NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA
END - SEM EXAMINATION, 2016 (AUTUMN)
B.Tech./Dual Degree 5th Semester

Subject code: CS331 **Subject Name:** Theory of Computation
Full Marks: 50 **Duration:** 3 Hours

All parts of a question should be answered at one place.
Question paper contains two pages.Question# 1 to 7 carries 6 marks each and Question#8 carries 8 marks

1) **a)** Construct an NPDA corresponding to the grammar given below: [3+3]

$$S \rightarrow aABB|aAA;\ A \rightarrow aBB|b;\ B \rightarrow bBB|A.$$

b) Find a context-free grammar that generates the language accepted by the NPDA

$M = (\{q_0, q_1\}, \{a, b\}, \{A, z\}, \delta, q_0, z, \{q_1\})$, with transitions,

$$\delta(q_0, a, z) = \{(q_0, Az)\}$$
$$\delta(q_0, b, A) = \{(q_0, AA)\}$$
$$\delta(q_0, a, A) = \{(q_1, \lambda)\}$$

2) **a)** Design a Turing machine that will accept the following language over $\sum = \{a, b\}$ [3+3]

$$L = \{a^n b^m; n \geq 2, n = m\}.$$

Plot the state transition graph and validate the string "***aaaabbbb***".

b) Design a Moore machine to detect a run in the input, that is, sequences of two or more identical symbols. For example, given the input ***abaabbbabaa***, the output shall be 00010110001.

3) **a)** Convert the following grammar into Greibach normal form. [3+3]

$$S \rightarrow ABb|a|b;\ A \rightarrow aaA|B;\ B \rightarrow bAb$$

b) Remove all unit-productions, all useless productions, and all λ-productions from the grammar

$$S \rightarrow aA|aBB;\ A \rightarrow aaA|\lambda;\ B \rightarrow bB|bbC;\ C \rightarrow B.$$

4) **a)** Show that the following grammar is ambiguous. [3+3]

$$S \rightarrow AB|aaaB;\ A \rightarrow a|Aa\ ;\ B \rightarrow b$$

b) Find the context-free grammar for $\sum = \{a, b\}$ for the language

$$L = \{a^n w w^R b^n\ (w \in \sum^*, n \geq 1\}$$

5) a) Find regular expressions for the languages accepted by the following automata. [3+3]

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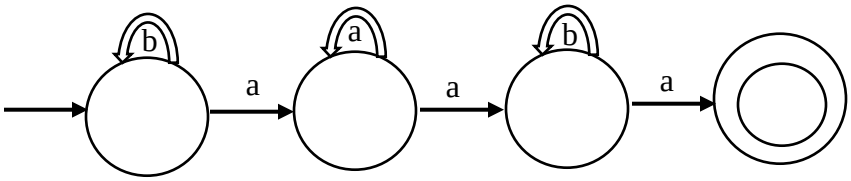


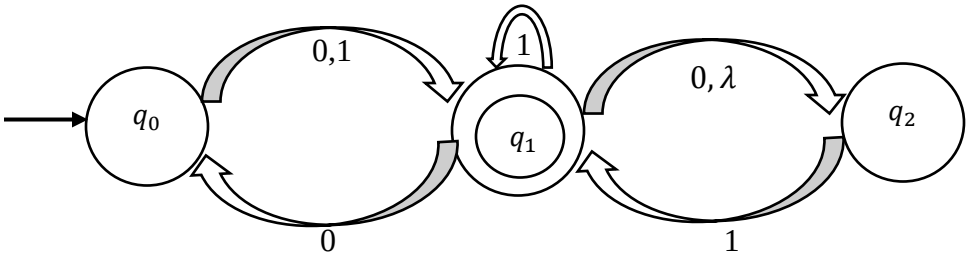
b) Find the index of a symbol in a given string, that is, given a symbol followed by a blank (blank is represented by #) and a string, for example, ***a#bcbaaba***, it finds the first position where the given symbol occurs in the string, for instance 4 in this example, and outputs the number in unary. The final content of the tape should be ***a#bcbaaba***#1111.

6) a) Find an NFA with three states that accepts the language. [3+3]

$$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}.$$

Do you think this language can be accepted by an equivalent NFA with fewer than three states? If so, then plot the state transition diagram.

b) Find the equivalent DFA for the NFA with epsilon transition given below. Which of the strings 00, 01001, 10010,000, and 0000 are accepted by the following NFA?



7) a) Convert the following CFG to an equivalent PDA. [3+3]

$$S \to aA|bB|cC \; ; A \to Sa \; ; B \to Sb, C \to \lambda \; .$$

b) Give a verbal description of the language generated by this grammar. Show that the following two grammars are equivalent.

$$a)\; S \to abAB|ba \; ; A \to aaa \; ; B \to aA|bb$$

$$b)\; S \to abAaA|abAbb|ba; A \to aaa$$

8) a) Determine whether or not the following language are regular [2+2+4]

$$i)\; L = \{a^n b^n : n \geq 1\} \cup \{a^n b^m : n \geq 1, m \geq 1\}.$$

$$ii)\; L = \{a^n b^n : n \geq 1\} \cup \{a^n b^{n+2} : n \geq 1\}$$

b) Determine whether or not the following languages are context free:

$$i)\; L = \{a^n b^j a^n b^j : n \geq 0, j \geq 0\}$$

$$ii)\; L = \{a^n b^j a^j b^n : n \geq 0, j \geq 0\}$$

c) Design a Mealy machine that takes a binary string as input and outputs after each symbol the remainder obtained when the input thus far is divided by 5 (Consider a non-negative number). For example, for the input sequence 011010, the output should be 013131.

********************ALL THE BEST********************