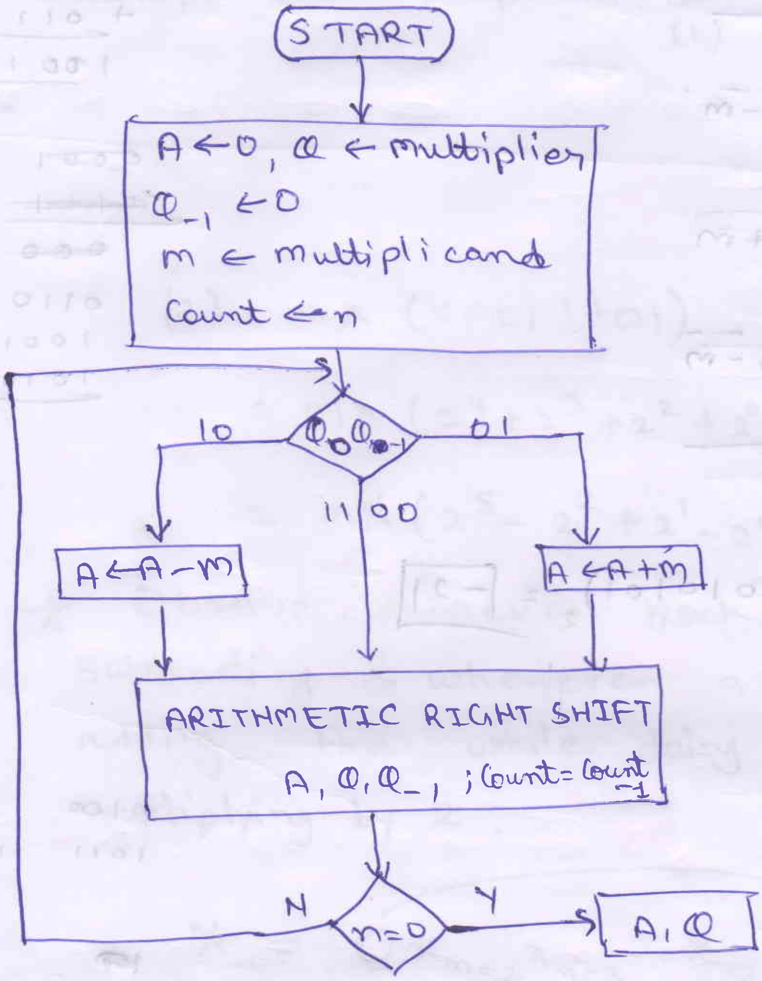
Booth's Algorithm for Signed Numbers :-



Eg: − 1001 (−7) → M
0011 (+3) → Q

A	Q	Q_{-1}	Count
0000	0011	0	4;
0111	0011	0	4; A ← A
0011	1001	1	3; ARS A,
0001	1100	1	2; ARS A,
1010	1100	1	2; A ← A
1101	0110	0	1; ARS A
1110	1011	0	0

Answer = $\boxed{(1110\ 1011) = (-}$

(1) Perform (7 X 3) using Booth's Algorithm,

(2) multiply 0111 by 1101 using Booth's Algorithm,

(3) multiply 1001 by 1101 using Booth's Algorithm,

(1)

A	Q	Q_{-1}	Count
0000	0011	0	4;
1001	0011	0	4; A ← A − m
1100	1001	1	3;
1110	0100	1	2;
0101	0100	1	2; A ← A + m
0010	1010	0	1;
0001	0101	0	0;

7 = 0111 = m

3 = 0011 = Q

1110
+ 0111
10101

$(10101)_2 = \boxed{21}$

(2) $\overset{M}{(0111)} \times \overset{Q}{(1101)}$ 0011

$7 \times (-3)$

A	Q	Q_{-1}	Count
0000	1101	0	4;
1001	1101	0	4; A ← A − M
1100	1110	1	3;
0011	1110	1	3; A ← A + M
0001	1111	0	2;
1010	1111	0	2; A ← A − M
1101	0111	1	1;
1110	1011	1	0;

$(11101011)_2 = -(00010101)_2 = \boxed{-21}$

(3) $\overset{M}{1001} \times \overset{Q}{1101}$

$(-7) \quad (-3)$

A	Q	Q_{-1}	Count
0000	1101	0	4;
0111	1101	0	4; A ← A − M ⇒ A ← A + (2'M)
0011	1110	1	3; ARS AQQ_{-1}, C = C − 1
1100	1110	1	3; A ← A + M
1110	0111	0	2; ARS AQQ_{-1}, C = C − 1
0101	0111	0	2; A ← A − M
0010	1011	1	1; ARS AQQ_{-1}, C = C − 1
0001	0101	1	0; ARS AQQ_{-1}, C = C − 1

~~00001~~

$(10101) = \boxed{21}$

→ Stallins

Why (and How) does Booth's Algorithm work?

Case I :- +ve multiplier :-

(1) $m \times (00011110)$

$= m \times (2^4 + 2^3 + 2^2 + 2^1)$

$= m \times (2^5 - 2^1)$

(2) $m \times (00011101)$

$= m \times (2^4 + 2^3 + 2^2 + 2^0)$

$= m \times (2^5 - 2^2 + 2^1 - 2^0)$

☆ Observe whenever block of '1' is coming we are subtracting & whenever a block '0' starts, we are adding. And while doing Right-shift, we are just multiplying by 2.

$X = 0\, x_{n-2} x_{n-3} \ldots x_0$; Starting-bit $= 0$ i.e. +ve

$$\boxed{\text{For Block of } (k+1)\text{ 1's} \quad 2^n + 2^{n-1} + 2^{n-2} + \ldots + 2^{n-k} = 2^{n+1} - 2^{n-k}}$$

Case II :- -ve multiplier :-

$1\, x_{n-2}\, x_{n-3} \ldots x_0$

$= -2^{n-1} + x_{n-2} \cdot 2^{n-2} + x_{n-3} \times 2^{n-3} + \ldots + x_1 \times 2^1 + x_0 \times 2^0$

Eg :- $11..11110\, x_{k-1} x_{k-2} \ldots x_0$

$= -2^{n-1} + 1 \times 2^{n-2} + 2^{n-3} + 2^{n-4} + \ldots + 2^{k+1} + x_{k-1} \cdot 2^{k-1} + x_{k-2} \cdot 2^{k-2} + \ldots + x_0 \cdot 2^0$

$= -2^{n-1} + (2^{n-1} - 2^{k+1}) + x_{k-1} \cdot 2^{k-1} + \ldots + x_0 \cdot 2^0$

$= -2^{k+1} + x_{k-1} \cdot 2^{k-1} + \ldots + x_0 \cdot 2^0$