Introduction to Probability and Statistics
Course ID:MA2203

Lecture-1

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Syllabus

Mid-Semester: Axiom of Probability(Motivation, Various types of definitions), Some basic results on probability (Boole's and Bonferoni's inequalities), Conditional probability, Bayes theorem, Probability for independent events, Random variable, types of random variables(discrete and continuous), Cumulative distribution function, probability mass function, probability density function, Mean, Variance, Standard deviation, Moments (central and about origin), Moment generating function, Special types of random variables, discrete: uniform, binomial, geometric, Poisson, Hyper geometric, Continuous: Uniform, Normal, Gamma, Exponential (one parameter).

After Mid-Semester: Two dimensional random variable, joint CDF, joint PDF/PMF, Marginal distribution, Conditional distribution, Calculating probabilities using two dimensional random variable, Sampling (with and without replacement), Distribution of sample mean and variance in the case of normal distribution, Estimation: Point estimation, Method of moments, method of maximum likelihood, Confidence Intervals for mean and variance in the case of normal, Testing of hypothesis (parameters of normal distribution), Goodness of fit Chi-square test, Regression and Correlation analysis, Rank correlation coefficient.

Motivation

- If an experiment is repeated under essentially homogeneous and similar conditions, we generally come across two types of situations. Note that, an experiment is a process of measurement or observation, in a laboratory, in a factory, in nature or wherever; so experiment is used in a more general sense.

1. The results (or outcome), is unique or certain: deterministic or predictable (a) For a perfect gas, $PV = Constant$ (b) The velocity v of a particle after time t is given by $v = u + at$ etc.
2. The result is not unique, but may be one of the several possible outcomes: Unpredictable or Probabilistic. This is also known as Random or Statistical Experiment.

Random Experiment

- An experiment is called random or statistical experiment, if it satisfies the following three conditions. (1) All outcomes of the experiment are known in advance, (2) Any performance of the experiment results in an outcome that is not known in advance, (3) The experiment can be repeated under identical conditions.
- In probability theory, we study this uncertainty of a random experiment.
- Examples of Random Experiments: (1) Tossing of a coin once. (2) Rolling of a six faced die. (3) Inspecting a light bulb. (4) Asking for opinion about a new electronic product. (5) Counting daily traffic accidents. (6) Measuring copper content of brass. (7) Picking a card from a well snuffled pack of cards. (8) Measuring tensile strength of wire.

Basic Terminologies

- Sample space: The set of all possible outcomes of an experiment (random). The elements of a sample space is known as sample points. We will denote the sample space by S. The result of an experiment is called an outcome.
- Trial and Event: Any particular performance of an random experiment is known as a trial. Outcomes or combinations of outcomes are called events. More formally, any subset of a sample space is called an event. The events will be denoted by capital letters, such as A, B, C, D etc.
- Example: Tossing of a coin once, here sample space is $S = \{Head = H, Tail = T\}$, Events are $A = \{Head\}$, $B = \{Tail\}$, $C = \{Head, Tail\}$, $E = \emptyset$.
- σ-algebra or σ-field on S: A σ-field is a non-empty class of subsets of S that is closed under countable unions and complements and contains the null set $\emptyset$.

Basic Terminologies

- Examples of σ-field: Let $A = \{a, b, c\}$, then define the class of subsets of A as $\mathcal{A}_1 = \{\{a\}, \{b\}, \{c\}\}$, $\mathcal{A}_2 =$ Power set of A. $\mathcal{A}_2$ is a σ-filed on A, whereas $\mathcal{A}_1$ is not.
- Exhaustive Events: The total number of possible outcomes of an random experiment.
- Favorable events: The number of cases favorable to an event.
- Mutually exclusive events: Events are said to be mutually exclusive or incompatible if the happening of any one of them precludes the happening of all the others. That is no two or more of them can happen simultaneously.
- Equally likely events: Outcomes are called equally likely if taking into consideration all the relevant evidences, there is no reason to expect one in preference to the others.

Basic Terminologies

- Independent Events: Several events are said to be independent if the happening or non-happening of an event is not affected by the supplementary knowledge concerning the occurrence of any number of the remaining events.
- Probability(Mathematical or Classical Definition): If a random experiment results in n exhaustive, mutually exclusive and equal likely outcomes, out of which m are favorable to the occurrence of an event A, then the probability of occurance or happening of A, usually denoted by $P(A)$, is given by

$$P(A) = \frac{m}{n} = \frac{\textit{No. of favourable cases}}{\textit{Total no. of exhaustive cases}}.$$

Continue....

- Observations: We can see, that $P(A) \geq 0$, also $0 \leq P(A) \leq 1$. $P(A) + P(A^c) = 1$.
- Here we can compute the Probability by logical reasoning, without conducting any experiment. Since the probability can be computed prior to obtaining any experimental data, it is also known as 'a priori' or Mathematical probability.
- In rolling a fair die, what is the probability of obtaining at least a 5, and probability of getting an odd number. We can use the above definition to compute this probability, since here we can assume that the sample points are equally likely, exhaustive and and finite. So, $P(\textit{at least } 5) = P(\{5,6\}) = 2/6 = 1/3$, and probability of getting odd number $P(\{1,3,5\}) = 3/6 = 1/2$.
- Limitations:(a) If the various outcomes of the random experiment are not equally likely or have equal chance of occurrence. (b) If the exhaustive number of outcomes of the random experiment is infinite or unknown.