**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

## Sol: 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

## Sol: Mean = 38, SD = 6, Z score = (value – mean)/SD

## Z score for age 44 =(44-38)/6

## =6/6 =1

## = (84.13% according to standard normal table)

## Z score for age 38 =(38-38)/6= 0

## =50% according to standard normal table

## People above 44 age = 100 – 84.13

## =15.87%

## So the probability of the number of employees between 38-44 years of age is = 84.13 – 50 = 34.13% (137 out of 400)

## Hence the statement is **False**.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

## Sol: Z score for age 30 =(30-38)/6

## =-1.333

## =91.76% according to the standard normal table.

## Hence the statement is **TRUE**.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.
2. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
3. 90.5, 105.9
4. 80.2, 119.8
5. 22, 78
6. 48.5, 151.5
7. 90.1, 109.9

## Sol: The probability of getting a value between a and b should be 0.99

## So the probability of going wrong is 0.01 that is (1-0.99)

## The probability towards left from a = 0.01/2= 0.005

## The probability towards Right from b = 0.01/2= 0.005

## For probability 0.005 the z value according to the z table is -2.57

## Z(-0.005)\*20+100=-(-2.57)\*20+100=151.4

## Z(0.005)\*20+100=-(2.57)\*20+100=48.6

## **So the option D 48.5, 151.5 is the right option**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains a 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

## Sol- import numpy as np

## From spicy import stats

## From spicy.stats import norm

Mean profits from both the divisions = mean 1 + mean 2

## Mean =5+7

## Print(‘mean profit is Rs’, mean\*45, ‘million’)

O/P- Mean Profit is Rs 540 Million

Variance of profits from both the divisions SD^2 = SD1^2 + SD2^2

## SD = np.sqrt((9)+(16))

## print('Standard Deviation is Rs', SD\*45,'Million')

O/P- Standard Deviation is Rs 225.0 Million

1. Specify a Rupee range (centered on the mean) such that it contains a 95% probability for the annual profit of the company.

## Sol- print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

O/P- Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

## Sol- X= 540+(-1.645)\*(225)

## print('5th percentile of profit (in Million Rupees) is',np.round(X,))

O/P- 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

## Sol- 1 making loss p(x<0)

## stats.norm.cdf(0 ,5, 3)

O/P- 0.0477903522728147

## 2- making loss p(x<0)

## stats.norm.cdf(0 ,7, 4)

O/P- 0.040059156863817086