**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Sol:

the work begin after 10 mins, so the average time increases from 45 min to 55 min for normal distribution :

Z=(X-mean)/Standard deviation

Z=(60-55)/8

Z=0.625

M

In R software for probability finding we use function called pnorm

1. pnorm(0.625)

= 0.2659



1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans :

Mean=38

SD=6

Z score = (Value- Mean)/SD

ZScore for 44 = (44-38)/6=1 =>84.13%

==>people above 44 age =100-84.13=15.87%==63 out of 400

ZScore for 38 = (38-38)/6=0==>50%

Hence people between 38 and 44 age=84.13 -50=34.13%==137 out of 400

Hence more employees at the processing center are older than 44 than between 38 and 44 is False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

Z Score for 30 =(30-38)/6=-1.33 =>9.15% ==36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees

True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

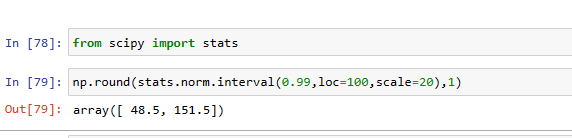
Ans :

The normal distribution has its link with the central limit theorem , which states that any large sum of independent identically distribution random variables are approximately normal then (X1+X2) and 2X1 tends to have normal distribution only if X1 and X2 are iid and n is large

The difference between 2x1 and (x1 + x2) is the magnitude they hold of two different sample subsets (x1 and x2) from the same source (population) x1 and x2 can be different subset of sample from similar source( population ) but if *X1* ~ *N*(μ, σ2) then 2*X1* ~ *N*(2μ, 4σ2) if *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are iid normal variables then (x1+x2) ~ *N*(2μ, 2σ2) hence 2x1-(x1+x2) ~ *N*(2μ-2μ, 4σ2 + 2σ2) the distribution remains same for every sample subset of similar source , it tends to fall under Normal distribution and slight deviations in parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans : D



1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee ranges between(9.9 to 98.1) crore rupees , 95% of the time for the annual profit of the company .

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: the 5th percentile of the profit for the company is 17 crore rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: the division #2 (profit 2 ~ N (7,42)) has larger probability of making a loss in a given year