SIG787 - MATHEMATICS OF ARTIFICIAL INTELLIGENCE

ASSIGNMENT - 3

Name: B. Priyankaa

Student Id: s224207654

Email Id: priyayj2016@gmail.co

1. (a) What type of graph does M Represent?

Graph $G = (V, E)$

Incidence Matrix $M =$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 1 & -1 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

The incidence Matrix is the representation of graph with

Vertices $V = \{a, b, c, d\}$

Edges $E = \{e_1, e_2, e_3, e_4, e_5\}$

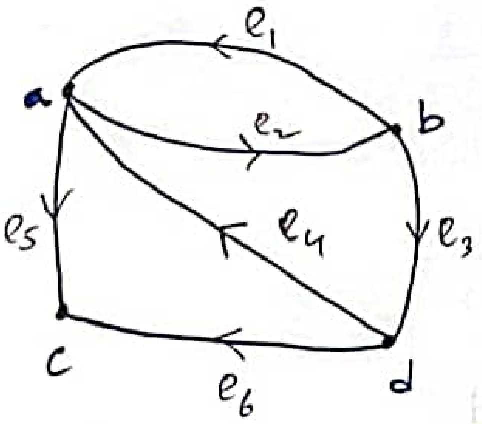
Each row of the Matrix M corresponds to a vertex and the columns correspond to the edges.

The entries of M represent the relationship between V & E

$-1 \rightarrow$ indicates edge is directed away from Vertex

$1 \rightarrow$ indicates edge is directed towards Vertex

$0 \rightarrow$ Indicates there is no path



b) Find the Adjcency Matrix A for this graph

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Edge e_1 connects to vertex b and a

Edge e_2 connects to vertex a and b

Edge e_3 connects to vertex b and d

Edge e_4 connects to vertex d and a

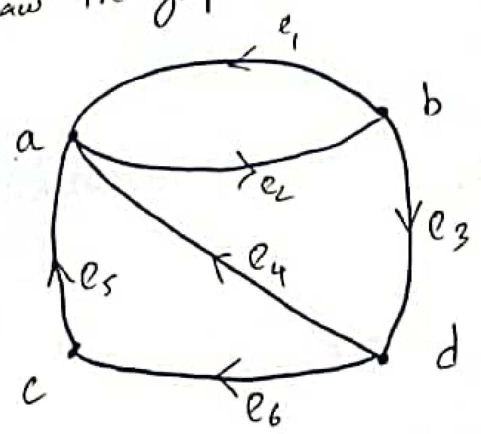
Edge e_5 connects to vertex a and c

Edge e_6 connects to vertex d and c

This gives the following Adjcency Matrix

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(c) Draw the graph:



(d) How Many paths of length 2 are there between nodes b & c

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

To find the number of paths, Multiply the Adjacency matrix the times of the length. The length specified here is 2

So compute A^2

$$A \cdot A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{array}{c} a \\ b \\ c \\ d \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & \boxed{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \\ \begin{array}{cccc} a & b & c & d \end{array} \end{array}$$

There are two paths of length 2 between b and c

(c) In terms of connectivity of graph, what is your interpretation of $tr(A^2)$?

In graph theory trace of Square of Adjacency matrix has specific interpretation related to connectivity and structure of the graph

Trace of Square of Adjacency Matrix

$$tr(A^2) = \sum_{i=1}^{n} (A^2)_{ii}$$

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Interpretation of $tr(A^2)$ → In the context of graph theory $tr(A^2)$ represents sum of number of paths of length 2 that starts and ends at the same vertex.

So,

For Vertex 1 : $A^2[0][0] = 1$ → The number of paths for the length 2 which starts and ends at same vertex

For Vertex 2 : $A^2[1][1] = 1 \rightarrow$ no. of paths $= 1$

For Vertex 3 : $A^2[2][2] = 0 \rightarrow$ No paths present

For Vertex 4 : $A^2[3][3] = 0 \rightarrow$ No paths present

Thus $tr(A^2)$ gives insights into local connectivity of the graph

- Higher the value of $tr(A^2)$, more closer walk of length 2 indicating a denser local connectivity around the vertices involved.

- Vertices 3 & 4 do not have any path of length 2 returning to itself, indicating it may be less interconnected.

) Without direct calculations, find one of the eigen values of A based on the information you can get from A. Then calculate the eigen vectors

$$tr(A) = A[0][0] + A[1][1] + A[2][2] + A[3][3]$$
$$= 0 + 0 + 0 + 0 = 0$$

Sum of the eigen values of A is 0

Determinant of A :-

A has row of all 0 (3^{rd} Row), so $det(A) = 0$

$\det(A) = 0$ hence eigen value $= 0$

Eigen Vector :

Finding the eigen vector for the corresponding eigen value 0

Eigen vector for $\lambda = 0$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Swapping R_1 & R_2

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$R_4 \rightarrow R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Swap R_3 & R_4

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 + V_4 = 0$$

$$V_2 + V_3 = 0$$

$$V_3 - V_4 = 0 \Rightarrow V_3 = V_4$$

$$V_1 + V_4 = 0 \Rightarrow V_1 = -V_4 \Rightarrow V_4 = -V_2$$

$$V_2 + V_3 = 0 \Rightarrow V_3 = -V_2$$

~~So~~

$$V_2 = -V_3 = -(-V_2) = V_2$$

$$V = \begin{bmatrix} V_2 \\ V_2 \\ -V_2 \\ -V_2 \end{bmatrix} = V_2 \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

So, The Eigenvector corresponding to eigenvalue 0 is

$$V = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

2. (a) Find the Probability distributions of having a social media account and obesity.

Individual	Having Social Media Account	Obesity
Ind 1	Yes	Yes
Ind 2	Yes	Yes
Ind 3	Yes	Yes
Ind 4	No	No
Ind 5	No	Yes

Constructing Contigency Table

	Obesity (Yes)	Obesity (No)	Total
Social Media (Yes)	3	0	3
Social Media (No)	1	1	2
	4	1	5

Probability distribution of having Social Media Account

Let it be X

$$P(X = Yes) = \frac{\text{No. of individuals with Social Media Account}}{\text{Total No. of individuals}}$$

$$P(X = Yes) = \frac{3}{5} = 0.6$$

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$$P(X = No) = \frac{\text{No. of individuals w/o Social Media Account}}{\text{Total no. of individuals}}$$

$$P(X = No) = \frac{2}{5} = 0.4$$

Probability distribution of having Obesity

Let it be Y

$$P(Y = Yes) = \frac{\text{No. of individuals with obesity}}{\text{Total no. of individuals}}$$

$$P(Y = Yes) = \frac{4}{5} = 0.8$$

$$P(Y = No) = \frac{\text{No. of individuals w/o obesity}}{\text{Total no. of individuals}}$$

$$P(Y = No) = \frac{1}{5} = 0.2$$

(b) Joint Probability Distribution

Defined as $P(X, Y)$ which is the combination of both the events X and Y.

$$P(X = Yes, Y = Yes) = 3/5 = 0.6$$

$$P(X = Yes, Y = No) = 0/5 = 0$$

$P(X = No, Y = Yes) = 1/5 = 0.2$

$P(X = No, Y = No) = 1/5 = 0.2$

(c) Are Two distributions independent

To determine if having social media account and obesity are independent, we need to check if joint probability is equal to the product of Marginal distribution for all the combination of the variables.

$$P(X, Y) = P(X) \; P(Y) \quad \text{for all combinations}$$

Probability Distribution of having Social Media account

Let it be X

$P(X = Yes) = 0.6$

$P(X = No) = 0.4$

Probability Distribution of Obesity

Let it be Y

$P(Y = Yes) = 0.8$

$P(Y = No) = 0.2$

Joint Probability Distribution

$P(X = Yes, Y = Yes) = 3/5 = 0.6$

$P(X = Yes, Y = No) = 0$

$P(X = No; Y = Yes) = 0.2$

$P(X = No, Y = No) = 0.2$

~~Chec~~ Checking the independance

$$P(X, Y) = P(X)\,P(Y)$$

$P(X = Yes, Y = Yes) = 0.6$

$P(X = Yes) = 0.6$

$P(Y = Yes) = 0.8$

$P(X)\,P(Y) = 0.6 \times 0.8 = 0.48$

$P(X, Y) \neq P(X)\,P(Y)$ Since $0.6 \neq 0.48$

So the variables are not independent

(d) Mutual information of having Social Media Account & Obesity

$$I(X:Y) = \sum_x \sum_y P(X=x, Y=y)\, \cancel{\log}\ \ln\left(\frac{P(X=x, Y=y)}{P(X=x), P(Y=y)}\right)$$

① $P(X = Yes, Y = Yes)$:

$P(X = Yes, Y = Yes) = 0.6$

$P(X = Yes) \cdot P(Y = Yes) = 0.6 \times 0.8 = 0.48$

$\log\left(\frac{0.6}{0.48}\right) = \log(1.25) \simeq 0.2231$

$0.6 \times 0.2231 = 0.13386$

② For $P(X = Yes, Y = No)$:

$P(X = Yes, Y = No) = 0$

$0 \times \log\left(\frac{0}{P(X = Yes)\, P(Y = No)}\right) = 0$

③ For

$P(X = No, Y = Yes)$:

$P(X = No, Y = Yes) = 0.2$

$P(X = No) \cdot P(Y = Yes) = 0.4 \times 0.8 = 0.32$

$\log\left(\frac{0.2}{0.32}\right) = \log(0.625) \simeq -0.470$

$I(X:Y) = 0.2 \times (-0.470)$

$= -0.094$

For

$P(X = No, Y = No)$:

$P(X = No, Y = No) = 0.2$

$P(X = No) \cdot P(Y = No) = 0.4 \times 0.2 = 0.08$

$$\log\left(\frac{0.2}{0.08}\right) = \log(2.5) = 0.916$$

$I(X:Y) = 0.2 \times 0.916 = 0.1832$

$I(X,Y) = 0.13386 + 0 + (-0.094) + 0.1832$
$= 0.22306$

Ans Mutual information between having a Social Media account and obesity is approx 0.22306

The value qualifies the amount of information gained about obesity by knowing whether someone has social media account

The value being positive indicates that there is some dependency between the two variables

(e) What is Mutual information of two independent random variables?

Mutual Information $I(X:Y)$ between two independent variables X and Y is theoritically 0.

This is because mutual information measures the amount of information gained about one random variable through observing the other.

If X and Y are independent, knowing X provides no information about Y & vice versa

~~If~~ $I(x:y) = \sum_x \sum_y P(X=x, Y=y) \ln\left(\frac{P(X=x, Y=y)}{P(X=x)\,P(Y=y)}\right)$

For independent variables :-

$$P_{X,Y}(x,y) = P_X(x) \cdot$$

Substituting it in the formula

$$I(x:y) = \sum_x \sum_y P_{X,Y}(x,y)$$

$$= \sum_x \sum_y P_{X,Y}(x,y) \ln(1)$$

$$I(x:y) = 0 \quad (\text{Since } \ln(1) = 0)$$

Mutual information is 0. This means knowing the value of one variable doesn't provide any information about the other variable.

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3. (a) Construct the joint probability distribution table, Are X & Y independent?

$$X = \{-2, -1, 0, 1, 2\}$$

Each event occurs with the probability of $1/5$.

$$Y = |X|$$

$$Y = \{0, 1, 2\}$$

The Probabilities are

$$P(Y=0) = P(X=0) = 1/5$$

$$P(Y=1) = P(X=1) + P(X=-1)$$

$$= 1/5 + 1/5 = \frac{2}{5}$$

$$P(Y=2) = P(X=2) + P(X=-2)$$

$$= \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

Joint Probability :-

For $Y=0$, we have $X=0$

$$P(X=0, Y=0) = P(X=0) = \frac{1}{5}$$

For $Y=1$, we have $X=1$ or $X=-1$

$$P(X=1, Y=1) = P(X=1) = \frac{1}{5}$$

$$P(X=-1, Y=1) = P(X=-1) = \frac{1}{5}$$

For $Y=2$, we have $X=2$ or $X=-2$

$$P(X=2, Y=2) = P(X=2) = 1/5$$

$$P(X=-2, Y=2) = P(X=-2) = \frac{1}{5}$$

Joint ~~Prob~~ Probability Distribution Table

$P(x,y)$	$y=0$	$y=1$	$y=2$	
$x=-2$	0	0	1/5	1/5
$x=-1$	0	1/5	0	1/5
$x=0$	1/5	0	0	1/5
$x=1$	0	1/5	0	1/5
$x=2$	0	0	1/5	1/5
	1/5	2/5	2/5	1

Checking If X & Y are independent :-

We need to check the following

$$P(X=x, Y=y) = P(X=x)\, P(Y=y)$$

Checking product of marginal probability against the joint probability

$$P(X=-2) \cdot P(Y=2) = \frac{1}{5} \cdot \frac{2}{5} = \frac{2}{25}$$

$P(X=-2, Y=2) = \frac{1}{5}$

$\frac{1}{5} \neq \frac{2}{25}$

The product of marginal probabilities doesn't match the joint probability for all (x, y) pair. So,

X and Y are Not independent

(b) Find $Corr(X, Y)$

$$Corr(X, Y) = \frac{Cov(x, y)}{\sigma_x \sigma_y}$$

To find the correlation between X & Y, we need to Compute the Covariance $cov(x, y)$ and Standard deviation σ_x & σ_y.

$$X = \{-2, -1, 0, 1, 2\}$$

$$Y = \{0, 1, 2\}$$

To get the Covariance we need to get the expected value of $E(x)$. $E(y)$ and $E(x, y)$.

$$Cov(X, Y) = E(X, Y) - E(X) E(Y)$$

$$E(x) = \sum_x x \cdot P(X=x)$$

$$= (-2)\frac{1}{5} + (-1)\frac{1}{5} + (0)\left(\frac{1}{5}\right) + (1)\frac{1}{5} + (2)\frac{1}{5}$$

$$= -\frac{2}{5} - \frac{1}{5} + \frac{1}{5} + \frac{2}{5}$$

$$E(x) = 0$$

$$E(y) = \sum_y y \cdot P(Y=y)$$

$$= (0)\left(\frac{1}{5}\right) + (1)\left(\frac{2}{5}\right) + (2)\left(\frac{2}{5}\right)$$

$$= 0 + \frac{2}{5} + \frac{4}{5}$$

$$E(y) = \frac{6}{5}$$

$$E(X,Y) = (-2)(2)\frac{1}{5} + (-1)(1)\frac{1}{5} + 0 + (1)(1)\frac{1}{5} + (2)(2)\frac{1}{5}$$

$$= -\frac{4}{5} - \frac{1}{5} + 0 + \frac{1}{5} + \frac{4}{5}$$

$$E(X,Y) = 0$$

So,
$$\operatorname{Cov}(X,Y) = E(X,Y) - E(X)E(Y)$$

$$= 0 - 0\left(\frac{6}{5}\right) = 0$$

Computing Standard Deviation

$$\sigma_x = \sqrt{E(x^2) - (E(x))^2}$$

$$E(x^2) = \sum_x x^2 P(X=x) = (-2)^2\frac{1}{5} + (-1)^2\frac{1}{5} + 0 + (1)^2\frac{1}{5} + (2)^2\frac{1}{5}$$

$$= \frac{4}{5} + \frac{1}{5} + \frac{1}{5} + \frac{4}{5} = \frac{10}{5}$$

$E(x^2) = 2$

$\sigma_x = \sqrt{2 - 0^2} = \sqrt{2}$

$\sigma_y = \sqrt{E(y^2) - (E(y))^2}$

$E(y^2) = \sum_y y^2 \cdot P(Y = y)$

$= 0 + (1)^2 \frac{2}{5} + (2)^2 \cancel{+} \frac{2}{5}$

$= 0 + \frac{2}{5} + \frac{8}{5} = \frac{10}{5}$

$E(y^2) = 2$

$\sigma_y = \sqrt{2 - (6/5)^2} = \sqrt{2 - \frac{36}{25}}$

$= \sqrt{\frac{50 - 36}{25}} = \sqrt{\frac{14}{25}} = \frac{\sqrt{14}}{5}$

$\sigma_x = \sqrt{2}$ & $\sigma_y = \frac{\sqrt{14}}{5}$

$\text{Corr}(x, y) = \frac{0}{\sqrt{2} \times \frac{\sqrt{14}}{5}} = 0$

The correlation between X & Y is 0 So, there is no linear relationship between x & y

(c) Based on your answer to part (a), can you explain the result in part (b)?

We have seen in part (a) and concluded that

$P(X=x, Y=y) \neq P(X=x)\,P(Y=y)$ for values of X & Y

Hence X & Y are not independent.

Now correlation result of X & Y is $\mathrm{Cov}(X,Y) = 0$

So the 2 variables have no relationship between them.

It would require that the joint distribution factorizes into the product of the marginals for all pairs $(X=x, Y=y)$, which is observed in our case.

So X and Y are not independent, but they are not correlated.

Nature of X & Y → Since $Y = |x|$, this non linear transformation can cause X and Y to have a relationship that is not captured by linear correlation.

(d) Fundamental difference between Q_2 & Q_3

From $Q_2 \rightarrow$ Y is defined as linear equation like $y = mx + c$ which explicitly defines the linear relationship between the variables. The correlation of two linearly related variables would be non-Zero.

From $Q_3 \rightarrow$ Y is defined as an absolute value of X like $Y = |x|$ in a non linear transformation. This means while Y is dependent on X, the transformation is non linear, this implies that the Standard Linear correlation might be Zero. It is exactly 0 in this case even though there is a strong non linear relationship.

In $Q_2 \rightarrow$ the concept of independence is not typically applicable in the same way because the variables are related through the equations (Linear equation)

In $Q_3 \rightarrow$ It is observed that the probability and distributions which might be independent mean that knowing the value of on variable gives no information about the other variable.

Correlation measures strength and direction of linear relationship between two variables. In our Q_3 X and Y are not independent & ~~but~~ they are not correlated as well thus indicating no linear relationship.

So, Probability and distribution focuses on probabilistic relationships and dependencies between random variables & Linear equation focuses on deterministic, explicitly defined linear relationship.

Thus the fundamental difference lies on how the random variables Y is defined in relation to the other variable X.

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4. (a) Linearize the function

$$f(x,y) = \sqrt{2x+3y} - \frac{x}{y} \text{ at the Point } (3,1)$$

Finding the linear approximation at that point or the first order Taylor expansion of $f(x,y)$ around (a,b). It is given by

$$L(x,y) = f(a,b) + (x-a)\, f_x(a,b) + (y-b)\, f_y(a,b) \rightarrow ①$$

Here f_x and f_y are partial derivatives w.r.t x and y

So, Computing for $f(3,1)$

$$f(3,1) = \sqrt{2(3)+3(1)} - \frac{3}{1}$$

$$= \sqrt{6+3} - 3$$

$$= \sqrt{9} - 3$$

$$= 3-3$$

$$= 0$$

Now the partial derivatives of f_x :-

$$f_x(x,y) = \frac{d}{dx}\left(\sqrt{2x+3y} - \frac{x}{y}\right)$$

$$\frac{d}{dx}\left(\sqrt{2x+3y}\right) = \frac{1}{2}(2x+3y)^{-1/2} \cdot \frac{d(2x+3y)}{dx}$$

$$= \frac{1}{2\sqrt{2x+3y}} \frac{d(2x+3y)}{dx}$$

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$$= \frac{2}{2\sqrt{2x+3y}}$$

$$= \frac{1}{\sqrt{2x+3y}}$$

$$\frac{d}{dx}\frac{(-x)}{(y)} = \frac{-1}{y}$$

Combining them

$$f_x(x,y) = \frac{1}{\sqrt{2x+3y}} - \frac{1}{y}$$

Partial derivative of Y w.r.t y:

$$\frac{d(\sqrt{2x+3y})}{dy} = \frac{1}{2\sqrt{2x+3y}} \frac{d}{dy}(2x+3y)$$

$$= \frac{1}{2\sqrt{2x+3y}} \times 3 = \frac{3}{2\sqrt{2x+3y}}$$

$$\frac{d}{dy}\left(\frac{-x}{y}\right) = -x \frac{d}{dy}\left(\frac{1}{y}\right)$$

$$= -x\left(-1y^{-2}\right)$$

$$= x/y^2$$

Combining them

$$f_x(x,y) = \frac{3}{2\sqrt{2x+3y}} + \frac{x}{y^2}$$

Evaluating $f_x(3,1)$:

$$f_x(3,1) = \frac{1}{\sqrt{2(3)+3(1)}} - \frac{1}{1}$$

$$= \frac{1}{\sqrt{9}} - 1 = \frac{1}{3} - 1 = \frac{-2}{3}$$

$$f_x(3,1) = \frac{-2}{3}$$

Evaluating $f_y(3,1)$:-

$$f_y(x,y) = \frac{3}{2\sqrt{2(3)+3(1)}} + \frac{3}{(1)^2}$$

$$= \frac{3}{2\sqrt{9}} + 3 = \frac{\cancel{3}}{2\times\cancel{3}} + 3$$

$$= \frac{1}{2} + 3 = \frac{7}{2} = 3.5$$

$$f_y(x,y) \Rightarrow f_y(3,1) = 3.5$$

Substituting the values in 1

$$L(x,y) = f(3,1) + f_x(3,1)(x-3) + f_y(3,1)(y-1)$$

$$= 0 + \left(\frac{-2}{3}\right)(x-3) + 3.5(y-1)$$

$$= \left(\frac{-2}{3}\right)(x-3) + 3.5(y-1)$$

$$= \frac{-2x}{3} + 2 + \frac{7}{2}y - \frac{7}{2}$$

$$= \frac{-2}{3}x + \left(\frac{4-7}{2}\right) + \frac{7}{2}y$$

$$L(x,y) = \frac{-2}{3}x + \frac{7}{2}y - \frac{3}{2}$$

This is the linearization of function

$$f(x,y) = \sqrt{2x+3y} - \frac{x}{y} \text{ at point } (3,1)$$

(b) Find Second order taylor polynomial for

$$f(x,y) = e^{5x} \ln(1+y) \text{ at the point } (0,0)$$

We need to calulate the function value, the first order partial derivative and Second order partial derivative.

The general form of the second order Taylor polynomial of a function $f(x,y)$ around the point (a,b)

$$Q(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b) + f_{xx}\frac{(a,b)}{2}(x-a)^2 + f_{xy}(a,b)(x-a)(y-b) + \frac{1}{2}f_{yy}(a,b)(y-b)^2 \rightarrow (1)$$

Function value at point $(0,0)$:-

$$f(0,0) = e^{5(0)}\ln(1+0) = e^0 \ln(1) = 1(0) = 0$$

$$f(0,0) = 0$$

Evaluating first order partial derivative

$$f_x(x,y) = \frac{d}{dx}\left(e^{5x}\ln(1+y)\right)$$

$$= \frac{d e^{5x}}{dx}$$

Using power rule of diff

$$= 5e^{5x} \qquad \frac{d e^{ax}}{dx} = ae^{ax}$$

$\ln(1+y) \rightarrow y$ is constant here

$$f_x(x,y) = \ln(1+y)\, 5e^{5x}$$

$$f_y(x,y) = \frac{d}{dy}\left(e^{5x} \ln(1+y)\right)$$

$$\frac{de^{5x}}{dy} = e^{5x}$$ (It doesn't change with 'y')

$$\frac{d}{dy}\ln(1+y) = 1\,(1+y)^{-1}$$
$$= \frac{1}{1+y}$$

So,

$\cancel{f_{yy}}\ \cancel{f_{yy}=}$

$$f_y(x,y) = \frac{1}{1+y}\, e^{5x}$$

Evaluating $f_x(x,y)$ and $f_y(x,y)$ for $(0,0)$

$$f_x(0,0) = \ln(1+0)\, 5e^{5(0)}$$
$$= \ln(1) \times 5e^0$$
$$= 5 \times 0 \times 1 = 0$$

$$\boxed{f_x(0,0) = 0}$$

$$f_y(0,0) = \frac{1}{1+0}\, e^{5(0)} = \frac{1}{1}\, e^0 = 1$$

$$\boxed{f_y(0,0) = 1}$$

Second order partial derivative

$$f_{xx}(x,y) = \frac{d}{dx}\left[5e^{5x} \ln(1+y)\right]$$

$$= \frac{d}{dx}(5e^{5x}) . \ln(1+y)$$

$$\boxed{f_{xx}(x,y) = 25e^{5x} \ln(1+y)}$$

$$f_{yy}(x,y) = \frac{d}{dy}\left[\frac{1}{1+y} e^{5x}\right]$$

$$\boxed{f_{yy}(x,y) = \frac{-e^{5x}}{(1+y)^2}}$$

$$f_{xy}(x,y) = \frac{d}{dy}\left[5e^{5x} \ln(1+y)\right]$$

$$= 5e^{5x}\left(\frac{1}{1+y}\right)$$

$$\boxed{f_{xy}(x,y) = \frac{5e^{5x}}{(1+y)}}$$

Evaluating $f_{xx}(x,y)$, $f_{yy}(x,y)$ & $f_{xy}(x,y)$

For $(0,0)$

$$\frac{f_{xx}(0,0)}{2} = \frac{25e^{5(0)} \ln(1+0)}{2} = 0$$

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$$f_{yy}(0,0) = \frac{-e^{5(0)}}{2(1+0)^2} = \frac{-1}{2}$$

$$f_{xy}(0,0) = \frac{5e^{5(0)}}{(1+0)} = \frac{5}{1} = 5$$

Substituting these values in ①

$$Q(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0) + \frac{1}{2}f_{xx}(0,0)(x-0)^2 + f_{xy}(0,0)(x-0)(y-0) + \frac{1}{2}f_{yy}(0,0)(y-0)^2$$

$$= 0 + 0(x-0) + 1(y-0) + 0(x-0)^2 + 5(x-0)(y-0) + \left(\frac{-1}{2}\right)(y-0)^2$$

$$= 0 + 0 + y + 0 + 5xy - \frac{y^2}{2}$$

$$\boxed{Q(x,y) = y + 5xy - \frac{y^2}{2}}$$

(C) For the Multivariate function

(i) $f(x,y,z) = yx^2 + zy^2 + z^2 - 2yx + 2zy + y - z$

The critical points are defined by solving the partial derivative equations

$$f_x \rightarrow \frac{\partial}{\partial x} f(x,y,z) = 0 \rightarrow (a)$$

$$f_y \rightarrow \frac{\partial}{\partial y} f(x,y,z) = 0 \rightarrow (b)$$

$$f_z \rightarrow \frac{\partial}{\partial y} f(x,y,z) = 0 \rightarrow (c)$$

Thus, let us solve for $f_x(x,y,z) = 0$

$$\Rightarrow \frac{\partial}{\partial x}\left[x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right] = 0$$

$$\Rightarrow 2xy + 0 + 0 - 2y + 0 + 0 + 0 = 0$$

$$2xy - 2y = 0 \quad \text{or} \quad y(x-1) = 0 \rightarrow (1)$$

Now for $f_y(x,y,z) = 0$

$$\Rightarrow \frac{\partial}{\partial y}\left[x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right] = 0$$

$$x^2 + 2yz + 0 - 2x + 2z + 1 - 0 = 0$$

$$x^2 + 2yz - 2x + 2z + 1 = 0 \rightarrow (2)$$

Now for $f_z(x,y,z) = 0$

$$\Rightarrow \frac{\partial}{\partial z}\left[x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right] = 0$$

$$0 + y^2 + 2z - 0 + 2y + 0 - 1 = 0$$

$$y^2 + 2z + 2y - 1 = 0 \rightarrow ③$$

From Equation ① we can infer that

$$y = 0 \text{ or } x = 1$$

1. Taking $y=0$ and ~~[illegible]~~ Sub in ③

$$y^2 + 2z + 2y - 1 = 0$$

$$0 + 2z + 2(0) - 1 = 0$$

$$2z = 1 \Rightarrow z = 1/2$$

Sub these values in ② we get

$$x^2 + 2yz - 2x + 2z + 1 = 0$$

$$x^2 + 2(0)(1/2) - 2x + 2(1/2) + 1 = 0$$

$$x^2 - 2x + 2 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4(1)(2)}}{2}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2} \text{ or } x = (1 \pm i)$$

It is a complex number from which we can infer that at $y=0$, there is no real root or Critical Point.

2. Taking $x=1$ as the other valid possibility, we get from (2)

$$x^2 + 2yz - 2x + 2z + 1 = 0$$

$$1^2 + 2yz - 2 + 2z + 1 = 0$$

$$2yz + 2z = 0$$

$$z(y+1) = 0 \quad \text{So } z=0 \text{ or } y=-1$$

(i) If $z=0$, $y^2 + 2z + 2y - 1 = 0$

$$y^2 + 0 + 2y - 1 = 0$$

$$y^2 + 2y - 1 = 0$$

$$y = \frac{-2 \pm \sqrt{2^2 + 4(1)(1)}}{2} = \frac{-2 \pm \sqrt{8}}{2}$$

$$y = -1 \pm \sqrt{2}$$

(ii) If $y=-1$, $y^2 + 2z + 2y - 1 = 0$

$$(-1)^2 + 2z + 2(-1) - 1 = 0$$

$$2z = 2$$

$$z = 1$$

Thus, Since we have real points on partial derivatives at $x = 1$, we can obtain the critical points as

a $(1, -1-\sqrt{2}, 0) \rightarrow P_1$

b $(1, -1+\sqrt{2}, 0) \rightarrow P_2$

c $(1, -1, 1) \rightarrow P_3$

c (ii) Now the Hessian Matrix (H_f) for a multivariate function (f) as $f(x,y,z)$ is given as

$$H_f(x,y,z) = Q_f(x,y,z) = \begin{bmatrix} \frac{\partial^2}{\partial x^2} f & \frac{\partial^2}{\partial x \partial y} f & \frac{\partial^2}{\partial x \partial z} f \\ \frac{\partial^2}{\partial y \partial x} f & \frac{\partial^2}{\partial y^2} f & \frac{\partial^2}{\partial y \partial z} f \\ \frac{\partial^2}{\partial z \partial x} f & \frac{\partial^2}{\partial y \partial z} f & \frac{\partial^2}{\partial z^2} f \end{bmatrix}$$

For our given equation

$$f(x,y,z) = x^2y + y^2z + z^2 - 2xy + 2yz + y - z$$

$$\frac{\partial^2}{\partial x^2} f = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial x}\left[2xy - 2y\right] = 2y$$

$$\frac{\partial^2}{\partial x \partial y} f = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial x}\left[x^2 + 2yz - 2x + 2z + 1\right] = 2x - 2$$

$$\frac{\partial^2}{\partial x \partial z} = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial z}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial x}\left[y^2 + 2z + 2y - 1\right] = 0$$

$$\frac{\partial^2}{\partial y \partial x} = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial y}\left[2xy - 2y\right] = 2x - 2$$

$$\frac{\partial^2}{\partial y^2} f = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial y}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial y}\left[x^2 + 2yz - 2x + 2z + 1\right] = 2z$$

$$\frac{\partial^2}{\partial y \partial z} f = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial z}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial y}\left[y^2 + 2z + 2y - 1\right] = 2y + 2$$

$$\frac{\partial^2}{\partial z \partial x} f = \frac{\partial}{\partial z}\left[\frac{\partial}{\partial x}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial z}\left[2xy - y\right] = 0$$

$$\frac{\partial^2}{\partial z \partial y} f = \frac{\partial}{\partial z}\left[\frac{\partial}{\partial y}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial z}\left[x^2 + 2yz - 2x + 2z + 1\right] = 2y + 2$$

$$\frac{\partial^2}{\partial z^2} f = \frac{\partial}{\partial z}\left[\frac{\partial}{\partial z}\left\{x^2y + y^2z + z^2 - 2xy + 2yz + y - z\right\}\right]$$

$$= \frac{\partial}{\partial z}\left[2z + y^2 + 2y - 1\right] = 2$$

Subbing the Second order partial derivatives into the Hessian matrix, we get

$$H_f(x,y,z) = \begin{bmatrix} 2y & 2x-2 & 0 \\ 2x-2 & 2z & 2y+2 \\ 0 & 2y+2 & 2 \end{bmatrix}$$

This is a symmetric matrix verifying our derivatives as correct.

(C)

(iii) To classify the nature of the critical point for $f(x,y,z)$ we need to put the coordinates into H_f then find the eigen values of the resultant matrix and observe the nature of the eigen values of each matrix

$$H_f(1, -1-\sqrt{2}, 0) = \begin{bmatrix} 2y & 2x-2 & 0 \\ 2x-2 & 2z & 2y+2 \\ 0 & 2y+2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(-1-\sqrt{2}) & 2(1)-2 & 0 \\ 2(1)-2 & 2(0) & 2(-1-\sqrt{2})+2 \\ 0 & 2(-1-\sqrt{2})+2 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} -2-2\sqrt{2} & 0 & 0 \\ 0 & 0 & -2\sqrt{2} \\ 0 & -2\sqrt{2} & 2 \end{bmatrix}$$

The eigenvalue of A can be obtained by the computation of

$$\det[A - \lambda I] = 0$$

$$\begin{bmatrix} -2-2\sqrt{2} & 0 & 0 \\ 0 & 0 & -2\sqrt{2} \\ 0 & -2\sqrt{2} & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{vmatrix} -2-2\sqrt{2}-\lambda & 0 & 0 \\ 0 & -\lambda & -2\sqrt{2} \\ 0 & -2\sqrt{2} & 2-\lambda \end{vmatrix} = 0$$

Solving the det (A)

$$(-2-2\sqrt{2}-\lambda)\left[(-\lambda)(2-\lambda) - (2\sqrt{2})(2\sqrt{2})\right] = 0$$

$$(\lambda+2+2\sqrt{2})\left[(\lambda^2-2\lambda-8)\right] = 0$$

$$(\lambda+2+2\sqrt{2})\left[(\lambda^2-4\lambda+2\lambda-8)\right] = 0$$

$$(\lambda+2+2\sqrt{2})(\lambda-4)(\lambda+2) = 0$$

$$\lambda_A = (-2-2\sqrt{2}), 4, -2$$

Since the eigen values for $H_f(1, -1-\sqrt{2}, 0)$ are Non Zero but with different signs, we can conclude that this a Saddle Point

Now for $H_f(1, -1+\sqrt{2}, 0) = \begin{bmatrix} 2y & 2x-2 & 0 \\ 2x-2 & 2z & 2y+2 \\ 0 & 2y+2 & 2 \end{bmatrix}$

Let this be B

$$B = \begin{bmatrix} 2(\sqrt{2}-1) & 2(1)-2 & 0 \\ 2(1)-2 & 2(0) & 2(\sqrt{2}-1)+2 \\ 0 & 2(\sqrt{2}-1)+2 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2(\sqrt{2}-1) & 0 & 0 \\ 0 & 0 & 2\sqrt{2} \\ 0 & 2\sqrt{2} & 2 \end{bmatrix}$$

Eigen values for $B(\lambda_B)$ is given by

$$\det[B - \lambda I] = 0$$

$$\left| \begin{bmatrix} 2(\sqrt{2}-1) & 0 & 0 \\ 0 & 0 & 2\sqrt{2} \\ 0 & 2\sqrt{2} & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2(\sqrt{2}-1)-\lambda & 0 & 0 \\ 0 & 0-\lambda & 2\sqrt{2} \\ 0 & 2\sqrt{2} & 2-\lambda \end{vmatrix} = 0$$

$$\left(2\sqrt{2}-2-\lambda\right)\left[(-\lambda)(2-\lambda)-(2\sqrt{2})(2\sqrt{2})\right]=0$$

$$(2\sqrt{2}-2-\lambda)(\lambda^2-2\lambda-8)=0$$

$$(2\sqrt{2}-2-\lambda)(\lambda^2-4\lambda+2\lambda-8)=0$$

$$(2\sqrt{2}-\lambda-2)(\lambda-4)(\lambda+2)=0$$

$$\lambda_B=(2\sqrt{2}-2),\ 4,\ -2$$

Here the eigenvalues for $H_f(1, -1+\sqrt{2}, 0)$ are Non Zero, but of different Signs, indicating this is also a Saddle Point

Now for $H_f(1,-1,1) = C = \begin{bmatrix} 2y & 2x-2 & 0 \\ 2x-2 & 2z & 2y+2 \\ 0 & 2y+2 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -2 & 2(1)-2 & 0 \\ 2(1)-2 & 2(1) & [2(-1)+2] \\ 0 & 2(-1)+2 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Eigenvalues for C is given by $\det[(C-\lambda I]=0$

$$\left|\begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} -2-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)(2-\lambda)(2-\lambda) = 0$$

$$\lambda_c = 2, 2, (-2)$$

This indicates non Zero eiganvalues but with different signs which means this matrix $H_f(1, -1, 1)$ is also a Saddle Point.

$$\left.\begin{aligned} P_1 &= (1, -1-\sqrt{2}, 0) \\ P_2 &= (1, -1+\sqrt{2}, 0) \\ P_3 &= (1, -1, 1) \end{aligned}\right\} \text{Saddle Points}$$

4(d) Find all values for k So that

$f(x,y) = x^4 + kxy + y^4$ has local maximum at $(1,1)$. Give your Answer in the form of interval

Evaluating the 1st & 2nd order condition at Point $(1,1)$

$$f_x = \frac{df}{dx} = 4x^3 + Ky$$

$$f_y = \frac{df}{dy} = 4y^3 + kx$$

So, If f has local maximum or minimum at (a,b)

and first order partial derivative exists, then

$$f_x(a,b) = 0 \text{ and } f_y(a,b) = 0$$

So, $f_x(1,1) = 4(1)^3 + k(1) = 4 + k = 0$

$f_y(1,1) = k(1) + 4(1)^3 = k + 4 = 0$

We get $\boxed{k = -4}$

The 1st order condition is satisfying if $k = -4$

Now 2nd order condition using Hessian Matrix,

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

$$f_{xx} = \frac{d^2 f}{dxx} = \frac{d}{dx}(4x^3 + ky) = 12x^2$$

$$f_{xy} = \frac{d^2 f}{dxy} = \frac{d}{dx}(4y^3 + kx) = k$$

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$$f_{yy} = \frac{d^2f}{dy^2} = \frac{d}{dy}(4y^3 + kx) = 12y^2$$

At $(1,1) \rightarrow$ $f_{xx}(1,1) = 12$

$f_{xy}(1,1) = k$

$f_{yy}(1,1) = 12(1)^2 = 12$

$$H = \begin{bmatrix} 12 & k \\ k & 12 \end{bmatrix}$$

For $(1,1)$ to be local minimum, the Hessian matrix must be positive definition at that point.

Det of Hessian Matrix

$$\det(H) = 12 \times 12 - k^2$$

H_{11} or $H_{22} = 12 > 0$

So for the Hessian Matrix to be +ve we need

$$144 - k^2 > 0$$

$$144 > k^2$$

$$k < \sqrt{144} \Rightarrow |k| < 12$$

Hence value of k for which $f(x,y) = x^2 + kxy + y^4$ is local minimum at $(1,1)$ are in interval $\boxed{k \in (-12, 12)}$

(e) Find the maximum and Minimum values of

$f(x,y,z) = -x-y+z$ Subject to $x^2+y^2=2$ and

$x+y+z=1$

Given objective function $\rightarrow$ $f(x,y,z) = -x-y+z$

Constraints :-

$g(x,y,z) \Rightarrow x^2+y^2-2=0$

$h(x,y,z) \Rightarrow x+y+z-1=0$

For this constrained optimiZation problem, we will be finding where on a graph are constraints of g and h perpendicular. This is provided by finding ∇f, ∇g and ∇h which turn out to be parallel, non Zero Scalar multiplier of each other

So, $\nabla f = \lambda \nabla g + \mu \nabla h$

Such that $\nabla f(x,y,z) = 0$ or

$\nabla f(x,y,z)$ does not exist.

Solving

$f_x = \lambda g_x + \mu h_x \rightarrow (1)$

$f_y = \lambda g_y + \mu h_y \rightarrow (2)$

$f_z = \lambda g_z + \mu h_z \rightarrow (3)$

$$f_x = \lambda g_x + \mu h_x$$

$$\frac{d}{dx} f(x,y,z) = \lambda \left[\frac{d}{dx} g(x,y,z)\right] + \mu \left[\frac{d}{dx} h(x,y,z)\right]$$

$$-1 - 0 + 0 = \lambda(2x + 0 - 0) + \mu(1 + 0 + 0 - 1)$$

$$-1 = \lambda(2x) + \mu \longrightarrow 4$$

$$f_y = \lambda g_y + \mu h_y$$

$$\frac{d}{dy} f_y = \lambda \frac{dg}{dy} + \mu \frac{dh}{dy}$$

$$-1 = \lambda[2y + 0 - 0] + \mu[0 + 1 + 0 - 0]$$

$$-1 = 2y\lambda + \mu \longrightarrow 5$$

$$f_z = \lambda g_z + \mu h_z$$

$$1 = \lambda[0 + 0 + 0] + \mu[0 + 0 + 1 - 0]$$

$$1 = 0 + \mu \longrightarrow 6$$

Since $\mu = 1$, Sub in 4 & 5

$$\left.\begin{aligned} \lambda(2x) + 1 &= -1 \\ \lambda(2y) + 1 &= -1 \end{aligned}\right\}$$

$$\left.\begin{aligned} \lambda(2x) &= -2 \\ \lambda(2y) &= -2 \end{aligned}\right\}$$

$$\lambda x = \lambda y$$

$$x = y \rightarrow (7)$$

Sub $x = y$ in $g(x, y, z)$

$$x^2 + y^2 - 2 = 0$$

$$x^2 + x^2 = 2$$

$$2x^2 - 2 = 0$$

$$2(x^2 - 1) = 0$$

$$(x+1)(x-1) = 0$$

So, $x = -1$ or $x = 1$

From (7) $\Rightarrow$ $y = -1$ or $y = 1$

Sub in $h(x, y, z)$

$$x + y + z = 1$$

$$2x + z = 1$$

$$z = 1 - 2x \rightarrow (8)$$

Now, if ~~xxx~~ $x = -1$, $z = 1 - 2(-1)$

$$= 1 + 2$$

$$\boxed{z = 3}$$

if $x = 1$, $z = 1 - 2(1)$

$$= 1 - 2$$

$$\boxed{z = -1}$$

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Since $x = y$, we will get only two points

$$P_1 = (-1, -1, 3)$$
$$P_2 = (1, 1, -1)$$

Let us Sub P_1 and P_2 in $f(x, y, z)$

$$f_1(-1,-1,3) = -x_1 - y_1 + z_1$$
$$= -(-1) - (-1) + 3$$
$$= \cancel{1+1+} \; 1 + 1 + 3$$
$$= 5$$

$$f_2(1, 1, -1) = -x_2 - y_2 + z_2$$
$$= -(1) - (1) + (-1)$$
$$= -1 \; -1 \; -1$$
$$= -3$$

$f_1(-1,-1,3) > f_2(1,1,-1)$, we can conclude that $(-1,-1,3)$ is the ~~Glog~~ Global Maximum while $(1,1,-1)$ is the Global Minimum for $f(x,y,z) = -x - y + z$