

# DATA STRUCTURES & ALGORITHMS

## 25: HASH TABLES (COLLISION RESOLUTION)

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# HASH TABLES

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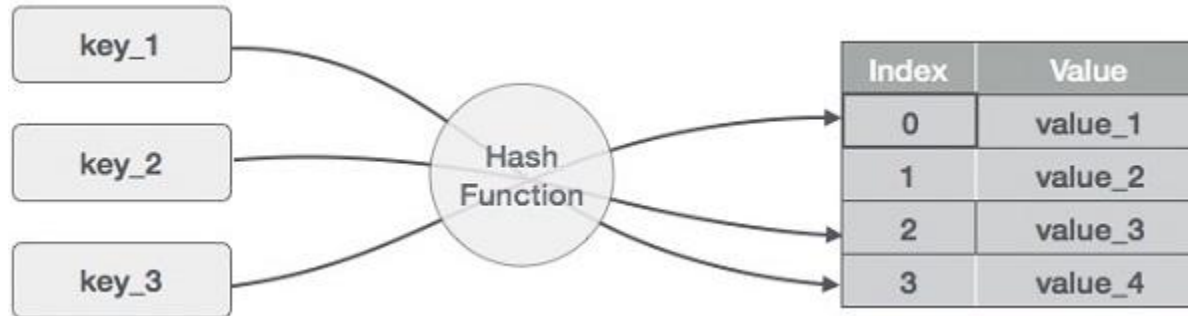
## Hash Tables

- A hash table is an effective data structure for implementing dictionaries.
- The built-in dictionaries of Python are implemented with hash tables.
- Indices can be integers, floats, chars, strings.

# HASH TABLES

## Hashing Function

- Function which transforms a key to an index.
- If **h** is a hash function that takes key as a parameter, it will compute index where value is placed.
- It is important to note that hash function returns the same index value every time for same key.



# DIVISION METHOD

# HASH TABLES

## Division Method for hashing

- Hashing an integer **k** is to divide **k** by **M** and then to use the remainder **modulo M**.
  - This is called the **division method** of hashing.
  - **$h(k) = k \bmod M$** 
    - **k** is the key value
    - **M** is the hash table size

# HASH TABLES

## Division Method for hashing

- Hashing an integer **k** is to divide **k** by **M** and then to use the remainder **modulo M**.
  - This is called the **division method** of hashing.
  - **$h(k) = k \bmod M$** 
    - **k** is the key value
    - **M** is the hash table size

### Example:

*Hash table size  $M = 10$*

*Key value,  $k = 112$*

*Hash function =  $h(k) = k \bmod M = 112 \bmod 10 = 2$*

COLLISION




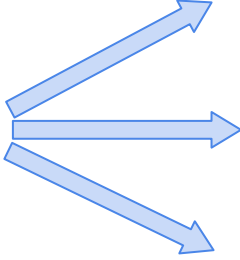
# HASH TABLES

## Collisions

- Collision occurs when one key value stores more than one value. Hash function produces the same position for two values.
- Hash table cannot ensure to have single value corresponding to a key. This is because a hash table has fixed number of keys.
- Any new entry in hash table, after all the slots are filled, causes collision. However, good hash tables use some mechanism to avoid the situations of collisions or to handle them efficiently.

# HASH TABLES

## Resolving Collisions

- **Closed addressing**  Chaining
- **Open addressing** 
  - Linear Probing
  - Quadratic Probing
  - Double Hashing

# HASH TABLES

```
code = ['CS', 'CDS', 'MATH', 'STAT']
```

```
for elem in code:
```

```
    val = 0
```

```
    for ch in elem:
```

```
        print(f'{ch} --> {ord(ch)}')
```

```
        val = val + ord(ch)
```

```
    print(f'ASCII sum for {elem} is {val}\n')
```

```
C --> 67
```

```
S --> 83
```

```
ASCII sum for CS is 150
```

```
C --> 67
```

```
D --> 68
```

```
S --> 83
```

```
ASCII sum for CDS is 218
```

```
M --> 77
```

```
A --> 65
```

```
T --> 84
```

```
H --> 72
```

```
ASCII sum for MATH is 298
```

```
S --> 83
```

```
T --> 84
```

```
A --> 65
```

```
T --> 84
```

```
ASCII sum for STAT is 316
```

CHAINING

# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	
2	
3	
4	
5	

key

index (u)

# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	
2	
3	
4	
5	

**key**

**index (u)**

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	
2	
3	
4	5
5	

**key**

5

**index (u)**

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	
2	
3	
4	5
5	

**key**

**index (u)**

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

1

$$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$$



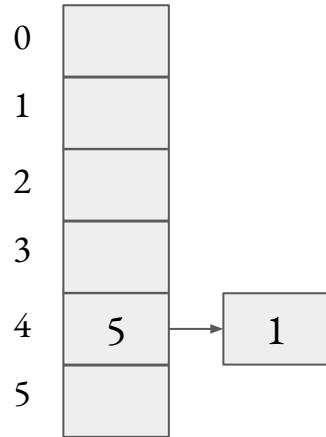
# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



**key**

**index (u)**

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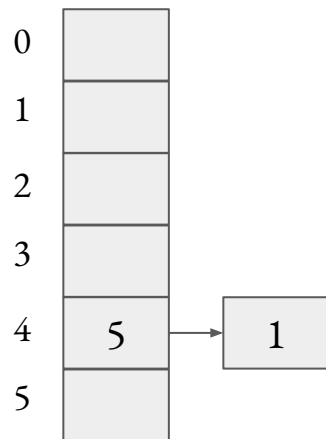
# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



**key**

**index (u)**

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

1

$$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$$

6

$$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$$

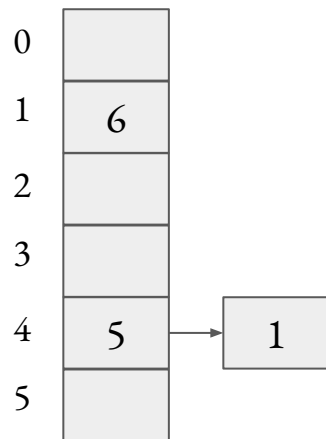
# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



**key**

**index (u)**

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

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6

$$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$$

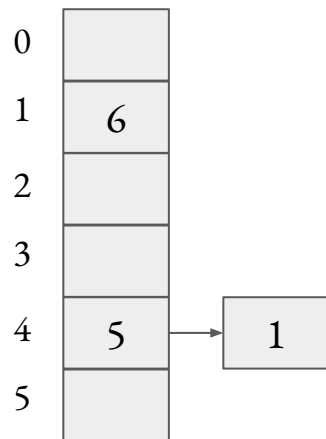
# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



**key**

**index (u)**

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

1

$$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$$

6

$$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$$

11

$$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$$

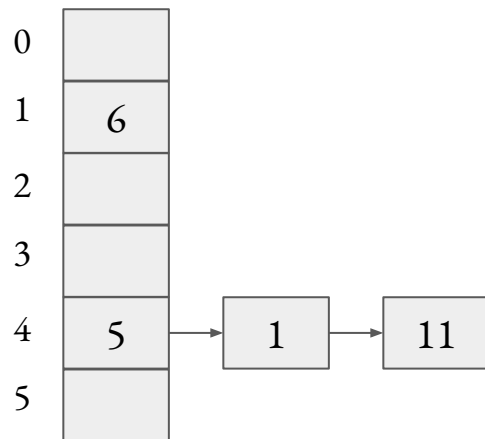
# HASH TABLES

## Closed addressing – Chaining

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



**key**

**index (u)**

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

1

$$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$$

6

$$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$$

11

$$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$$

# LINEAR PROBE

# HASH TABLES

## Open addressing – Linear Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	
2	
3	
4	5
5	

**key**

5

**index (u)**

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

**probe**

1

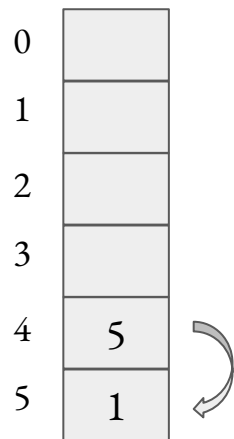
# HASH TABLES

## Open addressing – Linear Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



key	index (u)	probe
5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2



# HASH TABLES


## Open addressing – Linear Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	6
2	
3	
4	4
5	1



key	index (u)	probe
5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1

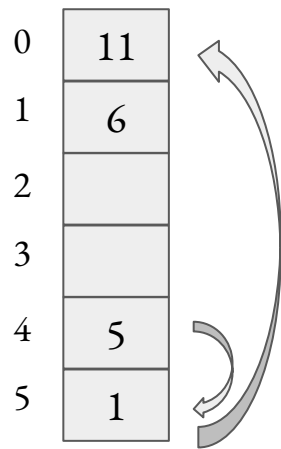
# HASH TABLES

## Open addressing – Linear Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



key	index (u)	probe
5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	3

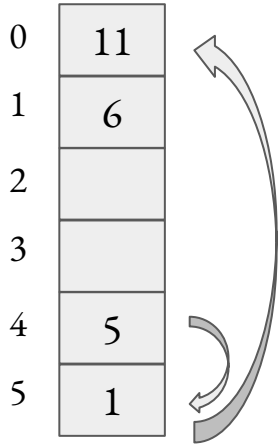
# HASH TABLES

## Open addressing – Linear Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



*probing*  
 $(u + i) \% M$   
 $i = 0 \text{ to } M-1$

key	index (u)	probe
5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	3

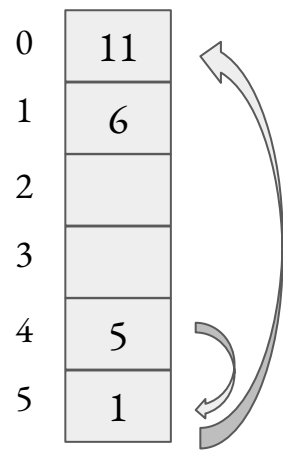
# HASH TABLES

## Open addressing – Linear Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$



*probing*

$$(u + i) \% M$$

$i = 0 \text{ to } M-1$

key

index (u)

probe

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

1

1

$$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$$

2

6

$$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$$

1

11

$$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$$

3

Order of elements in hash table: 11, 6, \_\_, \_\_, 5, 1

# QUADRATIC PROBE

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	
2	
3	
4	
5	

$$(u + i^2) \% M$$

$i = 0 \text{ to } M-1$

key	index (u)	probe
5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	
1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

		key	index (u)	probe
0		5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
2		6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5			
5				

*Order of elements in hash table:*

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## Open addressing – Quadratic Probe

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$$M = 6$$

0	
1	
2	
3	
4	5
5	

$$(u + i^2) \% M$$

$i = 0 \text{ to } M-1$

key	index (u)	probe
5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
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1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
2	$(4 + 0^2) \% 6 = 4$	6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5			
5				

*Order of elements in hash table:*

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## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

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		key	index (u)	probe
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1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
2		6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5			
5				

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

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1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
2	$(4 + 0^2) \% 6 = 4$ $(4 + 1^2) \% 6 = 5$	6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5			
5	1			

*Order of elements in hash table:*

# HASH TABLES

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		$(u + i^2) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	probe
0			5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	6		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
2			6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
3			11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5				
5	1				

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

		$(u + i^2) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	probe
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1	6		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
2			6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
3			11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5				
5	1				

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

		$(u + i^2) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	probe
0			5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	6	$(4 + 0^2) \% 6 = 4$	1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
2			6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
3			11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5				
5	1				

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

		$(u + i^2) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	probe
0			5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	6	$(4 + 0^2) \% 6 = 4$	1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
2		$(4 + 1^2) \% 6 = 5$	6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
3			11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5				
5	1				

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

		$(u + i^2) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	probe
0			5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	6	$(4 + 0^2) \% 6 = 4$	1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
2		$(4 + 1^2) \% 6 = 5$	6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
3		$(4 + 2^2) \% 6 = 2$	11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5				
5	1				

*Order of elements in hash table:*



# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

		$(u + i^2) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	probe
0			5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	1
1	6	$(4 + 0^2) \% 6 = 4$	1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	2
2	11	$(4 + 1^2) \% 6 = 5$	6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	1
3		$(4 + 2^2) \% 6 = 2$	11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	3
4	5				
5	1				

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Quadratic Probe

keys = 5, 1, 6, 11

$$h(k) = 3k + 1$$

$$M = 6$$

0	
1	6
2	11
3	
4	5
5	1

$$(u + i^2) \% M$$

$i = 0 \text{ to } M-1$

key

index (u)

probe

5

$$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$$

1

1

$$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$$

2

6

$$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$$

1

11

$$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$$

3

Order of elements in hash table: \_\_, 6, 11, \_\_, 5, 1

# DOUBLE HASH

# HASH TABLES

## Open addressing – Double Hash

keys = 5, 1, 6, 11

$$h_1(k) = 3k + 1$$

$$M = 6$$

$$h_2(k) = 2k + 1$$

	$(u + v \cdot i) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	index (v)
0		5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	
1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
2		6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4				
5				

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Double Hash

keys = 5, 1, 6, 11

$$h_1(k) = 3k + 1$$

$$M = 6$$

$$h_2(k) = 2k + 1$$

			key	index (u)	index (v)
0			5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	
1			1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
2			6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3			11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4					
5					

*Order of elements in hash table:*

# HASH TABLES

## Open addressing – Double Hash

keys = 5, 1, 6, 11

$$h_1(k) = 3k + 1$$

$$M = 6$$

$$h_2(k) = 2k + 1$$

	$(u + v \cdot i) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	index (v)
0		5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	
1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
2		6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5			
5				

*Order of elements in hash table:*

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## Open addressing – Double Hash

keys = 5, 1, 6, 11

$$h_1(k) = 3k + 1$$

$$M = 6$$

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	$(u + v \cdot i) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	index (v)
0		5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	
1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	
2		6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5			
5				

*Order of elements in hash table:*

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## Open addressing – Double Hash

keys = 5, 1, 6, 11

$$h_1(k) = 3k + 1$$

$$M = 6$$

$$h_2(k) = 2k + 1$$

	$(u + v \cdot i) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	index (v)
0		5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	
1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	$(2 \times 1 + 1) \% 6 = 3$
2		6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4	5			
5				

*Order of elements in hash table:*



# HASH TABLES

## Open addressing – Double Hash

keys = 5, 1, 6, 11

$$h_1(k) = 3k + 1$$

$$M = 6$$

$$h_2(k) = 2k + 1$$

	$(u + v \cdot i) \% M$ $i = 0 \text{ to } M-1$	key	index (u)	index (v)
0		5	$(3 \times 5 + 1) \% 6 = 16 \% 6 = 4$	
1		1	$(3 \times 1 + 1) \% 6 = 4 \% 6 = 4$	$(2 \times 1 + 1) \% 6 = 3$
2	$(4 + 3 \cdot 0) \% 6 = 4$	6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
3		11	$(3 \times 11 + 1) \% 6 = 34 \% 6 = 4$	
4				
5				

*Order of elements in hash table:*

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## Open addressing – Double Hash

keys = 5, 1, 6, 11

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2	$(4 + 3 \times 0) \% 6 = 4$ $(4 + 3 \times 1) \% 6 = 1$	6	$(3 \times 6 + 1) \% 6 = 19 \% 6 = 1$	
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