Integer Programming Assignment

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Installing and loading the required libraries

#install.packages("lpSolve")  
library(lpSolve)  
#install.packages("lpSolveAPI")  
library(lpSolveAPI)

This table below provides an estimate of the number of workers needed each day of the week.

Day\_of\_the\_week = c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")  
Workers\_required = c(18,27,22,26,25,21,19)  
Worker\_per\_day = data.frame(Day\_of\_the\_week, Workers\_required)  
Worker\_per\_day

## Day\_of\_the\_week Workers\_required  
## 1 Sunday 18  
## 2 Monday 27  
## 3 Tuesday 22  
## 4 Wednesday 26  
## 5 Thursday 25  
## 6 Friday 21  
## 7 Saturday 19

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is $750 per week. Workers working on Saturday or Sunday receive an additional $25 per day. The possible shifts and salaries for package handlers are:

Shifts = c(1,2,3,4,5,6,7)  
Days\_off = c("Sunday & Monday", "Monday & Tuesday", "Tuesday & Wednesday", "Wednesday & Thursday", "Thursday & Friday", "Friday & Saturday", "Saturday & Sunday")  
Wages = c(775, 800, 800, 800, 800, 775, 750)  
shift\_wage = data.frame(Shifts, Days\_off, Wages)  
shift\_wage

## Shifts Days\_off Wages  
## 1 1 Sunday & Monday 775  
## 2 2 Monday & Tuesday 800  
## 3 3 Tuesday & Wednesday 800  
## 4 4 Wednesday & Thursday 800  
## 5 5 Thursday & Friday 800  
## 6 6 Friday & Saturday 775  
## 7 7 Saturday & Sunday 750

Defining the decision variables:

w1 = no.of workers in shift1

w2 = no.of workers in shift2

w3 = no.of workers in shift3

w4 = no.of workers in shift4

w5 = no.of workers in shift5

w6 = no.of workers in shift6

w7 = no.of workers in shift7

The Objective function is

Z = 775(w1) + 800(w2) + 800(w3) + 800(w4) + 800(w5) + 775(w6) + 750(w7)

Constraints:

Sunday : 0(w1)+1(w2)+1(w3)+1(w4)+1(w5)+1(w6)+0(w7) >= 18

Monday : 0(w1)+0(w2)+1(w3)+1(w4)+1(w5)+1(w6)+1(w7) >= 27

Tuesday : 1(w1)+0(w2)+0(w3)+1(w4)+1(w5)+1(w6)+1(w7) >= 22

Wednesday : 1(w1)+1(w2)+0(w3)+0(w4)+1(w5)+1(w6)+1(w7) >= 26

Thursday : 1(w1)+1(w2)+1(w3)+0(w4)+0(w5)+1(w6)+1(w7) >= 25

Friday : 1(w1)+1(w2)+1(w3)+1(w4)+0(w5)+0(w6)+1(w7) >= 21

Saturday : 1(w1)+1(w2)+1(w3)+1(w4)+1(w5)+0(w6)+0(w7) >= 19

Non-negativity Constraints: w1,w2,w3,w4,w5,w6,w7 >= 0

solvelp = make.lp(7, 7)  
#Setting the objective function  
set.objfn(solvelp, c(775, 800, 800, 800, 800, 775, 750))  
#Setting the objective function to "min"  
lp.control(solvelp, sense = 'min')

## $anti.degen  
## [1] "fixedvars" "stalling"   
##   
## $basis.crash  
## [1] "none"  
##   
## $bb.depthlimit  
## [1] -50  
##   
## $bb.floorfirst  
## [1] "automatic"  
##   
## $bb.rule  
## [1] "pseudononint" "greedy" "dynamic" "rcostfixing"   
##   
## $break.at.first  
## [1] FALSE  
##   
## $break.at.value  
## [1] -1e+30  
##   
## $epsilon  
## epsb epsd epsel epsint epsperturb epspivot   
## 1e-10 1e-09 1e-12 1e-07 1e-05 2e-07   
##   
## $improve  
## [1] "dualfeas" "thetagap"  
##   
## $infinite  
## [1] 1e+30  
##   
## $maxpivot  
## [1] 250  
##   
## $mip.gap  
## absolute relative   
## 1e-11 1e-11   
##   
## $negrange  
## [1] -1e+06  
##   
## $obj.in.basis  
## [1] TRUE  
##   
## $pivoting  
## [1] "devex" "adaptive"  
##   
## $presolve  
## [1] "none"  
##   
## $scalelimit  
## [1] 5  
##   
## $scaling  
## [1] "geometric" "equilibrate" "integers"   
##   
## $sense  
## [1] "minimize"  
##   
## $simplextype  
## [1] "dual" "primal"  
##   
## $timeout  
## [1] 0  
##   
## $verbose  
## [1] "neutral"

Adding constraints

set.row(solvelp,1,c(0,1,1,1,1,1,0))  
set.row(solvelp,2,c(0,0,1,1,1,1,1))   
set.row(solvelp,3,c(1,0,0,1,1,1,1))  
set.row(solvelp,4,c(1,1,0,0,1,1,1))   
set.row(solvelp,5,c(1,1,1,0,0,1,1))   
set.row(solvelp,6,c(1,1,1,1,0,0,1))   
set.row(solvelp,7,c(1,1,1,1,1,0,0))

Adding the right hand side coefficients.

rhs<-c(18,27,22,26,25,21,19)  
set.rhs(solvelp,rhs)

Setting the Constraint types

set.constr.type(solvelp,c(">=", ">=", ">=", ">=", ">=", ">=", ">="))  
set.bounds(solvelp,lower = rep(0,7))

Giving row names

lp.rownames <- c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")  
dimnames(solvelp) = list(lp.rownames,(1:7))

Column names

lp.colnames <- c("w1","w2","w3","w4","w5","w6","w7")  
set.type(solvelp, columns = 1:7, type = "integer")  
solvelp

## Model name:   
## 1 2 3 4 5 6 7   
## Minimize 775 800 800 800 800 775 750   
## Sunday 0 1 1 1 1 1 0 >= 18  
## Monday 0 0 1 1 1 1 1 >= 27  
## Tuesday 1 0 0 1 1 1 1 >= 22  
## Wednesday 1 1 0 0 1 1 1 >= 26  
## Thursday 1 1 1 0 0 1 1 >= 25  
## Friday 1 1 1 1 0 0 1 >= 21  
## Saturday 1 1 1 1 1 0 0 >= 19  
## Kind Std Std Std Std Std Std Std   
## Type Int Int Int Int Int Int Int   
## Upper Inf Inf Inf Inf Inf Inf Inf   
## Lower 0 0 0 0 0 0 0

Solving the LP Problem

solve(solvelp)

## [1] 0

Finding the objective function of the LP Problem

get.objective(solvelp)

## [1] 25675

round(get.variables(solvelp))

## [1] 2 4 5 0 8 1 13

get.constraints(solvelp)

## [1] 18 27 24 28 25 24 19

Availability = matrix(c(0,4,5,0,8,1,0,  
 0,0,5,0,8,1,13,  
 2,0,0,0,8,1,13,  
 2,4,0,0,8,1,13,  
 2,4,5,0,0,1,13,  
 2,4,5,0,0,0,13,  
 2,4,5,0,8,0,0),ncol=7,byrow=TRUE)  
  
colnames(Availability) = c("Shift1", "Shift2", "Shift3", "Shift4", "Shift5", "Shift6", "Shift7")  
  
rownames(Availability) = c('Sunday', 'Monday','Tuesday','Wednesday','Thursday','Friday','Saturday')  
  
Availability

## Shift1 Shift2 Shift3 Shift4 Shift5 Shift6 Shift7  
## Sunday 0 4 5 0 8 1 0  
## Monday 0 0 5 0 8 1 13  
## Tuesday 2 0 0 0 8 1 13  
## Wednesday 2 4 0 0 8 1 13  
## Thursday 2 4 5 0 0 1 13  
## Friday 2 4 5 0 0 0 13  
## Saturday 2 4 5 0 8 0 0

This shows the availability of workers each day.

rowSums(Availability)

## Sunday Monday Tuesday Wednesday Thursday Friday Saturday   
## 18 27 24 28 25 24 19

CONCLUSION

From the above formulation of the problem we can observe that

The total cost is $25,675

The optimal no.of workers available each day is:

Sunday : 18

Monday : 27

Tuesday : 24

Wednesday : 28

Thursday : 25

Friday : 24

Saturday : 19