**Topics: Normal distribution, Functions of Random Variables**

**1. The time required for servicing transmissions is normally distributed with**  **= 45**

# minutes and  = 8 minutes. The service manager plans to have work begin on the

**transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**

1. **0.3875**
2. **0.2676**
3. **0.5**
4. **0.6987**

**Answer: Option B is correct (Solution in Visual Studio Code (Basic Stat (Set-2)))**

**2. The current age (in years) of 400 clerical employees at an insurance claims**

**processing center is normally distributed with mean**  **= 38 and Standard deviation**  **=6. For each statement below, please specify True/False. If false, briefly explain why.**

1. **More employees at the processing center are older than 44 than between 38 and**

**44.**

**Answer: A.** Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6

                          = 6/6

                          =  1

                          =  84.13 %

People above 44 age = 100 - 84.13

                                     =  15.87% = 63  out of 400

Z score for 38  = (38 - 38)/6 = 0/6 = 0

                          = 50%

Hence People between 38 & 44  age = 84.13 - 50

                                                                 =  34.13%  = 137 out of 400

**Therefore, More employees at the processing center are older than 44 than between 38 and 44 is False.**

1. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

**Answer:** Probabilty of employees less than age of 30 = Pr(X<30).

Z = (X -)/ = (30 -38)/6

Thus, the question can be answered by using the normal table to find Pr(X ≤ 30) = Pr(Z ≤ (30 -38)/6) = Pr(Z ≤ -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under age 30=0.0912\*400=36.48(or 36 employees).

Therefore, the statement B of the question is TRUE

* 1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

**Sol:** As we know that if X  N(µ1, σ1^2 ), and Y  N(µ2, σ2^2 ) are two independent random variables then X + Y  N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y  N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and

Y , then Z  N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N 4µ,6 σ^2)

* 1. **Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**

* 1. **90.5, 105.9**
  2. **80.2, 119.8**
  3. **22, 78**
  4. **48.5, 151.5**
  5. **90.1, 109.9**

**Answer: Option D is correct ((Solution in Visual Studio Code (Basic Stat (Set-2)))**

**5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**

1. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

**Answer:** Range (99, 980) in millions

1. **Specify the 5th percentile of profit (in Rupees) for the company? Answer:** 5th percentile of profit (in million) is 170

1. **Which of the two divisions has a larger probability of making a loss in a given year?**

**Answer:** Two divisions are 0.047 and 0.0400