

## Lecture 08

# Model Evaluation Part 1: Introduction to Overfitting and Underfitting

STAT 479: Machine Learning, Fall 2019

Sebastian Raschka

<http://stat.wisc.edu/~sraschka/teaching/stat479-fs2019/>

# Announcements First



Source: <https://pittnews.com/article/149994/news/starship-food-delivery-robots/>

Thu, Oct 10	Day 11	- L07: Ensemble Methods	<a href="#">[L07 Slides]</a> <a href="#">[L07 Notes]</a>	
Tue, Oct 15	Day 12	L07: cont'd		
Thu, Oct 17	Day 13	<b>Midterm Exam</b>		Takes place in the regular class room (VAN HISE 114) 4:00-5:15 pm. Please bring a <a href="#">scientific calcutor</a> .
Tue, Oct 22	Day 14			<b>Project Proposal due 6:00 pm. PDF submission via Canvas. Use the LaTeX report template available <a href="#">here</a>. Assessment criteria are explained <a href="#">here</a> and <a href="#">here</a>.</b>
Thu, Oct 24	Day 15			

<http://pages.stat.wisc.edu/~sraschka/teaching/stat479-fs2019/#calendar>



# L<sup>A</sup>T<sub>E</sub>X Template for STAT479 Project Proposal

First Author

firstauthor@wisc.edu

Second Author

secondauthor@wisc.edu

Third Author

thirdauthor@wisc.edu

- This template is based on the CVPR conference template<sup>1</sup>.
- The information in this template is very minimal, and this file should serve you as a framework for writing your proposal. You may prefer to use a more collaboration-friendly tool while drafting the report with your class mates before you prepare the final report for submission. Remember that you should **submit both the report and code** you used for this project via Canvas. Also, **only one member per team** needs to submit the project material.
- The project proposal is a 2-4 page document excluding references<sup>2</sup>.
- You are encouraged (not required) to use 1-2 figures to illustrate technical concepts.
- The proposal must be formatted and submitted as a PDF document on Canvas (the submission deadline will be later announced via the schedule & email).
- Please check out the text in these sections for further information.

## 1. Introduction

In this section, describe what you are planning to do. Also, briefly describe related work.

When discussing related work, do not forget to include appropriate references. This is an example of a citation [1]. To format the citations properly, put the corresponding references into the bibliography.bib file. You can obtain BibTeX-formatted references for the "bib" file from Google Scholar (<https://scholar.google.com>), for example, by clicking on the double-quote character under a citation and then selecting "BibTeX" as shown in Figure 1 and Figure 2.

<sup>1</sup>[http://statcourse2018.thecvf.com/submission/main\\_conference/author\\_guidelines](http://statcourse2018.thecvf.com/submission/main_conference/author_guidelines)

<sup>2</sup>This means, references should of course be included but do not count towards the page limit

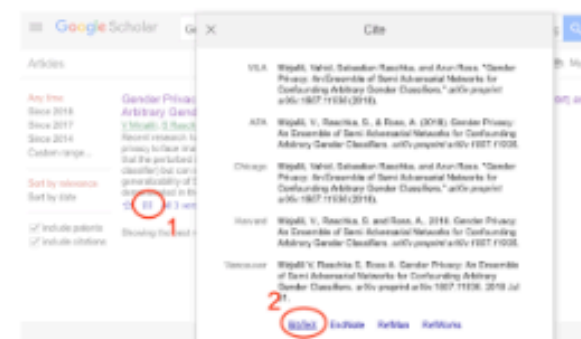


Figure 1. Example illustrating how to get BibTeX references from Google Scholar as a 1-column figure.

## 2. Motivation

Describe why your project is interesting. E.g., you can describe why your project could have a broader societal impact. Or, you may describe the motivation from a personal learning perspective.

## 3. Evaluation

What would the successful outcome of your project look like? In other words, under which circumstances would you consider your project to be "successful?"

How do you measure success, specific to this project, from a technical standpoint?

## 4. Resources

What resources are you going to use (datasets, computer hardware, computational tools, etc.)?

## 5. Contributions

You are expected to share the workload evenly, and every group member is expected to participate in both the experiments and writing. (As a group, you only need to submit one proposal and one report, though. So you need to work together and coordinate your efforts.)

Clearly indicate what computational and writing task each member of your group will be participating in.

2-4 pages (references do not count towards the page limit)

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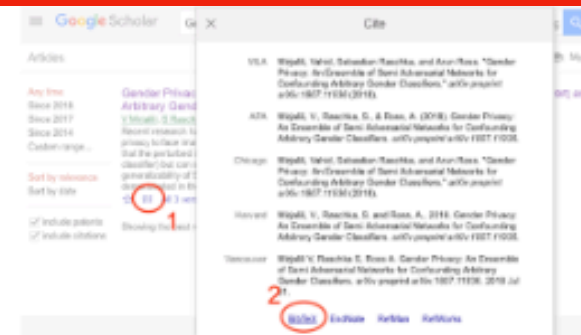


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The information in this template is very minimal, and

only one team member needs to submit the PDF

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Cite other work!

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# Example Proposal

## 1. Introduction

We aim to develop a deep learning model that effectively and efficiently predicts a blogger's age and gender based solely on their writing. Prior work by Schler et al. [3] has examined this topic in depth. They performed a thorough analysis of differences in writing across varying demographics and made interesting conclusions, e.g., female bloggers tend to show a pattern of more "personal" writing while male bloggers commonly discuss politics and technology. Their analysis provided them with reasonable predictions, achieving 80.1% for gender prediction and 76.2% for age prediction.

However, Schler et al. published their work in 2006 and as such used an outdated learning algorithm known as Multi-Class Real Winnow. We believe that recent advancements in natural language processing (NLP) using deep learning will allow us to build upon their work and achieve superior predictions. Our primary focus will be on the development of 2 models commonly used for NLP.

The first will be a convolutional neural network (CNN). Lopez et al. [2] illustrate one commonly used CNN architecture where sentences are tokenized into words, which are further transformed into a word embedding matrix. Convolutional filters are applied on this input layer, followed by a max-pooling operation to produce a sentence representation that can be used for analysis.

Next, we will develop a recurrent neural network (RNN). There are a few commonly used architectures for for natural language processing but we anticipate the Long-Short Term Memory (LSTM) introduced by Hochreiter et al. [1] will perform the best and as such will serve as our starting point.

We may also attempt other model development depending on time constraints and the scope of the material covered in class. Specifically, attention mechanisms and reinforcement learning appear to be regularly-used paradigms that none of us are particularly familiar with. If time permits, experimenting with these model types will be an intellectual curiosity at the very least.

that's nice, if time permits.  
I hope we will have enough time to cover this in class as well, but it may be very, very late in the course.

I assume you have access to this dataset!? Or do you plan to use a different one?

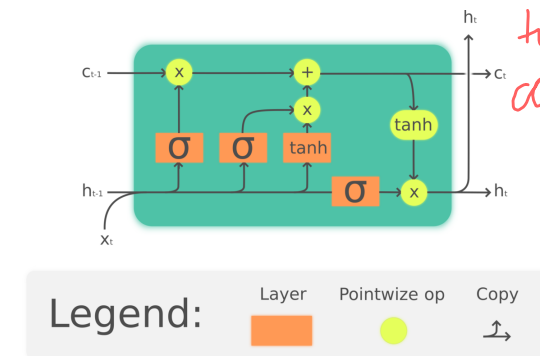


Figure 1. LSTM Architecture

## 2. Motivation

Identification of demographics based on writing is, in our opinion, a very interesting topic to explore. For most people, it is pretty easy to tell the age of an author based on the writing, and we think that using neural networks to do this would be an interesting application of a task that is very simple in most cases for a human brain to do but could be very challenging for a neural network. This idea also has very practical applications, such as deciding what kind of ads to display based on the age of the writer, as generally someone writes to people that are of similar age. It would be very useful to have relevant ads for websites such as blogs that cater based on age. Furthermore, examining the traits of writing across demographics could lead to interesting insights about internet blogging in general or society at large. Finally, we believe natural language processing to be a field that allows for a great variety of interesting models and this project will allow us to work with many methods of deep learning that are regularly used in research and industry.

## 3. Evaluation

Our primary focus is on developing a prediction engine that can reliably beat the model developed by Schler et al. [3]. Therefore, comparisons to their results will serve as our baseline for evaluation. Our simplest evaluation metric will be overall accuracy of predictions of both age and gender. We hope to improve at least 5% accuracy on both

Would you have detailed labels for exact ages or just rough categories?

In addition, you can also try logistic regression to start with as a baseline.



# Example Proposal

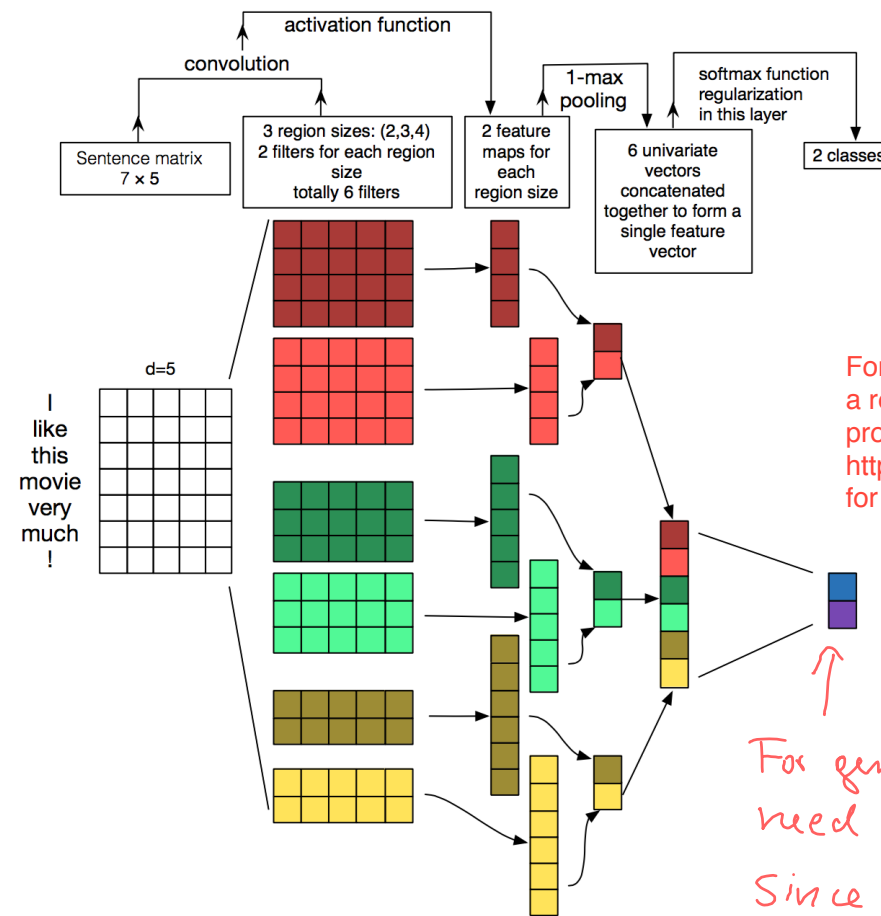


Figure 2. CNN Architecture for NLP.

demographics. For more detailed analysis, we will develop confusion matrices and compare directly to the results given in [3]. Our model may perform better or worse depending on what features are deemed to be relevant so careful analysis of these matrices may serve to judge feature importance. For example, Schler et al. [3] had a large number of author's in their 30's misclassified as in their 20's. Attempting to determine what features led to these errors could significantly help in improving performance.

## 4. Resources

We will use a dataset compiled and preprocessed by Schler et al. [3] as we hope to directly compare our results to their work. We will be using Python with the PyTorch machine learning library to construct our neural networks. Although the amount of data we are using is not particularly large, if necessary we will make use of Google Cloud computing if speed of training and running the neural networks becomes an issue.

If you are familiar with linux, I can also help getting you access to the department cluster.

## 5. Contributions

Our computational development will be split into three major components: data loading/preprocessing, our CNN model, and our RNN model. Drew will be focusing primarily on the data loading, on the CNN model, and on the RNN model. We each have an assigned responsibility but we plan to keep everyone involved in each section at least partially. For the report will take lead on the introduction and conclusion, will focus on related work and our proposed method, and will manage the experiment write-up and results discussion. Again, we will keep everyone involved in each section.

## References

- [1] S. Hochreiter and J. Schmidhuber. Long short-term memory. *Neural computation*, 9(8):1735–1780, 1997.
- [2] M. M. Lopez and J. Kalita. Deep learning applied to nlp. *arXiv preprint arXiv:1703.03091*, 2017.

# Citing Verbatim

Some text. "Methods based on voting can reduce variance" [1].  
Some text.

## References

[1] Dietterich, T. G., & Kong, E. B. (1995). Machine learning bias, statistical bias, and statistical variance of decision tree algorithms. Technical report, Department of Computer Science, Oregon State University.

# Paraphrasing

Some text. It has been shown that the variance of a learning algorithm can be lowered by combining multiple models via voting [1]. Some text.

## References

[1] Dietterich, T. G., & Kong, E. B. (1995). Machine learning bias, statistical bias, and statistical variance of decision tree algorithms. Technical report, Department of Computer Science, Oregon State University.

# "Citing" Images

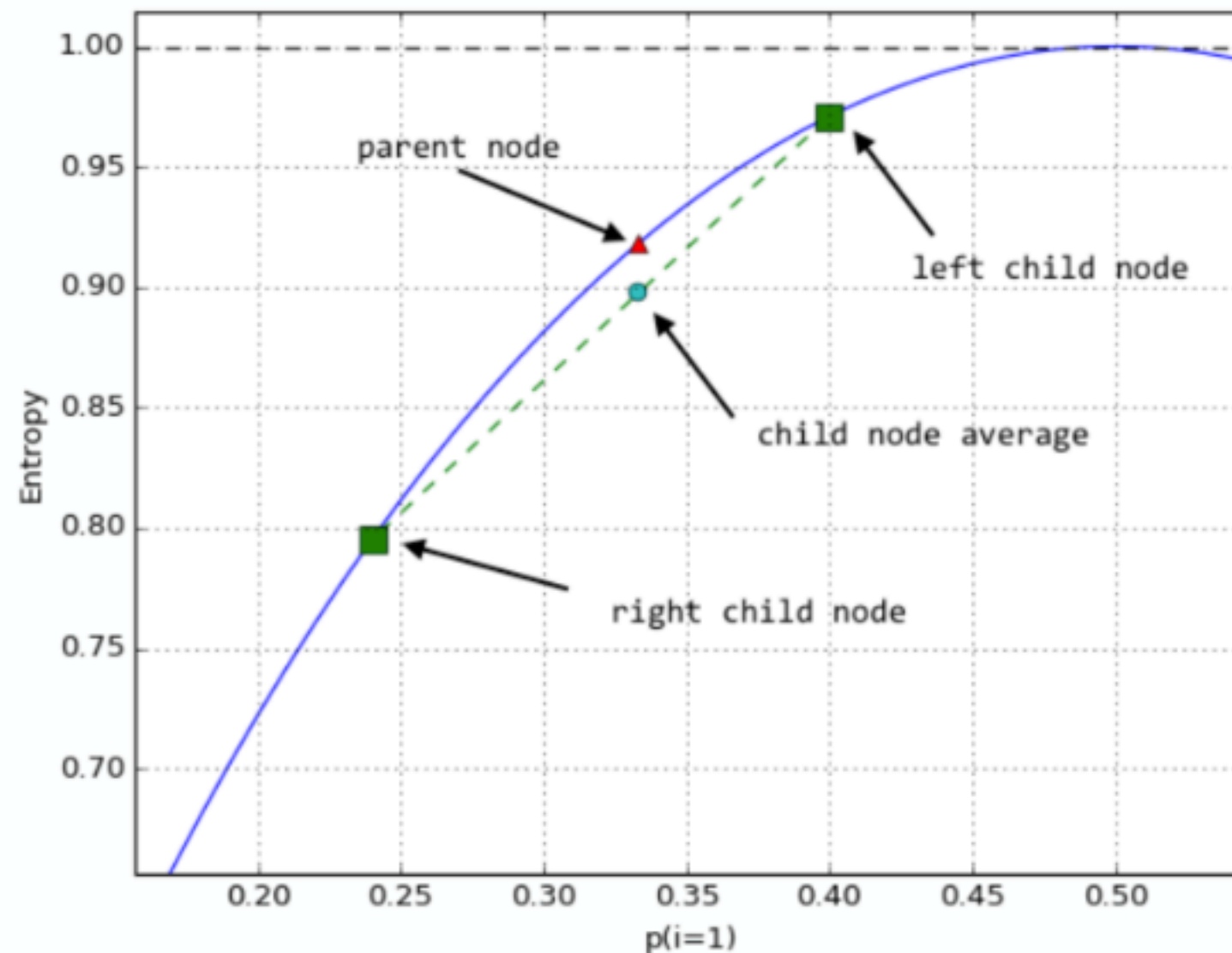
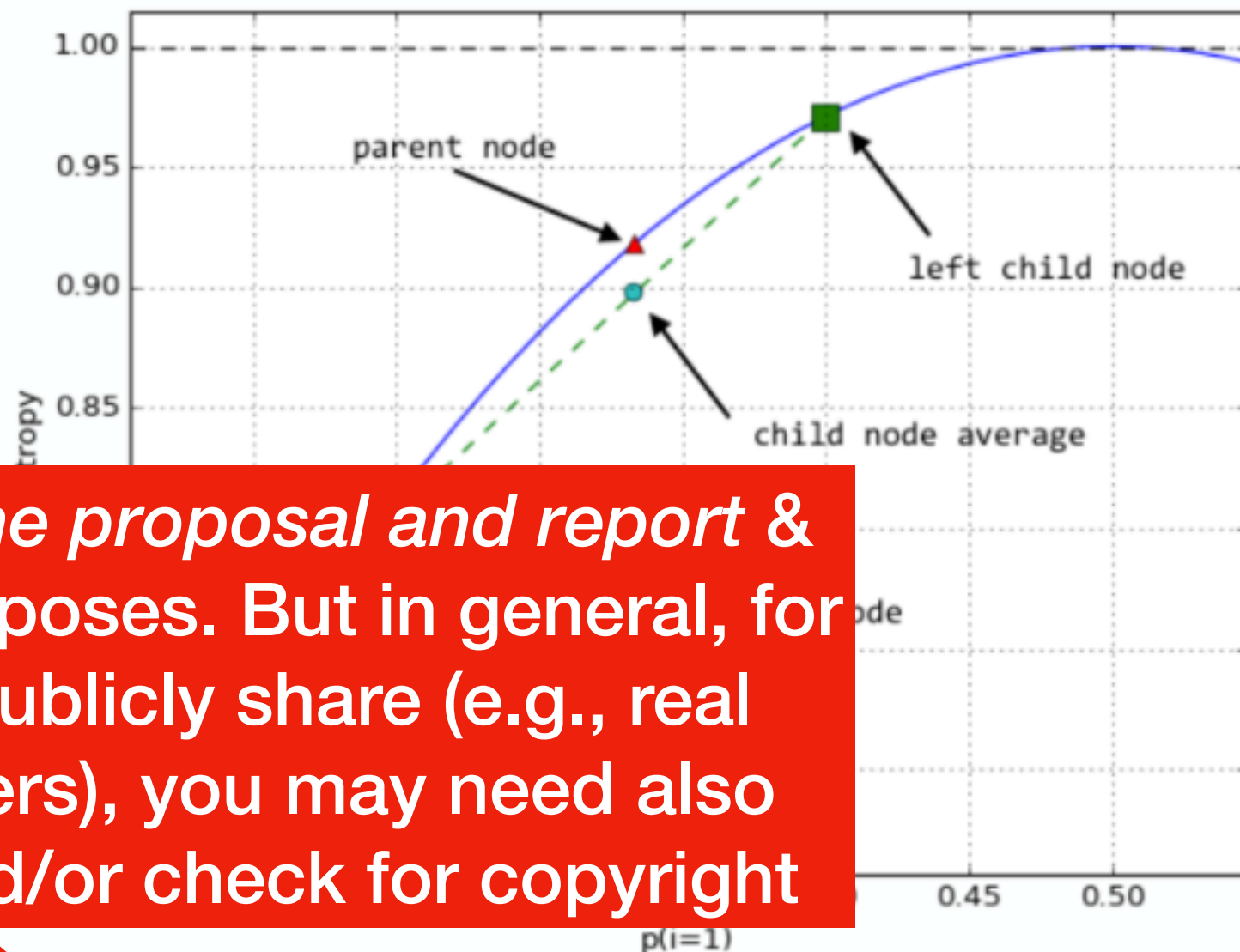


Figure XX: Some figure description.

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# "Citing" Images



Sufficient for *the proposal and report* & educational purposes. But in general, for papers you publicly share (e.g., real research papers), you may need also permission and/or check for copyright

Figure XX: Some figure description.

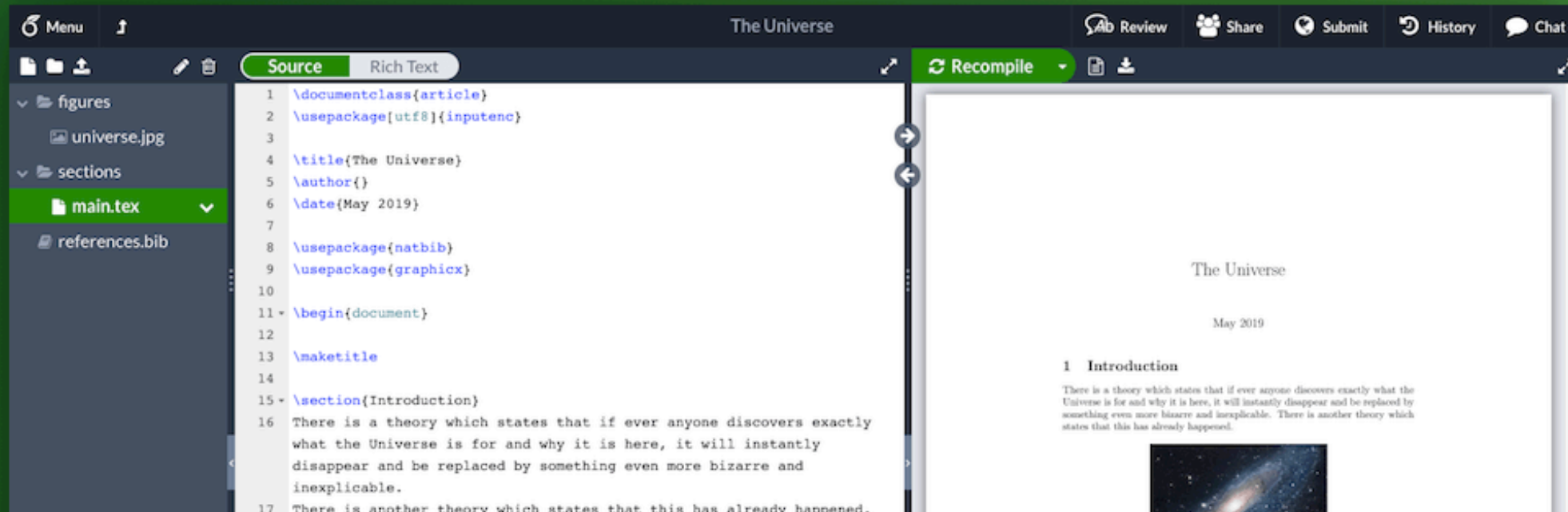
Source: [https://github.com/rasbt/stat479-machine-learning-fs19/blob/master/06\\_trees/06-trees\\_slides.pdf](https://github.com/rasbt/stat479-machine-learning-fs19/blob/master/06_trees/06-trees_slides.pdf)

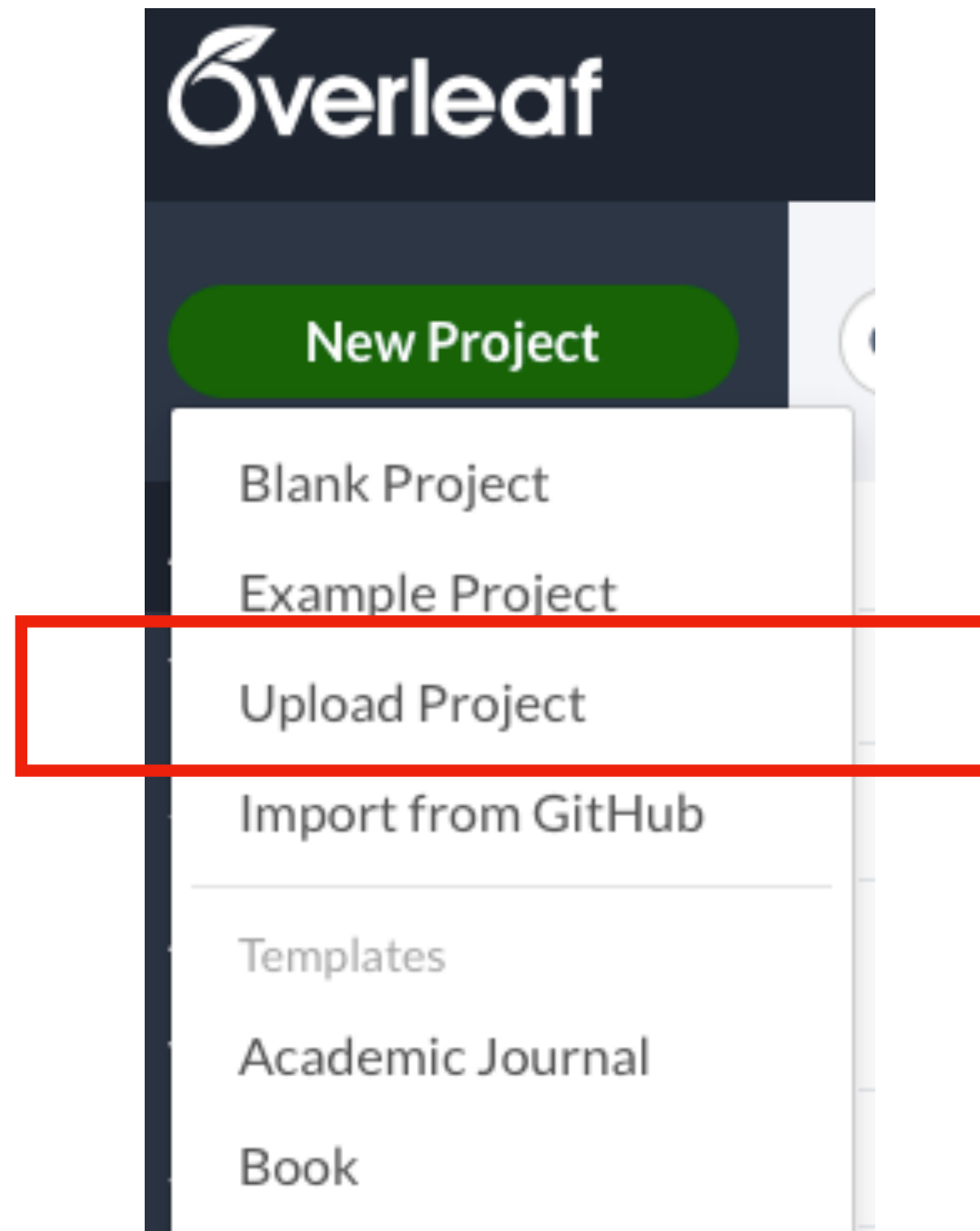
# Optional Tool/Platform for collaborative writing in LaTeX

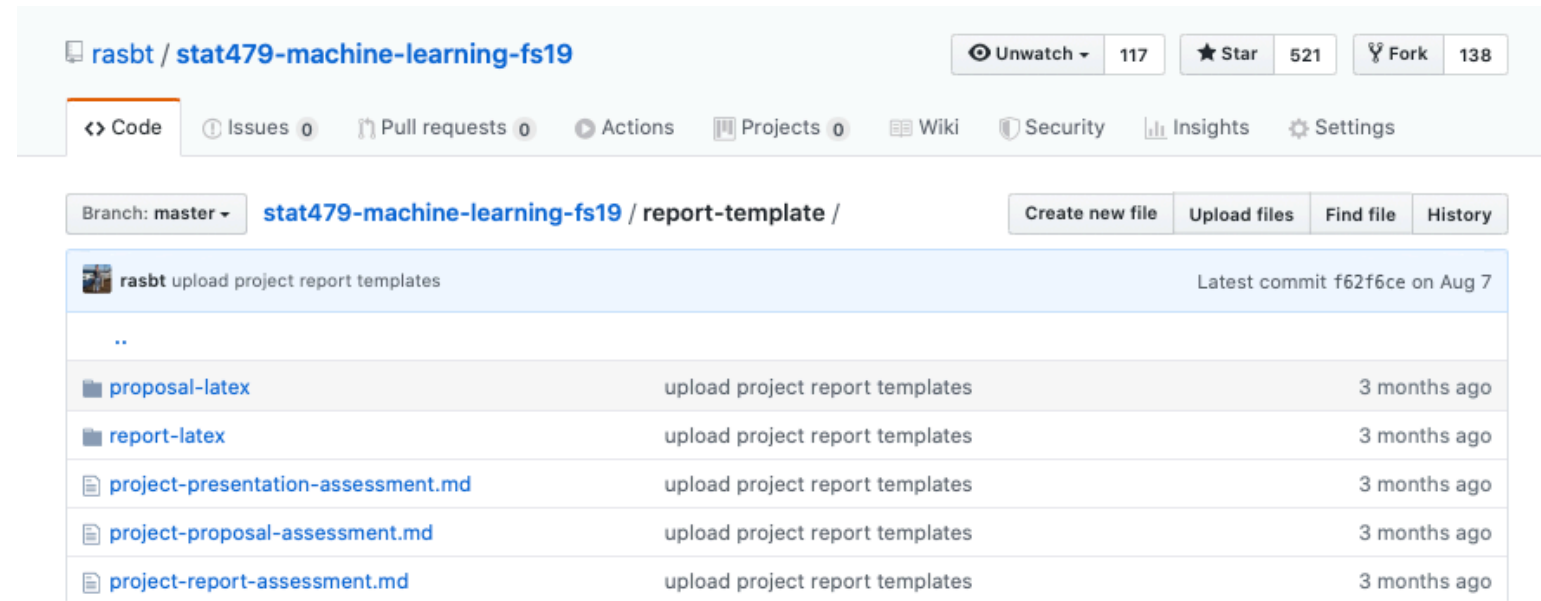
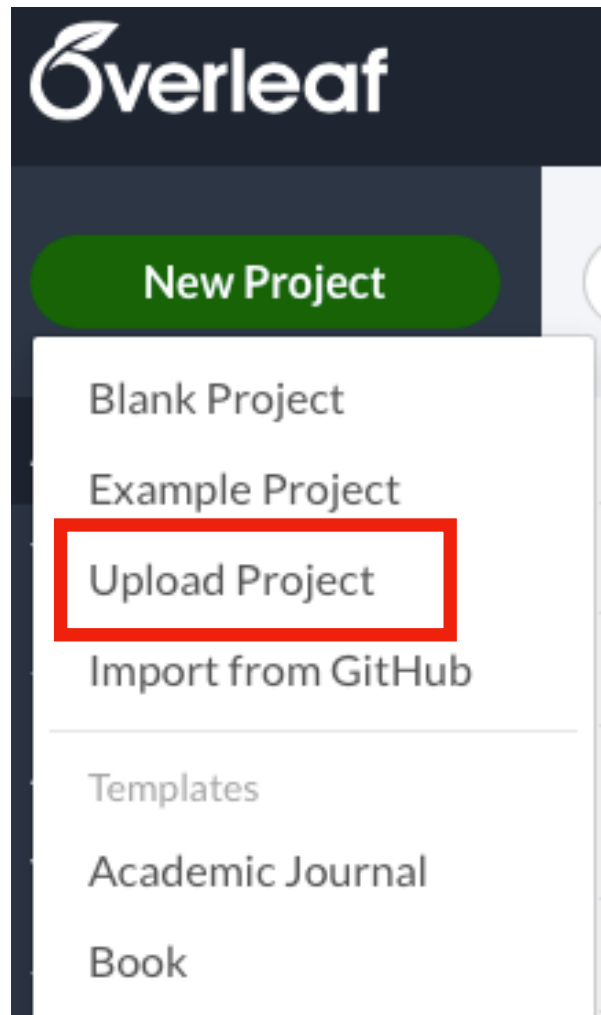
[Features & Benefits](#)[Templates](#)[Plans & Pricing](#)[Help](#)[Register](#)[Log In](#)

## LaTeX, Evolved

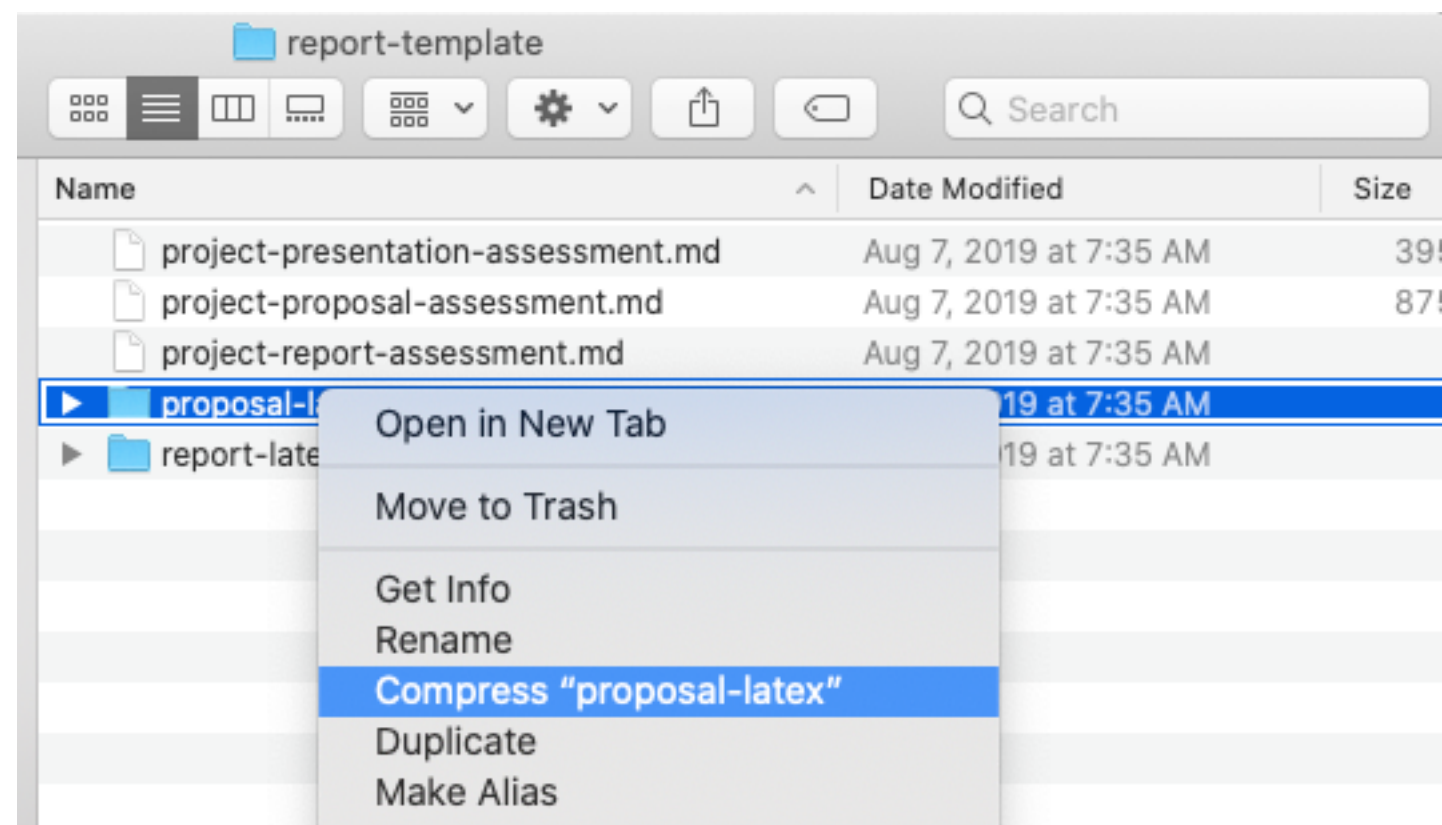
The easy to use, online, collaborative LaTeX editor







<https://github.com/rasbt/stat479-machine-learning-fs19/tree/master/report-template>







## Lecture 08

# Model Evaluation Part 1: Introduction to Overfitting and Underfitting

STAT 479: Machine Learning, Fall 2019

Sebastian Raschka

<http://stat.wisc.edu/~sraschka/teaching/stat479-fs2019/>

# Where we are in this course

[http://pages.stat.wisc.edu/  
~sraschka/teaching/stat479-  
fs2019/#topics-summary-  
planned](http://pages.stat.wisc.edu/~sraschka/teaching/stat479-fs2019/#topics-summary-planned)

## Part I: Introduction

- Lecture 1: What is Machine Learning? An Overview.
- Lecture 2: Intro to Supervised Learning: KNN

## Part II: Computational Foundations

- Lecture 3: Using Python, Anaconda, IPython, Jupyter Notebooks
- Lecture 4: Scientific Computing with NumPy, SciPy, and Matplotlib
- Lecture 5: Data Preprocessing and Machine Learning with Scikit-Learn

## Part III: Tree-Based Methods

- Lecture 6: Decision Trees
- Lecture 7: Ensemble Methods

## Part IV: Evaluation

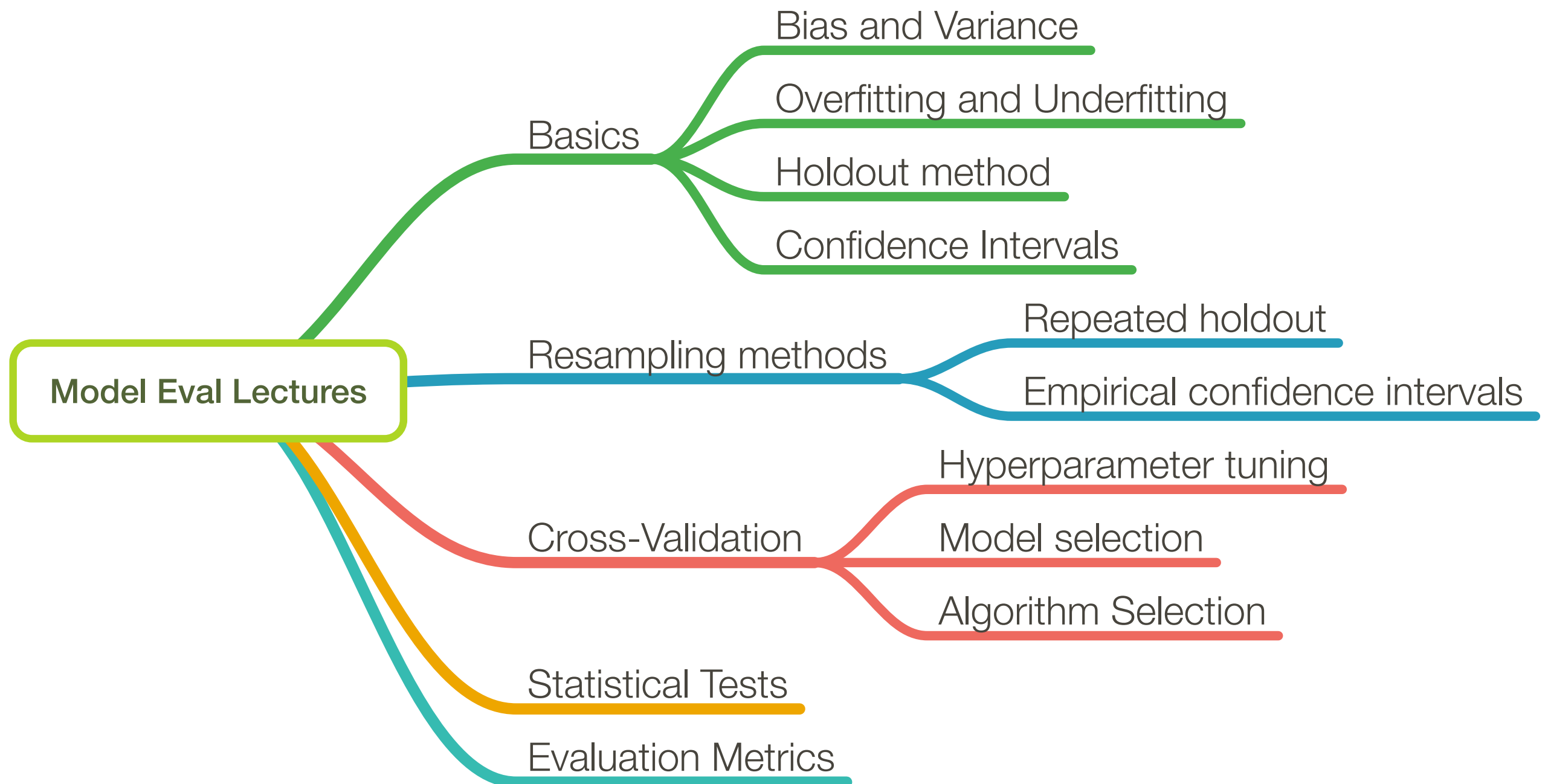
- Lecture 8: Model Evaluation 1: Introduction to Overfitting and Underfitting
- Lecture 9: Model Evaluation 2: Uncertainty Estimates and Resampling
- Lecture 10: Model Evaluation 3: Model Selection and Cross-Validation
- Lecture 11: Model Evaluation 4: Algorithm Selection and Statistical Tests
- Lecture 12: Model Evaluation 5: Performance Metrics

## Part V: Dimensionality Reduction

- Lecture 13: Feature Selection
- Lecture 14: Feature Extraction

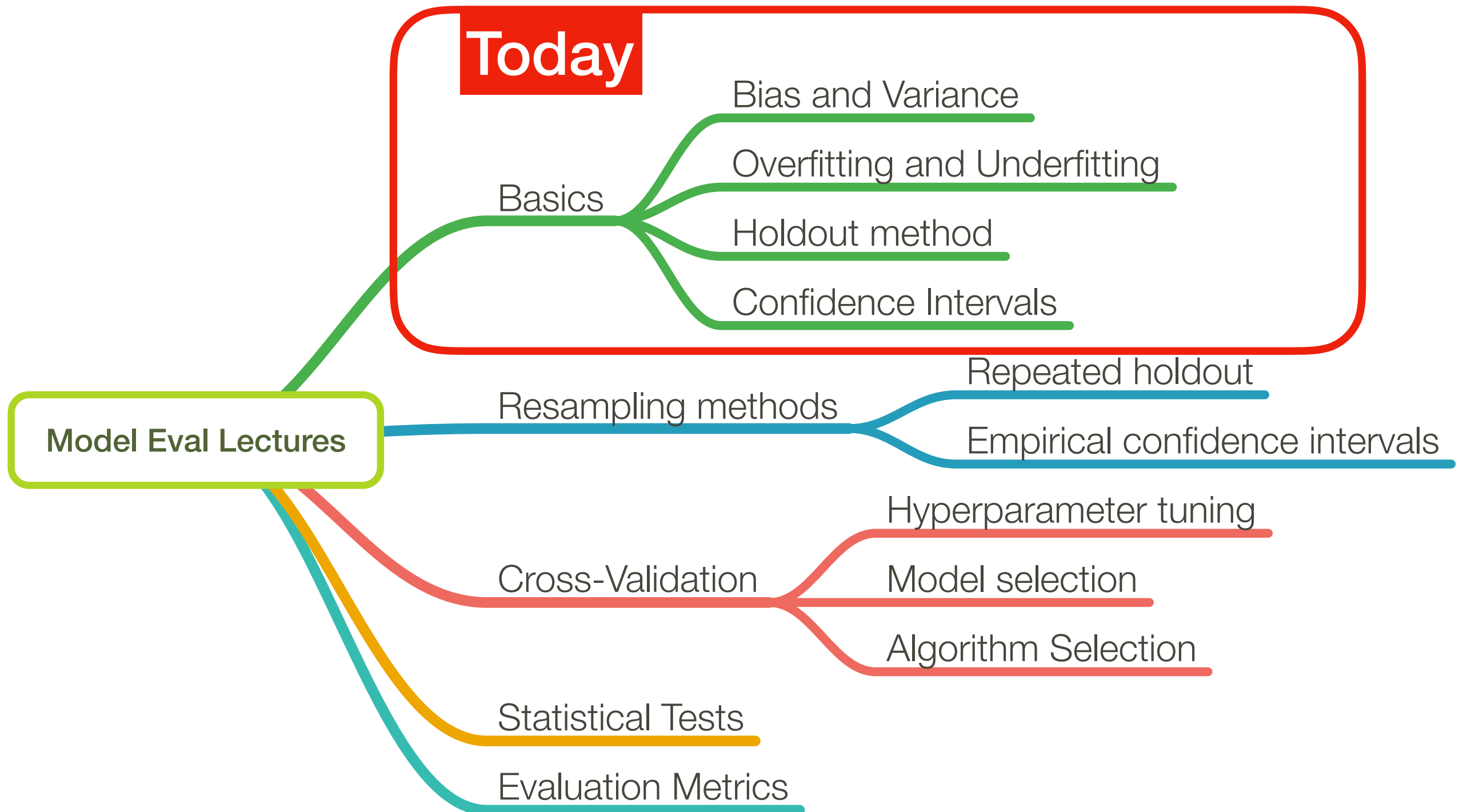
## Part VI: Bayesian Learning

# Overview





# Overview



# Overfitting and Underfitting

# Overfitting and Underfitting

## Generalization Performance

Want a model to "generalize" well to \_\_\_\_\_ data  
(Want "high generalization accuracy" or "low generalization error")

# Overfitting and Underfitting

## Assumptions

- i.i.d. assumption: training and test examples are independent and identically distributed (drawn from the same joint probability distribution,  $P(\mathbf{X}, y)$  )
- For some random model that has not been fitted to the training set,  
we expect **the training error is \_\_\_\_\_ the test error**
- The training error or accuracy provides  
**an \_\_\_\_\_imistically biased** estimate of the generalization performance

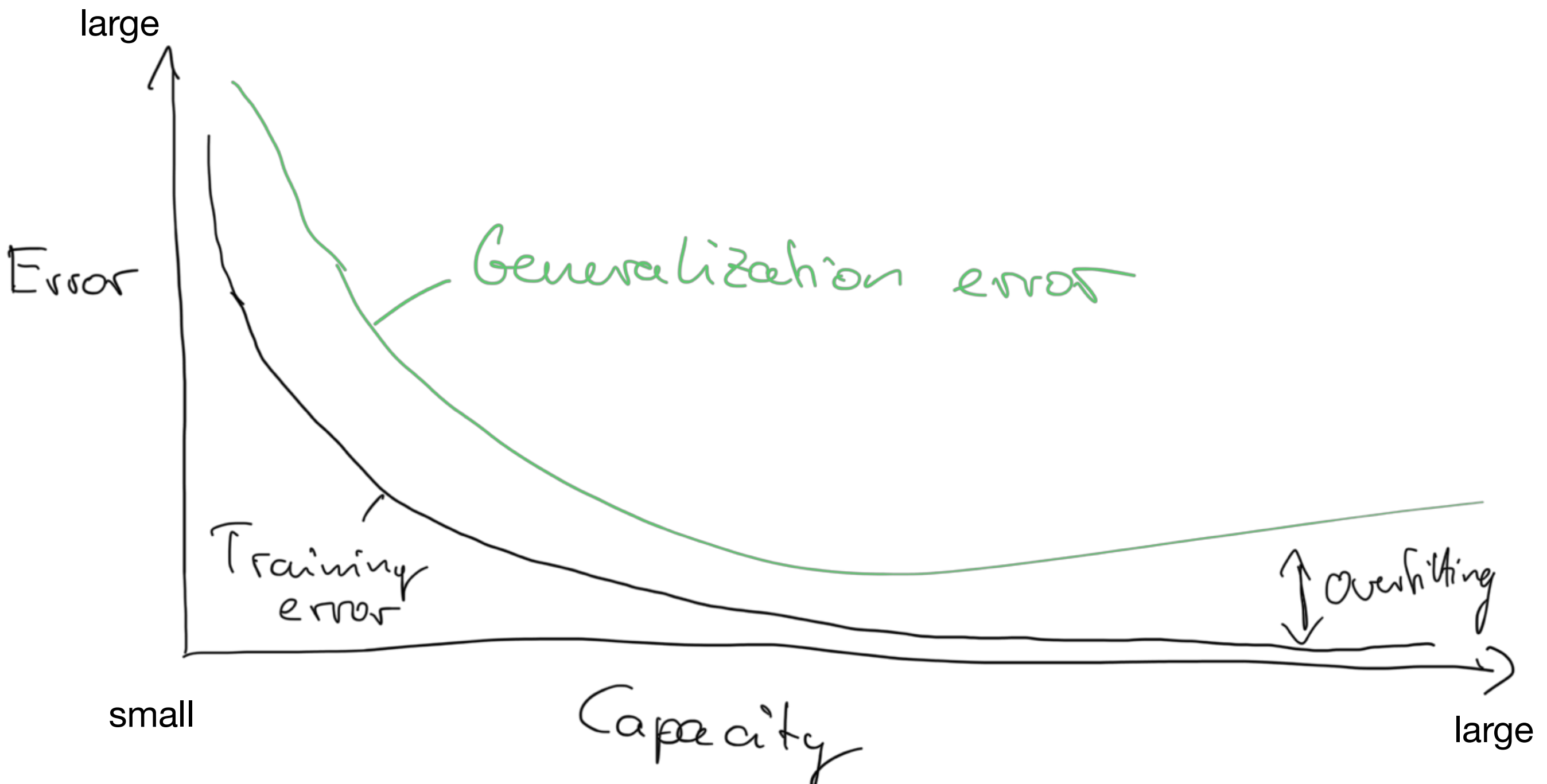
# Overfitting and Underfitting

## Model Capacity

- Underfitting: both the training and test error are \_\_\_\_\_
- Overfitting: gap between training and test error (where test error is larger)
- Large hypothesis space being searched by a learning algorithm  
-> high tendency to \_\_\_\_\_fit



# Overfitting and Underfitting





Articles

About 44 results (0.24 sec)

Any time

Since 2019

Since 2018

Since 2015

Custom range...

Sort by relevance

Sort by date

☒ include patents

☒ include citations

Create alert

[PDF] Prediction of Yelp Review Star Rating using Sentiment Analysis

C Li, J Zhang - 2014 - cs229.stanford.edu

... Final Report Figure 4: Ablative Analysis for 5-star Classification. As we can see, removing features may lead to higher mean square error, which supported our hypothesis that the resulted **model has high bias** and needs more features. 5.2 Recommendation Model ...

☆ Cited by 4 Related articles All 2 versions

Evaluation of Regression Models: Model Assessment, Model Selection and Generalization Error

F Emmert-Streib, M Dehmer - Machine Learning and Knowledge ..., 2019 - mdpi.com

When performing a regression or classification analysis, one needs to specify a statistical model. This model should avoid the overfitting and underfitting of data, and achieve a low generalization error that characterizes its prediction performance. In order to identify such a model ...

☆ Cited by 1 Related articles All 4 versions

[PDF] A Comparative Simulation Study of ARIMA and Fuzzy Time Series Model for Forecasting Time Series Data

HA Haji, K Sadik, AM Soleh - 2018 - researchgate.net

... increases for ARIMA model especially when  $\phi = 0.1$  and  $\theta = 0.9$  for both , which is to be expected. But for Yu **model has high bias** for that condition. The relationship between the bias and other forecasting accuracy measures is roughly linear for all methods ...

☆ Related articles All 2 versions

[PDF] Automatic recognition of handwritten digits using multi-layer sigmoid neural network

SK Katungunya, X Ding... - International Journal of ..., 2016 - pdfs.semanticscholar.org

... regularization parameter ( $\lambda$ ). Regularization add a penalty term that depends on the characteristics of the parameters. If a **model has high bias**, decreasing the effect of regularization can lead to better results. A high variance ...

☆ Cited by 2 Related articles

[PDF] Fuzzy mixture model for heaping data

H Jung, H Choi, T Park - Proceedings of the 9th NAUN international ..., 2015 - wseas.us

... The estimates of the FZIP model and the PZIP are similar to the true parameter values even if they have different heaping mechanism. However, the MP **model has high bias** for parameters of the count model. Table 2: Results of estimates of true parameters ...

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### Any time

Since 2019

Since 2018

Since 2015

Custom range...

### Sort by relevance

Sort by date

- ☒ include patents
- ☒ include citations

 Create alert

### [PDF] Model-based motion planning

B Burns, O Brock - Computer Science Department Faculty ..., 2004 - scholarworks.umass.edu

... random. Cohn et al. [10] note that hill-climbing may also be used to find  $\tilde{x}$ , but we have not found this to be necessary. The result is a sampling strategy that only queries sample points at which the **model has high variance**. A ...

☆  Cited by 12 Related articles All 10 versions

### Signalling and the pricing of new issues

M Grinblatt, CY Hwang - The Journal of Finance, 1989 - Wiley Online Library

... Nanda (1988), 2 2 In Nanda's model, firms with high mean returns also have low variances. Since this **model has high-variance** low-mean firms issuing debt, high-mean firms are penalized by issuing debt that is perceived as being riskier than it really is ...

☆  Cited by 1601 Related articles All 4 versions 

### [HTML] Bias-variance decomposition of errors in data-driven land cover change modeling

J Gao, AC Burnicki, JE Burt - Landscape ecology, 2016 - Springer

... AdaBoosting is expected to noticeably reduce modeling error only if the base **model has high variance**; if the base model performs poorly, boosting may transform it into a worse model (Breiman 1996; Domingos 2000). Results. Interpreting error component maps ...

☆  Cited by 1 Related articles All 6 versions

### Robust Bayesian Regularized Estimation Based on Regression Model

Z Li, W Zhao - Journal of Probability and Statistics, 2015 - hindawi.com

... does not provide an estimate of the variance parameter . In fact, the variance parameter plays an important role especially when the error in the regression **model has high variance**. On the other hand, the Lasso estimate in (2 ...

☆  Cited by 1 Related articles All 8 versions 

**"[...] model has high bias/variance"  
-- What does that mean?**

# **Bias-Variance Decomposition and Bias-Variance Trade-off**

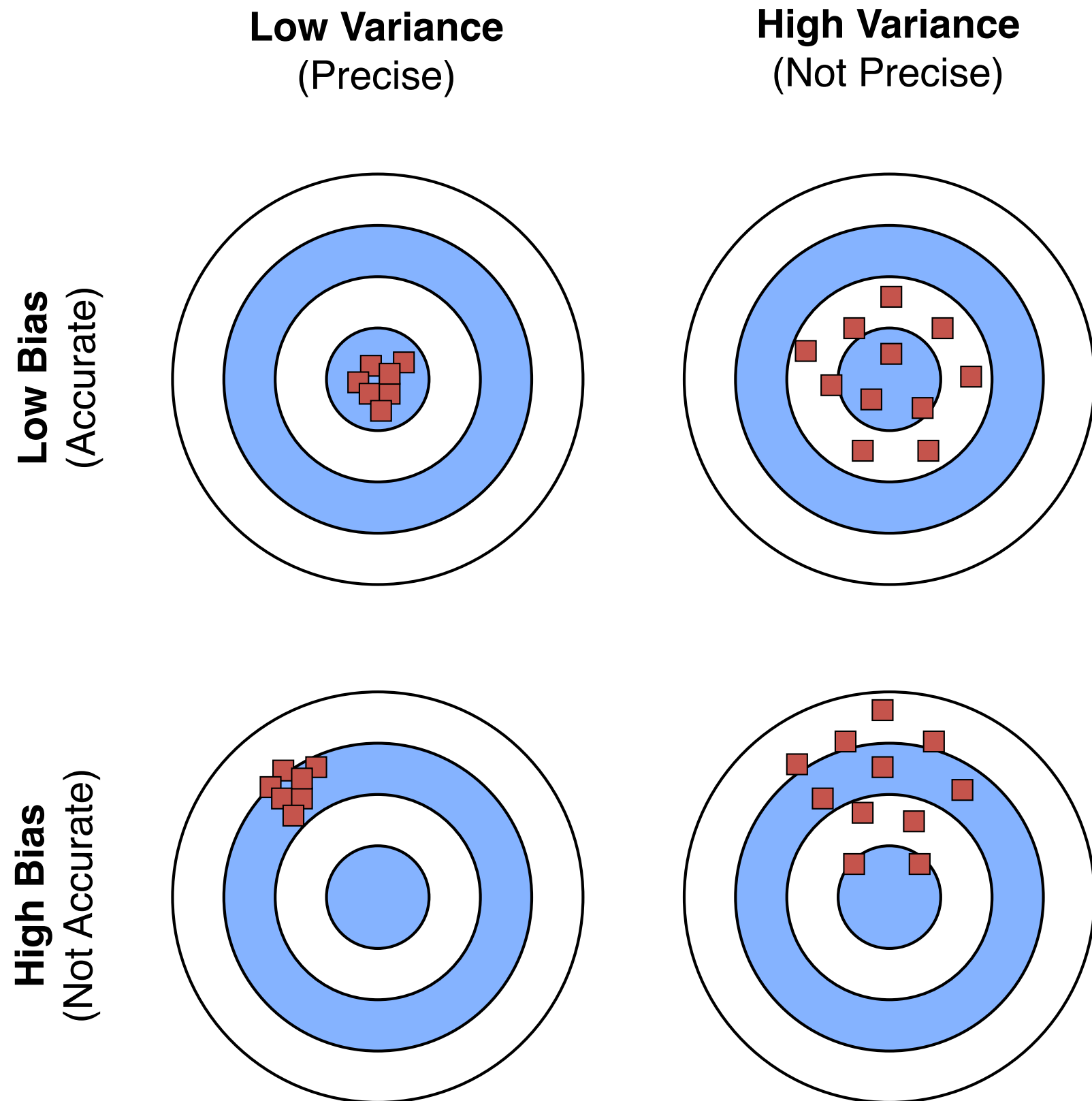
**(and how it related to overfitting and underfitting)**



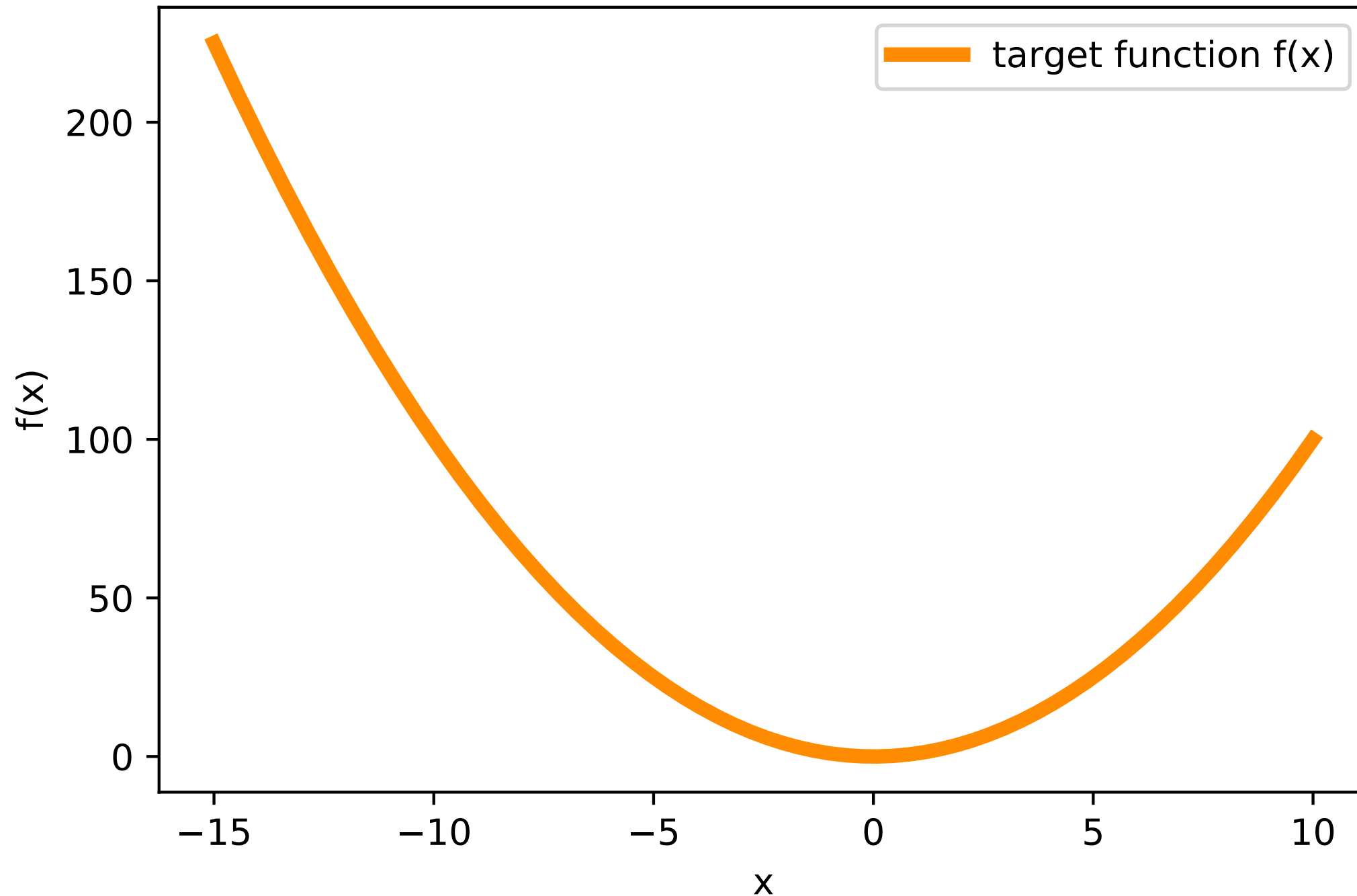
# Bias-Variance Decomposition

- Decomposition of the loss into bias and variance help us understand learning algorithms, concepts are related to underfitting and overfitting
- Helps explain why ensemble methods (last lecture) might perform better than single models

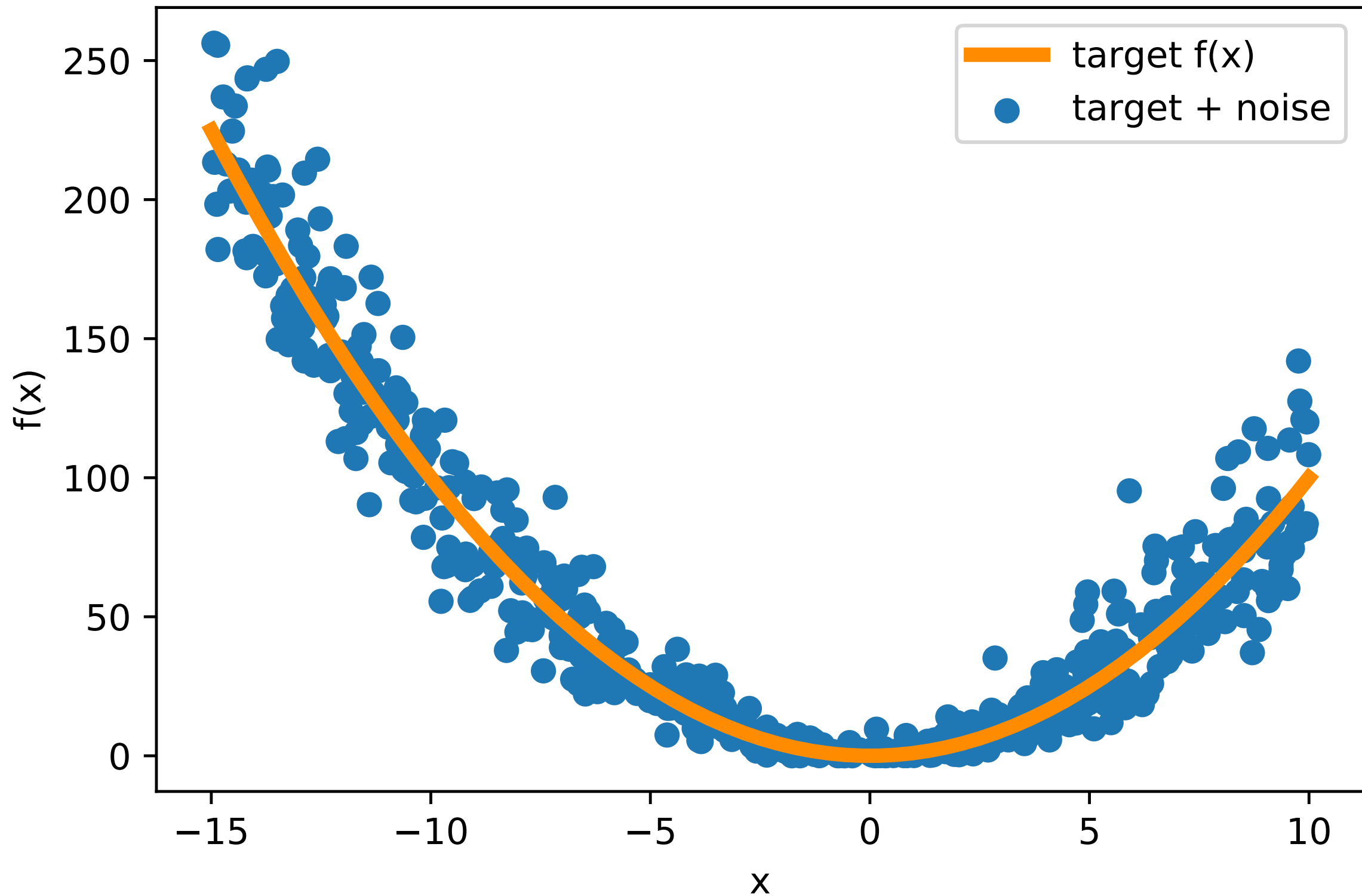
# Bias-Variance Intuition



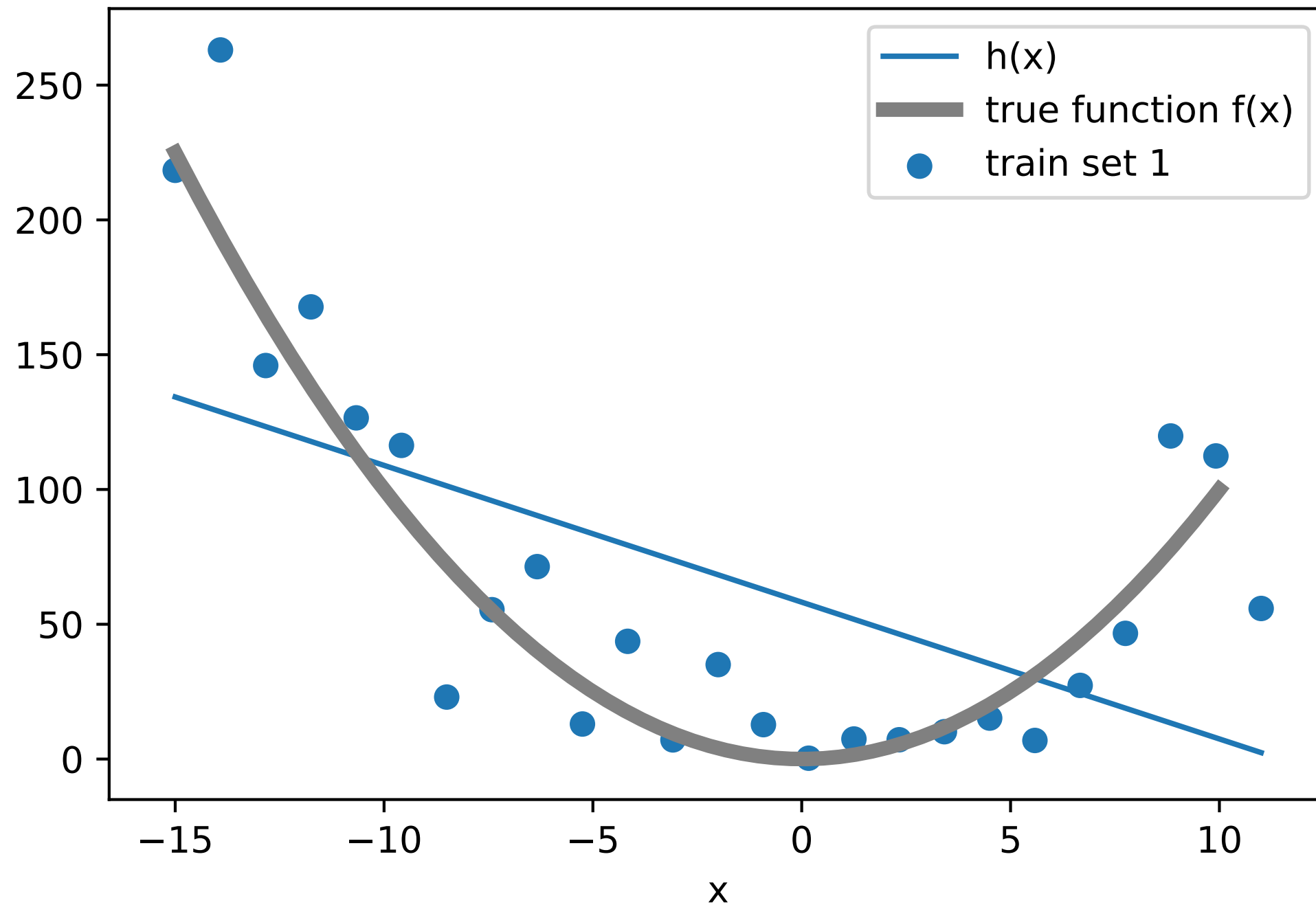
# Bias-Variance Intuition



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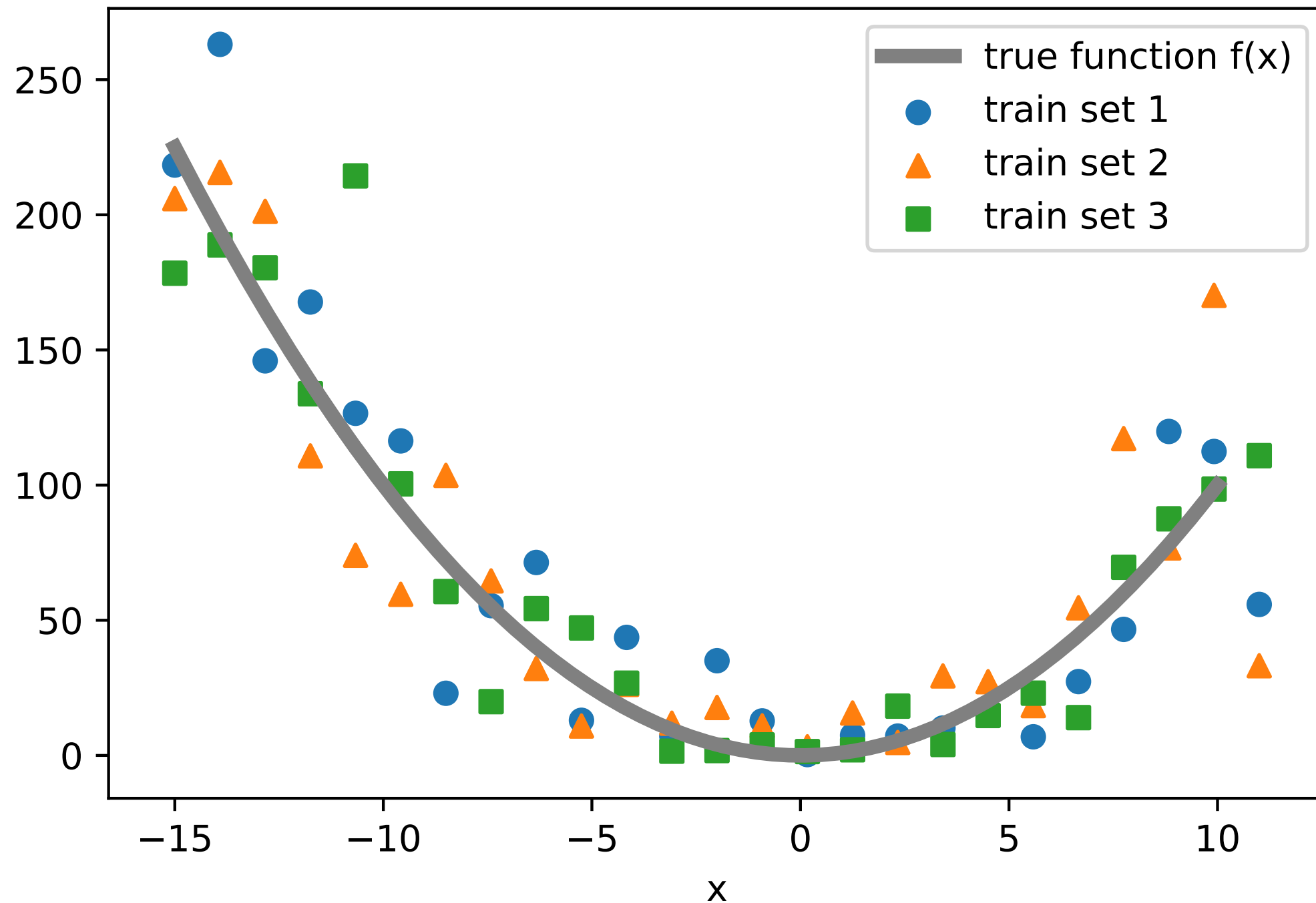


# Bias-Variance Intuition



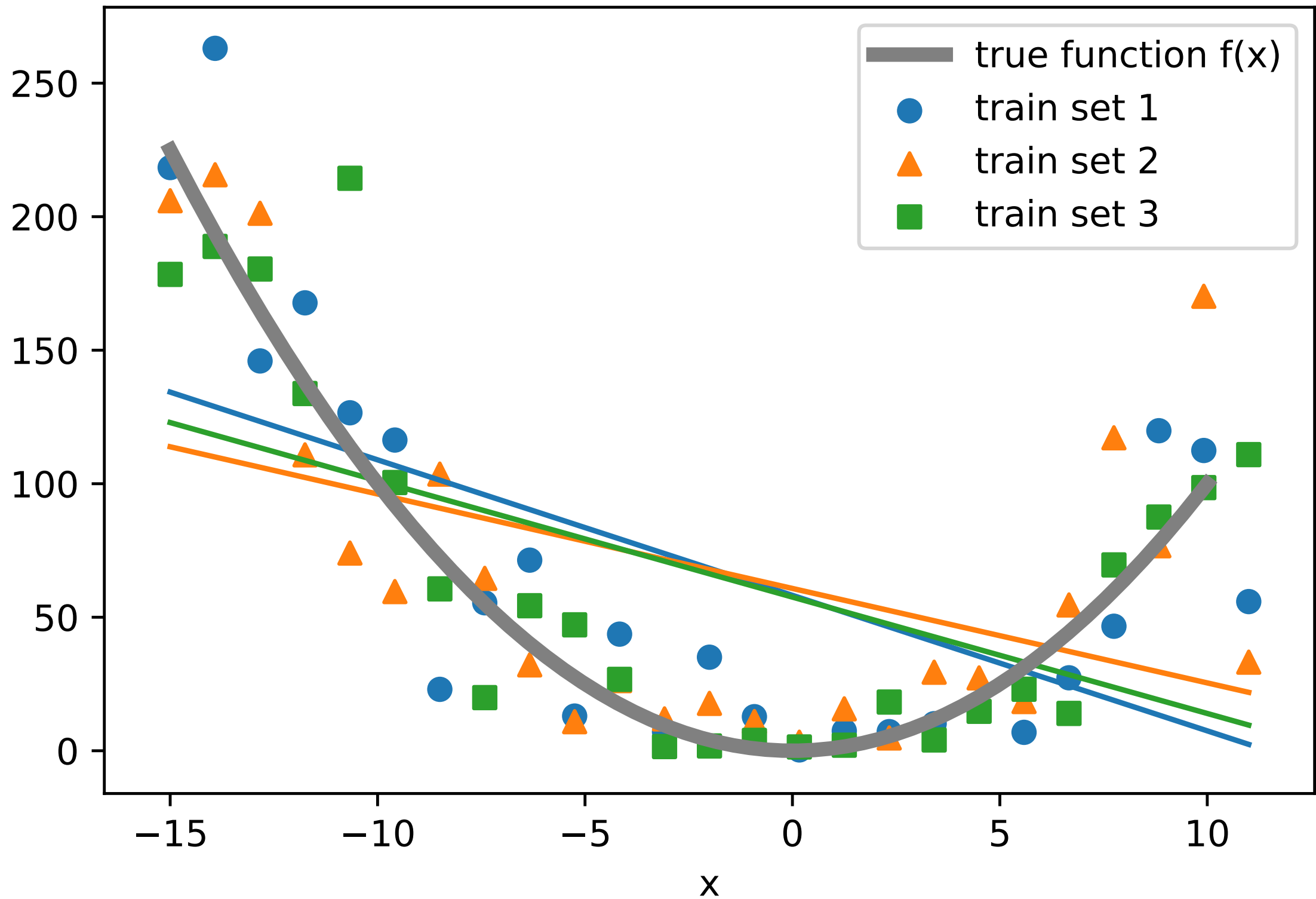
# Bias-Variance Intuition

suppose we have multiple training sets

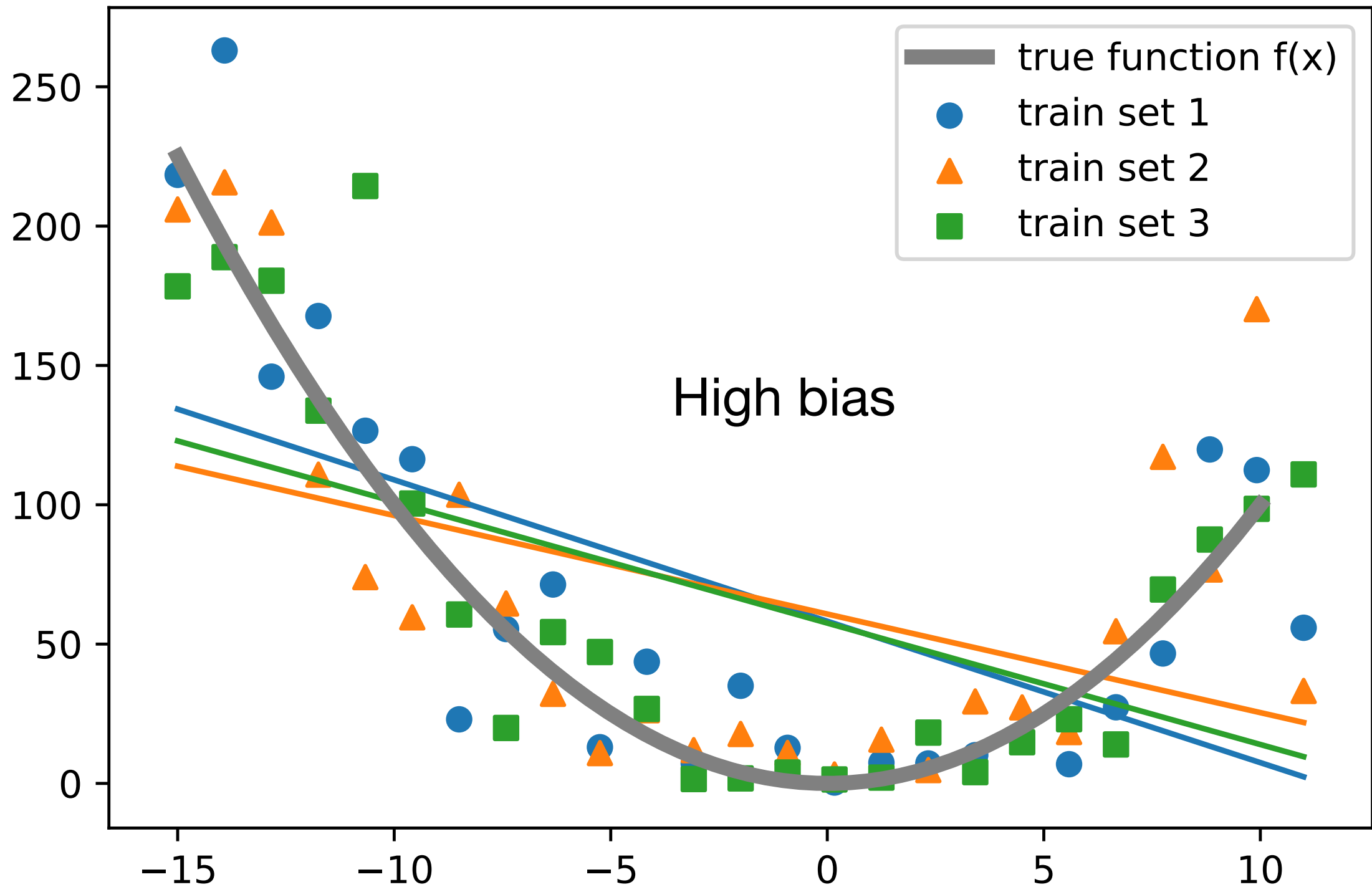




# Bias-Variance Intuition



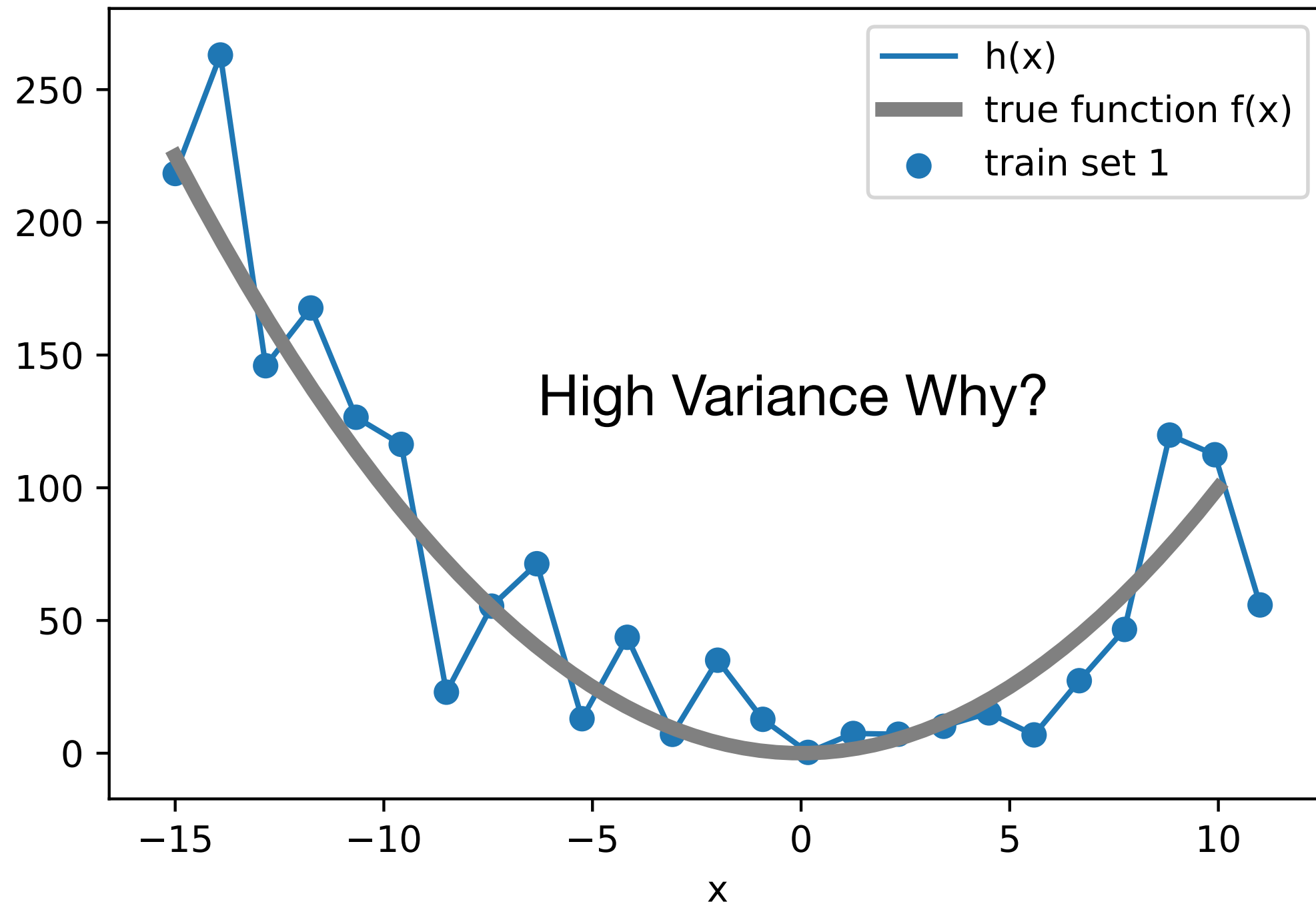
# Bias-Variance Intuition



(There are points where the bias is zero ...)

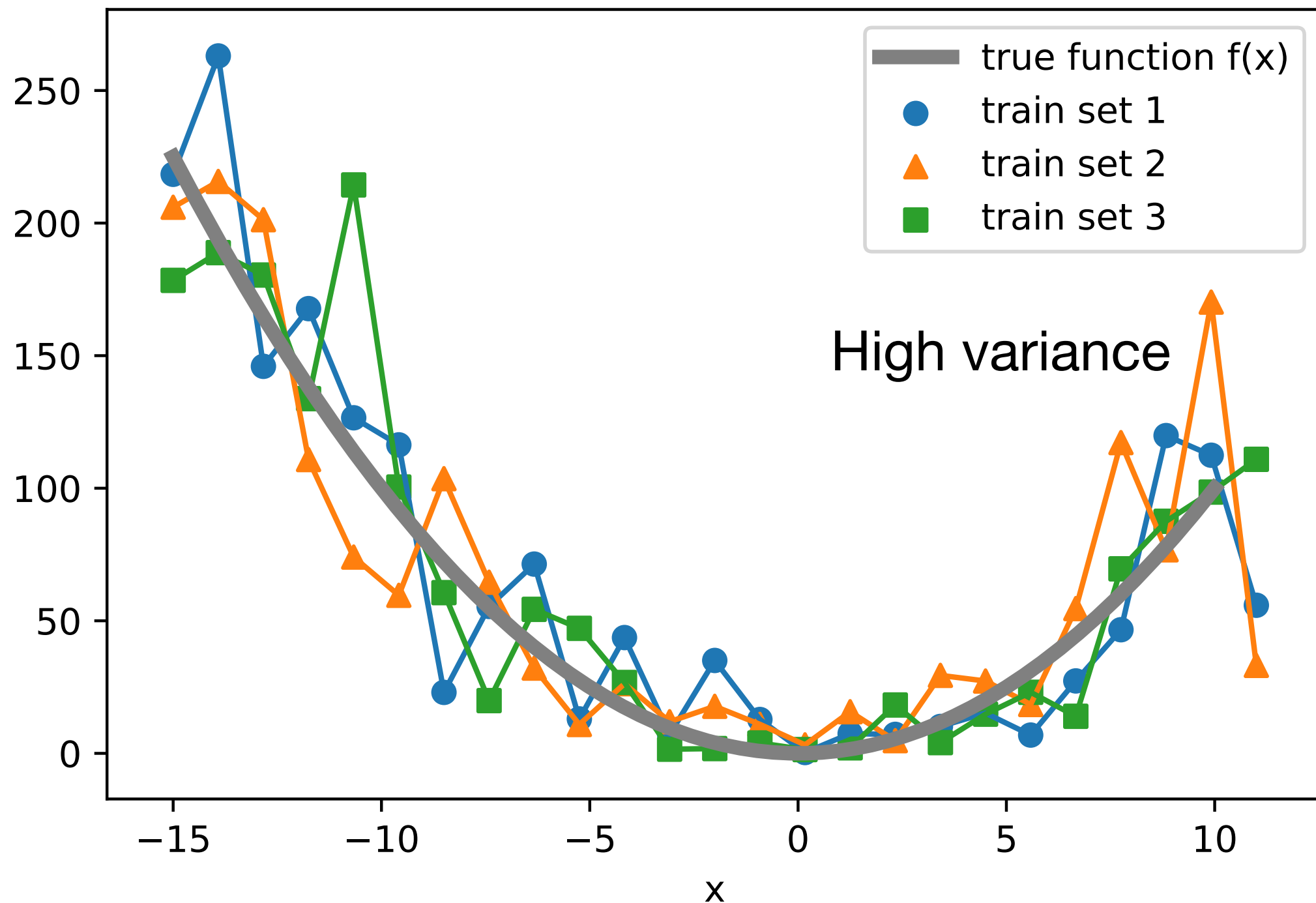
# Bias-Variance Intuition

(here, I fit an unpruned decision tree)



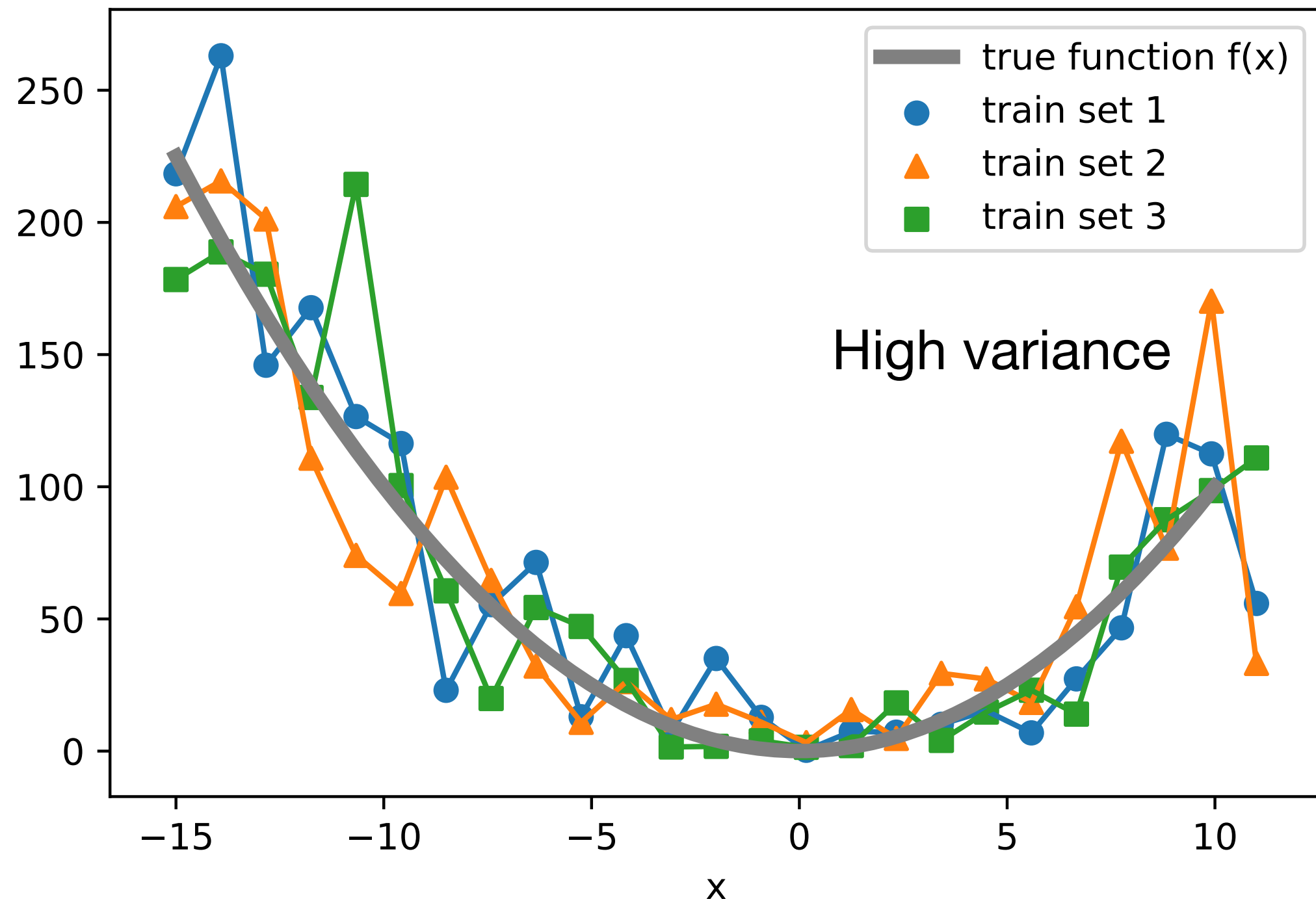
# Bias-Variance Intuition

suppose we have multiple training sets



# Bias-Variance Intuition

What happens if we take the average?  
Does this remind you of something?



# Terminology

Point estimator  $\hat{\theta}$  of some parameter  $\theta$

(could also be a function, e.g., the hypothesis is  
an estimator of some target function)



# Terminology

Point estimator  $\hat{\theta}$  of some parameter  $\theta$

(could also be a function, e.g., the hypothesis is an estimator of some target function)

$$\text{Bias} = E[\hat{\theta}] - \theta$$

# Terminology

## General Definition

$$\text{Bias}[\hat{\theta}] = E[\hat{\theta}] - \theta$$

$$\text{Var}[\hat{\theta}] = E[\hat{\theta}^2] - (E[\hat{\theta}])^2$$

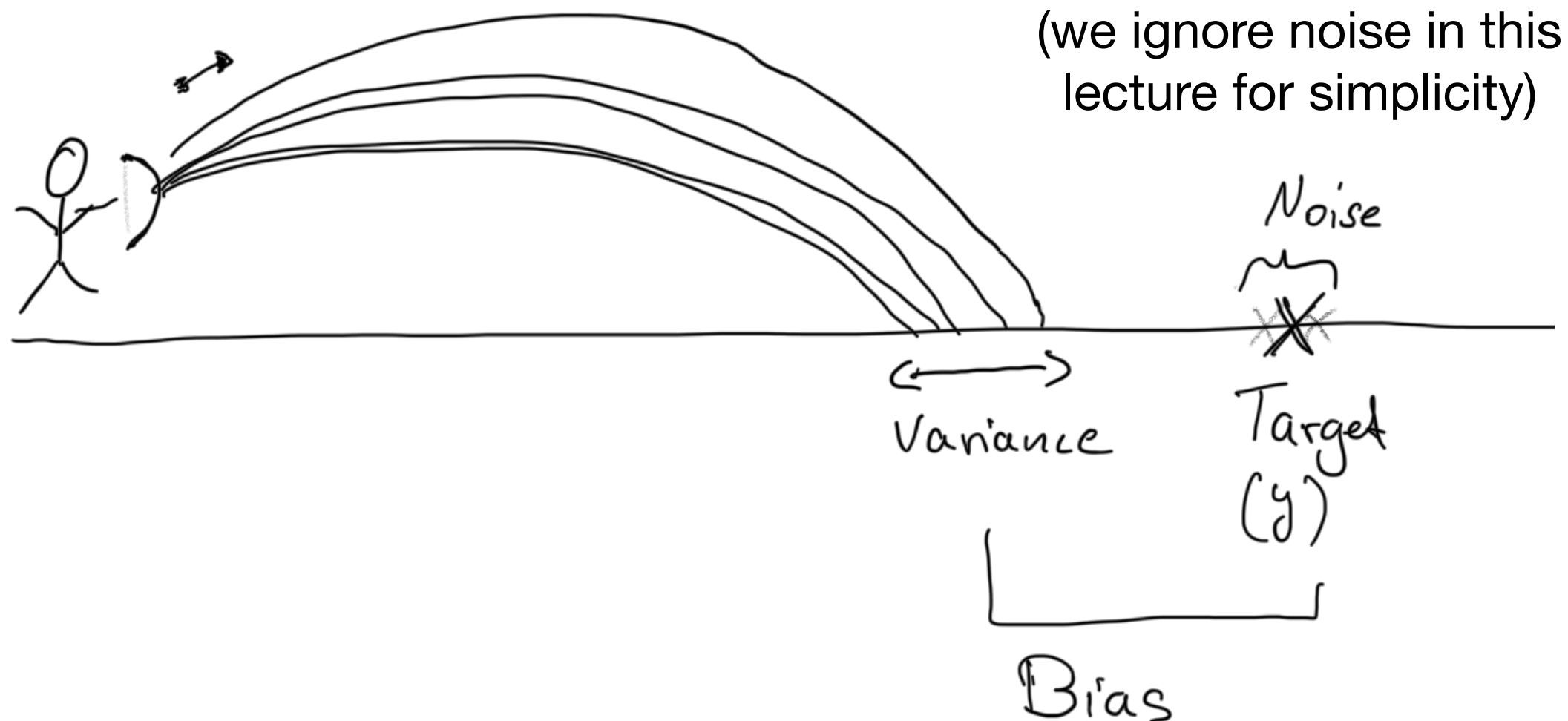
$$\text{Var}[\hat{\theta}] = E \left[ (E[\hat{\theta}] - \hat{\theta})^2 \right]$$

# Terminology

$$\text{Bias}[\hat{\theta}] = E[\hat{\theta}] - \theta$$

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## Intuition



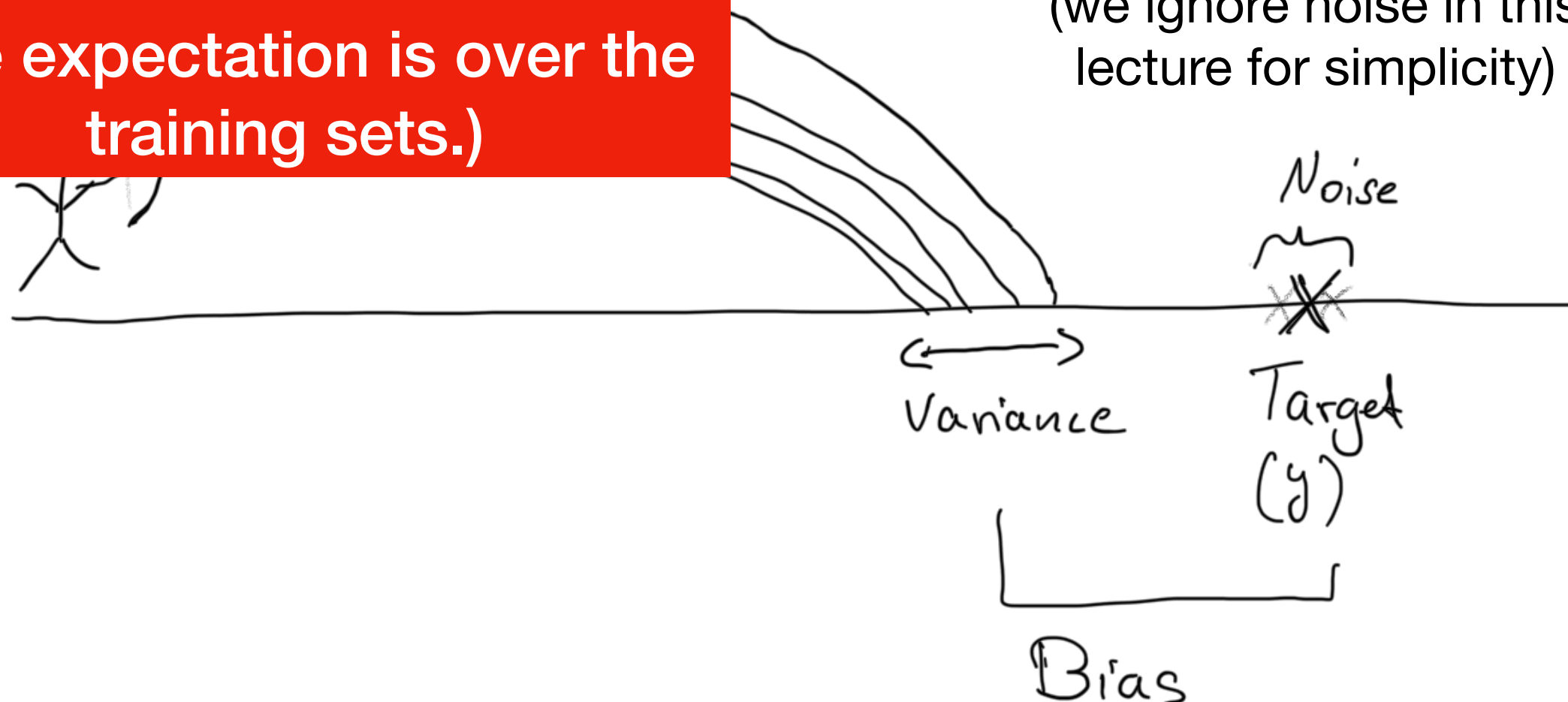
# Terminology

$$\text{Bias}[\hat{\theta}] = E[\hat{\theta}] - \theta$$

$$\text{Var}[\hat{\theta}] = E \left[ (E[\hat{\theta}] - \hat{\theta})^2 \right]$$

**Bias is the difference between the average estimator from different training samples and the true value.**  
**(The expectation is over the training sets.)**

(we ignore noise in this lecture for simplicity)



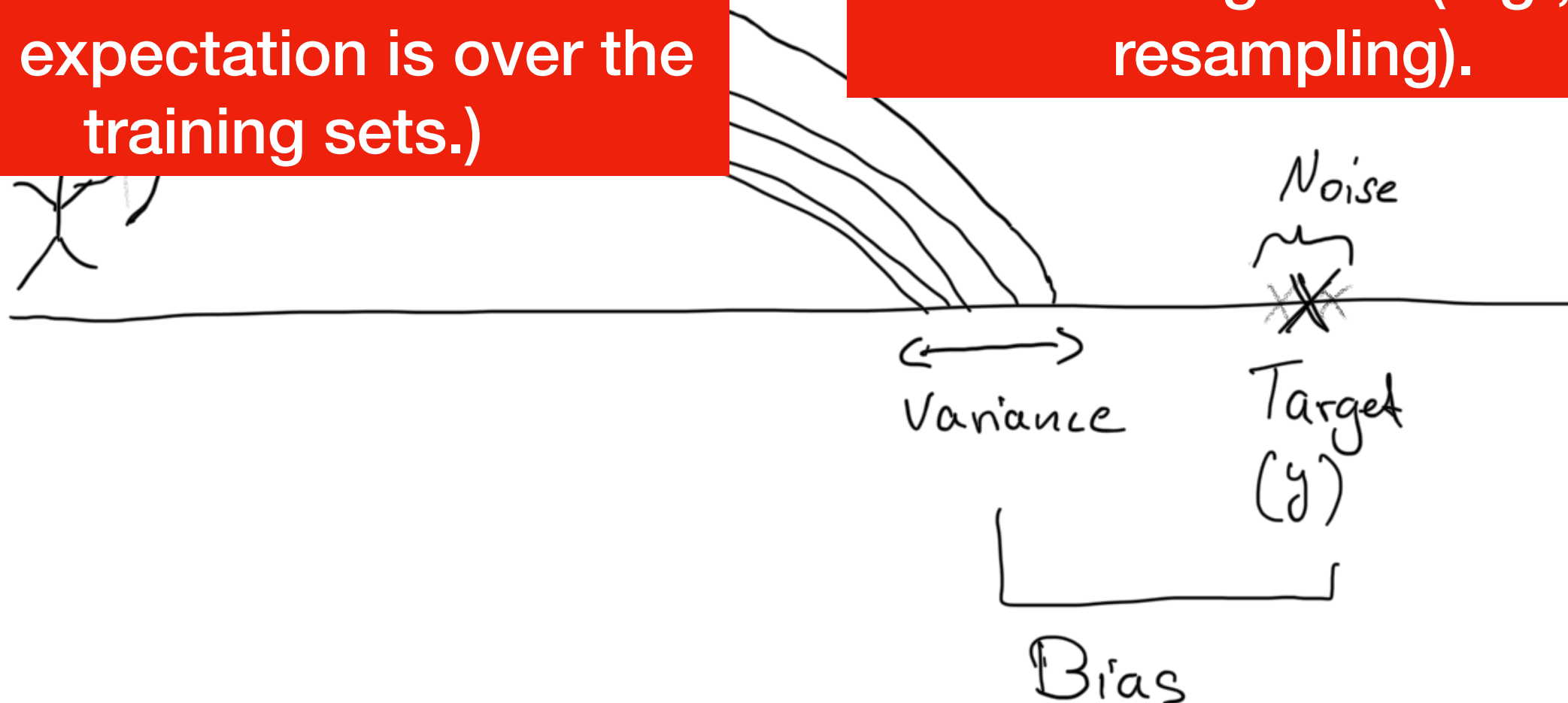
# Terminology

$$\text{Bias}[\hat{\theta}] = E[\hat{\theta}] - \theta$$

$$\text{Var}[\hat{\theta}] = E \left[ (E[\hat{\theta}] - \hat{\theta})^2 \right]$$

Bias is the difference between the average estimator from different training samples and the true value.  
(The expectation is over the training sets.)

The variance provides an estimate of how much the estimate varies as we vary the training data (e.g., by resampling).



# Bias-Variance Decomposition

$$\text{Loss} = \text{Bias} + \text{Variance} + \text{Noise}$$

# Bias-Variance of the Squared Error

$$\text{Bias}[\hat{\theta}] = E[\hat{\theta}] - \theta$$

$$\text{Var}[\hat{\theta}] = E[\hat{\theta}^2] - (E[\hat{\theta}])^2$$

$$\text{Var}[\hat{\theta}] = E \left[ (E[\hat{\theta}] - \hat{\theta})^2 \right]$$

## "ML Notation" for Squared Error Loss

$y = f(x)$  target

$\hat{y} = \hat{f}(x) = h(x)$  prediction

$S = (y - \hat{y})^2$  squared error

for simplicity, we ignore  
the noise term

(Next slides: the expectation is over the training data, i.e, the average estimator from different training samples)



# Bias-Variance of the Squared Error

$$y = f(x) \text{ target}$$

**"ML Notation" for  
Squared Error Loss**

$$\hat{y} = \hat{f}(x) = h(x) \text{ prediction}$$

$$S = (y - \hat{y})^2 \text{ squared error}$$

$$S = (y - \hat{y})^2$$

$$\begin{aligned}(y - \hat{y})^2 &= (y - E[\hat{y}] + E[\hat{y}] - \hat{y})^2 \\ &= (y - E[\hat{y}])^2 + (E[\hat{y}] - \hat{y})^2 + 2(y - E[\hat{y}])(E[\hat{y}] - \hat{y})\end{aligned}$$

# Bias-Variance of the Squared Error

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$$E[S] = E[(y - \hat{y})^2]$$

$$\begin{aligned}E[(y - \hat{y})^2] &= (y - E[\hat{y}])^2 + E[(E[\hat{y}] - \hat{y})^2] \\ &= \text{Bias}^2 + \text{Var}\end{aligned}$$

# Bias-Variance of the Squared Error

$$S = (y - \hat{y})^2$$

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???

# Bias-Variance of the Squared Error

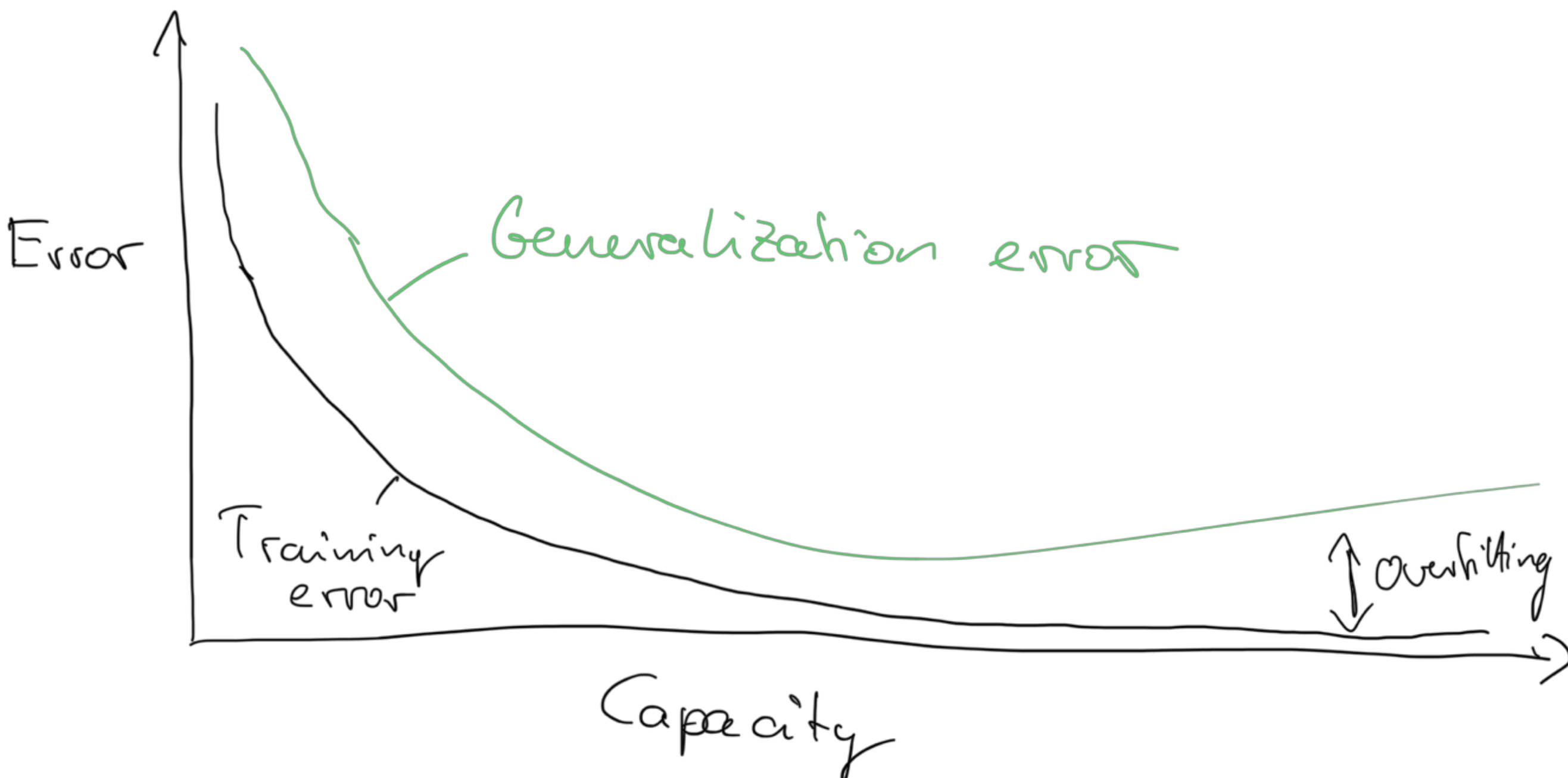
$$S = (y - \hat{y})^2$$

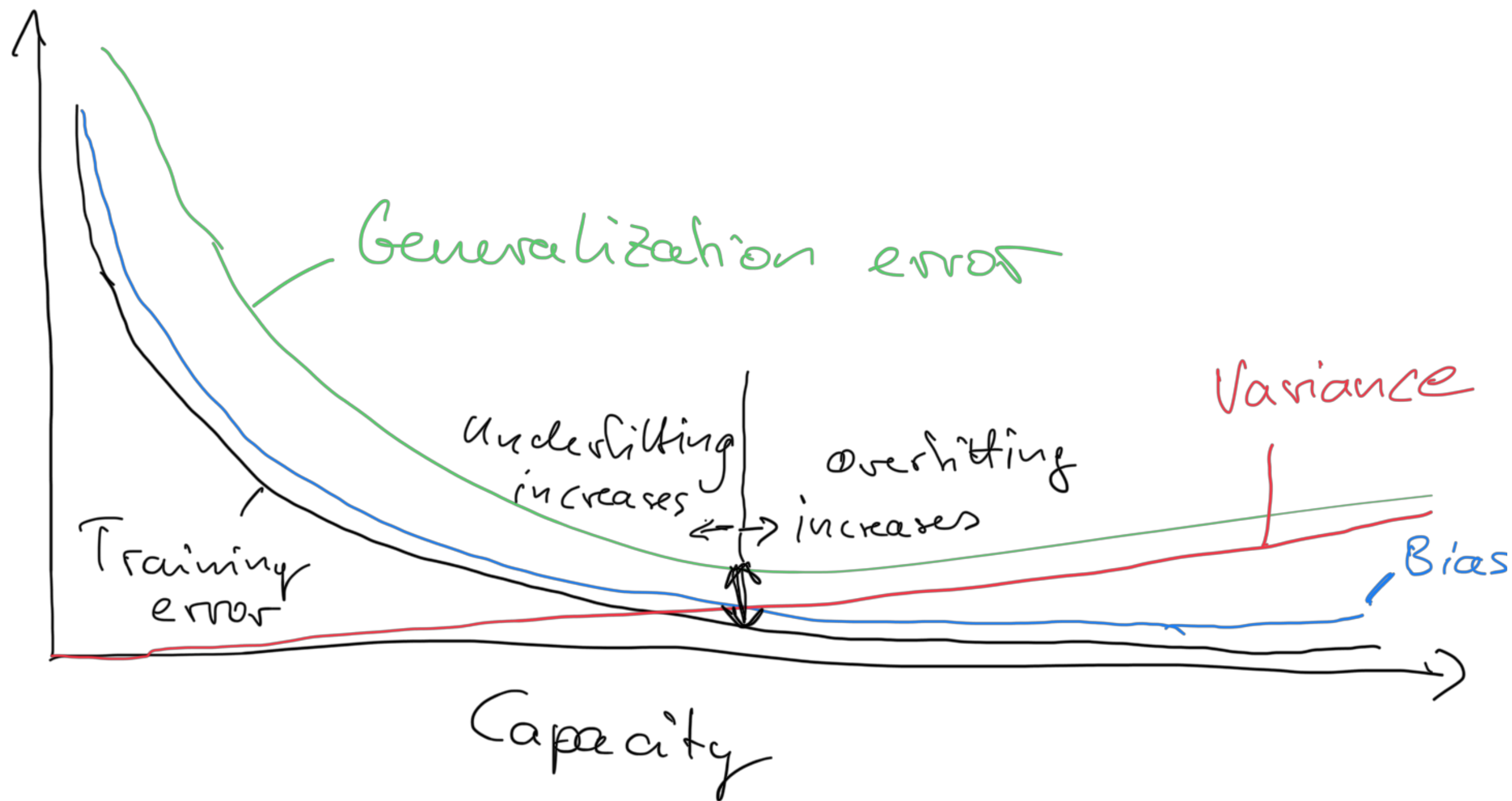
$$\begin{aligned}(y - \hat{y})^2 &= (y - E[\hat{y}] + E[\hat{y}] - \hat{y})^2 \\ &= (y - E[\hat{y}])^2 + (E[\hat{y}] - \hat{y})^2 + 2(y - E[\hat{y}])(E[\hat{y}] - \hat{y})\end{aligned}$$

???

$$\begin{aligned}E[2(y - E[\hat{y}])(E[\hat{y}] - \hat{y})] &= 2E[(y - E[\hat{y}])(E[\hat{y}] - \hat{y})] \\ &= 2(y - E[\hat{y}])E[(E[\hat{y}] - \hat{y})] \\ &= 2(y - E[\hat{y}])(E[E[\hat{y}]] - E[\hat{y}]) \\ &= 2(y - E[\hat{y}])(E[\hat{y}] - E[\hat{y}]) \\ &= 0\end{aligned}$$

**Now, how is this related to overfitting and underfitting?**







**How can we think of the bias-variance decomposition in the context of the classification error (0/1 loss)?**

Domingos, P. (2000). A unified bias-variance decomposition.  
In Proceedings of 17th International Conference on Machine Learning  
(pp. 231-238).

"several authors have proposed bias-variance decompositions related to zero-one loss (Kong & Dietterich, 1995; Breiman, 1996b; Kohavi & Wolpert, 1996; Tibshirani, 1996; Friedman, 1997). However, each of these decompositions has significant shortcomings."

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**Detailed explanation of the 0/1 loss decomposition in  
the lecture notes**

# Other "Biases"

# Statistical Bias vs "Machine Learning Bias"

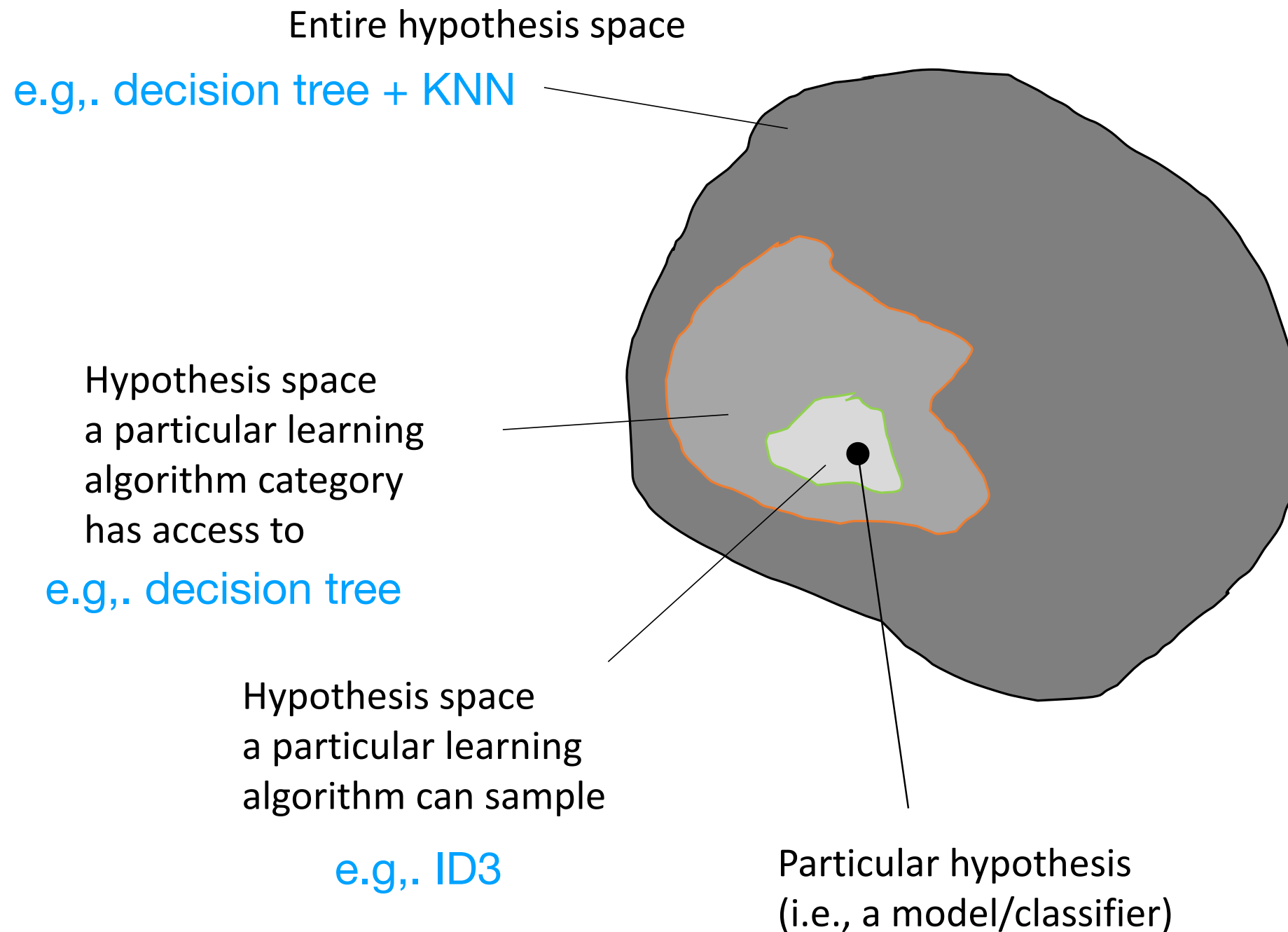
"Machine learning bias" sometimes also called "inductive bias"

e.g., decision tree algorithms consider small trees before they consider large trees

(if training data can be classified by small tree, large trees are not considered)

# Hypothesis Space

(From Lecture 1)



# "Fairness" Bias

"The term bias is often used to refer to demographic disparities in algorithmic systems that are objectionable for societal reasons. "

Barocas, S., Hardt, M., & Narayanan, A. Fairness and Machine Learning.  
<https://fairmlbook.org/introduction.html>

## Gender Shades: Intersectional Accuracy Disparities in Commercial Gender Classification

[\[edit\]](#)

**Joy Buolamwini, Timnit Gebru ; *Proceedings of the 1st Conference on Fairness, Accountability and Transparency, PMLR 81:77-91, 2018.***

### Abstract

Recent studies demonstrate that machine learning algorithms can discriminate based on classes like race and gender. In this work, we present an approach to evaluate bias present in automated facial analysis algorithms and datasets with respect to phenotypic subgroups. Using the dermatologist approved Fitzpatrick Skin Type classification system, we characterize the gender and skin type distribution of two facial analysis benchmarks, IJB-A and Adience. We find that these datasets are overwhelmingly composed of lighter-skinned subjects (79.6% for IJB-A and 86.2% for Adience) and introduce a new facial analysis dataset which is balanced by gender and skin type. We evaluate 3 commercial gender classification systems using our dataset and show that darker-skinned females are the most misclassified group (with error rates of up to 34.7%). The maximum error rate for lighter-skinned males is 0.8%. The substantial disparities in the accuracy of classifying darker females, lighter females, darker males, and lighter males in gender classification systems require urgent attention if commercial companies are to build genuinely fair, transparent and accountable facial analysis algorithms.

<http://proceedings.mlr.press/v81/buolamwini18a.html>



# Reading Assignments

## Lecture notes

[https://github.com/rasbt/stat479-machine-learning-fs19/tree/master/08\\_model-eval-1](https://github.com/rasbt/stat479-machine-learning-fs19/tree/master/08_model-eval-1)