

# DATA STRUCTURES & ALGORITHMS

## 23: QUICK SORT

(DIVIDE & CONQUER: PARTITION-EXCHANGE SORT)

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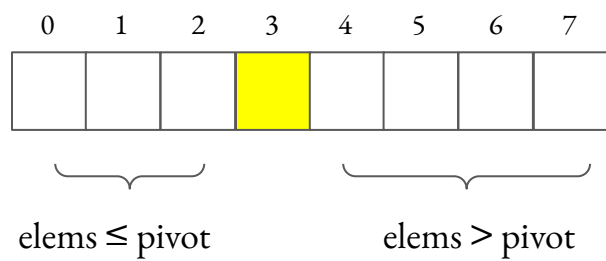
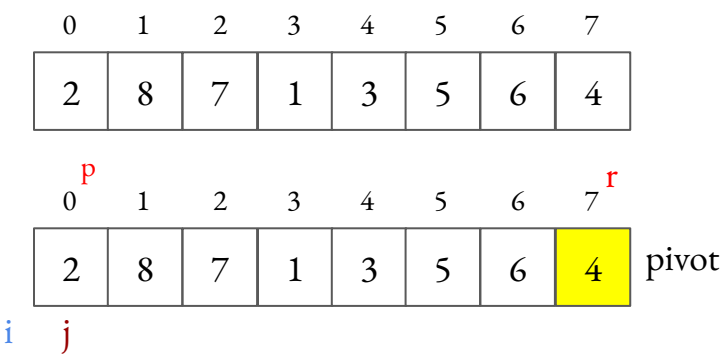
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$i$   $j$   $p$   $r$  pivot

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$x = A[r]$

$x = 4$

for  $j = p$  to  $r-1$

if( $A[j] \leq x$ )

$i = i + 1$

swap( $A[i]$ ,  $A[j]$ )

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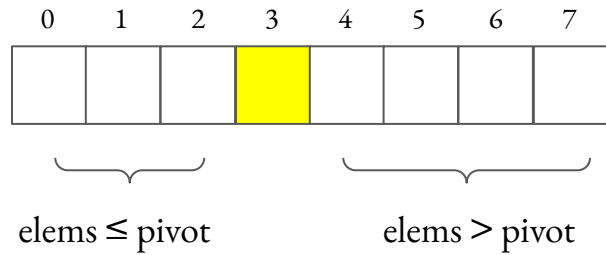
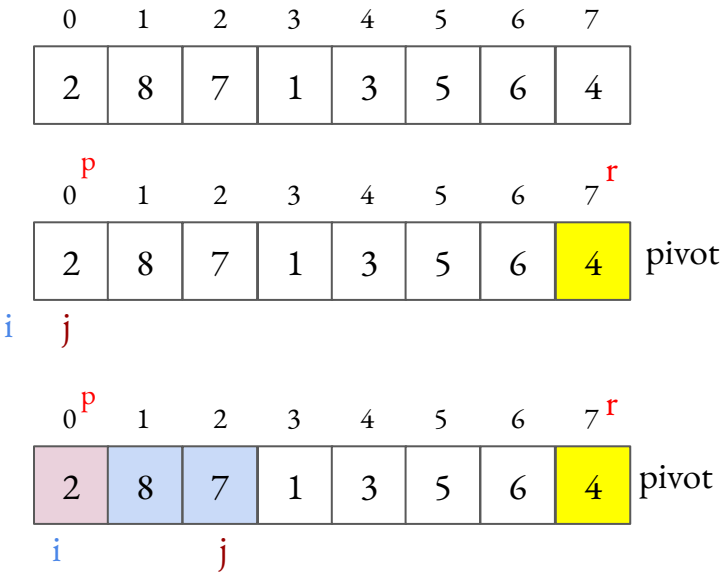
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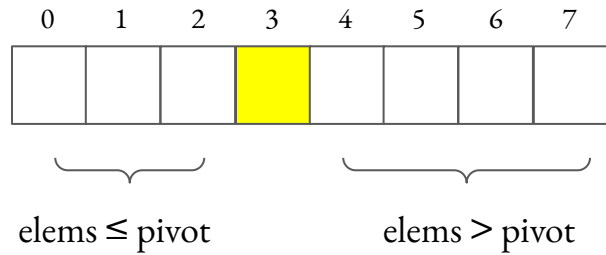
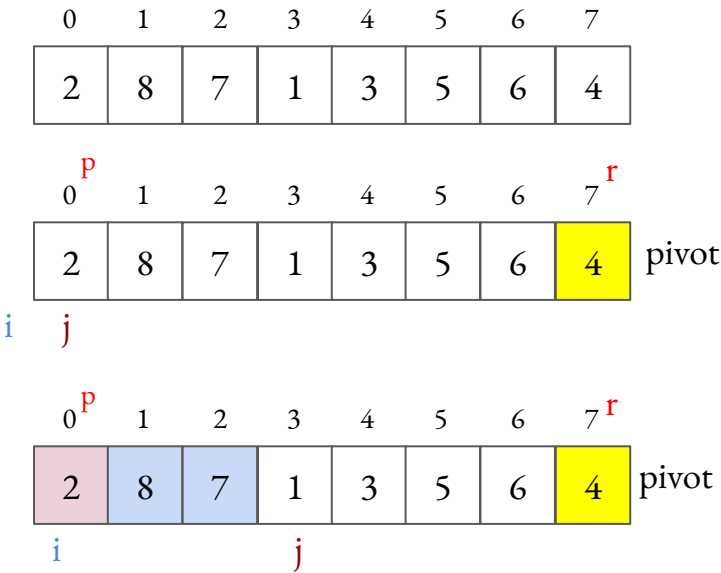
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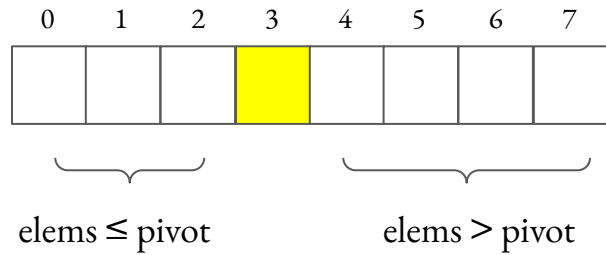
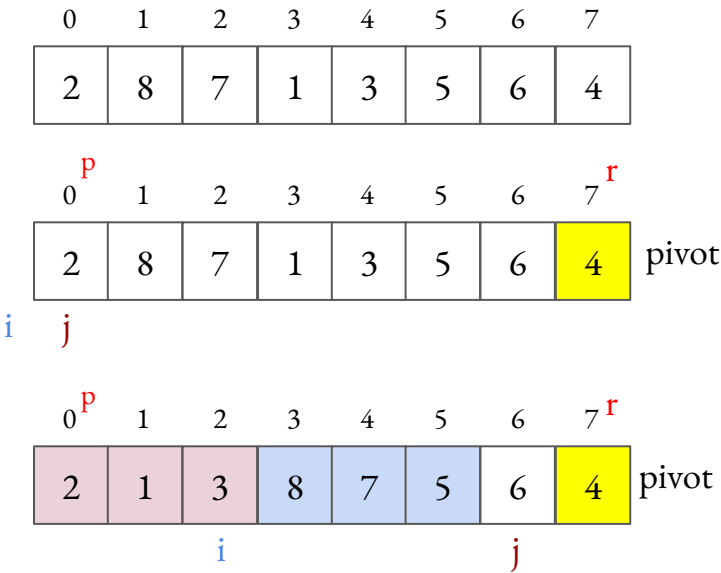
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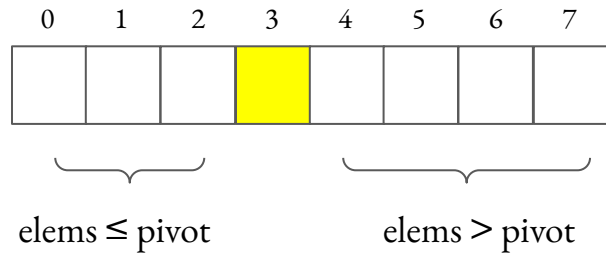
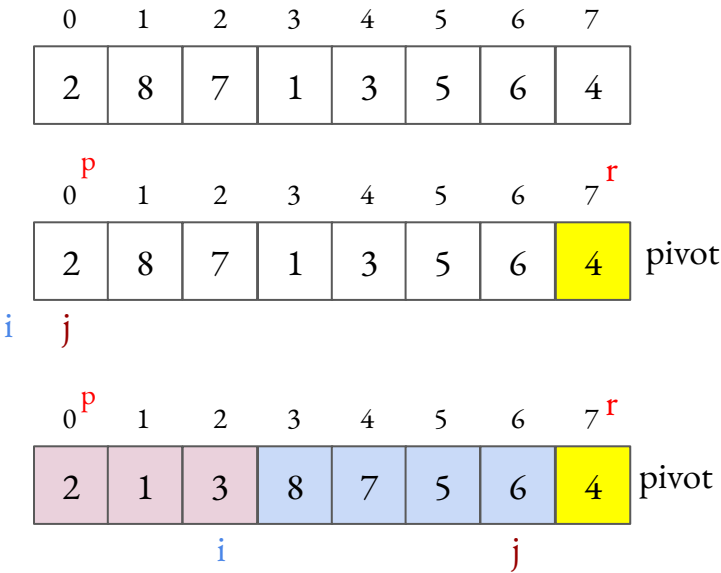
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  if( $A[j] \leq x$ )

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    swap( $A[i], A[j]$ )

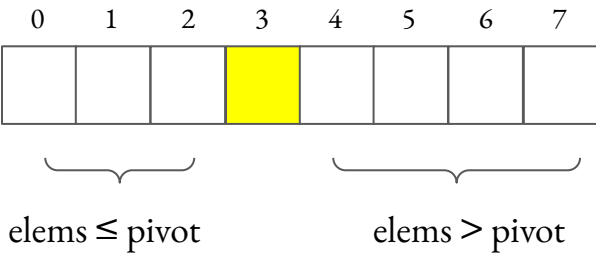
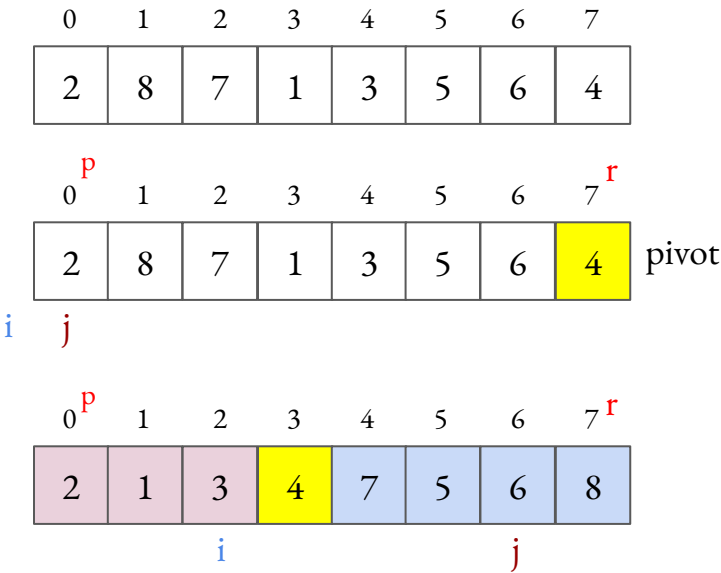
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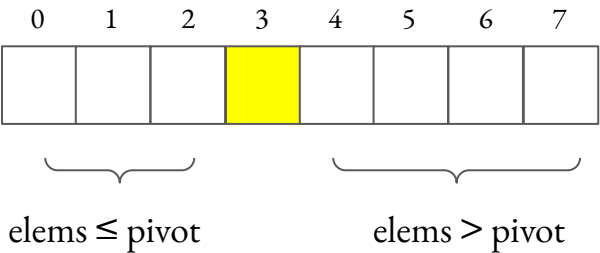
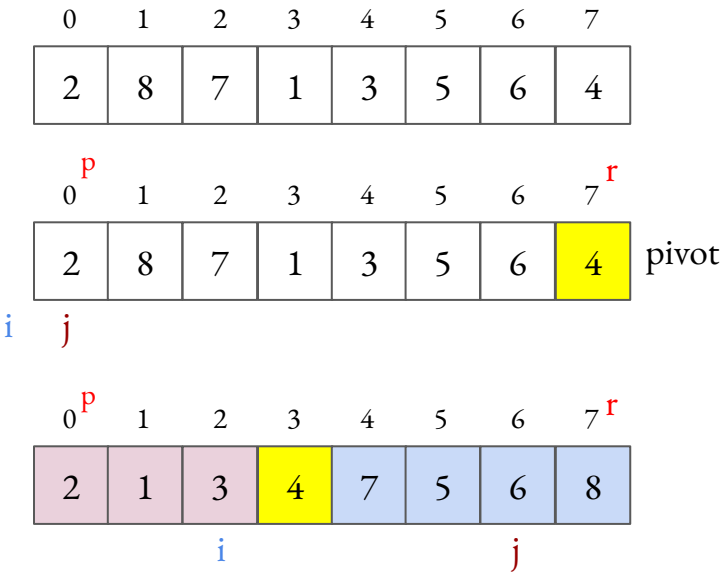
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for j = p to r-1
  if(A[j] ≤ x)
    i = i + 1
    swap(A[i], A[j])

swap(A[i+1], A[r])
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x = 4
for j = p to r-1
    if(A[j] ≤ x)
        i = i + 1
        swap(A[i], A[j])

swap(A[i+1], A[r])
return i+1
```

# QUICK SORT

Quicksort is based on the three-step process of divide-and-conquer.

- To sort the subarray  $A[p : r]$ :

**Divide:** Partition  $A[p : r]$ , into two (possibly empty) subarrays  $A[p : q - 1]$  and  $A[q + 1 : r]$ , such that each element in the first subarray  $A[p : q - 1]$  is  $\leq A[q]$  and  $A[q]$  is  $\leq$  each element in the second subarray  $A[q + 1 : r]$ .

**Conquer:** Sort the two subarrays by recursive calls to QUICKSORT.

**Combine:** No work is needed to combine the subarrays, because they are sorted in place.

- Perform the divide step by a procedure PARTITION, which returns the index  $q$  that marks the position separating the subarrays.

# QUICK SORT

PARTITION( $A, p, r$ )

```
 $x = A[r]$  // the pivot
 $i = p - 1$  // highest index into the low side
for  $j = p$  to  $r - 1$  // process each element other than the pivot
    if  $A[j] \leq x$  // does this element belong on the low side?
         $i = i + 1$  // index of a new slot in the low side
        exchange  $A[i]$  with  $A[j]$  // put this element there
exchange  $A[i + 1]$  with  $A[r]$  // pivot goes just to the right of the low side
return  $i + 1$  // new index of the pivot
```

# QUICK SORT

QUICKSORT( $A, p, r$ )

**if**  $p < r$

    // Partition the subarray around the pivot, which ends up in  $A[q]$ .

$q = \text{PARTITION}(A, p, r)$

    QUICKSORT( $A, p, q - 1$ ) // recursively sort the low side

    QUICKSORT( $A, q + 1, r$ ) // recursively sort the high side

Initial call is QUICKSORT( $A, 1, n$ ).

**Best case:**  $\Omega(n \log n)$

**Worst case:**  $O(n^2)$

**Average case:**  $\Theta(n \log n)$