Data Structures & Algorithms

09: Graphs; Part-I

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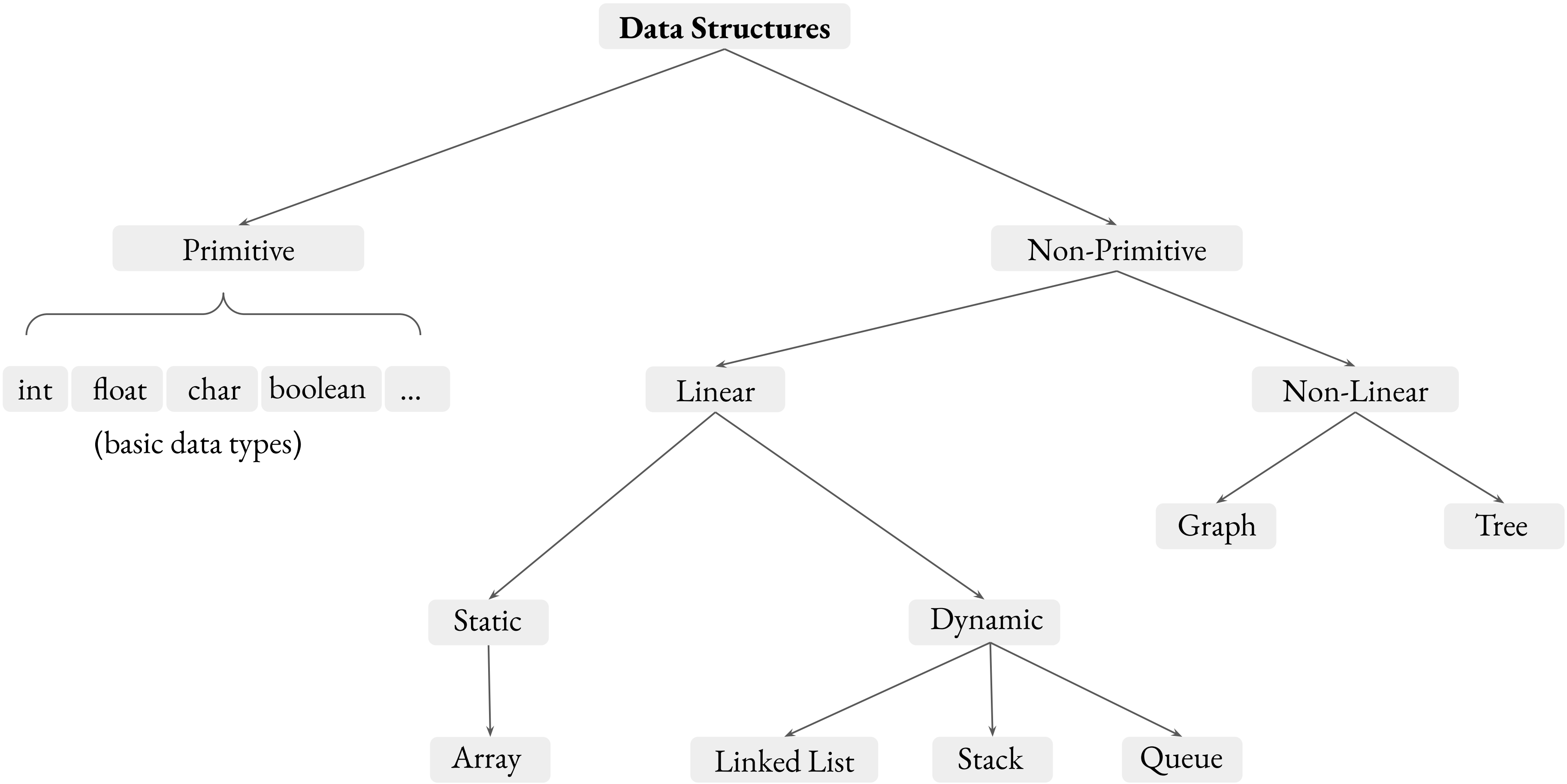

Data Structures
Primitive
Non-Primitive
int
float
char
boolean
...
(basic data types)
Linear
Non-Linear
Graph
Tree
Static
Dynamic
Array
Linked List
Stack
Queue

GRAPHS

Graphs, directed graphs, trees and binary trees are important data structures in the field of Computer Science.

Graphs

A graph G consists of two things:

(i) A set $V = V(G)$ whose elements are called vertices, points, or nodes of G.

(ii) A set $E = E(G)$ of unordered pairs of distinct vertices called edges of G.

We denote such a graph by $G(V, E)$ when we want to emphasize the two parts of G.

G(V, E) where:

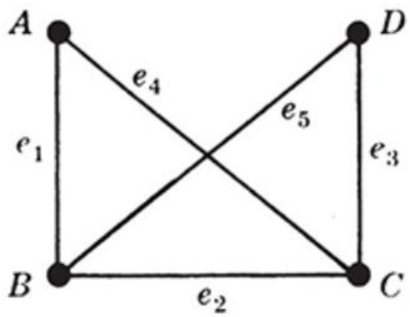
(i) V consists of vertices: {A, B, C, D}

(ii) E consists of edges: $\{e_1, e_2, e_3, e_4, e_5\}$

$e_1 = \{A, B\}$, $e_2 = \{B, C\}$,

$e_3 = \{C, D\}$, $e_4 = \{A, C\}$,

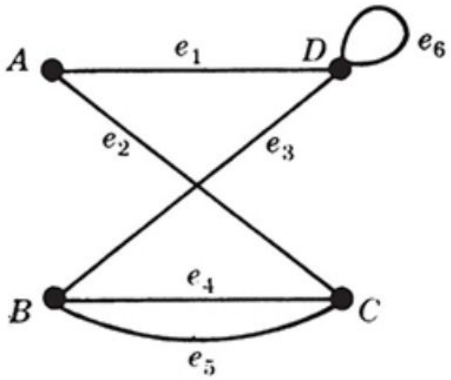
$e_5 = \{B, D\}$.



Multigraphs

A **multigraph** *G(V, E)* also contain set *V* of vertices and set *E* of edges, except that *E* may contain:

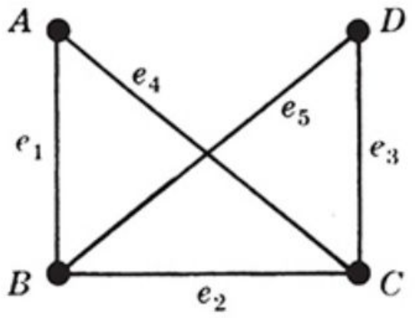
- **multiple edges**, i.e., edges connecting the same endpoints, and
- **one or more loops**, i.e., an edge whose endpoints are the same vertex.



Multigraph

In the example, $V = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

$e_1 = \{A, D\}$ $e_2 = \{A, C\}$ $e_3 = \{B, D\}$
$e_4 = \{B, C\}$ $e_5 = \{B, C\}$ $e_6 = \{D, D\}$



Graph

Note:

A **graph** is a **multigraph** without *multiple edges or loops*.

GRAPHS

Trivial Graph

A graph with one vertex and no edges

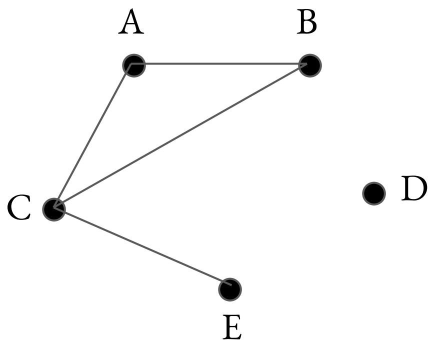
Empty or Null graph

Graph with no vertices and no edges.

Isolated vertex

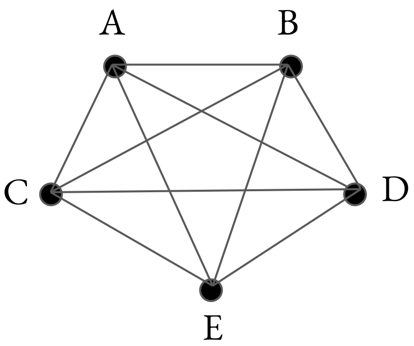
A vertex which do not belong to any edges.

D is isolated vertex in the example shown right.



Graphs

Assume *G(V, E)* is a **graph** which has five vertices.
What is the maximum number m of edges in E?

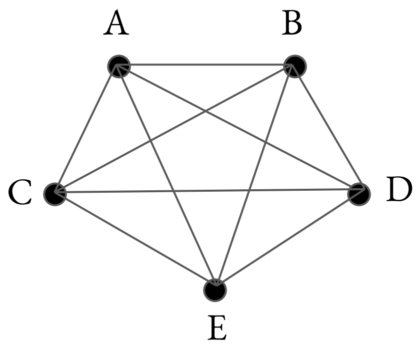


Graphs

Assume *G(V, E)* is a **graph** which has five vertices.

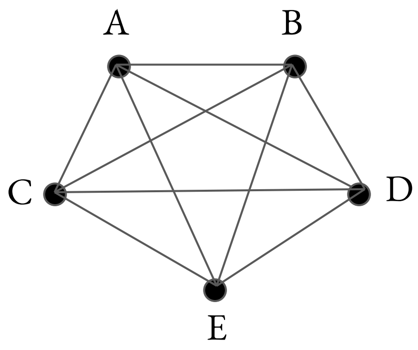
What is the maximum number m of edges in E?

m = 10 edges



$$C(n, 2) = \binom{n}{2} = \frac{n(n-1)}{2}$$

Graphs



Assume *G(V, E)* is a **graph** which has five vertices.

What is the maximum number m of edges in E?

m = 10 edges

$$C(n, 2) = \binom{n}{2} = \frac{n(n-1)}{2}$$

Assume *G(V, E)* is a **multigraph** which has five vertices.

What is the maximum number m of edges in E?

m cannot be calculated.

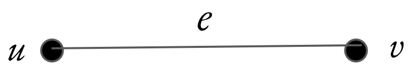
Multigraphs can contain finite or infinite number of edges and loops.

GRAPHS

Adjacency and incidence

$e = \{u, v\}$ is an edge.

- u and v are adjacent vertices (they are endpoints of e).
- e is incident on u and on v.



Degree of a vertex

$\deg(v)$ = number of edges incident on v

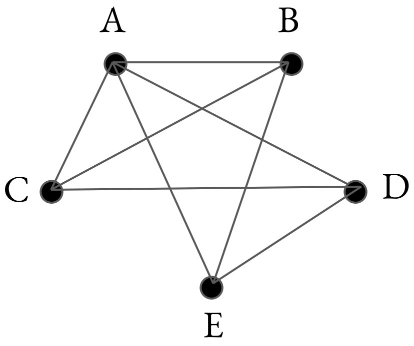
Parity of vertex

The parity of vertex v is said to be odd or even according as $\deg(v)$ is odd or even.

Note:

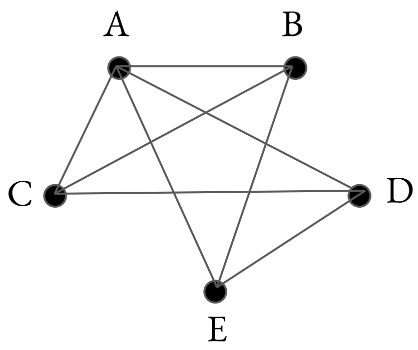
- The sum of degrees of the vertices of a graph is equal to twice the number of edges.

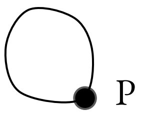
Graphs



deg(A) = 4	parity(A) = even
deg(B) = 3	parity(B) = odd
deg(C) = 3	parity(C) = odd
deg(D) = 3	parity(D) = odd
deg(E) = 3	parity(E) = odd

Graphs





deg(A) = 4	parity(A) = even
deg(B) = 3	parity(B) = odd
deg(C) = 3	parity(C) = odd
deg(D) = 3	parity(D) = odd
deg(E) = 3	parity(E) = odd

deg(P) = 2	parity(P) = even

Subgraph

Consider a graph $\mathbf{G} = \mathbf{G}(\mathbf{V}, \mathbf{E})$. A graph $\mathbf{H} = \mathbf{H}(\mathbf{V'}, \mathbf{E'})$ is called a subgraph of $\mathbf{G}$ if the vertices and edges of $\mathbf{H}$ are contained in the vertices and edges of $\mathbf{G}$, that is, if $\mathbf{V'} \subseteq \mathbf{V}$ and $\mathbf{E'} \subseteq \mathbf{E}$.

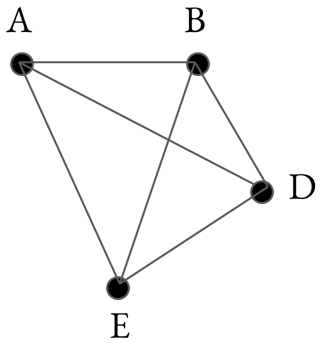
In particular:

(i) A subgraph $\mathbf{H}(\mathbf{V'}, \mathbf{E'})$ of $\mathbf{G}(\mathbf{V}, \mathbf{E})$ is called the subgraph induced by its vertices $\mathbf{V'}$ if its edge set $\mathbf{E'}$ contains all edges in $\mathbf{G}$ whose endpoints belong to vertices in $\mathbf{H}$.

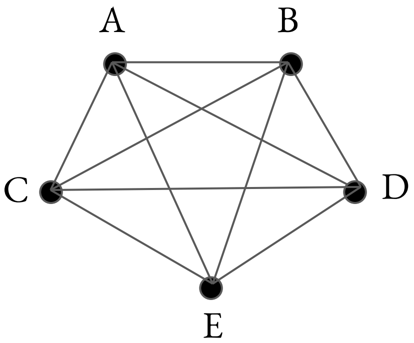
(ii) If $\boldsymbol{v}$ is a vertex in $\mathbf{G}$, then $\mathbf{G} - \boldsymbol{v}$ is the subgraph of $\mathbf{G}$ obtained by deleting $\boldsymbol{v}$ from $\mathbf{G}$ and deleting all edges in $\mathbf{G}$ which contain $\boldsymbol{v}$.

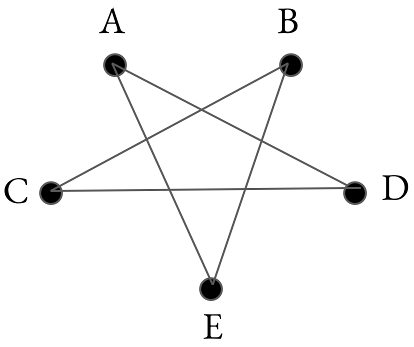
(iii) If $\boldsymbol{e}$ is an edge in $\mathbf{G}$, then $\mathbf{G} - \boldsymbol{e}$ is the subgraph of $\mathbf{G}$ obtained by simply deleting the edge $\boldsymbol{e}$ from $\mathbf{G}$.

Graphs

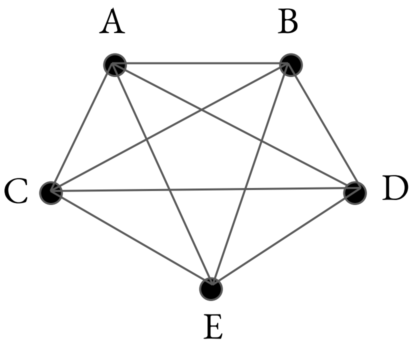


$\subseteq$





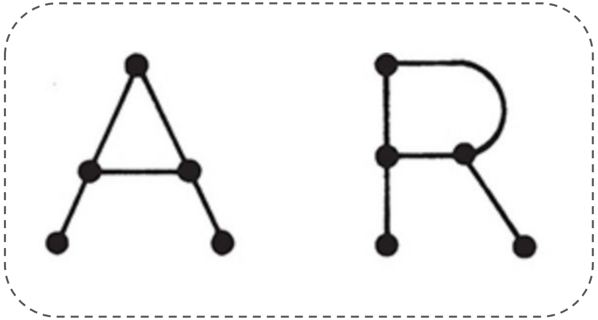
$\subseteq$

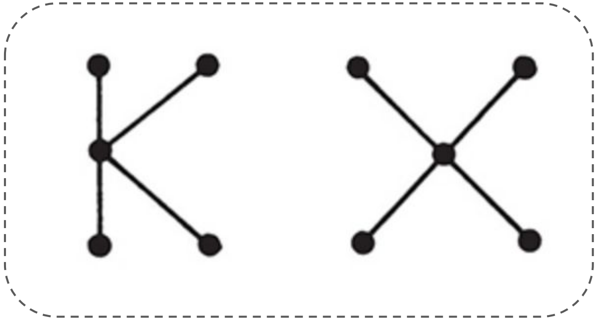


GRAPHS

Isomorphic Graphs

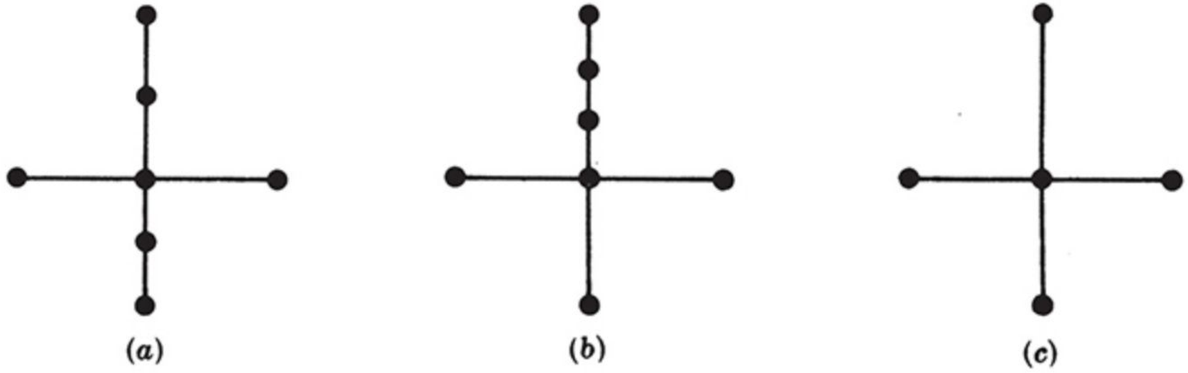
Graphs $\mathbf{G(V, E)}$ and $\mathbf{G^*(V^*, E^*)}$ are said to be **isomorphic** if there exists a one-to-one correspondence $\mathbf{f : V \to V^*}$ such that $\{u, v\}$ is an edge of $\mathbf{G}$ if and only if $\{f(u), f(v)\}$ is an edge of $\mathbf{G^*}$.





Homeomorphic Graphs

Given any graph **G**, we can obtain a new graph by dividing an edge of **G** with additional vertices.
Two graphs **G** and **G*** are said to homeomorphic if they can be obtained from the same graph or isomorphic graphs by this method.



(a) and *(b)* are not isomorphic but they are homeomorphic since each can be obtained from *(c)* by adding appropriate vertices.

Paths

A **path** in a multigraph **G** consists of an alternating sequence of vertices and edges of the form

$$v_0, e_1, v_1, e_2, v_2, \ldots, e_{n-1}, v_{n-1}, e_n, v_n$$

where each edge $\boldsymbol{e_i}$ contains the vertices $\boldsymbol{v_{i-1}}$ and $\boldsymbol{v_i}$ (which appear on the sides of $\boldsymbol{e_i}$ in the sequence).

The number $\boldsymbol{n}$ of edges is called the **length** of the path.

When there is no ambiguity, we denote a path by its sequence of vertices $(v_0, v_1, \ldots, v_n)$.

GRAPHS

The path is said to be **closed** if $\boldsymbol{v_0} = \boldsymbol{v_n}$.
Otherwise, we say the path is from $\boldsymbol{v_0}$ to $\boldsymbol{v_n}$ or between $\boldsymbol{v_0}$ and $\boldsymbol{v_n}$, or connects $\boldsymbol{v_0}$ to $\boldsymbol{v_n}$.

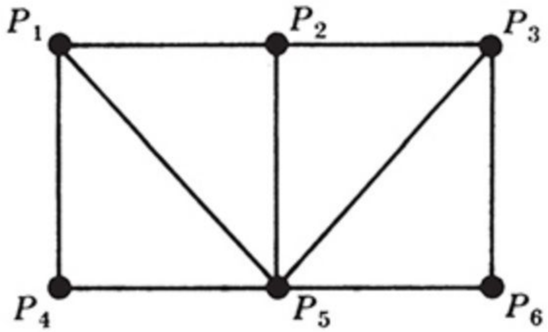
A **trail** is when no edge is used twice.

A **simple path** is a path in which all vertices are distinct.

A **cycle** is a closed path of **length 3 or more** in which all vertices are distinct except $\boldsymbol{v_0} = \boldsymbol{v_n}$.

GRAPHS

Consider the following sequence:



$$\alpha = (P_4, P_1, P_2, P_5, P_1, P_2, P_3, P_6), \quad \beta = (P_4, P_1, P_5, P_2, P_6),$$
$$\gamma = (P_4, P_1, P_5, P_2, P_3, P_5, P_6), \quad \delta = (P_4, P_1, P_5, P_3, P_6).$$

α is a path from P_4 to P_6 and not a trail since the edge $\{P_1, P_2\}$ is used twice.

β is not a path since there is no edge $\{P_2, P_6\}$.

γ is a trail since no edge is used twice; but it is not a simple path since the vertex P_5 is used twice.

δ is a simple path from P_4 to P_6.