

## Sets and Parameters

- $J = \{1, \dots, N\}$ : set of jobs indexed by  $i$
- $M = \{1, \dots, L\}$ : set of machines indexed by  $m$
- $O_i = \{1, \dots, n_i\}$ : sequence of operations for job  $i$
- $p_{i,t}$ : processing time of operation  $t \in O_i$  of job  $i$
- $m(i, t) \in M$ : machine required for operation  $t$  of job  $i$
- $K = \sum_{i \in J} |O_i|$ : total number of operations (also number of stages)

## Decision Variables

- $\eta_{k,i,t} \in \{0, 1\}$ : 1 if job  $i$ 's operation  $t$  is scheduled at stage  $k$ , 0 otherwise
- $\Theta_{k,m} \geq 0$ : availability time of machine  $m$  at stage  $k$
- $C_{\max} \geq 0$ : makespan, the maximum completion time across all machines

## Optimization Problem

minimize  $C_{\max}$

$$\begin{aligned}
 \text{subject to} \quad & \sum_{(i,t)} \eta_{k,i,t} = 1, & \forall k = 0, \dots, K-1, \\
 & \sum_{k=0}^{K-1} \eta_{k,i,t} = 1, & \forall (i,t) \in J \times O_i, \\
 & \Theta_{k+1,m} \geq \Theta_{k,m} + \sum_{(i,t)} p_{i,t} \eta_{k,i,t} \mathbf{1}_{m(i,t)=m}, & \forall k = 0, \dots, K-1, m \in M, \\
 & C_{\max} \geq \Theta_{K,m}, & \forall m \in M, \\
 & \sum_{k_1=0}^{k_2} \eta_{k_1,i,t-1} \geq \eta_{k_2,i,t}, & \forall i, t = 1, \dots, |O_i| - 1, k_2 = 0, \dots, K-1
 \end{aligned}$$