National Institute of Technology Rourkela

Department of Computer Science and Engineering

B.Tech/Dual Degree 5th Semester

End Semester Examination (Autumn) 2017

Subject: **Theory of Computation** Subject Code: **CS 331**

Full Marks: **50** Duration: **3 Hours**

Answer any ***FIVE*** questions.

Figures at the right margin indicate marks.

All parts of a question must be answered at one place.

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1. (a) Draw the state diagram for a Turing machine that increments a binary number. Assume that the input tape contains at least one non-blank symbol. For example, if the initial tape contains the binary representation of 7 i.e., $..b111b..$ then the output tape should be the binary representation of 8, i.e., $..b1000b..$ (where b represents the blank symbol). [4]

   (b) Find the language generated by the following grammars: [4]

   i. S → 0S1 | 0A1, A→1A |1

   ii. S→0S1 | 0A |0 | 1B |1, A→0A |0, B→1B|1

   iii. S→0S1 | 0A1, A→1A0 |10

   iv. S→0A | 1S | 0 | 1, A→1A|1S|1

   (c) Construct a regular grammar which can generate the set of all strings starting with a letter (A to Z) followed by a string of letters or digits (0 to 9). [2]

2. (a) Let G be the grammar with the following rules [2]
   S→0B|1A, A→0|0S|1AA, B→1|1S|0BB. For the string 00110101 find

   i. Left most derivation,

   ii. Right most derivation

   (b) Show that the grammar S→a | abSb | aAb, A →bS|aAAb is ambiguous. [2]

   (c) Construct a reduced grammar equivalent to the grammar S→aAa, A→Sb|bCC|DaA, C→abb|DD, E→aC, D→aDA [2]

   (d) State the pumping lemma for context free grammar. Show that the language $L = \{a^{n^2} | n \geq 1\}$ is not a context-free. [4]

3. (a) Convert the following grammar into it's equivalent Greibach Normal Form. [4]
   $S \rightarrow AB$
   $A \rightarrow BS|b$
   $B \rightarrow SA|a$

   (b) Consider a language $L = \{a^m b^n|$ where n and m are positive integers and $n < m\}$. [6]

   i. Find the context-free grammar for L.

   ii. Construct the state transition diagram of the PDA accepting L by empty store.

   iii. Construct the state transition diagram of the PDA accepting L by final state.

4. (a) Find a context free grammar with minimum number of production rules possible for the language given below and also, construct the PDA. [3]
$\{1^m 1^n 1^{m+n} 0^p 0^q 0^{p+q} \mid m, n, p, q \geq 0\}$
(b) Consider the grammar [3]
$S \rightarrow abScB|\lambda$
$B \rightarrow bB|b$
What language does it generate?
(c) Construct a deterministic finite automaton equivalent to the grammar [4]
$S \rightarrow aS|bS|aA$
$A \rightarrow bB$
$B \rightarrow aC$
$C \rightarrow \in$

5. (a) Construct a DFA equivalent to an NFA whose transition table is defined in the following Table 1: [4]

Table 1: State transition table for the NFA,. **Here q_3 is the final state.**

State	Input a	Input b
$\rightarrow q_0$	$\{q_1, q_3\}$	$\{q_2, q_3\}$
q_1	$\{q_1\}$	$\{q_3\}$
q_2	$\{q_3\}$	$\{q_2\}$
q_3	Φ	Φ

(b) Design a Turing machine to compute $f(x) = x/2$ if x is even and $f(x) = (x+1)/2$ if x is odd, where x is positive integer represented in unary. [6]

6. (a) The state transition function of a NPDA is given below which accepts the language by empty stack. Find the Context Free Grammar.(N:B- *Here q_0 is the initial state of the machine*) [4]
$\delta(q_0, a, Z) \rightarrow (q_1, XZ)$
$\delta(q_1, a, X) \rightarrow (q_2, Y)$
$\delta(q_2, a, Y) \rightarrow (q_1, XY)$
$\delta(q_2, b, Y) \rightarrow (q_3, \lambda)$
$\delta(q_3, b, Y) \rightarrow (q_3, \lambda)$
$\delta(q_3, \lambda, Z) \rightarrow (q_3, \lambda)$

(b) Write a regular expression for the following languages: [6]
i. The language consisting of all odd integers *without* leading zeros over the alphabet $\Sigma = \{0, 1, 2, ..., 9\}$.
ii. The set of all strings over $\{0, 1\}$ which has at most two zeros.
iii. The set of all strings over $\{0, 1\}$ such that any block of five consecutive symbols contains at least two 0's.

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