National Institute of Technology Rourkela
Department of Mathematics
Third Semester(Autumn), Academic Year 2020-2021

Course: (MA 2203) (Assignment)Tutorial: I
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1. Define what is an random experiment. Further discuss various possible ways of defining probability for an event in a sample space.

2. Let A and B are any two events of a sample space S such that $A \subset B$. Then show that $P(A) \leq P(B)$.

3. Prove that if the events A and B are independent then (i) A^c and B^c, (ii)A and B^c,(iii)A^c and B are independent.

4. For any arbitrary events A and B show that $P(A \cup B) \leq P(A) + P(B)$. Further generalize the result to any number of events.

5. If six dice are rolled what is the probability that all show different faces.

6. Show that if the events A, B and C are independent, then (i)A and $B \cup C$ and (ii) $A - B$ and C are independent.

7. Three screws are drawn at random from a lot of 100 screws, 10 of which are defective. Find the probability of the event that all 3 screws drawn are non defective, assuming that we draw (a) with replacement, (b) without replacement.

8. What is the probability that in a group of 20 people(that includes no twins) at least two have the same birthday, if we assume that the probability of having birthday on a given day is 1/365 for every day.

9. A die is loaded in such a manner that for $n = 1, 2, 3, 4, 5, 6$ the probability of the face marked n, landing on top when the die is rolled is proportional to n. Find the probability that an odd number will appear on tossing the die.

10. For any three events A, B and C, prove that $P((A \cup B)|C) = P(A|C) + P(B|C) - P((A \cap B)|C)$.

11. A problem in Mathematics is given to three students A, B and C whose chances of solving it are 1/2, 3/4 and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently.

12. From a city population, the probability of selecting (i) a male or a smoker is 7/10 (ii) a male smoker is 2/5 and (iii) a male, if a smoker is already selected is 2/3. Find the probability of selecting (a) a non-smoker (b)a male and (c) a smoker if a male is first selected.

13. A slip of paper is given to person A, who marks it with either a plus or minus sign; the probability of her writing a plus sign is 1/3. A passes the slip to B, who may either leave it alone or change the sign before passing it to C. Next, C passes the slip to D after perhaps changing the sign; finally, D passes it to a referee after perhaps changing the sign. The referee sees a plus sign on the slip. It is known that B,C and D each change the sign with probability 2/3. Compute the probability that A originally wrote a plus sign.

14. The sum of two non-negative quantities is equal to 2n. find the chance that their product is not less than 3/4 times their greatest product.

15. Each coefficient in the equation $ax^2 + bx + c = 0$, is determined by throwing a fair die. Find the probability that the equation will have real roots.

16. What is pairwise independent and independent of events? Is it true that pairwise independent implies independent and the vice-versa? Justify with a suitable example.

17. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA?

18. A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. What is the chance that actually there was a six?

19. The outcomes of an experiment are represented by the points in the square bounded by $x = 0$, $y = 0$, $x = a$, $y = a$ in the xy-plane. If the probability is distributed uniformly, determine the probability that $x^2 + y^2 > a^2$.

20. Two points are taken at random on the given straight line of length a. Show that the probability of their distance exceeding a given length $c(< a)$ is equal to $(1 - \frac{c}{a})^2$.

21. Three students A, B and C are in a running race. A and B have the same probability of winning and each it twice as likely to win as C. Find the probability that B or C wins.

22. Companies C_1, C_2 and C_3 produce 30%, 45% and 25% of the cars respectively. It is known that 2%, 3% and 2% of these cars produced from C_1, C_2 and C_3 are defective. (a) What is the probability that a car purchased is defective? (b) If a car purchased is found to be defective, what is the probability that this car produced by company C_1.

23. An interval of length 1, say $(0, 1)$ is divided into three intervals by choosing two points at random. What is the probability that the three line segments form a triangle?

24. Let $S = \mathcal{R}$, and $\mathcal{A}$: the set of all subsets of $\mathcal{R}$. For each interval I in $\mathcal{A}$ let us define the function P as $P(I) = \int_I \frac{1}{\pi} \frac{1}{1+x^2} dx$. Does P defines a probability on the sample space S.

25. In Q.no 24, let A be the event $A = \{x : x \geq 0\}$. Find $P(A)$ and $P(\{x : x > 0\})$.

26. Urn 1 contains one white and two black marbles, urn 2 contains one black and two white marbles, and urn 3 contains three black and three white marbles. A die is rolled. If a 1,2,or 3 shows up, urn 1 is selected; if a 4 shows up, urn 2 is selected; and if a 5 or 6 shows up urn 3 is selected. A marble is then drawn at random from the urn selected. Let A be the event that the marble drawn is white. If U, V, W respectively denote the events that the urn selected is 1,2,3 then find $P(A)$. Also calculate the conditional probabilities $P(V|A)$, $P(W|A)$.

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