PERFORMANCE

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Practical-01

Topic : Limits & Continuity

Q1 $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

Soln:- $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[\dfrac{(\sqrt{a+2x} - \sqrt{3x})(\sqrt{3a+x} + 2\sqrt{x})}{3a+x-4x} \right]$

$= \lim_{x \to a} \left[\dfrac{(\sqrt{a+2x} - \sqrt{3x})(\sqrt{3a+x} + 2\sqrt{x})}{3a-3x} \right]$

$= \lim_{x \to a} \left[\dfrac{(\sqrt{a+2x} - \sqrt{3x})(\sqrt{3a+x} + 2\sqrt{x})}{3(a-x)} \right]$

$= \lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{3(a-x)} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \right] \times \sqrt{3a+x} + 2\sqrt{x}$

$= \lim_{x \to a} \left[\dfrac{(a+2x-3x)\sqrt{3a+x} + 2\sqrt{x}}{3(a-x)(\sqrt{a+2x} + \sqrt{3x})} \right]$

$= \lim_{x \to a} \left[\dfrac{\cancel{a-x}}{3\cancel{(a-x)}} \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}} \right]$

$= \lim_{x \to a} \dfrac{1}{3} \left[\dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}} \right]$

$$= \frac{1}{3}\left[\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}\right]$$

$$= \frac{1}{3}\left[\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}\right]$$

$$= \frac{1}{3}\left[\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}\right]$$

$$= \frac{1}{3}\left[\frac{2\sqrt{a}(\cancel{2})}{\sqrt{3}\sqrt{a}(\cancel{2})}\right]$$

$$= \frac{1}{3}\times\frac{2\sqrt{a}}{\sqrt{3}\sqrt{a}}$$

$$= \frac{2\sqrt{a}}{3\sqrt{3a}}$$ Ans

Q2 $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

Soln:- $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$$= \lim_{y\to a}\left[\frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y})+\sqrt{a})}\right].$$

$$= \lim_{y\to a}\left[\frac{1}{\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}\right]$$

$$= \lim_{y\to a}\left[\frac{1}{a+y+\sqrt{a}\sqrt{a+y}}\right]$$

$$= \left[\frac{1}{\sqrt{a+0}+\sqrt{a}\sqrt{a+0}}\right]$$

$$= \left[\frac{1}{a+a}\right] = \frac{1}{2a} \text{ Ans}$$

Q3 $\lim_{x\to\pi/6}\left[\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}\right]$

Soln: $\lim_{x\to\pi/6}\left[\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}\times\frac{\cos x+\sqrt{3}\sin x}{\cos x+\sqrt{3}\sin x}\right]$

$$= \lim_{x\to\pi/6}\left[\frac{\cos^2 x-3\sin^2 x}{\pi-6x(\cos x+\sqrt{3}\sin x)}\right]$$

$$= \lim_{x\to\pi/6}\left[\right.$$

$\rightarrow$ By substituting $x-\frac{\pi}{6}=h$

$x = h+\frac{\pi}{6}$

where $h\to 0$

$$\lim_{h\to 0}\frac{\cos\left(h+\frac{\pi}{6}\right)-\sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi-6\left(h+\frac{\pi}{6}\right)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\cos\frac{\pi}{6}-\sin h\sin\frac{\pi}{6}-\sqrt{3}\sin h\cos\frac{\pi}{6}+\cos h\sin\frac{\pi}{6}}{\pi-6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\frac{\sqrt{3}}{2}-\sin h\frac{1}{2}-\sqrt{3}\left(\sin h\frac{\sqrt{3}}{2}+\cos h\frac{1}{2}\right)}{\cancel{\pi}-6h+\cancel{\pi}}$$

$$\lim_{h\to 0}\frac{\cos\frac{\sqrt{3}}{2}h-\sin\frac{h}{2}-\sin\frac{3h}{2}-\cos\frac{\sqrt{3}h}{2}}{-6h}$$

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$$\lim_{h\to 0} \frac{\cancel{+}\sin\frac{4h}{2}}{\cancel{+}6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{12h}$$

$$\frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h} = \frac{1}{3}\times 1 = \frac{1}{3}$$ Ans

(4) $$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

→ By rationalizing both Numerator and Denominator

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{8}{2}\frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After Applying Limit we get

$= 4$ Ans

(5) Examine the continuity of the following at given points.

(5)(i) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0 < x \le \pi/2 \\ = \dfrac{\cos x}{\pi - 2x}, & \text{for } \pi/2 < x < \pi \end{cases} \quad \text{at } x = \pi/2$$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}} \quad \therefore f(\pi/2) = 0$$

f at $x = \pi/2$ define.

(ii) $\lim\limits_{x \to \pi/2^+} f(x) = \lim\limits_{x \to \pi/2^+} \frac{\cos x}{\pi - 2x}$

By Substituting Method

$x - \pi/2 = h$

$x = h + \pi/2$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left(h + \pi/2\right)}$$

$$\lim_{h \to 0} \frac{\cos(h+\pi/2)}{\pi - \cancel{2}\left(\frac{2h+\pi}{\cancel{2}}\right)}$$

$$\lim_{h \to 0} \frac{\cos(h+\pi/2)}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos\pi/2 - \sin h \sin\pi/2}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h \to 0} \frac{+\sin h}{+2h}$$

$$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

(iii) $\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x \to \pi/2^-} \frac{2\sin x \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x \to \pi/2^-} \frac{2\sin x \cos x}{\sqrt{2}\sin x}$

$\lim_{x \to \pi/2^-} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ f is not continous at $x = \pi/2$.

(ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \leq x \leq 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$

at $x=3$ & $x=6$.

Soln:- (i) $f(3) = \frac{x^2-9}{x-3} = 0$

~~f at~~ $x=3$ define

(ii) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$.

$\lim_{x\to3^+} f(x) = \lim_{x\to3^+} (x+3) = 6$

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$\lim_{x\to3^-} f(x) = \lim_{x\to3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 6$

$\therefore$ L.H.L = R.H.L

$\therefore$ f is continuous at $x=3$.

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

(ii) $\lim_{x\to6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to6^+} \frac{(x-3)(x+3)}{x+3}$

$\lim_{x\to6^+} (x-3) = 3$

$\lim_{x\to6^-} x+3 = 3+6 = 9$

$\therefore$ L.H.L $\neq$ R.H.L

function is not continuous.

(6) Find value of K, so that the function $f(x)$ is cts at the indicated Point.

(i) $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x<0 \\ = K & x=0 \end{cases}$ at $x=0$

Soln:- f is continuous at $x=0$

$\lim_{x\to0} f(x) = f(0)$

$\lim_{x\to0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x\to0} \frac{2\sin^2 2x}{x^2} = K$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = K$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$$

$$2(2)^2 = K \quad K = 8$$ Ans

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$= K$ $\quad x = 0$ $\Big\}$ at $x = 0$

Soln :- $f(x) = (\sec^2 x)^{\cot^2 x}$

Using $\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

&

$\cot^2 x = \frac{1}{\tan^2 x}$

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1 + \tan^2 x)^{1/\tan^2 x}$

we know that

$\lim_{x\to 0} (1 + Px)^{1/Px} = e$

$= e$

$\boxed{K = e}$ Ans

(iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$
$= K$ $\quad x = \pi/3$ $\Big\}$ at $x = \pi/3$

$x - \pi/3 = h$

$x = h + \pi/3$

where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

Using $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan\pi/3 + \tan h}{1 - \tan\pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\sqrt{3}(1 - \tan\pi/3 \cdot \tan h) - (\tan\pi/3 + \tan h)}{1 - \tan\pi/3 \cdot \tan h}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3}\left(\frac{1}{1 - \sqrt{3}(0)}\right)$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

Ans

(7) Discuss the continuity of the following functions which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity

(1) $$f(x) = \begin{cases} \dfrac{1 - \cos 3x}{x \tan x} & x \neq 0 \\ 9 & x = 0 \end{cases} \quad \text{at } x = 0$$

Soln:- $f(x) = \dfrac{1 - \cos 3x}{x \tan x}$

$$\lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x \tan x}$$

$$\lim_{x\to 0} = \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\frac{x \cdot \tan x}{x^2} \times x^2}$$

$$= \frac{2\lim_{x\to 0}\left(\frac{3}{2}\right)^2}{1} = 2 \times \frac{9}{4} = \frac{9}{2}$$ Ans

$\lim_{x\to 0} f(x) = 9/2 \quad g = f(0)$

$\therefore$ f is not continuous at $x=0$.

Redefine function

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$.

f has removable discontinuity at $x=0$

(11) $$f(x) = \begin{cases} \frac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases}$$ at $x=0$

Soln:— $$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x \to 0} 3 \cdot \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \log e \frac{\pi}{180} = \pi/60 = f(0)$$

f is continuous at $x=0$.

(8) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is cts at $x=0$ find $f(0)$.

Soln :- ∴ Given f is continuous at $x=0$.

$$\lim_{x \to 0} f(x) = f(0)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x \to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 on Num & Denominator.

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0) \text{ Ans}$$

(9) If $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is cts at $x = \pi/2$ find $f(\pi/2)$.

$$\rightarrow \lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2-1+\sin x}{\cos^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x \to \pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$\therefore\ f(\pi/2) = \frac{1}{4\sqrt{2}}$ Ans

Ak
6/12/19

Practical - 02

TOPIC :- Derivative.

Q1 Show that the following function defined from IR to IR are differentiable.

(i) $\cot x$

Soln:-

$f(x) = \cot x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$\lim_{h \to 0} \frac{\cot x - \cot a}{x - a}$

Put $x - a = h$

$x = h + a$ as $\to a$

$h \to 0$

$\lim_{h \to 0} \frac{\cot(a+h) - \cot a}{a + h - a}$

$= \lim_{h \to 0} \frac{\cot(a+h) - \cot a}{h}$

$= \lim_{h \to 0} \frac{\frac{\cos(a+h)}{\sin(a+h)} - \frac{\cos a}{\sin a}}{h}$

$\lim_{h \to 0} \frac{\cos(a+h)\sin a - \cos a \cdot \sin(a+h)}{\sin(a+h)(\sin a) \cdot h}$

$\lim_{h \to 0} \frac{\cancel{\cos}(a \sin(\cancel{a} - \cancel{a} - h)}{\sin(a+h)\sin a \cdot h}$

$\lim_{h \to 0} \frac{-\sin h}{\sin(a+h)\sin a \cdot h}$ =

$$\lim_{h\to 0} - \frac{1}{\sin(a+h)\sin a} \lim_{h\to 0} \frac{\sin h}{h}$$

$$\lim_{h\to 0} = \frac{-1}{\sin(a+h)\sin a} \times 1$$

$$= \frac{-1}{\sin a \cdot \sin a} = \frac{1}{\sin^2 a} = -\operatorname{cosec}^2 a$$

Ans

(11) $\operatorname{cosec} x$

Soln:- $f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$\lim_{h\to 0} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

Put $x-a=h$

$x = h+a$ as $\to a$ $h \to 0$

$$\lim_{h\to 0} \frac{\operatorname{cosec}(h+a) - \operatorname{cosec} a}{h+\cancel{a}-\cancel{a}}$$

$$\lim_{h\to 0} \frac{\cancel{\operatorname{cosec}}\frac{1}{\sin(h+a)} - \frac{1}{\sin a}}{h}$$

$$\lim_{h\to 0} \frac{\sin a - \sin(h+a)}{h \times (\sin(h+a)) \cdot \sin a}$$

$$\lim_{h\to 0} \frac{2\cos(a+h/2) \cdot \sin(-h/2)}{h \times \sin(h+a) \cdot \sin a}$$

$$= \lim_{h \to 0} \frac{2\cos(a+h/2) \cdot \sin(-h/2)}{h \times \sin(h+a) \cdot \sin a}$$

$$= \lim_{h \to 0} \frac{\cancel{2} \cdot \cos(a+h/2)}{\sin(h+a) \cdot \sin a} \times -\frac{1}{\cancel{2}} \lim_{h \to 0} \frac{\sin(-h/2)}{-h/2}$$

$$= \lim_{h \to 0} \frac{\cos(a+h/2)}{\sin(h+a)\sin a} \times -1$$

$$= \cancel{\lim_{h \to 0}} - \frac{\cos(a+0/2)}{\sin a \cdot \sin a} = -\frac{\cos a}{\sin a} \cdot \frac{1}{\sin a}$$

$$= -\cot a \cdot \operatorname{cosec} a$$

Ans

(iii) $\sec x$

Soln:- $f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$\lim_{h \to 0} \frac{\cancel{co}\sec x - \cancel{co}\sec a}{x-a}$$

put $x - a = h$

$x = h+a$ as $\to a$ $h \to 0$

$$\lim_{h \to 0} \frac{\sec(h+a) - \sec a}{h+a-a}$$

$$\lim_{h \to 0} \frac{\sec(h+a) - \sec a}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{\cos(h+a)} - \frac{1}{\cos a}}{h}$$

$$\lim_{h \to 0} \frac{\cos a - \cos(h+a)}{h \times \cos(h+a)\cos a}$$

$$\lim_{h \to 0} \frac{-2\sin\frac{(a+h+a)}{2}\sin\frac{(a-a+h)}{2}}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(a+\frac{h}{2}\right)\cdot \sin\left(-\frac{h}{2}\right)}{h \times \sin(h+a)\cdot \sin a}$$

$$= \lim_{h \to 0} \frac{\cancel{2}\cdot\cos\left(a+\frac{h}{2}\right)}{\sin(h+a)\cdot \sin a} \times -\frac{1}{\cancel{2}} \lim_{h \to 0} \frac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}}$$

$$= \lim_{h \to 0} \frac{\cos\left(a+\frac{h}{2}\right)}{\sin(h+a)\sin a} \times -1$$

$$= \cancel{\lim_{h \to 0}} - \frac{\cos\left(a+\frac{0}{2}\right)}{\sin a \cdot \sin a} = -\frac{\cos a}{\sin a}\cdot\frac{1}{\sin a}$$

$$= -\cot a \cdot \operatorname{cosec} a$$

Ans

(iii) $\sec x$

Soln:- $f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$\lim_{h \to 0} \frac{\cancel{co}\sec x - \cancel{co}\sec a}{x-a}$$

Put $x-a=h$

$x = h+a$ as $\to a$ $h \to 0$

$$\lim_{h \to 0} \frac{\sec(h+a)-\sec a}{h+a-a}$$

$$\lim_{h \to 0} \frac{\sec(h+a)-\sec a}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{\cos(h+a)} - \frac{1}{\cos a}}{h}$$

$$\lim_{h \to 0} \frac{\cos a - \cos(h+a)}{h \times \cos(h+a)\cos a}$$

$$\lim_{h \to 0} \frac{-2\sin\frac{(a+h+a)}{2}\sin\frac{(a-a+h)}{2}}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2}}{\cos a\cos(a+h)\times -\frac{h}{2}} \times -\frac{1}{2}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a\cos(a+0)}$$

$$= \frac{-1}{2} \times -2\,\frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \cdot \sec a \quad \underline{\text{Ans}}$$

Q2 If $f(x) = 4x+1 \quad , x \leq 2$

$\qquad\qquad = x^2+5 \quad , x > 0$, at $x = 2$ then find function is differentiable or not.

Soln:-

L.H.D:

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-2}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{x-2} = 4$$

RHD

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 2+2 = 4$$

$$Df(2^+) = 4$$

RHD = LHD

f is differentiable at $x=2$.

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Q3 If $f(x) = 4x+7$, $x < 3$

$= x^2+3x+1$, $x \geq 3$ at $x=3$, then

Find f is differentiable or not?

Soln :-

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6=9$$

$Df(3^-) = 9$

LHD $= Df(3^-)$

$$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$Df(3^+) = 4$

RHD $\neq$ LHD f is not differentiable at $x=3$.

Q4 If $f(x) = 8x - 5$, $x \leq 2$
$= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$, Then find f is differentiable or not.

Soln :-

$$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$$

RHD :

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$\lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$\lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 3 \times 2 + 2 = 8$$

$$Df(2^+) = 8$$

LHD =

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{x-2}} = 8$$

L.H.D = RHD f is differentiable at x=2.

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Practical no - 03

Topic : Application of Derivative

(i) Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$

$\rightarrow f'(x) = 3x^2 - 5$

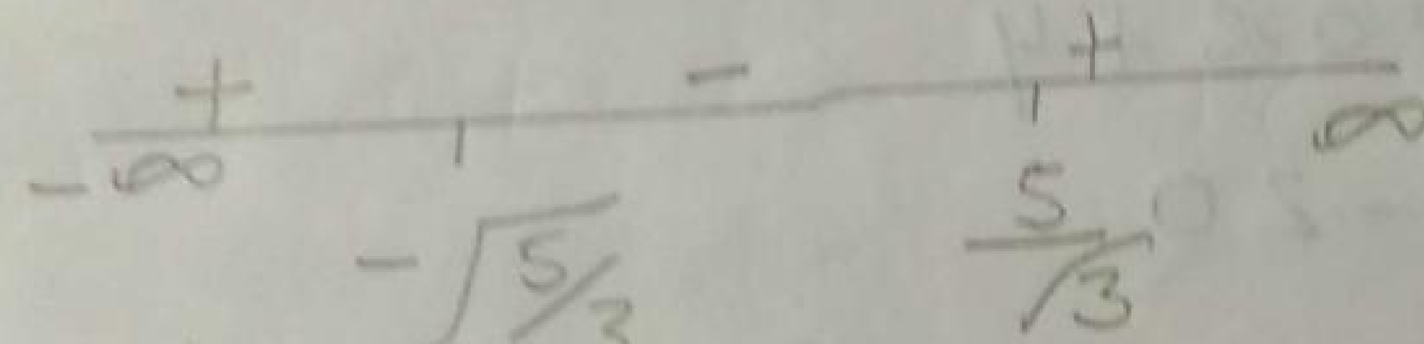
f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > 5/3$

$x > \pm\sqrt{\frac{5}{3}}$



$\therefore x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{\frac{5}{3}}, \infty)$

$\therefore$ f is decreasing iff $f'(x) < 0$

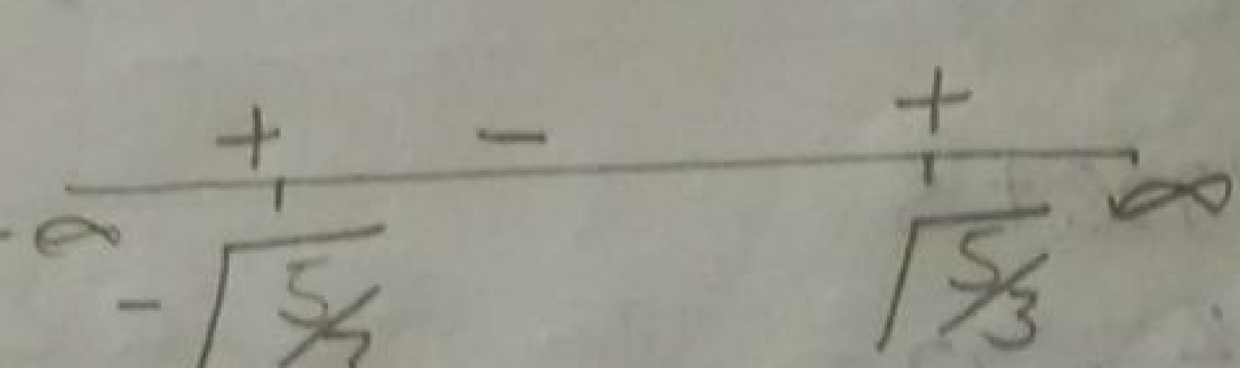
$\therefore 3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < 5/3$

$x < \pm\sqrt{5/3}$

$\therefore x \in (-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}})$



(ii) $f(x) = x^2 - 4x$

$\rightarrow f'(x) = 2x - 4$

$\therefore$ f is increasing iff $f'(x) > 0$

$2x - 4 > 0$

$= 2(x-2) > 0$

$= x-2 > 0$

$x > 2$

$\therefore x \in (2, \infty)$

$\therefore$ f is decreasing iff $f'(x) < 0$

$2x-4 < 0$

$2(x-2) < 0$

$x-2 < 0$

$x < 2$

$\therefore x \in (-\infty, 2)$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$\longrightarrow f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f'(x) > 0$

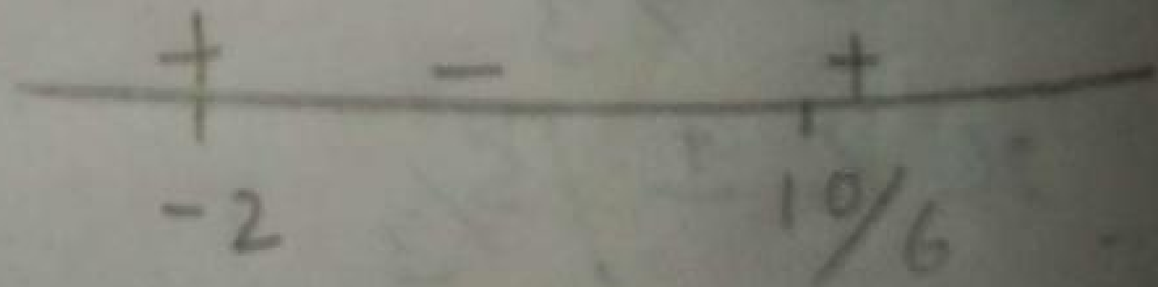
$\therefore 3x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x-10)(x+2) > 0$

$\therefore x \in (-2, 10/6)$



(iv) $f(x) = x^3 - \cancel{27x} + 5$

$\longrightarrow f'(x) = \cancel{3x^2} - 27x$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2-9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

$\overset{+}{\underset{3}{|}} \quad - \quad \overset{+}{\underset{3}{|}}$

f is decreasing iff $f'(x) < 0$

$3(x^2 - a) < 0$

$x^2 - a < 0$

$(x-3)(x+3) < 0$

$\therefore x \in (-3, 3)$

~~$f(x) =$~~

$\overset{+}{\underset{-3}{|}} \quad - \quad \overset{+}{\underset{3}{|}}$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$\Rightarrow f(x) = -24 - 18x + 6x^2$

i.e. $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6(x^2 - 3x - 4) > 6$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

$\overset{+}{\underset{-1}{|}} \quad - \quad \overset{+}{\underset{4}{|}}$

f is decreasing iff $f'(x) < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4)$

$(x+1)(x-4) < 0$

$\overset{+}{\underset{-1}{|}} \quad - \quad \overset{+}{\underset{4}{|}}$

$\therefore x \in (1, 4)$

Q2 find the intervals in which function is concave upwards and concave downwards.

(1) $y = 3x^2 - 2x^3$

Sol:- Let,

$$f(x) = y = 3x^2 - 2x^3$$

$$\therefore f'(x) = 6x - 6x^2$$

$$f''(x) = 6 - 12x$$

$$6(1-2x)$$

$f''(x)$ is concave upwards iff

$$f''(x) > 0$$

$$6(1-2x) > 0$$

$$1 - 2x > 0$$

$$-2x > -1$$

$$2x < 1$$

$$x < \frac{1}{2}$$

$$x \in \left(-\infty, \frac{1}{2}\right)$$

$f''(x)$ is concave downwards iff,

$$f''(x) < 0$$

$$6(1-2x) < 0$$

$$1 - 2x < 0 \quad = -2x < -1$$

$$2x > 1$$

$$x > \frac{1}{2}$$

$$\therefore x \in \left(\frac{1}{2}, \infty\right)$$

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Soln:- Let,

$f(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$f''(x)$ is concave upwards iff

$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

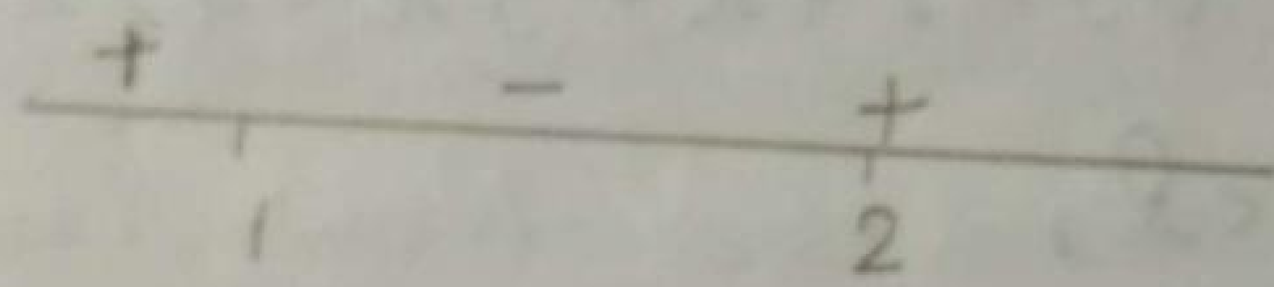
$x^2 - 3x + 2 > 0$

$x^2 - x - 2x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-2)(x-1) > 0$

$\therefore x \in (-\infty, 1) \cup (2, \infty)$



$f''(x)$ is concave downwards iff

$f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

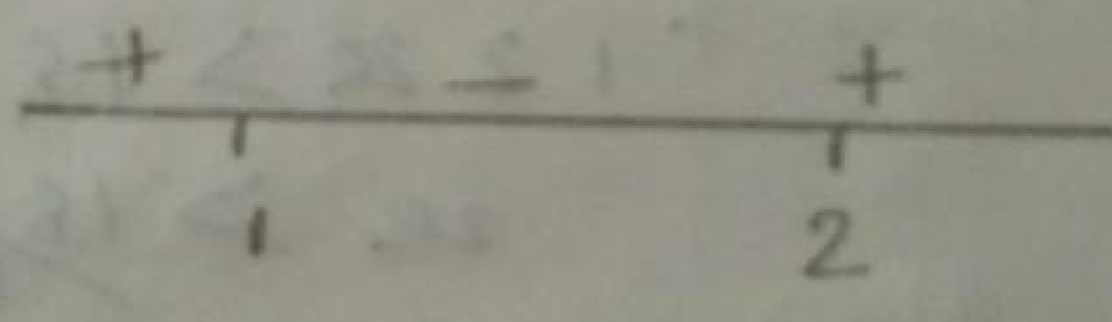
$x^2 - 3x + 2 < 0$

$x^2 - x - 2x + 2 < 0$

$x(x-1) - 2(x-2) < 0$

$(x-2)(x-1) < 0$

$\therefore x \in (1, 2)$



(iii) $y = x^3 - 27x + 5$

Soln:- Let, $f(x) = y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave upwards iff

$f''(x) > 0$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x \in (-\infty, 0)$

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

Soln:- Let,

$f(x) = y = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$f''(x)$ is concave upwards iff,

$f''(x) > 0$

$-18 + 12x > 0$

$12x > 18$

$x > 18/12$

$\therefore x \in (\frac{3}{2}, \infty)$

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$-18 + 12x < 0$

$12x < 18$

$x < 18/12$

$\therefore x \in (-\infty, 3/2)$

(v) $y = 2x^3 + x^2 - 20x + 4$

Soln:- Let,

$f(x) = y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$2(6x+1)$

$\therefore f''(x)$ is concave upwards iff,

$f''(x) > 0$

$2(6x+1) > 0$

$6x+1 > 0$

$6x > -1$

$x > -1/6$

$\therefore x \in \left(\frac{-1}{6}, \infty\right)$

$\therefore f''(x)$ is concave downwards iff,

$f''(x) < 0$

$2(6x+1) < 0$

$6x+1 < 0$

$6x < -1$

$x < \frac{-1}{6}$

$\therefore x \in \left(-\infty, \frac{-1}{6}\right)$

A
20/12/19

Practical no : 01

Topic : Application of Derivative
&
Newtons Method

(1) Find maxima & minima value of following function

(1) $f(x) = x^2 + \frac{16}{x^2}$

Soln :- $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

~~Now consid~~ Now consider,

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$\therefore 2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$ $\quad x = \pm 2$

$\therefore f''(x) = 2 + \frac{96}{x^4}$

Now

$f''(2) = 2 + \frac{96}{(2)^4}$

$= 2 + \frac{96}{16} = 2 + 6 = 8$

$\therefore$ at $x = 2$ f has minimum value

$\therefore f(2) = 2\times 2 + \frac{16}{(2)^2}$

$= 4 + \frac{16}{4}$

$= 8 > 6$

f has minimum at $x = 2$.

$f''(-2) = 2 + \frac{96}{(-2)^4} = 2 + \frac{96}{16} = 2+6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2, 2$.

~~$\therefore$ Function~~

(ii) $f(x) = 3 - 5x^3 + 3x^5$

Soln:- $f'(x) = -15x^2 + 15x^4$

Now consider

$f'(x) = 0$

$= -15x^2 + 15x^4 = 0$

$= 15x^4 = 15x^2$

$= x^2 = \frac{15x^2}{15x^2}$

$= x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

Now $f''(1) = -30\times 1 + 60\times(1)^3 = -30 + 60 = 30$

$30 > 0$

$\therefore$ at $x = 1$ f has minimum value.

$\therefore f(1) = 3 - 5\times 1 + 3\times 1$

$= 3 - 5 + 3$

$= 6 - 5 = 1 > 0$

f has minimum value.

$f''(-1) = -30 \times (-1) + 60 \times (-1)^3$

$= 30 - 60$

$= -30$

$-30 < 0$

$\therefore$ at $x = -1$ f has maximum value.

$f(-1) = 3 - 5 \times (-1)^3 + 3 \times (-1)^5$

$= 3 + 5 - 3$

$= 8 - 3 = 5 > 0$

$\therefore$ f has minimum value at $x = -1$.

$\therefore$ f has the maximum value -30 at $x = -1$ and has the minimum value at $x = 1$.

(iii) $f(x) = x^3 - 3x^2 + 1$ is $[-\frac{1}{2}, 4]$

Soln:-

$f'(x) = 3x^2 - 6x$

Consider

$f'(x) = 0$

$3x^2 - 6x = 0 = 3x(x-2) = 0$

$\cancel{3x^2 = 6x} = x = 0 \quad x = 2$

$\cancel{x^2 = \frac{6x}{3}}$

$\boxed{x = 2} \quad \boxed{x = 0}$

$f''(x) = 6x - 6$

$f''(2) = 6 \times 2 - 6$

$= 6 > 0$

f has minimum at $x = 2$.

$f(2) = (2)^3 - 3\times(2)^2 + 1$

$= 8 - 12 + 1$

$= 9 - 12$

$-3 \cancel{<} 0$

f has maximum at $x = 2$.

$f''(0) = 6x - 6 = 6\times 0 - 6 = -6 < 0$

f has maximum at $x = 0$.

$f(0) = (0)^3 - 3\times(0)^2 + 1 = 1$

f has maximum -3 at $x = 2$ and -6 at $x = 0$.

f has minimum 6 at $x = 2$ and 1 at $x = 0$.

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Soln: $f'(x) = 6x^2 - 6x - 12$

Consider

$f'(x) = 0$

$6x^2 - 6x - 12$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$f''(x) =$

$12x - 6$

$f''(2) = 12\times 2 - 6$

$= 24 - 6 = 18 > 0$

f has minimum at $x = 2$.

$$f(2) = 2x^3 - 3x^2 - 12x + 1$$
$$= 2 \times (2)^3 - 3 \times (2)^2 - 12 \times 2 + 1$$
$$= 2 \times 8 - 3 \times 4 - 12 \times 2 + 1$$
$$= 16 - 12 - 24 + 1$$
$$= 17 - 36$$
$$= -19 < 0$$

f has maximum at $x = 2$.

$$f''(-1) = 12x - 6$$
$$= -12 - 6$$
$$= -18 < 0$$

f has maximum at $x = -1$

$f(-1)$
$$= 2 \times (-1)^3 - 3 \times (-1)^2 - 12 \times -1 + 1$$
$$= -2 - 3 + 12 + 1$$
$$= -5 + 13$$
$$= 8$$
$$8 > 0$$

f has minimum at $x = -1$

$\therefore$ f ~~has~~ minimum value 18 at $x = 2$ ~~and~~ 8 at $x = -1$.

f has maximum value -19 at $x = 2$ and -18 at $x = -1$.

Q2 Find the root of following equation by Newton's Method (Take ~~5~~ iteration only). 48
correct upto ~~5~~ decimal.

(1) $f(x) = 2x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

soln :-
$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = 0 + \frac{9.5}{55}$$

$$\therefore f(x_1) = (0.1727)^3 - (0.1727)^2 - 55(0.172) + 9.5$$
$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= -55.9467$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 0.1727 - \frac{0.0829}{55.9467} = 0.1712$$

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$
$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$
$$= -55.9393$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 0.1712 + \frac{0.0011}{55.9393}$$
$$= 0.1712$$

$\therefore$ The root of the equation is 0.1712.

(11) $f(x) = x^3 - 4x - 9$ in $[2,3]$

$\rightarrow$ $f'(x) = 3x^2 - 4$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$\therefore$ $f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$= 3 - \frac{6}{23} = 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 18.5096$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$= 2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

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$f(x_2) = (2.7071)^2 - 4(2.707) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.4851}$

$= 2.7065$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9$

$= -0.901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 21.8943 - 4$

$= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$

$2.7015 + 0.0050$

$= \cancel{2.70}65$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ [1, 2]

Soln :- $f(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(7)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$ be initial approximation

By newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_2) / f(x_2)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(15.77)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$= f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1) / f(x_1)$

$= 1.577 + 0.6755 / 8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.52 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 16$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f(x_2)$

$= 1.6592 + 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 16$

$= -7.6977$

$x_4 = x_3 - f(x_3) / f(x_3)$

$= 1.6618 + \frac{0.0004}{7.6977}$

$= 1.6618$

Practical no-05

Topic : Integration

Q1) Solve the following integration

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

Soln: $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x-3-1+1}}$

$= \int \frac{dx}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-4}}$ $\left(\int \frac{1}{\sqrt{x^2-a}} = \log\left|x+\sqrt{x^2-a}\right| \right.$

$= \int \frac{dx}{\sqrt{(x+1)^2-4}}$ = Let $(x+1) = u$ $\therefore du = dx$

$= \int \frac{du}{\sqrt{(u)^2-4}}$

$= \log\left|(x+1)+\sqrt{(x+1)^2-4}\right| + c$ Ans

(ii) $\int (4e^{3x}+1)\,dx$

Soln:- $\int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \left[\frac{4e^{3x}}{3} + x\right] + c$

$$= \left[\frac{4e^{3x}}{3} + x\right] + c$$

Ans.

(iii) $\int (x^2 - 3\sin x + 5\sqrt{x})\, dx$

Soln:- $\int 2x^2 dx \int 3\sin x\, dx + \int 5\sqrt{x}\, dx$

$$= 2\int x^2 dx - 3\int \sin x\, dx + 5\int x^{1/2} dx$$

$$= \left[2\frac{x^3}{3} + 3\cos x + 5\frac{x^{3/2}}{3/2}\right] + c$$

$$= \left[2\frac{x^3}{3} + 3\cos x + \frac{10}{3}\cdot x^{3/2}\right] + c$$

Ans.

(iv) $\int \left(\frac{x^3 + 3x + 4}{\sqrt{x}}\right) dx$

Soln:- $\int \frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\, dx$

$$= \int x^{3-1/2} + 3x^{1-1/2} + 4x^{-1/2}\, dx$$

$$= \int x^{5/2} + 3x^{1/2} + 4x^{-1/2}\, dx$$

$$= \left[\frac{x^{7/2}}{7/2} + \frac{3x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2}\right] + c$$

$$= \left[\frac{2}{7}x^{7/2} + \frac{2\times 3}{3}x^{3/2} + 8x^{1/2}\right] + c$$

$$= \left[\frac{2}{7}x^{7/2} + 2x^{3/2} + 8x^{1/2}\right] + c$$

$$= 2x^{1/2}\left[\frac{x^{5/2}}{7} + x^{1/2} + 4\right] + c$$

$= 2\sqrt{x}\left[\frac{x^{5/2}}{7} + \sqrt{x} + 4\right] + c$

$= 2\sqrt{x}\left[\frac{x^{5/2}}{7} + \sqrt{x} + 4\right] + c$ Ans

(v) $\int t^7 \sin(2t^4)\, dt$

$\rightarrow$ Put $u = 2t^4$ $du = 2 \times 4 t^3$

$= \int t^7 \times (\sin(2t^4)) \times \frac{1}{2 \times 4 t^3} du$

$= \int t^4 \sin(2t^4) . \times \frac{1}{} du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8} du = \frac{t^4 \times \sin(2t^4) du}{8}$

substitute t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)}{8} du$

$= \int \frac{u \times \sin(u)}{2} / 8 \; du$

$= \int \frac{u \times \sin(u)}{16} du$

$= \frac{1}{16} \int u \times \sin(u) du$

$\int u \, dv = uv - \int v \, du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1 \, du$ $v = -\cos(u)$

$= \frac{1}{16} \int (u \times (-\cos(u)) - \int -\cos(u) du$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \int \cos(u) du$

$= \frac{1}{16} \times \left[u \times (-\cos(u)) + \sin(u)\right]$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$ Ans

(vi) $\int \sqrt{x}(x^2-1)\,dx$

$\rightarrow \int x^{1/2}(x^2-1)\,dx$

$= \int (x^{2+1/2} - x^{1/2})\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= \left[\frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2}\right] + c$

$= \left[\frac{2}{7}x^{7/2} - \frac{2}{3}x^{3/2}\right] + c$ Ans

(vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$\Rightarrow$ Let $\frac{1}{x^2} = t$

$\frac{d\frac{1}{x^2}}{dx} = dt \quad = \frac{-2}{x^3} = dt \quad = \frac{1}{x^3} = \frac{-dt}{2}$

$= \int \sin(t)\,\frac{-dt}{2}$

$= -\frac{1}{2}\int \sin(t)\,dt$

$= -\frac{1}{2}\left[-\cos t\right] + c$

$= \frac{1}{2}\cos t + c \quad = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + c$ Ans

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin x}}\,dx$

Let $\sin x = t$

$\cos x\,dx = dt$

$\rightarrow \int \frac{1}{\sqrt[3]{t}}\,dt$

$= \int \frac{1}{(\sqrt{t})^3}\,dt$

$$= \int \frac{1}{(t^{1/2})^3} \, dt$$

$$= \int \frac{1}{t^{3/2}} \, dt$$

$$= \int t^{-3/2} \, dt$$

$$= \left[\frac{t^{-3/2+1}}{-3/2+1} \right] + c = \left[\frac{t^{-1/2}}{-1/2} \right] + c$$

$$= -\frac{1}{2} t^{-1/2} + c$$

$$= -\frac{1}{2} \sin x^{-1/2} + c$$

(ix) $\int e^{\cos^2 x} \sin 2x \, dx$

$\rightarrow \int e^{\cos^2 x} \, 2 \sin x \cos x \, dx$

Let $\cos^2 x = t$

$$\frac{d \cos^2 x}{dx} = dt$$

$$\cancel{2 \cos x} \cdot - \sin x = dt$$

$$2 \cos x \sin x \, dx = - dt$$

$$= \int e^t - dt$$

$$= - \int e^t \, dt$$

$= -\left[e^{t} + c\right]$

$= -e^{\cos^2 x} + c$

$$\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right)$$

$\rightarrow$ Let $x^3 - 3x^2 + 1 = t$

$3x^2 - 6x\,dx = dt$

$3(x^2 - 2x) = dt$

$(x^2 - 2x) = \frac{dt}{3}$

$$= \int \frac{\frac{dt}{3}}{t}$$

$$= \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\left[\log(t) + c\right]$$

$$= \frac{1}{3}\left[\log(x^3 - 3x^2 + 1) + c\right]$$

AK
03/01/2020

Practical :- 06

Topic : Application of integration & Numerical integration

Q1 Find the length of the following curve.

(1) $x = t\sin t$, $y = 1-\cos t$ $t \in [0, 2\pi]$

Soln :- $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t\sin t$ $\therefore \frac{dx}{dt} = \cos t$

$y = 1-\cos t$ $\therefore \frac{dy}{dt} = 0-(-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1-\cos t + \sin t)^2 + (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1-2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2-2\cos t}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)}$

$= \int_0^{2\pi} \sqrt{2 \times 2\sin^2 t/2} = \sqrt{4\sin^2 t/2}$

$= \int_0^{2\pi} 2\,|\sin t/2)\, dt$ $\therefore \sin^2 t/2 = \frac{1-\cos t}{2}$

$= \int_0^{2\pi} 2\sin t/2\, dt$

$\left(-4\cos(t/2)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0) = 4+4 = 8$

$y = \sqrt{4-x^2} \quad x \in [-2, 2]$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$y = \sqrt{4-x^2} \quad \therefore \frac{dy}{dx} = 2\int_0^2 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$

$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$

$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$

$= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

(11) $y = x^{3/2}$ in $[0, 4]$

Soln:- $f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = \frac{9}{4}x$

$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_a^b \sqrt{1 + \frac{9}{4}x}\, dx$

$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$

$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$

$= \frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$

$= \frac{1}{24}\left[(4)^{3/2} - (40)^{3/2}\right]$

(iv) $x = 3\sin t \quad y = 3\cos t$

$\rightarrow \dfrac{dx}{dt} = 3\cos t \quad \dfrac{dy}{dt} = -3\sin t$

$$I = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3\, dt = 3\int_0^{2\pi} dt = 3[x]_0^{2\pi} = 3(2\pi - 0) = 6\pi$$

(v) $x = \frac{1}{6} y^3 + \frac{1}{2y} \quad y = [1,2]$

$\rightarrow \dfrac{dx}{dy} = \dfrac{x^2}{2} - \dfrac{1}{2y^2} = \dfrac{y^4 - 1}{2y^2}$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$$= \frac{17}{12}$$

Q2 Using Simpson's Rule solve the following

(1) $\int_0^2 e^{x^2} dx$ with $n=4$

→ $a = 0 \quad b = 2 \quad n = 4$

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1 + 54.5981) + 4(1.2840 + 9.4877 + 2(2.7182) + 54.5981)\right]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 14.6326\right]$$

$$= 1.1779$$

ii) $\int_0^4 x^2\,dx$

$\rightarrow L = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$= \int_0^4 x^2\,dx = \frac{1}{3}\left[(16+4(10)+8)\right]$

$= 64/3$

$\int_0^4 x^2\,dx = 21.533$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\,dx \quad n=6$

$\rightarrow L = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.80687	0.8727	0.9907

$\int_0^{\pi/3} \sqrt{\sin(x)} = \pi/54 \times 12.1163$

$= \int_0^{\pi/3} \sqrt{\sin x}\,dx$

$= 0.7049$

Practical - 07

TOPIC : Differential Equation

Solve the following differential Equation

1) $x\frac{dy}{dx} + y = e^x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$= I(x) = 1/x \quad Q(x) = \frac{e^x}{x}$

$I \cdot F = e^{\int 1/x\, dx}$

$= e^{\int 1/x\, dx}$

$= e^{\log(x)}$

$I \cdot F = x$

$y(I \cdot F) = \int Q(x)(I \cdot F)\, dx + C$

$= \int \frac{e^x}{x} \cdot x \cdot dx + C$

$= \int e^x dx + 1$

$xy = e^x + C$

$y(I \cdot F) = \int Q(x)\,(I \cdot F)\, dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3\, dx + C$

$= \int \sin x\, dx + C$

$x^3 y = -\cos x + C$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\rightarrow \frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x} \quad (\div \text{ by } e^x)$

$= \frac{dy}{dx} + 2y = 1/e^{x}$

$= \frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad Q(x) = e^{-x}$

$IF = e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)\,dx + C$

$= y e^{2x} = \int e^{-x+2x}\,dx + C$

$= \int e^{x}\,dx + C$

$y \cdot e^{2x} = e^{x} + C.$

(3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\longrightarrow x \cdot \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad Q(x) = \frac{\cos x}{x^2}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= e^{\int 2/x\,dx}$

$= \log(x^2$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$= \int \frac{\cos x}{x^2} \cdot x^2 \, dx + C$

$= \int \cos x + C$

$x^2 y = \sin x + C$

(iv) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\Rightarrow \frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int P(x) \, dx}$

$= e^{\int 3/x \, dx}$

$= e^{3/x \, dx}$

$= e^{\log x^3}$

$I \cdot F = x^3$

$y (I \cdot F) = \int Q(x) \, (I \cdot F) \, dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3 \, dx + C$

$= \int \sin x \, dx + C$

$x^3 y = -\cos x + C$

(5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$$\rightarrow \frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2x\, e^{-2x}$

$$I.F = e^{\int P(x)dx}$$
$$= e^{\int 2dx}$$
$$= e^{2x}$$

$$y(IF) = \int Q(x)\,(IF)\,dx + C$$

$$= \int 2x e^{-2x} e^{2x}\, dx + C$$

$$= \int 2x\, dx + c$$

$$y e^{2x} = x^2 + c$$

(6) $\sec^2 x \cdot \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\rightarrow \sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$

$$\frac{\sec^2 x}{\tan x}dx = -\frac{\sec^2 y\, dx}{\tan y}$$

$$= \int \frac{\sec^2 x\, dy}{\tan x} = -\int \frac{\sec^2 y}{\tan y} dy$$

$$= \log|\tan x| = -\log|\tan y| + c$$

$$= \log|\tan x - \tan y| = c$$

$$= \tan x - \tan y = e^{x}$$

$\frac{dy}{dx} = \sin^2(x-y+1)$

$\Rightarrow$ put $x-y+1 = v$

Differentiating on both sides

$= x-y+1 = v$

$= 1-\frac{dy}{dx} = \frac{dv}{dx}$

$= 1-\frac{dv}{dx} = \frac{dy}{dx}$

$= 1-\frac{dy}{dx} = \sin^2 v$

$= \frac{dv}{dx} = 1-\sin^2 v$

$= \frac{dv}{dx} = \cos^2 v$

$= \frac{dv}{\cos^2 x} = dx$

$= \int \sec^2 v \, dv = \int dx$

$= \tan v = x+c$

$= \tan(x+y-1) = x+c$

(8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

$\rightarrow$ put $2x+3y = v$

$2 + \frac{3dy}{dx} = \frac{dv}{dx}$

$= \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$

$= \frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$= \frac{dv}{dx} = \frac{v-1}{v+2}+2$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= 3\frac{(v+1)}{v+2}$$

$$\therefore \int \frac{(v+2)}{v+1} dv = 3dx$$

$$= \cancel{v + \log|x|} =$$

$$\therefore \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = 3x$$

$$\therefore v + \log|x| = 3x + C$$

$$\therefore 2x + 3y + \log|2x+3y+1| = 3x + C$$

$$\therefore 3x = x - \log|2x+3y+1| + c$$

AK
10/01/2020

PRACTICAL :- 08

TOPIC: Euler's Method

Using Euler's Method find the following

(1) $\frac{dy}{dx} = y + e^{x} - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$:

→ $f(x, y) = y + e^{x} - 2$; $y_0 = 2$, $x_0 = 0$; $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615

$$\boxed{y_{n+1} = y_n + h\, f(x_n, y_n)}$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

∴ By Euler's formula,
$y(2) = 9.2831$

(2) $\frac{dy}{dx} = 1 + y^2$

→ $f(x, y) = 1 + y^2$, $y_0 = 0$, $x_0 = 0$; $h = 0.2$

Using Euler's iteration Formula:
$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2442		

∴ By Euler's Formula,
$y(1) = 1.2942$

(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$ $x_0=0$, $h=0.2$

→ Using Euler's iteration Formula,

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$
0	0	1	0
1	0.2	0	
2	0.4		
3	0.6		
4	0.8		
5	1		

(4) $\frac{dy}{dx} = 3x^2+1$ $y_0=2$, $x_0=1$, $h=0$

For $h=0.5$.

→ Using Euler's iteration Formula,

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

∴ By Euler's Formula

$y(2) = 28.5$

For $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

∴ By Euler's Formula

$y(2) = 8.9048$

(5) $\frac{dy}{dx} = \sqrt{xy} + 2 \quad y_0 = 1, \; x_0 = 1, \; h = 0.2$

→ Using Euler's iteration Formula

$y_{n+1} = y_n + hf(x_n, y_n)$

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7/10/2020

n	x_n	y_n	$f(x_n, y_n)$
0	1	1	3
1	1.2	1.6	

$\therefore$ By Euler's Formula

$y(1.2) = 1.6$ Ans

Practical-09

~~Topic~~ : Evaluate the following limits

(1) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3x + y^2 - 1}{xy + 5}$

$\to \lim_{(x,y)(-4,-1)} \dfrac{x^3 - 3x - y^2 - 1}{xy + 5}$

$\lim_{(x,y)(-4,-1)} \dfrac{x^3 - 3x - y^2 - 1}{xy + 5}$

Applying limits

$= \dfrac{(-4)^3 - 3(-4) + (-1)^2 - 1}{(-4)(-1) + 5}$

$= \dfrac{-64 + 12 + 1 - 1}{4 + 5} = \dfrac{-52}{9}$

(2) $\lim_{(x,y)(2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$\to \lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

Applying limit

$= \dfrac{(0+1)\left[(2)^2 + (0)^2 - 4(2)\right]}{2 + (3)(0)}$

$= \dfrac{1(4 + 0 - 8)}{2} = \dfrac{4-8}{2} = \dfrac{-4}{2} = -2$

(iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$\rightarrow \lim_{(x,y,z)(1,1,1)} \frac{x^3-y^2z^2}{x^3-x^2yz}$

Applying limit

$= \frac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)}$

$= \frac{1-1}{1-1} = 0$ Ans

Q2 Find f_x, f_y for each of the following function.

1) $f(x,y) = xye^{x^2}+y^2$

$\rightarrow f_x = \frac{\partial}{\partial x}(f(x,y))$

$\cancel{= \frac{\partial}{\partial x}(xye^{x^2}+y^2)}$

$\cancel{= ye^{x^2}+y^2\cdot(2x)}$

$f_x = 2xye^{x^2}+y2$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2}+y^2)$

$= xe^{x^2} + y^2(2y)$

$= 2y\cancel{x}e^{x^2} + y^2$

(ii) $f(x,y) = e^{x}\cos y$

$\rightarrow f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^{x}\cos y)$

$= e^{x}(\cos y)$

$f_y = \frac{\partial}{\partial y}(e^{x}\cos y)$

$= -e^{x}\sin y$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$\rightarrow f_x = \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$= 2x^2y - 3x^2 + 3y$

Q3 Using definition find values of f_x, f_y of $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

$\rightarrow f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y-0}{(1+y^2)^2}$

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$$= \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y^2)}$$

$$= \frac{2}{1+y^2} \quad At(0,0) = \frac{2}{1+0} = 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

$$At(0,0) = \frac{-4(0)(0)}{(0)^2} = 0$$

Q4 Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$.

(i) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$$\rightarrow f_x = \frac{x^2 \cdot \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y-x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2-xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

$$= \frac{x^2(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6} \quad \text{--- (1)}$$

$$= f_y = \frac{\partial}{\partial x}\left(\frac{2y+x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2} = 0 \quad \text{--- (2)}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{--- (3)}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{x^2\frac{\partial}{\partial x}(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2 - 4xy + 2x^2}{x^4} \quad \text{--- (4)}$$

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from (3) & (4)

$f_{xy} = f_{yx}$.

(ii) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos xy + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f_y = x\cos xy + e^{x+y}(1)$
$= x\cos xy + e^{x+y}$

$\therefore f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)(y) + e^{x+y}$

$= -y^2\sin(xy) + e^{x+y}$ — (1)

$f_{yy} = \frac{\partial}{\partial x}(x\cos x(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}$
$= -x^2\sin xy + e^{x+y}$ — (2)

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ (3)

$f_{yx} = \frac{\partial}{\partial x}(x\cos xy) + e^{x+y})$

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$= -x^2 \sin(xy) + \cos(xy) + e^{x+y}$ — (4)

$\therefore$ from (3) & (4)

$f_{xy} \neq f_{yx}$ A

Q5 Find the linearization of at given point.

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\qquad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\qquad = \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}} + x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$ A

(ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$\rightarrow f(\pi/2, 0) = 1 - \pi/2 + 0$

$= 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$ $\qquad f_y = 0 - 0 + \sin x$

$f_x(\pi/2, 0) = -1 + 0$ $\qquad f(\pi/2, 0) = \sin \pi/2 = 1$

$= -1$

$$f(x,y) = f(a,b) + f_x(a,b)(x-1) + f_y(a,b)$$
$$(y-b)$$

$$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$$

$$= 1 - \pi/2 - x + \pi/2 + y$$

$$= 1 - xy$$

(ii) $f(x,y) = \log x + \log y$ at $(1,1)$

$\longrightarrow f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad f_y = 0 + \frac{1}{y}$

f_x (at) $(1,1) = 1$ $\quad = 1$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= 0 + 1(x-1) + 1(y-1)$$

$$= x - 1 + y - 1$$

$$= x + y - 2$$

AK
24/01/2020

Practical-10

TOPIC: Directional derivative gradient vector & maxima, minima Tangent & normal vectors.

Q. Find the directional derivative of the following function of given points & in the direction of given vector.

(1) $f(x,y) = x+2y-3$. $a=(1,-1)$, $u=3i-j$

Soln:- Here

$u = 3i - j$ is not a unit vector.

$\bar{u} = 3i - j$

$|u| = \sqrt{10}$

$\therefore$ Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3i-j)$

$= \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Now, $f(a+hu) = f\left[(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right]$

$= 1 + 2(-1) - 3 + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) +$

$\left(\frac{1+3h}{\sqrt{10}}, \frac{-1-h}{\sqrt{10}}\right)$

$$= -4 + \frac{3h}{\sqrt{10}} - 4 + 1 + \frac{3h}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$$

$$= -4 + \frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$$

$$= \lim_{h \to 0} \frac{h}{\sqrt{10}} \cdot \frac{1}{h}$$

$$= \frac{1}{\sqrt{10}}$$

Ans

(11) $f(x, y) = y^2 - 4x + 1 \quad a = (3, 4) \quad u = i + 5j$

Soln:- Here

$u = i + 5j$ is not a unit vector.

$\vec{u} = i + 5j$

$|\vec{u}| = \sqrt{26}$

Unit vector along u is $\frac{\vec{u}}{|\vec{u}|} = \frac{1}{\sqrt{26}}(i + 5j)$

$$= \frac{1}{\sqrt{26}}(1, 5)$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

Now

$$f(a+hu) = f\left[(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right]$$

$$= f\left[3+\frac{h}{\sqrt{26}}, 4+\frac{5h}{\sqrt{26}}\right]$$

$$= \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right)+1$$

$$= 16+\frac{25h^2}{\sqrt{26}}+\frac{40h}{\sqrt{26}}-12-\frac{4h}{\sqrt{26}}+1$$

$$= \frac{25h^2}{26}+\frac{36h}{\sqrt{26}}+5$$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{25h^2}{26}+\frac{36h}{\sqrt{26}}+5-5}{h}$$

$$= \lim_{h\to 0} \frac{25h^2}{26}+\frac{36h}{\sqrt{26}}$$

$$= \lim_{h\to 0} \frac{\left(\frac{25h}{26}+\frac{36}{\sqrt{26}}\right)\cancel{h}}{\cancel{h}}$$

$$= \frac{25(0)}{26}+\frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

(iii) $f(x,y) = 2x+3y$ $\quad a=(1,2)$ $\quad u = 3i+4j$

Soln:- Here

$u = 3i+4j$ is not a unit vector.

$\overline{u} = 3i+4j$

$|\overline{u}| = \sqrt{25} = 5$

Unit vector along $u = \frac{u}{|u|} = \frac{1}{5}(3\hat{i}+5\hat{j})$

$= \frac{1}{5}(3, 4)$

$= \frac{1}{5}$

Now

$f(a+hv) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4h}{5}\right)$

$= 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 8 + \frac{18h}{5}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hv) - f(a)}{h}$

$= \lim_{h \to 0} \frac{8 + \frac{18h}{5} - 8}{h} = \frac{18}{5}$

Q2 Find gradient vector for the following function at given point.

(1) $f(x,y) = x^y + y^x \quad a = (1,1)$

$\rightarrow f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(y x^{y-1} + y^x \log y, \ x y^{x-1} + x^y \log x\right)$

$\nabla f(x,y)$ at $(1,1)$

$= (1, 1^{1-1}) + (1^1 \log 1), (1(1)^{1-1} + 1^1 \log 1)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1}x) \cdot y^2 \quad a = (1,-1)$

$\to \quad f_x = y^2 \left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1}x\right)$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1}x\right)$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2} \times \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, -\pi/2\right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z} \quad a = (1,1,0)$

$\to \quad f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$
$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+0})$

$= (-1, -1, -2)$

Q3 find the equation of tangent & normal to each of the following curves at given points.

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$\rightarrow$ $f(x,y) = x^2 \cos y + e^{xy} - 2$
$f_x = 2x\cos y + ye^{xy}$
$f_y = -x^2 \sin y + xe^{xy}$

$(x_0, y_0) = (1, 0)$

f_x at $(1,0) = 2(1)\cos 0 + 0$
$= 2$

f_y at $(1,0) = -((1)^2 \sin 0 + 1 e^1 (0))$
$= 1$

$f_x (x - x_0) + f_y (y - y_0) = 0$
$2(x-1) + 1(y-0) = 0$
$2x - 2 + y = 0$
$2x + y - 2 = 0$
Equation of tangent.

for equation of normal
$bx + ay + d = 0$
$x + 2y + d = 0$
$(1) + 2(0) + d = 0$ at $(1,0)$
$= 1 + d = 0$
$d = -1$

$x + 2y - 1 = 0$ Equation of normal.

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$
$\rightarrow f(x,y) = x^2 + y^2 - 2x + 3y + 2$
$f_x = 2x + 0 - 2 + 0 + 0$
$= 2x - 2$
$f_y = 0 + 2y - 0 + 3 + 0$
$= 2y + 3$

f_x at $(2,-2) = 2(2) - 2 = 2$
f_y at $(2,-2) = 2(-2) + 3 = -1$

Equation of tangent
$f_x (x - x_0) + f_y (y - y_0) = 0$
$2(x-2) + -1(y+2) = 0$
$\cancel{2x+}$ $2x - 4 - y - 2 = 0$

$2x - y - 6 = 0$

for equation of Normal

$bx + ay + d = 0$

$-x + 2y + d = 0$

$-2 + 2(-2) + d = 0$

$d = 6$

$-x + 2y + 6 = 0$

equation of Normal.

Q4 Find the equation of tangent & normal line to each of the following surface.

(1) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$\longrightarrow f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + 2 - 0$

$= 2x + 2$

$f_y = -2z + 3 + 0 - 0$

$= -2z + 3$

$f_z = 0 - 2y + 0 + x - 0$

$= -2y + x$

f_x at $(2,1,0) = 2(2(1) + 0)$

$= 4$

f_y at $(2,1,0) = -2(1) + 2$

$= 6$

Equation of tangent

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x + 3y - 11 = 0$

Equation of normal

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0}$$

2) $3xyz - x - y + z = -4$

Sol:- $f(x,y,z) = 3xyz - x - y + z + 4$

$f_x = 3yz - 1 - 0 + 0 + 0 = 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 + 0 + 1 - 0$

$= 3xy + 1$

f_x at $(1,-1,2) = 3(-1)(2) - 1$

$= -7$

f_y (at) $(1,-1,2) = 3(1)(2) - 1$

$= 5$

f_z at $(1,-1,2) = 3(1)(-1) + 1$

$= -2$

Equation of tangent

$$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$$

$$= 7(x-1) + 5(y+1) + (-2)(z-2) = 0$$

$$= -7x + 5y - 2z + 16 = 0$$

Equation of Normal

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q5 (a) Find the local maxima & minima for the following function.

(1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$ — (1)

$f_y = 2y - 3x + 0 - 4$

$= 2y - 3x - 4$ — (2)

$f_x = 0$

$6x - 3y + 6 = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — (3)

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — (4)

Multiplying (3) by 2 and subtracting with y

70

$$2y - 3x = 4$$
$$\underline{-2y + 4x = -4}$$
$$x = 0$$

Put value of x in equation (3)

$$0 - y = -2$$
$$y = 2$$

Critical points are $(0, 2)$

Now

$$r = f_{xx} = 6$$
$$t = f_{yy} = 2$$
$$s = f_{xy} = 3$$

$rt - s^2 = 12 - 9 = 3 > 0$

Here $r > 0$ and $rt - s^2 > 0$

$\therefore f$ has minimum at $(0, 2)$

$$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$$
$$= 0 + 4 - 0 + 0 - 8$$
$$= -4$$

(ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$\rightarrow f_x = 8x^3 + 6xy$

$= 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y$

$= 3x^2 - 2y$

$f_y = 0$

$3x^2 - 2y = 0$ — (2)

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 6y) = 0$

$4x^2 + 6y = 0$ — (1)

Multiplying equation (2) with (3) and adding with (1)

$$9x^2 - 6y = 0$$
$$\underline{4x^2 + 6y = 0}$$
$$13x^2 = 0$$
$$x = 0$$

Put x in equation (1)

$$0 + 6y = 0$$
$$y = 0$$

Critical point are $(0,0)$

$r = f_{xx} = 24x^2 + 8y$

$t = f_{yy} = -2$

$s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -48x^2 - 12y - 36x^2$

$= -84x^2 - 12y$

<u>At $(0,0)$</u>

$r = 24(0)^2 + 6(0)$

$= 0$

$s = 6(0)$

$= 0$

$rt - s^2 = -84(0)^2 - 12(0)$

$= 0$

Nothing can be said.

(5) $f(x, y) = x^2 + y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$

$2x + 2 = 0$

$x = \frac{-2}{2}$

$x = -1$

$f_y = 0$

$-2y + 8 = 0$

$2y = 8$

$y = 4$

Critical points are $(-1, 4)$

Now

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$rt - s^2 = 2 \times -2 = 0$

$= -4$

Here

$r > 0$ and $rt - s^2 < 0$

$\therefore$

Nothing can be said.

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