Sets and Parameters

- $J = \{1, \ldots, N\}$: set of jobs indexed by i
- $M = \{1, \ldots, L\}$: set of machines indexed by m
- $O_i = \{0, \ldots, n_i - 1\}$: set of operations for job i
- $p_{i,k}$: processing time of operation k of job i
- $m(i,k) \in M$: machine required for operation k of job i
- bigM: a large constant for disjunctive constraints

Decision Variables

- $S_{i,k} \geq 0$: start time of operation k of job i
- $C_{i,k} \geq 0$: completion time of operation k of job i
- $C_{\max} \geq 0$: makespan (maximum completion time)
- $x_{i,k,j,l} \in \{0,1\}$: binary variable for each pair of operations (i,k) and (j,l) on the same machine, indicating which comes first

Optimization Problem

$$
\begin{aligned}
\text{minimize} \quad & C_{\max} \\
\text{subject to} \quad & C_{i,k} = S_{i,k} + p_{i,k}, && \forall i \in J, \forall k \in O_i, \\
& S_{i,k+1} \geq C_{i,k}, && \forall i \in J, \forall k = 0, \ldots, n_i - 2, \\
& S_{i,k} \geq C_{j,l} - \text{bigM}(1 - x_{i,k,j,l}), && \forall m \in M, \forall (i,k),(j,l) \text{ on } m, i < j, \\
& S_{j,l} \geq C_{i,k} - \text{bigM} x_{i,k,j,l}, && \forall m \in M, \forall (i,k),(j,l) \text{ on } m, i < j, \\
& C_{\max} \geq C_{i,n_i-1}, && \forall i \in J
\end{aligned}
$$