Sets and Parameters

- $J = \{1, \ldots, N\}$: set of jobs indexed by j

- $M = \{1, \ldots, L\}$: set of machines indexed by m

- $O_j = \{0, \ldots, n_j - 1\}$: set of operations for job j

- $T = \{0, \ldots, T_{\max}\}$: set of stages (time slots)

- $p_{j,k}$: processing time of operation k of job j

- $m(j, k) \in M$: machine required for operation k of job j

- bigM: a large constant for time constraints

Decision Variables

- $x_{j,k,t} \in \{0, 1\}$: 1 if operation k of job j starts at stage t, 0 otherwise

- $S_{j,k} \geq 0$: start time of operation k of job j

- $C_{\max} \geq 0$: makespan (maximum completion time)

Optimization Problem

$$\text{minimize} \quad C_{\max}$$

$$\text{subject to} \quad \sum_{t \in T} x_{j,k,t} = 1, \qquad \forall j \in J, \forall k \in O_j,$$

$$\sum_{j \in J} \sum_{k \in O_j : m(j,k)=m} x_{j,k,t} \leq 1, \qquad \forall m \in M, \forall t \in T,$$

$$\sum_{t' \in T} t' x_{j,k+1,t'} \geq \sum_{t \in T} (t+1) x_{j,k,t}, \qquad \forall j \in J, \forall k = 0, \ldots, n_j - 2,$$

$$S_{j,k} = \sum_{t \in T} t \cdot p_{j,k} x_{j,k,t}, \qquad \forall j \in J, \forall k \in O_j,$$

$$C_{\max} \geq S_{j,n_j-1} + p_{j,n_j-1}, \quad \forall j \in J$$