

## QMM Assignment 1

### 1. a. Clearly define the decision variables.

In the given example, the decision variables are quantities of each type of backpack to produce per week.

a = number of collegiate backpacks to produce per week.

b = number of mini backpacks to produce per week.

P = total profit

### b. What is the objective function?

The objective function is what we are trying to maximize or minimize. In this case, Back Savers aims to maximize their profit. So, the objective function to maximize profit (p) is:

$$P = 32a + 24b$$

### c. What are the constraints?

- Material Constraint: Back Savers receive a 5000 square-foot shipment of material each week. Each collegiate requires 3 square feet while each mini requires 2 square feet. The constraint can be written as

$$3a + 2b \leq 5000$$

- Sales Forecast Constraints: The sales forecasts indicate that at most 1000 collegiates and 1200 minis can be sold per week.

$$a \leq 1000$$

$$b \leq 1200$$

- Labor constraints: Back Savers has 35 laborers that each provides 40 hours of labor per week. Each collegiate requires 45 minutes of labor and mini requires 40 minutes of labor. Converting this into minutes as

$$45a + 40b \leq 35 * 40 * 60$$

- Non-negativity constraints:  $a \geq 0$ ,  $b \geq 0$

### d. Write down the full mathematical formulation for this Linear Programming problem.

Here is the full mathematical formulation for this Linear Programming problem.

$$P = 32a + 24b$$

Constraints:

$$3a + 2b \leq 5000$$

$$a \leq 1000$$

$$b \leq 1200$$

$$45a + 40b \leq 35 * 40 * 60$$

$$a \geq 0, b \geq 0.$$

### 2. a. Define the decision variables.

$J_{S1}$  = number of small-sized units produced at plant1.


$J_{M1}$  = number of medium-sized units produced at plant1.

$J_{L1}$  = number of large-sized units produced at plant1.

$J_{S2}$  = number of small-sized units produced at plant2.

$J_{M2}$  = number of medium-sized units produced at plant2.

$J_{L2}$  = number of large-sized units produced at plant2.



$J_{S3}$  = number of small-sized units produced at plant3.  
 $J_{M3}$  = number of medium-sized units produced at plant3.  
 $J_{L3}$  = number of large-sized units produced at plant3.

**b. Formulate a linear programming model for this problem.**

To formulate a linear programming model, we need to define objective function and constraints. The objective is to maximize profit. Here is the linear programming model,

Maximize profit(P) =  $420(J_{L1} + J_{L2} + J_{L3}) + 360(J_{M1} + J_{M2} + J_{M3}) + 300(J_{S1} + J_{S2} + J_{S3})$ .

Constraints:

Capacity Constraints:

- Plant1:  $J_{L1} + J_{M1} + J_{S1} \leq 750$
- Plant2:  $J_{L2} + J_{M2} + J_{S2} \leq 900$
- Plant3:  $J_{L3} + J_{M3} + J_{S3} \leq 450$

Storage Constraints:

- Plant1:  $20J_{L1} + 15J_{M1} + 12J_{S1} \leq 13000$
- Plant2:  $20J_{L2} + 15J_{M2} + 12J_{S2} \leq 12000$
- Plant3:  $20J_{L3} + 15J_{M3} + 12J_{S3} \leq 5000$

Sales Forecast Constraints:

- Large size:  $J_{L1} + J_{L2} + J_{L3} \leq 900$
- Medium size:  $J_{M1} + J_{M2} + J_{M3} \leq 1200$
- Small size:  $J_{S1} + J_{S2} + J_{S3} \leq 750$

Non negativity Constraints:

- $J_{S1}, J_{M1}, J_{L1}, J_{S2}, J_{M2}, J_{L2}, J_{S3}, J_{M3}, J_{L3} \geq 0$

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce new products.

Percentage to avoid layoff:  $(J_{L1} + J_{M1} + J_{S1})/750 * 100$   
 $(J_{L2} + J_{M2} + J_{S2})/900 * 100$   
 $(J_{L3} + J_{M3} + J_{S3})/450 * 100$