optimization problem with the objective function $J(c) = Re\, Tr\{G^{\dagger}U_T(c)\}$ with $U_T(c)$ being the unitary matrix at time T under control c. Gradient-based methods are observed to be able to find the global maximum almost always. The optimization landscape is termed trap-free if the global maximum can be reached almost always. Most of the landscape analysis is conducted for a fully controllable quantum system. During this summer internship, we focused on a 2-qubit system and study its landscape property. In particular, we wanted to know under what conditions the landscape is trap-free. To that end, we first study the case where we have full control on the system and show that it has trap-free landscape by applying known result from the literature. To make it more realistic, we then consider the case where the two seemingly isolated qubits suffer from unknown crosstalk. To study this case, a theoretical analysis and numerical simulation have been conducted.

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Introduction

This project is aimed at building a better understanding of quantum control which is a necessary component for quantum computing and quantum information processing. Being able to perform a quantum gate with high fidelity is still a challenge to current quantum computers, and hence, it is worthwhile to deepen our understanding of how to improve quantum control. Simulations and experiments are conducted to find the optimal control that achieves the highest fidelity [1]. It is notable that all these controls are able to achieve the global optimal value of the objective function. That is saying the control landscape does not contain any local extrema such that the gradient-based learning algorithm can find the optimal value. This is a rather surprising phenomenon and knowing the condition for when it occurs is valuable to the field of quantum computing.

Description of the Project

As indicated by the title, we are studying the landscape of a 2-qubit quantum system for that this is the smallest nontrivial system. The whole project contains 2 researching steps:

1. Study the landscape of the fully controlled 2-qubit system.

   A 2-qubit system simulator is built by using QuTiP. A Rabi experiment is successfully conducted to show the validity of the simulator.

   In the meantime, the landscape analysis is performed by following the work of Russell et. al. [5], where the main result relays on the parametric transversality theorem (PTT), a famous theorem in the field of differential topology. By verifying that the condition of PTT is satisfied in the 2-qubit system, we can show that the fully controllable 2-qubit system is indeed trap-free.

2. Study the landscape of the 2-qubit system with unknown crosstalk.

   A fully controllable system is not a practical setting, and hence, we then consider the system where controls on each qubit are available but because of the existence of the unknown crosstalk, the control would also affect the other qubit without it being noticed. Under this setting, we would like to know if we can still observe a trap-free landscape.

   The objective function for such a system is given by:

$$J(c) = \max_{\Phi,\Psi} Re\ Tr\{(W\otimes\Phi)^{\dagger}U_T(c)\} + Re\ Tr\{(\Psi\otimes V)^{\dagger}U_T(c)\}, \tag{1}$$

   which is generalized from the work by Robert et.al. [6]. Here, the unitary matrices W, V are the target quantum gates on each single qubit respectively, while $U_T(c)$ is the total unitary matrix of the system under control c at time T. This objective represents that we are controlling each qubit separately without knowing the crosstalk. By following [6], we achieve a similar sufficient condition for trap-free landscape associated with the objective function on such 2-qubit system.

However, the system being trap-free is surprising, and hence, numerical simulations using QuTiP are conducted to verify the result.

Contributions Made to the Project

This project is conducted by myself under the guidance of my mentors.

During this project, I set up a simulation environment on my computer and gained experience in using QuTiP to perform simulations of a 2-qubit quantum system. The theoretical part of the landscape analysis depends on the field of differential topology which was a new subject to me. During this project I acquired a firm understanding of the concepts used in differential topology. Moreover, I rederived several theorems founding the base of quantum landscape theory.

Skills and Knowledge Gained

Landscape analysis from the point of view of differential topology was completely new to me, and this experience enabled me to determine whether a gradient-based algorithm can converge to an extremum without traps.

Moreover, I am now able to conduct simulations of a quantum system under controls using QuTiP which is one of the most popular simulation tools in quantum control. Not only did it fill in my knowledge gaps of the underlying quantum physics, but also taught me the procedures of how to set up new coding environments and run complicated simulations.

Relevance to the Mission of the NSF

The NSF mission is to promote the progress of science, and this research project indeed fulfills this mission for we are deepening the understanding of the quantum control landscape and exploring an unknown territory in this field where the objective function is the sum of two correlated sub-objective functions. Due to the project's realistic setting, we believe that this research project can make a unique contribution to quantum control.

Experience Impact on My Academic and Career Planning

This experience allowed me to quickly acquire new tools for simulating a quantum system and learn differential topology, which solidates my interest in mathematics. Being able to learn new mathematical concepts and apply them to practical situation created a sense of achievement! And I would definitely like to learn more about differential topology and its applications.

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