

## ③ BIT MAGIC

Bitwise operator

• And &amp;

• Or |

• XOR ^

• Left shift operator &lt;&lt;

• Right shift operator &gt;&gt;

• Not ~

→ Bitwise And &amp;

x	y	x & y
0	0	0
0	1	0
1	0	0
1	1	1

→ Bitwise OR |

x	y	x   y
0	0	0
0	1	1
1	0	1
1	1	1

→ Bitwise XOR ^

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

→ Left shift operator &lt;&lt;

x = 3

x &lt;&lt; i → ignore i leading bits and add i trailing zeros.

★ If leading y bits are 0, then  $x \ll y == x * 2^y$ 

→ Right shift operator &gt;&gt;

x = 33

x &gt;&gt; i → ignore i trailing bits and add i leading zeros

★  $x \gg y == \left\lfloor \frac{x}{2^y} \right\rfloor$



→ Bitwise Not  $\sim$

$2^3$  complement of  $x$  in  $n$  bit representation  $= 2^n - x$

## To check  $k^{\text{th}}$  bit of a number  $n$  is set(1)

① if  $(n \& (1 \ll (k-1))) \neq 0$

cout << "Yes";

else

cout << "No";

② if  $((n \gg (k-1)) \& 1) == 1$

cout << "Yes";

else

cout << "No";

## Count Set bits

① Brian Kernighan's Algorithm  $\{ n \& (n-1) \}$

int countSetBit(int n) {

int res = 0;

while (n > 0) {

n = n & (n-1);

res++;

}

return res;

}

TC:  $O(\text{total no. of set bits})$

② Look up table for 32 bit number  $\{0-256 \text{ pre computed ans}\}$

int res = 0;

int table[256];

void initialize() {

table[0] = 0;

for (int i = 1; i < 256; i++)

table[i] = (i & 1) + table[i/2];

Priyansh



int count (int n) {

int res = table[n & 0xff];

n = n >> 8;

res += table[n & 0xff];

n = n >> 8;

res += table[n & 0xff];

n = n >> 8;

res += table[n & 0xff];

return res;

}

## Power of two (last power of 2 for given number n)  
return ((n != 0) and (n & (n-1) == 0));

## XOR (^) properties

- $x \wedge 0 = x$

- $x \wedge y = y \wedge x$

- $x \wedge (y \wedge z) = (x \wedge y) \wedge z$

- $x \wedge x = 0$

## To get the rightmost set bit

$x \& \sim(x-1)$

## find one odd appearing no. [given a vector where every no. comes even times  
find the odd appearing no.]

int find (int arr[], int n) {

int res = 0;

for (int i = 0; i < n; i++) {

res ^= arr[i];

return res;

}



## find two odd appearing no. [given a vector where ~~no~~ every no. comes even no. of time and only 2 no. are odd times appearing].

```
void find (int arr[], int n) {
    int res = 0, res1 = 0, res2 = 0;
    for (int i = 0; i < n; i++)
        res ^= arr[i];
    // rightmost set bit
    int Sn = res & ~(res-1);
    for (int i = 0; i < n; i++) {
        if ((arr[i] & Sn) != 0)
            res1 ^= arr[i];
        else
            res2 ^= arr[i];
    }
    cout << res1 << res2;
}
```

## No. of subset in a string of given length  $n = 2^n$

## Power Set {a,b,c}  $\Rightarrow$  {}, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}

```
void powerset (string str) {
    int n = str.length();
    int ext = pow(2, n);
    for (int i = 0; i < ext; i++) {
        for (int j = 0; j < n; j++) {
            if ((i & (1 << j)) != 0)
                cout << str[j];
        }
        cout << endl;
    }
}
```

Tc:  $\Theta(2^n * n)$



☆☆ find The Rightmost set bit

- ①  $\text{ffs}()$ ;
- ②  $\log_2 (n \& \sim(n-1) + 1)$ ;

☆☆ Total no. of set bit in a power of 2  $\Rightarrow$

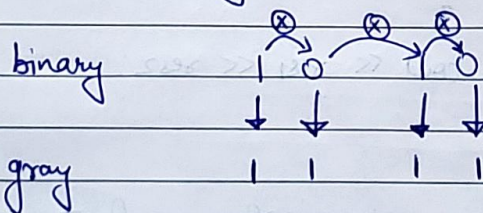
$$\text{total set bit in } 0 \text{ to } 2^n = n * 2^{(n-1)}$$

☆☆ If  $n$  is any given no. Then,

$$\text{total set bit in } 0 \text{ to } n = [x * 2^{(x-1)}] + [n - 2^x + 1] + \text{solve}(n - 2^x)$$

where  $x$  is max<sup>m</sup> power of 2 ie  $2^x \leq n$

## Binary Code  $\longrightarrow$  Gray Code

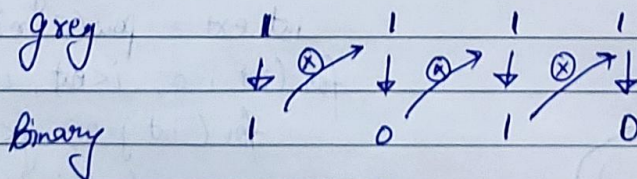


MSB same

rest do XOR  
with last digit

$$\text{code : } n \wedge (n \gg 1)$$

## Gray code  $\longrightarrow$  Binary code



MSB same

rest do XOR with  
last result

```
code: int res = n;
      while (n > 0) {
          n = n >> 1;
          res = res ^ n;
      }
```



★  $x = x \& (x-1)$  : turn off the rightmost set bit.

★ flip a given number: given no. (n) eg: ~~1111~~ 1011  
 take value (v) all set bit 1111  
 flip no. = (v) - (n) = 1111 - 1011 = 0100

★ Swap (a) & (b).

$$a \wedge = b$$

$$b \wedge = a$$

$$a \wedge = b$$

★★ find MSB position in given no. (n)  $\Rightarrow \log_2(n)$

★★ find value of XOR for 1. 2. 3  $\rightarrow N$

if $N \% 4 == 0$	then	$1 \wedge 2 \wedge 3 \dots \wedge N = N$
1	then	$1 \wedge 2 \wedge 3 \dots \wedge N = 1$
2	then	$1 \wedge 2 \wedge 3 \dots \wedge N = N+1$
3	then	$1 \wedge 2 \wedge 3 \dots \wedge N = 0$