Integer Programming Assignment

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Installing and loading the required libraries

```
#install.packages("lpSolve")
library(lpSolve)
#install.packages("lpSolveAPI")
library(lpSolveAPI)
```

This table below provides an estimate of the number of workers needed each day of the week.

```
Day_of_the_week = c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")
Workers_required = c(18,27,22,26,25,21,19)
Worker_per_day = data.frame(Day_of_the_week, Workers_required)
Worker_per_day
```

```
##
     Day_of_the_week Workers_required
## 1
               Sunday
## 2
               Monday
                                     27
## 3
              Tuesday
                                     22
## 4
           Wednesday
                                     26
## 5
             Thursday
                                     25
## 6
                                     21
               Friday
## 7
             Saturday
                                      19
```

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is \$750 per week. Workers working on Saturday or Sunday receive an additional \$25 per day. The possible shifts and salaries for package handlers are:

```
Shifts = c(1,2,3,4,5,6,7)

Days_off = c("Sunday & Monday", "Monday & Tuesday", "Tuesday & Wednesday", "Wednesday & Thursday", "Thursday", "Thursday", "Thursday", "Thursday", "Thursday", "Thursday", "Thursday", "Thursday", "Thursday", "Wednesday & Thursday", "Thursday", "Thursday", "Thursday", "Wednesday & Thursday", "Thursday", "Thursday", "Wednesday & Thursday", "Thursday", "Thursday", "Wednesday & Thursday", "Thursday", "Thursday & Wednesday", "Wednesday & Thursday", "Thursday", "Thursday & Wednesday", "Wednesday & Thursday", "Thursday & Wednesday", "Wednesday & Thursday", "Thursday & Wednesday", "Wednesday & Thursday", "Thursday", "Thursday & Wednesday & Thursday", "Thursday & Wednesday", "Wednesday & Thursday", "Thursday & Wednesday", "Wednesday & Thursday", "Thursday & Wednesday & Thursday & Thursday & Thursday & Wednesday & Thursday & Thursday & Wednesday & Thursday & T
```

```
##
     Shifts
                         Days_off Wages
## 1
          1
                 Sunday & Monday
                                    775
## 2
                Monday & Tuesday
                                    800
          3 Tuesday & Wednesday
## 3
                                    800
## 4
          4 Wednesday & Thursday
                                    800
                                    800
## 5
          5
               Thursday & Friday
## 6
               Friday & Saturday
                                    775
               Saturday & Sunday
## 7
                                    750
```

```
Defining the decision variables:
w1 = no.of workers in shift1
w2 = no.of workers in shift2
w3 = no.of workers in shift3
w4 = no.of workers in shift4
w5 = no.of workers in shift5
w6 = no.of workers in shift6
w7 = no.of workers in shift?
The Objective function is
Z = 775(w1) + 800(w2) + 800(w3) + 800(w4) + 800(w5) + 775(w6) + 750(w7)
Constraints:
Sunday: 0(w1)+1(w2)+1(w3)+1(w4)+1(w5)+1(w6)+0(w7) >= 18
Monday: 0(w1)+0(w2)+1(w3)+1(w4)+1(w5)+1(w6)+1(w7) >= 27
Tuesday: 1(w1)+0(w2)+0(w3)+1(w4)+1(w5)+1(w6)+1(w7) >= 22
Wednesday: 1(w1)+1(w2)+0(w3)+0(w4)+1(w5)+1(w6)+1(w7) >= 26
Thursday: 1(w1)+1(w2)+1(w3)+0(w4)+0(w5)+1(w6)+1(w7) >= 25
Friday: 1(w1)+1(w2)+1(w3)+1(w4)+0(w5)+0(w6)+1(w7) >= 21
Saturday: 1(w1)+1(w2)+1(w3)+1(w4)+1(w5)+0(w6)+0(w7) >= 19
Non-negativity Constraints: w1, w2, w3, w4, w5, w6, w7 >= 0
solvelp = make.lp(7, 7)
#Setting the objective function
set.objfn(solvelp, c(775, 800, 800, 800, 800, 775, 750))
#Setting the objective function to "min"
lp.control(solvelp, sense = 'min')
## $anti.degen
## [1] "fixedvars" "stalling"
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"
                                       "dynamic"
                                                       "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
```

```
## [1] -1e+30
##
## $epsilon
##
                                         epsint epsperturb
         epsb
                    epsd
                               epsel
                                                              epspivot
##
        1e-10
                   1e-09
                               1e-12
                                          1e-07
                                                     1e-05
                                                                 2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
## $infinite
## [1] 1e+30
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
      1e-11
##
             1e-11
##
## $negrange
## [1] -1e+06
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"
                  "adaptive"
## $presolve
## [1] "none"
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"
                     "equilibrate" "integers"
##
## $sense
## [1] "minimize"
##
## $simplextype
## [1] "dual"
                "primal"
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
Adding constraints
set.row(solvelp,1,c(0,1,1,1,1,1,0))
set.row(solvelp,2,c(0,0,1,1,1,1,1))
set.row(solvelp,3,c(1,0,0,1,1,1,1))
```

```
set.row(solvelp,4,c(1,1,0,0,1,1,1))
set.row(solvelp,5,c(1,1,1,0,0,1,1))
set.row(solvelp,6,c(1,1,1,1,0,0,1))
set.row(solvelp,7,c(1,1,1,1,1,0,0))
```

Adding the right hand side coefficients.

```
rhs<-c(18,27,22,26,25,21,19)
set.rhs(solvelp,rhs)
```

Setting the Constraint types

```
set.constr.type(solvelp,c(">=", ">=", ">=", ">=", ">=", ">=", ">="))
set.bounds(solvelp,lower = rep(0,7))
```

Giving row names

```
lp.rownames <- c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")
dimnames(solvelp) = list(lp.rownames,(1:7))</pre>
```

Column names

```
lp.colnames <- c("w1","w2","w3","w4","w5","w6","w7")
set.type(solvelp, columns = 1:7, type = "integer")
solvelp</pre>
```

```
## Model name:
                                         7
##
              1
                   2
                       3
                            4
                                5
                                     6
                                   775
## Minimize 775 800 800 800 800
                                       750
## Sunday
             0 1
                       1
                                1
                                                18
                                         0
## Monday
              0
                   0
                       1
                            1
                                1
                                     1
                                         1
                                            >=
                                                27
## Tuesday
              1
                   0
                       0
                            1
                                1
                                         1
                                            >=
                                                22
                                     1
## Wednesday
                       0
                            0
                                1
                                            >=
                                                26
              1
                  1
                                     1
                                         1
## Thursday
                            0
                                0
                                                25
              1
                 1
                     1
                                     1
                                         1 >=
## Friday
                      1
                           1
                                0
                                     0
                                         1 >= 21
              1
                  1
## Saturday
                                1
              1
                  1
                       1
                            1
                                         0
## Kind
            Std Std Std Std Std Std
## Type
            Int Int
                     Int Int Int Int Int
                     Inf
## Upper
                                   Inf
                                       Inf
            Inf Inf
                          Inf
                              Inf
## Lower
             0
                  0
                       0
                           0
                                0
                                     0
```

Solving the LP Problem

```
solve(solvelp)
```

```
## [1] 0
```

Finding the objective function of the LP Problem

```
get.objective(solvelp)
## [1] 25675
round(get.variables(solvelp))
## [1] 2 4 5 0 8 1 13
get.constraints(solvelp)
## [1] 18 27 24 28 25 24 19
Availability = matrix(c(0,4,5,0,8,1,0,
                 0,0,5,0,8,1,13,
                 2,0,0,0,8,1,13,
                 2,4,0,0,8,1,13,
                 2,4,5,0,0,1,13,
                 2,4,5,0,0,0,13,
                 2,4,5,0,8,0,0),ncol=7,byrow=TRUE)
colnames(Availability) = c("Shift1", "Shift2", "Shift3", "Shift4", "Shift5", "Shift6", "Shift7")
rownames(Availability) = c('Sunday', 'Monday', 'Tuesday', 'Wednesday', 'Thursday', 'Friday', 'Saturday')
Availability
             Shift1 Shift2 Shift3 Shift4 Shift5 Shift6 Shift7
##
## Sunday
                  0
                         4
                                5
                                       0
                                               8
                                                      1
                                                             0
                  0
                         0
## Monday
                                5
                                       0
                                               8
                                                      1
                                                            13
## Tuesday
                  2
                         0
                                0
                                       0
                                               8
                                                      1
                                                            13
## Wednesday
                  2
                         4
                                0
                                       0
                                               8
                                                      1
                                                            13
                  2
                         4
                                5
## Thursday
                                       0
                                               0
                                                      1
                                                            13
## Friday
                  2
                         4
                                5
                                       0
                                               0
                                                      0
                                                            13
                                5
## Saturday
                                               8
                                                             0
```

This shows the availability of workers each day.

rowSums(Availability)

##	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
##	18	27	24	28	25	24	19

CONCLUSION

From the above formulation of the problem we can observe that

The total cost is \$25,675

The optimal no.of workers available each day is:

Sunday: 18

Monday: 27

 ${\it Tuesday}:\, 24$

Wednesday : 28

Thursday : 25

Friday: 24

Saturday: 19