# Data Structures & Algorithms

## 13: Minimum Spanning Tree

### Part - I

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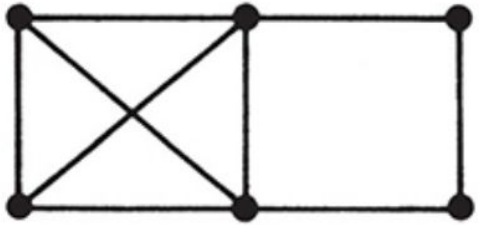
# Minimum Spanning Tree (MST)

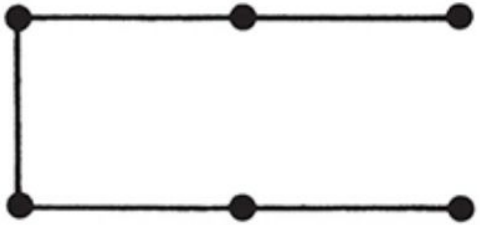
# Minimum Spanning Tree

## Spanning Tree

A subgraph **T** of a connected graph **G** is called a spanning tree of **G** if **T** is a tree and **T** includes all the vertices of **G**.

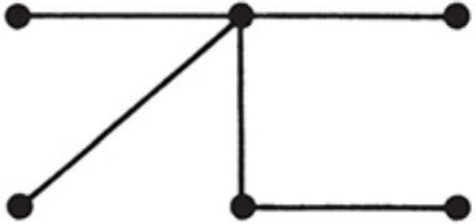


*G*



$T_1$

$T_2$

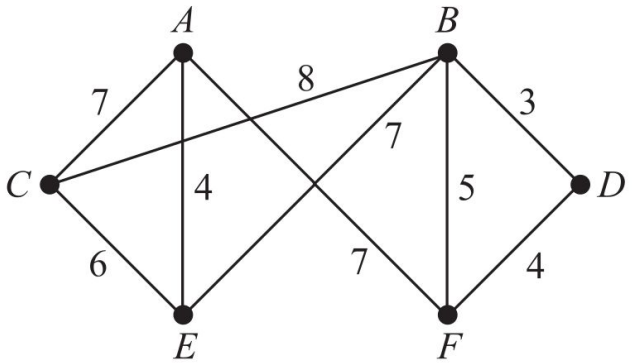


$T_3$

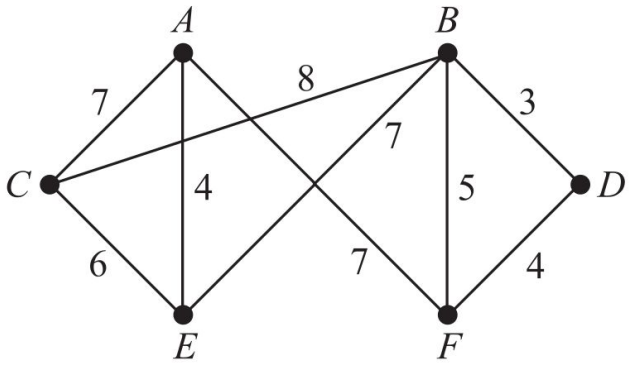
# Minimum Spanning Tree

## Minimum Spanning Tree (MST)

Suppose ***G*** is a *connected weighted graph*. That is, each edge of ***G*** is assigned a nonnegative number called the *weight* of the edge. Then any spanning tree ***T*** of ***G*** is assigned a total weight obtained by adding the weights of the edges in ***T***. A *minimal spanning tree* of ***G*** is a spanning tree whose *total weight* is as small as possible.



# Minimum Spanning Tree




- A town has a set of houses and a set of roads.
- A road connects 2 and only 2 houses.
- A road connecting houses $u$ and $v$ has a repair cost $w(u, v)$.
- ***Goal:*** Repair enough (and no more) roads such that

  1. everyone stays connected: can reach every house from all other houses, and
  2. total repair cost is minimum.

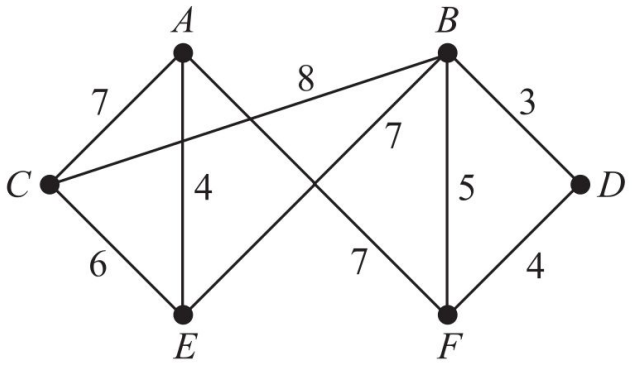
# Minimum SPanning Tree

## A simple approach to solve MST

Given a connected weighted graph **G** with **n** vertices.

- **Step 1:** Arrange the edges of G in the order of decreasing weights.
- **Step 2:** Proceed sequentially, delete each edge that does not disconnect the graph until n-1 edges remain.
- **Step 3:** Exit

# Minimum SPanning Tree



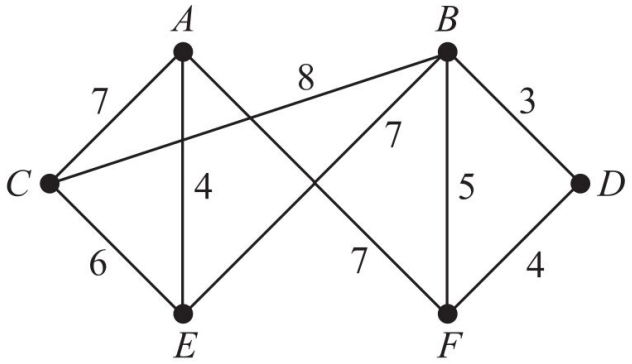

First we order the edges by decreasing weights, and then we successively delete edges without disconnecting Q until five edges remain. This yields the following data:

| Edges | $BC$ | $AF$ | $AC$ | $BE$ | $CE$ | $BF$ | $AE$ | $DF$ | $BD$ |
|---|---|---|---|---|---|---|---|---|---|
| Weight | 8 | 7 | 7 | 7 | 6 | 5 | 4 | 4 | 3 |
| Delete | Yes | Yes | Yes | No | No | Yes | | | |

Thus the minimal spanning tree of $Q$ which is obtained contains the edges

$$BE, \quad CE, \quad AE, \quad DF, \quad BD$$

# Minimum SPanning Tree

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