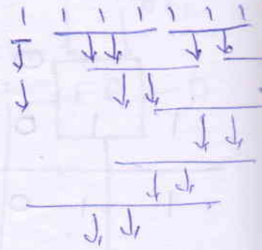
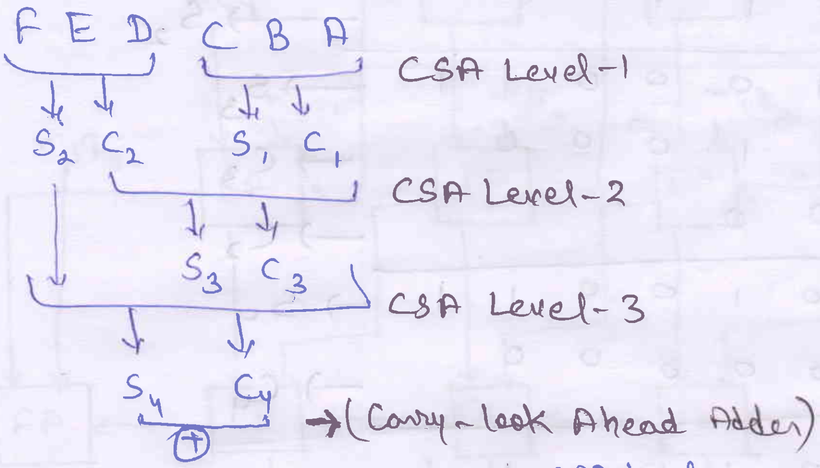
★ If we have ~~6~~ 6 bits, how to count CSA levels.





Gate-delay $= 1 + (2 \times \underbrace{3}_{\text{CSA level}}) + \underbrace{8}_{\text{CLA}}$

$= \boxed{15}$

07.02

Revisiting CLA & Higher level Generate & Propopogate function

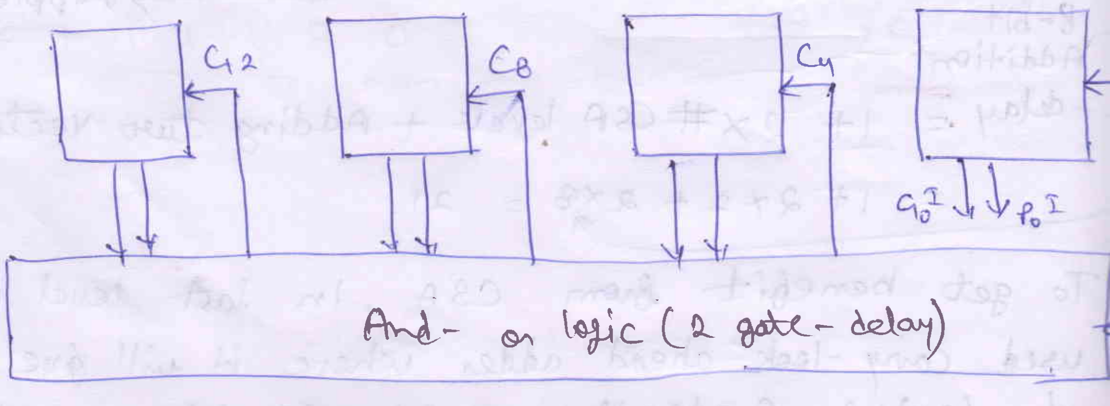
$C_1 = G_0 + P_0 C_0$

$C_2 = G_1 + P_1 C_1 = G_1 + P_1 G_0 + P_1 P_0 C_0$

$C_3 = G_2 + P_2 C_2 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$

$C_4 = G_3 + P_3 C_3 = \underbrace{G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0}_{G_0^I} + \underbrace{P_3 P_2 \ldots}_{P_0^I}$

Similarly, write C_8, C_{12}, C_{16} in terms of G_0^I & P_0^I



Gate-delays:

→ G_0^I, P_0^I : 3 gate-delays.

→ C_4, C_8, C_{12}, C_{16} : 5 gate-delays

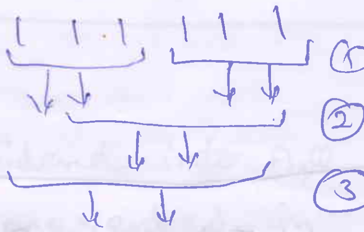
→ C_{13}, C_{14}, C_{15} : 7 gate-delays

→ S_{16} : 8 gate-delays,

$$G_{16} = \underbrace{G_3^I + P_3^I G_2^I + P_3^I P_2^I G_1^I + P_3^I P_2^I P_1^I G_0^I}_{G_0^{II}} + \underbrace{P_3^I P_2^I P_1^I P_0^I}_{P_0^{II}} C_0$$

In this case, for S_{64}, we will have $\boxed{12 \text{ gate-delays}}$.

Que:- What will be the gate-delay if you use CSA for multiplying ~~two~~ 6-bit number using Higher-level Generate & Propagate function / or adding ~~to~~ two vectors at last level :-



(12) Gate-delay = $\overset{m_i q_i}{1} + (2 \times 3) + 8 = 15$

For 32-bit using 2nd Higher-level G_0^{II} & P_0^{II} :-

11 111 111 111 111 111 111 111 111 111 111 — Level 1

$2+20 = 1+7$ groups — 2

$1+14 = 5$ groups — 3

$2 \times 5 = 10 = 1+3$ groups — 4

$1+2 \times 3 = 7 = 1+2$ groups — 5

$1+2 \times 2 = 5 = 2+1$ group — 6

$2+1 \times 2 = 4 = 1+1$ group — 7

$1+1 \times 2 = 3 = 1$ group — 8

2 ⟶ 2nd Higher-level CLA.

$1+(2 \times 8)+12 = \boxed{29}$

Approximating no. of levels :-

Let n be the no. of summands.

$$n$$
$$\downarrow$$
$$2\times\frac{n}{3} \quad \left(= 2\times\left\lfloor\frac{n}{3}\right\rfloor + n \bmod 3\right)$$
$$\downarrow$$
$$2^2\times\frac{n}{3^2}$$
$$\downarrow$$
$$\vdots$$
$$\downarrow$$
$$2^K\cdot\frac{n}{3^K}$$

$$\left(\frac{2}{3}\right)^K = \frac{2}{n}$$

$$\Rightarrow K\log_2\left(\frac{2}{3}\right) = \log_2\frac{2}{n}$$

$$\Rightarrow K = \frac{\log_2 2 - \log_2 n}{\log_2 2 - \log_2 3}$$

$$\Rightarrow \boxed{K = \frac{\log_2\frac{n}{2}}{\log_2\frac{3}{2}}}$$

$$\Rightarrow K = \log_{3/2}\left(\frac{n}{2}\right)$$

$$\Rightarrow K = \frac{\log_2\frac{n}{2}}{0.586}$$

$$\Rightarrow \boxed{K = 1.7(\log_2 n - 1)}$$

Integer - Division (Stallings) :-

Pencil & Paper Method :-

```
            00001101    → Quotient
Divisor → 1011 | 10010011  → Dividend
             - 1011
               001110
             -   1011
                 001111
             -     1011
                    100
                     ↑
                 Remainder
```